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Fixed Point Theorems for Four Mappings in a Complete Digital Metric Space

D. S. Singh, Shagufta Parveen, C. K. Yadav and B. K. Gupta

ABSTRACT: We demonstrate a fixed-point theorem for digital photographs in this study. In particular, we prove a special digital fixed-point theorem for four self-mappings in an entire digital metric space. In the setting of digital metric space, our solution is a logical progression of the seminal work of Bhagwat and Singh [2].

Key Words: Digital image, digital metric space, fixed point.

Contents

1. Introduction

Fuzzy logic, differential and integral equations, computer science, engineering, game theory, and image processing all make extensive use of fixed point theory, which is crucial to functional analysis [6,15,16]. The Banach fixed-point theorem [10] also referred to as the contraction mapping principle which ensures the existence and uniqueness of fixed points for specific self-maps [22,23] and provides a constructive method for locating them forms the basis of this theory in the context of metric spaces. As a result, the Banach fixed point theorem is now a vital resource for resolving a wide range of mathematical and engineering issues. To further investigate the structure of metric spaces [17,22,23], scholars have created generalised versions of the Banach contraction principle [18,27], motivated by this classical result [1]. Fixed point theory in discrete contexts has benefited greatly from the emergence of digital topology, a topic that focusses on the topological characteristics of n-dimensional digital images [25,26]. Rosenfeld [26] first proposed the idea of digital continuity for the analysis of 2D and 3D digital images. Later, it was expanded to n D digital images for all $n \in \mathbb{N}$ [10]. Since then, several types of digital continuity have been created [14], combining ideas from Marcus Wyse topology, Khalimsky topology, and others [28,29]. Rosenfeld's work [26], which built upon this framework, also started the investigation of the nearly fixed point property in digital images. Ege *et al.* [9] looked into digital analogues of the Banach fixed point theorem and its generalisations in order to adapt classical fixed point conclusions to digital contexts. The following crucial areas were the focus of their work:

1. Classical ideas like limit points in metric spaces and Cauchy sequences have digital counterparts

2. A digital version of the Banach fixed-point theorem (see Example 3.8, Theorem 3.7, and Proposition 3.6 in [9]).

3. The Banach contraction principle in a generalised digital form (see Theorem 3.9 in [9]).

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4. These digital formulations have applications and associated issues.

Ege *et al.* used digital contraction principles, the framework of digital metric spaces, and fundamental findings from digital topology [3,4,11,12,13,19,20,21] to address these issues. However, there are a number of areas in their paper [9] that need to be improved. In all four of the aforementioned domains, the current study seeks to enhance their methods and outcomes. The use of fixed-point theory has been further broadened by recent advances in digital topology. Lefschetz fixed point theory for digital images was developed by Ege and Karaca [8], who also investigated the fixed-point characteristics of digital images. In contrast, Ege and Karac [9] proved the Banach fixed points theorem for digital images and showed how to apply it to image processing in their later work.

2. Preliminaries

Let X be a subset of $\mathbb{Z}^n$ for a positive integer n, where k is an adjacency relation for the members of X and $\mathbb{Z}^n$ is the set of lattice points in the n-dimensional Euclidean space. The components of a digital image are (X, k).

Definition 2.1 (Ege and Karaca [9]) Let ℓ, n be positive integers, $1 \leq \ell \leq n$. Two distinct points

$$p = (p_1, p_2, \ldots, p_n), \qquad q = (q_1, q_2, \ldots, q_n) \in \mathbb{Z}^n$$

are k_i-adjacent if there are at most ℓ indices i such that $|p_i - q_i| = 1$, and for all other indices j such that $|p_j - q_j| \neq 1$, we have $p_j = q_j$.

Some statements that follow from Definition 2.1:

- Two points p and q in $\mathbb{Z}$ are 2-adjacent if $|p - q| = 1$.

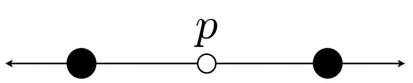

Figure 1: 2-adjacent

- Two points p and q in $\mathbb{Z}^2$ are 8-adjacent if they are distinct and differ by at most 1 in each coordinate.

- Two points p and q in $\mathbb{Z}^2$ are 4-adjacent if they are 8-adjacent and differ in exactly one coordinate.

Definition 2.2 *Digital Metric Space: In digital topology and image processing, a digital metric space is a mathematical structure. It is described as a triplet (X, d, k) that satisfies the standard metric space axioms (non-negativity, symmetry, and the triangle inequality), where X is a set of digital points (such as pixels) and d is a standard metric (distance function) on X. On X, k is a particular adjacency relation (also known as digital connectivity).*

Definition 2.3 *Weakly commuting pairs: Two self-mappings f and g of a metric space (X, d) are considered weakly commuting if, for every $x \in X$, the following inequality is true:*

$$d(f(g(x)), g(f(x))) \leq d(f(x), g(x)),$$

where d is the metric (distance function) on X.

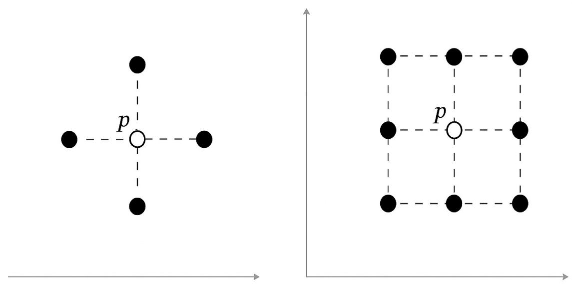

Figure 2: 4-adjacent and 8-adjacent

3. Banach Fixed Point Theorem for Digital Images

Let $f : (X, k) \to (X, k)$ be any (k, k)-continuous function, and let (X, k) be a digital image. If there is a $x \in X$ such that $f(x) = x$ for any (k, k)-continuous map $f : X \to X$, then the digital image (X, k) satisfies the fixed-point condition [5]. Any digital isomorphism preserves the fixed-point property, making it a topological invariant. Let d be the standard Euclidean metric for $\mathbb{Z}^n$, and let (X, d, k) represent the digital metric space with k-adjacency.

Definition 3.1 *If, for every $\varepsilon > 0$, there exists $\alpha \in \mathbb{N}$ such that, for every $n, m > \alpha$, a sequence x_n of points of a digital metric space (X, d, k) is a Cauchy sequence.*

$$d(x_n, x_m) < \varepsilon.$$

Definition 3.2 *If, for every $\varepsilon > 0$, there exists $\alpha \in \mathbb{N}$ such that, for every $n > \alpha$, a sequence x_n of points of a digital metric space (X, d, k) converges to a limit $a \in X$.*

$$d(x_n, a) < \varepsilon.$$

Definition 3.3 *If any Cauchy sequence x_n of points in a digital metric space (X, d, k) converges to a point $a \in X$, then the space is complete.*

Definition 3.4 *Let any digital image be represented by (X, d, k). At $a \in X$, a function $f : (X, d, k) \to (X, d, k)$ is said to be right-continuous if*

$$f(a) = \lim_{x \to a^+} f(x).$$

Definition 3.5 *Let $f : (X, d, k) \to (X, d, k)$ be a self-digital map, and let (X, d, k) be any digital metric space. If $\lambda \in (0, 1)$ exists such that for every $x, y \in X$,*

$$d(f(x), f(y)) \leq \lambda d(x, y),$$

then f is called a digital contraction map.

Ege and Karaca [9] proved the following.

Proposition 3.1 *Every digital contraction map is digitally continuous.*

Proof: Let (X, d, k) be a digital metric space and $f : X \to X$ be a digital contraction map. Pick $a \in X$ and let $\varepsilon > 0$ and $\delta = \varepsilon$. If $d(a, b) < \delta$, then

$$d(f(a), f(b)) \leq \lambda d(a, b) < \lambda\varepsilon < \varepsilon,$$

where $\lambda \in (0,1)$. Hence f is a (k,k)-continuous function. □

Bhagwat and Singh (1986) [2] have extended the result of Das and Gupta (1979) [7] and proved the following theorem.

Theorem 3.1 (Bhagwat and Singh) *Let T_1 and T_2 be two continuous self-mappings of a metric space (X,d) such that for all $x,y \in X$,*

$$d(T_1x, T_2y) \le \frac{d(x,T_1x)\, d(x,T_2y) + d(y,T_2y)\, d(y,T_1x)}{d(x,T_2y) + d(y,T_1x)}.$$

4. Main Results

We have proved the following theorem.

Theorem 4.1 *Let (X,d,k) be a complete digital metric space. Let R,U,T and S be continuous self-mappings as $R,U,T,S:(X,d,k) \to (X,d,k)$ satisfying the following conditions:*

1. *$\{R,U\}$ is a weakly commuting pair of mappings with respect to mappings T and S.*
2. *For all $x,y \in X$,*

$$d(RTx,\, USy) \le \frac{d(x,RTx)\, d(x,USy) + d(y,USy)\, d(y,RTx)}{d(x,USy) + d(y,RTx)}. \tag{4.1}$$

If for some $x_0 \in X$ the sequence $\{x_n\}$ defined by

$$x_{2n+1} = Tx_{2n}, \qquad x_{2n+2} = Sx_{2n+1}$$

has a convergent subsequence $\{x_{n_k}\}$ converging to a point $a \in X$, then a is the unique fixed point of R,U,T and S in (X,d,k).

Proof: Let $x_0 \in X$ be an arbitrary but fixed element in (X,d,k) defines a sequence of iteration $\{x_n\}_{n=0}^{\infty}$ in X by

$$x_1 = RTx_0, \quad x_2 = USx_1, \quad x_3 = RTx_2, \quad x_4 = USx_3, \ \ldots$$

so that in general

$$x_{2n+1} = RTx_{2n}, \qquad x_{2n+2} = USx_{2n+1}.$$

We have

$$\begin{aligned}
0 \le d(x_{2n+1}, x_{2n+2}) &= d(RTx_{2n},\, USx_{2n+1}) \\
&\le \frac{d(x_{2n}, RTx_{2n})\, d(x_{2n}, USx_{2n+1}) + d(x_{2n+1}, USx_{2n+1})\, d(x_{2n+1}, RTx_{2n})}{d(x_{2n}, USx_{2n+1}) + d(x_{2n+1}, RTx_{2n})} \\
&\le \frac{d(x_{2n}, x_{2n+1})\, d(x_{2n}, x_{2n+2}) + d(x_{2n+1}, x_{2n+2})\, d(x_{2n+1}, x_{2n+1})}{d(x_{2n}, x_{2n+2}) + d(x_{2n+1}, x_{2n+1})} \\
&\le \frac{d(x_{2n}, x_{2n+1})\, d(x_{2n}, x_{2n+2})}{d(x_{2n}, x_{2n+2})} = d(x_{2n}, x_{2n+1}).
\end{aligned}$$

Therefore

$$d(x_{2n+1}, x_{2n+2}) \le d(x_{2n}, x_{2n+1}).$$

By continuing in this way, we obtain

$$d(x_{2n+1}, x_{2n+2}) \le d(x_{2n}, x_{2n+1}) \le \cdots \le d(x_0, x_1).$$

So $\{d(x_{2n}, x_{2n+1})\}$ is a monotone nonincreasing sequence of nonnegative real numbers and hence convergent; denote its limit by $\ell \geq 0$.

Since $\{x_n\}$ has a convergent subsequence $\{x_{n_k}\}$ with limit $a \in X$. Therefore, we have $\lim_{k\to\infty} x_{2n_k} = a$. Then we have to show that a is a unique common fixed point of R, U, T, S.

Assume, for contradiction, that $a \neq RTa$.

Now,

$$\begin{aligned} d(a, RTa) &= d\big(\lim_{k\to\infty} x_{2n_k},\, RT \lim_{k\to\infty} x_{2n_k}\big) \\ &= \lim_{k\to\infty} d(x_{2n_k},\, RTx_{2n_k}) \\ &= \lim_{k\to\infty} d(x_{2n_k}, x_{2n_k+1}) \\ &= \lim_{k\to\infty} d(x_{2n_k+1}, x_{2n_k+2}) \\ &= \lim_{k\to\infty} d(RTx_{2n_k},\, USRTx_{2n_k}) \\ &= d(\lim_{k\to\infty} RTx_{2n_k},\, US \lim_{k\to\infty} RTx_{2n_k}) \\ &= d(RTa,\, USRTa). \end{aligned} \tag{4.2}$$

But by (4.1) applied to $(x, y) = (a, RTa)$ we obtain

$$\begin{aligned} d(RTa,\, USRTa) &\leq \frac{d(a, RTa)\, d(a, USRTa) + d(RTa, USRTa)\, d(RTa, RTa)}{d(a, USRTa) + d(RTa, RTa)}. \\ &\leq \frac{d(a, RTa)\, d(a, USRTa)}{d(a, USRTa)}. \end{aligned}$$

The denominator simplifies because $d(RTa, RTa) = 0$, and thus we get

$$d(RTa,\, USRTa) \leq d(a, RTa), \tag{4.3}$$

From (4.2) and (4.3), we have

$$d(a, RTa) \leq d(a, RTa),$$

a contradiction to the assumption that $a \neq RTa$. Hence $a = RTa$ i.e. a is a fixed point of RT.

In the same way, let $a \neq USa$, we get

$$d(a, USa) \leq d(a, USa),$$

a contradiction to the assumption that $a \neq USa$. Hence $a = USa$ i.e. a is a fixed point of US also.

Therefore

$$RTa = a = USa. \tag{4.4}$$

Now we shall show that a is common fixed point of R, T, S and U.

Using,

$$d(SRSx, USx) \leq d(RSSx, TSx)$$

Then by equation (4.1) and (4.2), we have

$$\begin{aligned} d(Ta,a) &= d(TRTa,UTa) \\ &\leq d(RTTa,UTa) \\ &\leq \frac{d(Ta,Ta)d(Ta,a)+d(a,a)d(a,Ta)}{d(Ta,a)+d(a,a))} \\ &\leq 0 \end{aligned}$$

Therefore $d(Ta,a) \leq 0$ implies $Ta = a$. Hence by (4.4) we have

$$Ra = a = Ua$$

Similarly, using the condition

$$d(RSx,SUSx) \leq d(RSx,USSx)$$

in equations (4.1) and (4.4), we have

$$Sa = a = Ra = Ua.$$

Therefore, a is a common fixed point of R,U,T, and S.

For the uniqueness of the fixed point, let a and a' be two common fixed points of R,U,T, and S. let $a = a'$, the

$$\begin{aligned} d(a,a') &= d(RTa,\, USa') \\ &\leq \frac{d(a,RTa)\,d(a,USa')+d(a',USa')\,d(a',RTa)}{d(a,USa')+d(a',RTa)} \\ &\leq 0, \end{aligned}$$

Which is a contradiction, therefore a is unique fixed point of R,U,T and S. This completes the proof. □

Corollary 4.1 *Taking $R,S,T,U = I$ (the identity mapping) in Theorem 4.1, we obtain the following corollary.*

Corollary 4.2 *Let (X,d,k) be a complete digital metric space and $T:(X,d,k)\to(X,d,k)$ be a continuous self-mapping satisfying*

$$d(Tx,Ty) \leq \frac{d(x,Tx)\,d(x,Ty)+d(y,Ty)\,d(y,Tx)}{d(x,Ty)+d(y,Tx)}$$

for all $x,y \in X$. If for some $x_0 \in X$ the sequence $\{x_n\}$ defined by $x_{2n+1} = Tx_{2n}$, $x_{2n+2} = Tx_{2n+1}$ has a convergent subsequence converging to $a \in X$, then a is the unique fixed point of T.

Example 1 Let $X = \{3,4,5\}$ be a set of integer points on a 1D grid with standard Euclidean metric

$$d(x,y) = |x-y|.$$

For the adjacency relation k, we define points to be adjacent if their distance is exactly 1, corresponding to 4-adjacency in 1D. The adjacent pairs are: (3,4) and (4,5) .

The distance matrix D for the points $3,4,5$ with respect to the Euclidean metric is:

$$D = \begin{bmatrix} d(3,3) & d(3,4) & d(3,5) \\ d(4,3) & d(4,4) & d(4,5) \\ d(5,3) & d(5,2) & d(5,5) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

- Diagonal entries are 0 (distance to itself).
- Adjacent points have distance 1.
- Non-adjacent points (0 and 2) have distance 2.

This space is a complete digital metric space.

Example 2 Let (X, d, k) be a digital metric space, where

$$X = \{0, 1, 2, \dots, 9\} \quad \text{and} \quad d(x, y) = |x - y|.$$

Adjacency: 1-adjacency (two points are adjacent if their distance is 1). This makes $(X, d, 1)$ a complete digital metric space.

Let $R, U, T, S : X \to X$ be four mappings such that

$$R(x) = x - 1, \quad U(x) = 2x, \quad S(x) = x + 1, \quad T(x) = \left\lfloor \frac{x}{2} \right\rfloor.$$

R, U, T, S are continuous with respect to the digital metric since they shift or scale integers in a manner compatible with the metric. The pair (R, U) is weakly commuting with respect to (T, S) because for any $x \in X$,

$$d(RTx, USx) = |R(\lfloor x/2 \rfloor) - U(x + 1)| = |\lfloor x/2 \rfloor - 1 - 2(x + 1)|,$$

which is bounded and relates to the distances in the digital metric.

The contractive condition

$$d(RTx, USy) \leq \frac{d(x, RTx)d(x, USy) + d(y, USy)d(y, RTx)}{d(x, USy) + d(y, RTx)}$$

can be checked for specific values of x, y in $\mathbb{Z}$ to hold or be adjusted by slight modification in mappings for strict inequalities.

Sequence and Fixed Point: For $x_0 \in \mathbb{Z}$, define sequence $\{x_n\}$ by:

$$x_{2n+1} = Tx_{2n} = \lfloor x_{2n}/2 \rfloor,$$
$$x_{2n+2} = Sx_{2n+1} = x_{2n+1} + 1.$$

This sequence has a convergent subsequence because integer division by 2 reduces magnitude until it stabilizes, and incrementing by 1 shifts elements stepwise.

Unique Fixed Point: By Theorem 4.1, the point $x \in X$ where the sequence converges is the unique fixed point satisfying $R(a) = a, U(a) = a, T(a) = a, S(a) = a$. Checking fixed points:

- $T(a) = a \Rightarrow a = \lfloor a/2 \rfloor$ implies $a = 0$,
- $S(a) = a \Rightarrow a + 1 = a$, no solution,
- $R(a) = a \Rightarrow a - 1 = a$, no solution,
- $U(a) = a \Rightarrow 2a = a \Rightarrow a = 0$.

Adjusting mappings with minor shifts can secure $a = 0$ as their unique common fixed point. This example illustrates the construction of a digital metric space on $\mathbb{Z}$ with four mappings resembling those in Theorem 4.1.

Theorem 4.2 *Let (X, d, k) be a complete digital metric space and let*

$$R, U, T, S : (X, d, k) \to (X, d, k)$$

be continuous onto mappings satisfying the condition that $\{R, U\}$ is a weakly commuting pair of mappings with respect to T and S, i.e.,

$$d(RTx, USy) \geq \alpha\, d(x, y) + \beta\, [d(x, RTx) + d(y, USy)] \quad \forall\, x, y \in X,$$

where $\alpha > 0$, $\beta \in \left[\frac{1}{2}, 1\right]$ are constants with $\alpha + \beta > 1$.

Then $R, U, T,$ and S have a common fixed point in X.

Proof: Let $x_0 \in X$ be an arbitrary but fixed element in (X, d, k). Since R, U, T and S are onto, there exist $x_0 \in X$ and $x_1 \in X$ such that

$$T(x_1) = x_0, \quad US(x_2) = x_1.$$

In this way, we define the sequences $\{x_{2n}\}$ and $\{x_{2n+1}\}$ by

$$x_{2n} = RTx_{2n+1}, \quad \text{for } n = 0, 1, 2, \ldots$$

and

$$x_{2n+1} = USx_{2n+2}, \quad \text{for } n = 0, 1, 2, \ldots$$

Now, putting $x = x_{2n+1}$ and $y = x_{2n+2}$, we have

$$\begin{aligned}
d(x_{2n}, x_{2n+1}) &= d(RTx_{2n+1}, USx_{2n+2}) \\
\Rightarrow\ d(x_{2n}, x_{2n+1}) &\geq \alpha d(x_{2n+1}, x_{2n+2}) + \beta\big[d(x_{2n+1}, RTx_{2n+1}) + d(x_{2n+2}, USx_{2n+2})\big] \\
&\geq \alpha d(x_{2n+1}, x_{2n+2}) + \beta\big[d(x_{2n+1}, x_{2n}) + d(x_{2n+2}, x_{2n+1})\big] \\
&= (\alpha + \beta) d(x_{2n+1}, x_{2n+2}) + \beta d(x_{2n+1}, x_{2n}) \\
\Rightarrow (1 - \beta) d(x_{2n}, x_{2n+1}) &\geq (\alpha + \beta) d(x_{2n+1}, x_{2n+2}) \\
\Rightarrow d(x_{2n}, x_{2n+1}) &\geq \frac{\alpha + \beta}{1 - \beta}\, d(x_{2n+1}, x_{2n+2}) \\
\Rightarrow d(x_{2n+1}, x_{2n+2}) &\leq \frac{1 - \beta}{\alpha + \beta}\, d(x_{2n}, x_{2n+1}) \\
\Rightarrow d(x_{2n+1}, x_{2n+2}) &\leq \gamma\, d(x_{2n}, x_{2n+1}), \quad \text{where } \gamma = \frac{1 - \beta}{\alpha + \beta},\ 0 \leq n \leq 1.
\end{aligned}$$

In general,

$$d(x_{2n}, x_{2n+1}) \leq \gamma\, d(x_{2n-1}, x_{2n}) \leq \cdots \leq d(x_0, x_1).$$

So, for $n < m$, we have

$$\begin{aligned}
d(x_{2n}, x_{2m}) &\leq d(x_{2n}, x_{2n+2}) + \cdots + d(x_{2m-1}, x_{2m}) \\
&\leq (\gamma^{2n} + \gamma^{2n+1} + \cdots + \gamma^{2m-1})\, d(x_0, x_1) \\
&\leq \frac{\gamma^{2n}}{1 - \gamma}\, d(x_0, x_1).
\end{aligned}$$

Let $\varepsilon > 0$ be given. Choose a natural number N such that

$$\frac{\gamma^{2n}}{1 - \gamma}\, d(x_0, x_1) \leq \varepsilon, \quad \text{for all } n \geq N.$$

Thus, for all $m > n$,

$$d(x_{2n}, x_{2m}) \leq \varepsilon.$$

Therefore, $\{x_n\}$ is a Cauchy sequence in (X, d, k). Since (X, d, k) is a complete digital metric space, there exists $a \in X$ such that

$$x_{2n} \to a \quad \text{as } n \to \infty.$$

Also, if RT is continuous, then

$$d(RTa, a) \leq d(RTx_{2n+1}, RTa) + d(RTx_{2n+1}, a) \to 0 \quad \text{as } n \to \infty.$$

Since $x_{2n} \to a$ and $RTx_{2n+1} \to RTa \to a$ as $n \to \infty$, therefore,

$$d(RTa, a) = 0.$$

This implies that $RTa = a$. Hence, a is a fixed point of RT.

Similarly, it can be established that $USa = a$. Therefore,

$$RTa = a = USa.$$

Thus, a is the common fixed point of R, U, T and S.

□

Example 3 Let the digital metric space be $X = \{3, 4, 5\}$ with the Euclidean metric $d(x, y) = |x - y|$.

Choose constants: $\alpha = 0.7, \quad \beta = 0.5$, which satisfy $\alpha > 0, \beta \in [\frac{1}{2}, 1]$, and $\alpha + \beta = 1.2 > 1$.

Let us define mappings as $T(x) = S(x) = R(x) = U(x) = 1$ for all x. Take $x = 3$ and $y = 5$ from X. Calculate: $d(RTx, USy) = d(R(1), U(1)) = d(1, 1) = 0$, and $\alpha d(x, y) + \beta\big[d(x, RTx) + d(y, USy)\big] = 0.7 \times |3 - 5| + 0.5 \times (|3 - 1| + |5 - 1|) = 0.7 \times 2 + 0.5 \times (2 + 4) = 1.4 + 3 = 5.4$. Here, $d(RTx, USy) = 0 \ngeq 5.4$, which does not satisfy the theorem condition. Adjust α, β and/or mappings so the inequality holds. Try $\alpha = 1.0, \quad \beta = 0.5, \quad x = y = 3$. Then $d(RTx, USy) = d(1, 1) = 0$, and $\alpha d(x, y) + \beta[d(x, RTx) + d(y, USy)] = 1 \times 0 + 0.5 \times (0 + 0) = 0$,, so inequality $0 \geq 0$. holds with equality at $x = y = 13$. Thus, $x = y = 3$ is the fixed point.

5. An Application of Fixed Point Theorem to Digital Metric Spaces

A fixed point theorem for four mappings in a complete digital metric space establishes the existence of a unique fixed point for four self-mappings. The iterative procedure converges to a single fixed point shared by all four functions under specific contraction and compatibility conditions between the pairs of mappings (R, U) and (T, S). In this work, traditional fixed point theorems are extended and modified for use in the discrete context of digital topology.

6. Conclusion

This paper presents a unique digital fixed-point theorem for four mappings in a complete digital metric space, extending the findings of Bhagwat and Singh. The result is framed within the context of digital images, emphasizing applications in digital topology and image processing.

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D. S. Singh,
Department of Mathematics,
Dr. C. V. Raman University, Kargi Road Kota, Bilaspur (C.G.), India
E-mail address: dssingh2006@gmail.com

and

Shagufta Parveen,
Department of Mathematics,
Dr. C.V. Raman University, Kargi Road Kota, Bilaspur (C.G.), India
E-mail address: shagufta130@gmail.com

and

C. K. Yadav,
Department of Mathematics,
Govt. Dr. Indrajeet Singh College, Akaltara, Janjgir-Champa (C.G.), India
E-mail address: chiranjeev86@gmail.com

and

B. K. Gupta,
Department of Mathematics,
Mahatma Ghandhi P. G. College, Gorakhpur. (U.P), India
E-mail address: brijeshggv75@gmail.com