



## Distance Graph of Jahangir Graph and its Laceability

Annapoorna M. S., Anitha Kiran

**ABSTRACT:** Numerous scientific disciplines have conducted extensive research on graph operations. This idea facilitates a new way of looking at graph attributes. Tensor product, cartesian product, strong product or normal product, modular product, lexicographical product, rooted product, brick product, etc. are some examples of the various types of graph operations. The concept of distance plays a very important role in graph theory and one interesting concept on distance is the distance graph. Many authors have obtained significant results related to these distance graphs. Consider two graphs  $G_1$  and  $G_2$  with some property. Then, a graph operation on  $G_1$  and  $G_2$  gives the new graph  $H$  with a different property. This change of property is used for several kinds of modeling in structures. In this article, the distance graph of the Jahangir graph is obtained and the results related to laceability are proved. If a connected graph  $G$  has a Hamiltonian path connecting at least one pair of distinct vertices  $u$  and  $v$ , with the property  $d(u, v) = t$ ,  $1 \leq t \leq \text{diam}(G)$ , then it is Hamiltonian- $t^*$ -laceable.  $G$  is termed  $t^*$ -connected if it is Hamiltonian- $t^*$ -laceable for all  $t$ . This work demonstrates the  $t^*$ -connectivity of the distance graph of the Jahangir graph  $D_{jr}(n, m)$  for  $2 \leq n \leq 13$  and  $m \geq 3$ .

**Keywords:** Distance graph, Hamiltonian- $t$ -laceable, Hamiltonian- $t^*$ -laceable, Jahangir graph,  $t^*$ -connectedness.

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### 1. Introduction

All graphs considered here are finite, simple, connected and undirected. Let  $u$  and  $v$  be two vertices in  $G$ . The distance between  $u$  and  $v$ , denoted by  $d(u, v)$ , is the length of a shortest  $u$ - $v$  path in  $G$ . A graph  $G$  is *Hamiltonian- $t$ -laceable* (*Hamiltonian- $t^*$ -laceable*) if there exists in it a Hamiltonian path between every pair (at least one pair) of distinct vertices  $u$  and  $v$  with the property  $d(u, v) = t$ ,  $1 \leq t \leq \text{diam}(G)$ . A graph  $G$  is termed  *$t^*$ -connected* if it is Hamiltonian- $t^*$ -laceable for all  $t$  such that  $1 \leq t \leq \text{diam}(G)$ .

Let  $D$  be the set of all distances between every pair of vertices in a graph  $G$  and let  $S$  be a subset of  $D$ . The distance graph associated with  $G$ , denoted by  $D(G, S)$ , is the graph having the same vertex set as that of  $G$ , with two vertices  $x$  and  $y$  being adjacent in  $D(G, S)$  whenever  $d(x, y) \in S$ . Laceability properties of modified distance graphs of grid graphs have been studied in [1]. The concept of distance in graphs has been explained in detail in [2]. In [3], Dirac worked on Hamiltonian properties of graphs. In [4], Eminjan Sabir et al. provided a synthesis of sufficient conditions for a bipartite graph to be spanning laceable in terms of extremal number of edges, bipartite independence number, bistability, and biclosure. In [5], Goodman and Hedetniemi have studied the Hamiltonian characteristics of graphs. Hamilton-laceable bi-powers of locally finite bipartite graphs are discussed in [6]. In [7], Hongwei Qiao and Jixiang Meng have shown that the double generalized Petersen graph is Hamilton-laceable for even  $n \geq 4$ . Jing Qu and Nanbin Cao have studied the edge metric dimension and mixed metric dimension of planar graphs in [8]. The results related to the properties of Domination David Derived Networks have been presented in [9,10]. The lower bound for pendant tree-connectivity has been derived in [11]. Laceability properties in the distance graphs of paths  $P_{2n}$  with distance set  $\{1, 2k\}$  have been studied by Murali and Harinath in [12].

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This study shows the  $t^*$ -connectivity of the distance graph of the Jahangir graph  $D_{jr}(n, m)$  for  $2 \leq n \leq 13$  and  $m \geq 3$ .

**Definition 1: Jahangir graph**

The Jahangir graph  $J_{n,m}$  is a graph with  $nm + 1$  vertices and  $m(n + 1)$  edges for all  $n \geq 2$  and  $m \geq 3$ . The graph  $J_{n,m}$  consists of a cycle  $C_{nm}$  together with one additional vertex that is adjacent to  $m$  vertices of  $C_{nm}$ , each at equal distance from one another. Figure 1 shows some particular cases of  $J_{n,m}$ .

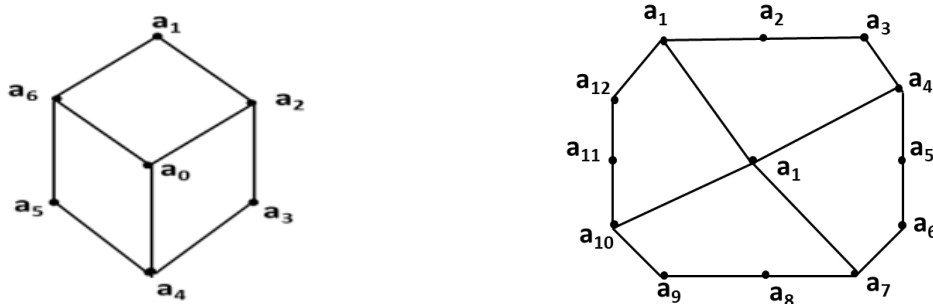


Figure 1: Jahangir graph  $J_{2,3}$  and  $J_{3,4}$

**Definition 2: The Distance Graph of the Jahangir Graph  $D_{jr}(n, m)$**

This graph is obtained as follows:

- Consider the Jahangir graph  $J_{n,m}$  with vertex set

$$V = \{a_0, a_1, a_2, \dots, a_{nm}\}$$

and the edge set

$$E = [\{a_i a_{i+1}, a_{i+1} a_{i+2}, \dots, a_{i+(mn-2)} a_{i+(mn-1)}\} \\ \cup \{a_0 a_i\} \cup \{a_0 a_{i+n}\} \cup \{a_0 a_{i+2n}\} \cup \dots \cup \{a_0 a_{i+(nm-n)}\}, \quad i = 1]$$

- Consider the distance set  $S = \{2, 3\}$  and construct the distance graph of the Jahangir graph  $D_{jr}(n, m)$ .

In Figure 2 below, the Jahangir graph  $J_{n,m}$  and the distance graph of the Jahangir graph  $D_{jr}(n, m)$  is shown.

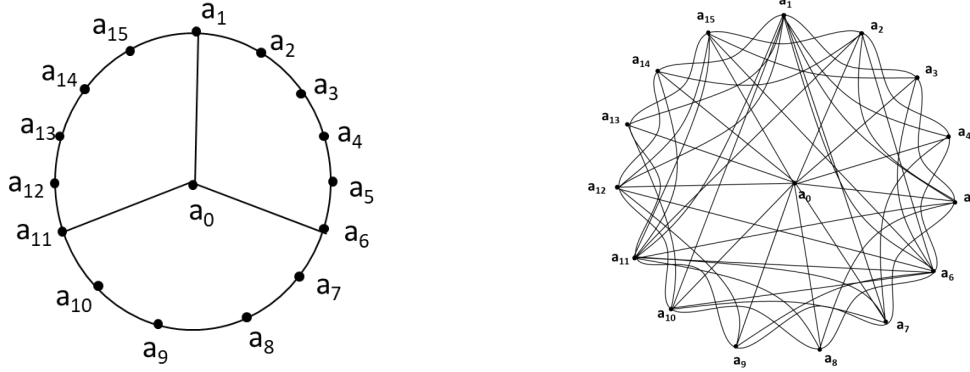


Figure 2: Jahangir graph  $J_{5,3}$  and the distance graph of Jahangir graph  $D_{j,r}(5,3)$

**Definition 3:** Let  $P$  be a path from the vertices  $a_i$  to  $a_j$  in a graph  $G$  and let  $P'$  be a path from  $a_j$  to  $a_k$ . Then the path  $P \cup P'$  is the path obtained by extending the path  $P$  from  $a_i$  to  $a_j$  to  $a_k$  through the common vertex  $a_j$ .

## 2. Results and Discussion

In this section, the distance graph of the Jahangir graph  $D_{jr}(n, m)$  for  $2 \leq n \leq 13$  and  $m \geq 3$  is examined for  $t^*$ -connectivity. The following terminologies are introduced in this context and will be used to demonstrate  $t^*$ -connectedness.

$$a_i^I [n] = a_i, a_{(i+m)}, a_{(i+2m)}, a_{(i+3m)}, \dots, a_n$$

$$a_r^{R_x} [y] = a_r, a_{(r+x)}, a_{(r+2x)}, a_{(r+3x)}, \dots, a_y \quad \text{where } r = nm.$$

**Theorem 1:** The graph  $D_{jr}(n, m)$  for  $2 \leq n \leq 5$  and  $m \geq 3$  is  $t^*$ -connected.

**Proof:** Let  $G = D_{jr}(n, m)$  be the distance graph of Jahangir graph vertex set

$$V = \{a_0, a_1, a_2, \dots, a_{mn}\} \text{ and edge set}$$

$$E = \left\{ \begin{aligned} & \{a_i a_{i+2}, a_{i+2} a_{i+4}, \dots, a_{i+(nm-4)} a_{i+(nm-2)}, a_{i+(nm-2)} a_i\} \\ & \cup \{a_j a_{j+2}, a_{j+2} a_{j+4}, \dots, a_{j+(nm-4)} a_{j+(nm-2)}, a_{j+(nm-2)} a_j\} \\ & \cup (\{a_0 a_2\} \cup \{a_0 a_{2+i}\} \cup \{a_0 a_{2+2i}\} \cup \dots \cup \{a_0 a_{2+(nm-2)i}\}) \\ & \setminus (\{a_0 a_{i+n}\} \cup \{a_0 a_{i+2n}\} \cup \{a_0 a_{i+3n}\} \cup \dots \cup \{a_0 a_{i+(nm-n)}\}), \quad i = 1, j = 1 \end{aligned} \right\}.$$

Evidently,  $d(G) = 2$ .

Consider the graph  $D_{jr}(3, m)$  which is described using the below connections

$$\begin{pmatrix} a_1 & \cdots & a_3 \\ \times & \cdots & \vdots \\ a_0, a_{3m}, a_2 & \cdots & \text{all} \end{pmatrix} \quad \begin{pmatrix} a_i & \cdots & a_{i+2}, a_{i+3}, a_{i+5} \\ \vdots & i = 2, 5, \dots, 3m-1 & \cdots \\ a_0 & \cdots & a_{3m-2} \end{pmatrix} \\
\begin{pmatrix} a_4 & \cdots & a_6 \\ \times & \cdots & \vdots \\ a_0, a_3, a_5 & \cdots & \text{all} \end{pmatrix} \quad \begin{pmatrix} a_j & \cdots & a_{j+2}, a_{j+3}, a_{j+4} \\ \vdots & j = 3, 6, \dots, 3m & \cdots \\ a_0 & \cdots & a_{3m-2} \end{pmatrix} \\
\begin{pmatrix} a_7 & \cdots & a_9 \\ \times & \cdots & \vdots \\ a_0, a_6, a_8 & \cdots & \text{all} \end{pmatrix} \\
\vdots \\
\begin{pmatrix} a_{3m-2} & \cdots & a_{3m} \\ \times & \cdots & \vdots \\ a_0, a_{3m-3}, a_{3m-1} & \cdots & \text{all} \end{pmatrix}$$

The following cases are examined in order to establish the outcome.

**Case (i):** When  $t = 1$

In the graph  $G = D_{jr}(n, m)$ , we have  $d(a_0, a_2) = 1$ . Again, there exist two subcases.

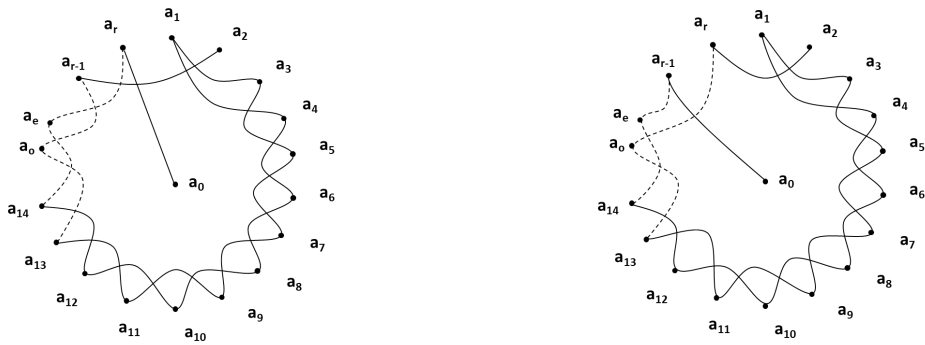
**Subcase (i):** For  $nm$  even, the path

$P : a_0 a_r R^{-2}[4] P^0 a_1 I^2[r-1] P^0 a_2$  is the Hamiltonian path between the vertices  $a_0$  and  $a_2$ .

**Subcase (ii):** For  $nm$  odd, the path

$P : a_0 a_{r-1} R^{-2}[4] P^0 a_1 I^2[r] P^0 a_2$  is the Hamiltonian path between the vertices  $a_0$  and  $a_2$ .

For  $t = 1$



**Figure 3:** Hamiltonian path from vertex  $a_0$  to  $a_2$  in the graph  $D_{jr}(3, m)$  for both cases  $nm = \text{even}$  and  $nm = \text{odd}$

**Case (ii):** When  $t = 2$ .

In the graph  $G = D_{jr}(n, m)$ , we have  $d(a_0, a_1) = 2$ , again here exists subcases.

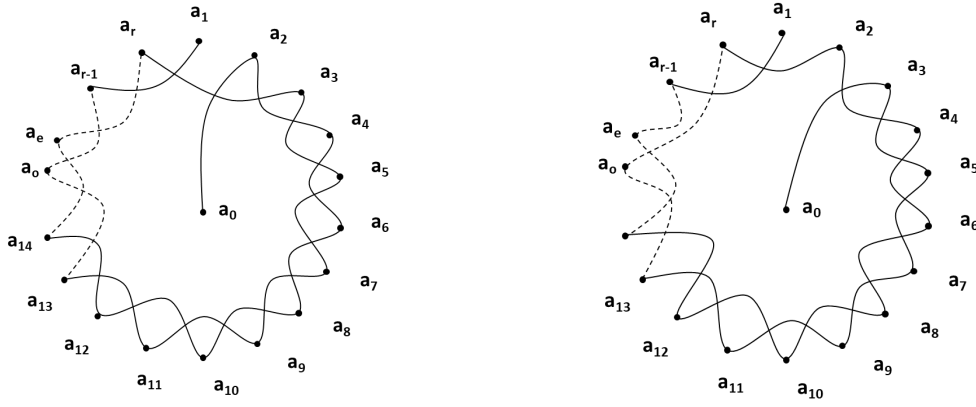
**Subcase (i):** For  $nm = \text{even}$  and the path

$$P : a_0 a_2 I^2[r] P^0 a_3 I^2[r-1] P^0 a_1 \text{ is the Hamiltonian path between the vertices } a_0 \text{ and } a_1.$$

**Subcase (ii):** For  $nm = \text{odd}$ , the path

$$P : a_0 a_3 I^2[r] P^0 a_2 I^2[r-1] P^0 a_1 \text{ is the Hamiltonian path between the vertices } a_0 \text{ and } a_1.$$

For  $t = 2$



**Figure 4: Hamiltonian path from vertex  $a_0$  to  $a_1$  in the graph  $D_{jr}(3, m)$  for both cases  $nm = \text{even}$  and  $nm = \text{odd}$**

**Theorem 2:** The graph  $D_{jr}(n, m)$  for  $6 \leq n \leq 9$  and  $m \geq 3$  is  $t^*$ -connected.

**Proof:** Clearly,  $d(G) = 3$ . The following cases are examined in order to establish the outcome.

**Case (i): when  $t = 1$**

In the graph  $G = D_{jr}(n, m)$ ,  $d(a_0, a_2) = 1$ , again here exists two sub cases.

**Sub case (i):** for  $nm = \text{even}$  and the path

$$P : a_0 a_r R^{-2}[4] P^0 a_1 I^2[r-1] P^0 a_2 \text{ is the Hamiltonian path between the vertices } a_0 \text{ and } a_2.$$

**Sub case (ii):** for  $nm = \text{odd}$  and the path

$$P : a_0 a_{(r-1)} R^{-2}[4] P^0 a_1 I^2[r] P^0 a_2 \text{ is the Hamiltonian path between the vertices } a_0 \text{ and } a_2.$$

**Case (ii): when  $t = 2$**

In the graph  $G = D_{jr}(n, m)$ ,  $d(a_0, a_1) = 2$ , again here exists two sub cases.

**Sub case (i):** for  $nm = \text{even}$

$$P : a_0 I^2[r] P^0 a_3 I^2[r-1] P^0 a_1 \text{ is the Hamiltonian path between the vertices } a_0 \text{ and } a_1.$$

**Sub case (ii):** for  $nm = \text{odd}$

$$P : a_0 a_3 I^2[r] P^0 a_2 I^2[r-1] P^0 a_1 \text{ is the Hamiltonian path between the vertices } a_0 \text{ and } a_1.$$

**Case (iii): when  $t = 3$**

In the graph  $G = D_{jr}(n, m)$ ,  $d(a_4, a_{11}) = 3$ .

For  $G = D_{jr}(6, m)$ , the path

$$P : a_4 a_2 a_r R^{-2}[12] P^0 a_9 I^{-2}[1] P^0 a_{(r-1)} R^{-2}[13] P^0 a_{10} I^{-2}[6] P^0 a_0 a_{11}$$

is the Hamiltonian path connecting the vertices  $a_4$  and  $a_{11}$ .

For  $G = D_{jr}(7, m)$ , again here exists two sub cases.

**Sub case (i):** for  $nm = \text{even}$

$$P : a_4 a_2 a_r R^{-2}[14] P^0 a_0 a_{13} a_{10} a_7 I^{-2}[1] P^0 a_{(r-1)} R^{-2}[15] P^0 a_{12} I^{-3}[6] P^0 a_8 a_{11}$$

is the Hamiltonian path connecting the vertices  $a_4$  and  $a_{11}$ .

**Sub case (ii):** for  $nm = \text{odd}$

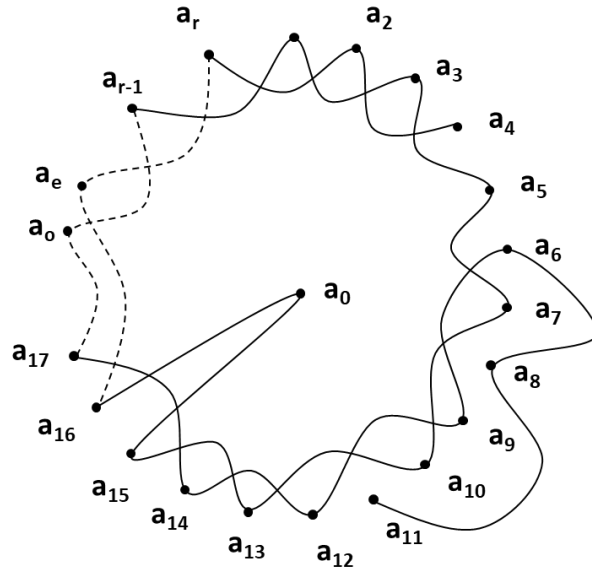
$$P : a_4 a_2 a_r R^{-2}[13] P^0 a_{10} a_7 I^{-2}[1] a_{(r-1)} R^{-2}[12] P^0 a_9 a_6 a_8 a_{11}$$

is the Hamiltonian path connecting the vertices  $a_4$  and  $a_{11}$ .

For  $G = D_{jr}(8, m)$ , the path

$$P : a_4 a_2 a_r R^{-2}[16] P^0 a_0 a_{15} a_{13} a_{10} a_7 I^{-2}[1] P^0 a_{(r-1)} R^{-2}[17] P^0 a_{14} a_{12} a_9 a_6 a_8 a_{11}$$

is the Hamiltonian path connecting the vertices  $a_4$  and  $a_{11}$ .



**Figure 5: Hamiltonian path between the vertices  $a_4$  and  $a_{11}$  in the graph  $D_{jr}(8, m)$ .**

$$\begin{aligned}
 & \left( \begin{array}{ccc} a_1 & \rightarrow & a_{k+2}, a_{k+3}, \\ (k = 1, 9, 17, \dots & & a_{k+7} \dots a_{k+9} \\ , 8m-7) & & a_{k+15} \dots a_{k+17}, \\ & & a_{k+21}, a_{k+22} \end{array} \right) & \left( \begin{array}{ccc} a_k & \rightarrow & a_{i-2}, a_{i-3}, \\ (k = 2, 8, 10, 16, \dots, 8m-8, & & a_{i+2}, a_{i+3} \\ 8m-6, 8m) & & a_{i+7}, a_{i+15}, a_0 \end{array} \right) \\
 & \left( \begin{array}{ccc} a_i & \rightarrow & a_{i-2}, a_{i-3}, \\ (k = 3, 7, 11, \dots & & a_{i+2}, a_{i+3}, a_0 \\ , 8m-1) & & \end{array} \right) & \left( \begin{array}{ccc} a_i & \rightarrow & a_{i-2}, a_{i-3}, \\ (k = 4, 6, 12, 14, \dots, 8m-10, & & a_{i+2}, a_{i+3}, a_0 \\ 8m-4, 8m-2) & & \end{array} \right) \\
 & \left( \begin{array}{ccc} a_i & \rightarrow & a_{i-2}, a_{i-3}, \\ (i = 5, 13, 21, \dots, 8m-1) & & a_{i+2}, a_{a+3} \end{array} \right)
 \end{aligned}$$

For  $G = D_{jr}(9, m)$ ,  $d(a_4, a_{12}) = 3$ , again here exist two subcases.

**Subcase (i):** For  $nm$  even, the path

$$P : a_4 I^2[8] P^0 a_5 I^{-2}[1] P^0 a_{r-1} R^{-2}[7] P^0 a_{10} a_2 a_r R^{-2}[20] P^0 a_0 a_{18} I^{-2}[12]$$

is the Hamiltonian path connecting the vertices  $a_4$  and  $a_{12}$ .

**Subcase (ii):** For  $nm$  odd, the path

$$P : a_4 a_2 a_r R^{-2}[13] P^0 a_{10} a_7 I^{-2}[1] P^0 a_{r-1} R^{-2}[14] P^0 a_{11} a_0 a_8 a_6 a_9 a_{12}$$

is the Hamiltonian path connecting the vertices  $a_4$  and  $a_{12}$ .

**Theorem 3.** The graph  $D_{jr}(n, m)$  for  $10 \leq n \leq 11$  and  $m \geq 3$  is  $t^*$ -connected.

**Proof.** Clearly,  $d(G) = 4$ . The following cases are examined to establish the result.

For the graph  $G = D_{jr}(10, m)$ ,  $d(a_4, a_{13}) = 3$  and the path

$$P : a_4 I^2[8] P^0 a_5 I^{-2}[1] P^0 a_{r-1} R^{-2}[15] P^0 a_{12} a_{10} a_7 a_9 a_0 a_2 a_r R^{-2}[14] P^0 a_{11} a_{13}$$

is the Hamiltonian path connecting  $a_4$  and  $a_{13}$ .

For  $G = D_{jr}(11, m)$ ,  $d(a_4, a_{14}) = 3$ , and again there exist two subcases.

**Subcase (i):** For  $nm$  even, the path

$$P : a_4 I^2[10] P^0 a_7 I^{-2}[1] P^0 a_{r-1} R^{-2}[15] P^0 a_{12} a_9 I^2[13] P^0 a_0 a_2 a_r R^{-2}[14]$$

is the Hamiltonian path connecting  $a_4$  and  $a_{14}$ .

**Subcase (ii):** For  $nm$  odd,

$$P : a_4 I^2[12] P^0 a_2 a_r R^{-2}[1] P^0 a_{r-1} R^{-2}[24] P^0 a_0 a_{22} I^{-2}[14]$$

is the Hamiltonian path connecting  $a_4$  and  $a_{14}$ .

For  $G = D_{jr}(10, m)$ ,  $d(a_4, a_{16}) = 4$  and the path

$$P : a_4 I^2[14] P^0 a_{11} I^{-2}[1] P^0 a_{r-1} R^{-2}[13] P^0 a_0 a_2 a_r R^{-2}[16]$$

is the Hamiltonian path connecting  $a_4$  and  $a_{16}$ .

For  $G = D_{jr}(11, m)$ ,  $d(a_4, a_{17}) = 4$ , again there exist two subcases.

**Subcase (i):** For  $nm$  even,

$$P : a_4 I^2[14] P^0 a_0 a_2 a_r R^{-2}[16] P^0 a_{19} I^2[r-1] P^0 a_1 I^2[17]$$

is the Hamiltonian path between  $a_4$  and  $a_{17}$ .

**Subcase (ii):** For  $nm$  odd,

$$P : a_4 I^2[12] P^0 a_{15} I^{-2}[1] P^0 a_{r-1} R^{-2}[14] P^0 a_0 a_2 a_r R^{-2}[17]$$

is the Hamiltonian path between  $a_4$  and  $a_{17}$ .

**Theorem 4.** The graph  $D_{jr}(n, m)$  for  $12 \leq n \leq 13$  and  $m \geq 3$  is  $t^*$ -connected.

**Proof.**

For  $G = D_{jr}(12, m)$ ,  $d(a_4, a_{15}) = 3$  and the path

$$P : a_4 I^2[r] P^0 a_2 a_0 a_3 I^2[13] P^0 a_1 a_{r-1} R^{-2}[15]$$

is the Hamiltonian path connecting  $a_4$  and  $a_{15}$ .

For  $G = D_{jr}(13, m)$ ,  $d(a_4, a_{16}) = 3$ , again two subcases exist:

**Subcase (i):** For  $nm$  even,

$$P : a_4 I^2[14] P^0 a_2 a_r a_0 a_{r-1} R^{-2}[1] P^0 a_1 a_{r-2} R^{-2}[16]$$

is the Hamiltonian path connecting  $a_4$  and  $a_{16}$ .

**Subcase (ii):** For  $nm$  odd, the path is (text missing).

For  $G = D_{jr}(12, m)$ ,  $d(a_4, a_{18}) = 4$  and the path

$$P : a_4 I^2[14] P^0 a_0 a_2 a_r R^{-2}[20] P^0 a_{17} I^{-2}[1] P^0 a_{r-1} R^{-2}[19] P^0 a_{16} a_{18}$$

is the Hamiltonian path between  $a_4$  and  $a_{18}$ .

For  $G = D_{jr}(13, m)$ ,  $d(a_4, a_{19}) = 4$ , again with subcases:

**Subcase (i):** For  $nm$  even,

$$P : a_4 I^2[18] P^0 a_{21} I^2[r-1] P^0 a_1 a_3 a_0 a_{15} I^{-2}[2] P^0 a_r R^{-2}[20] P^0 a_{17} a_{19}$$

is the Hamiltonian path.

**Subcase (ii):** For  $nm$  odd,

$$P : a_4 I^2[16] P^0 a_0 a_2 a_r R^{-2}[21] P^0 a_{18} a_{15} I^{-2}[1] P^0 a_{r-1} R^{-2}[20] P^0 a_{17} a_{19}$$

is the Hamiltonian path.

For  $G = D_{jr}(12, m)$ ,  $d(a_9, a_{22}) = 5$  and the path

$$P : a_9 I^{-2}[1] P^0 a_{r-1} R^{-2}[23] P^0 a_0 a_{11} I^2[21] P^0 a_{24} I^2[r] P^0 a_2 I^2[22]$$

connects  $a_9$  and  $a_{22}$ .

For  $G = D_{jr}(13, m)$ ,  $d(a_9, a_{22}) = 5$ , again two subcases exist.

**Subcase (i):** For  $nm$  even,

$$P : a_9 I^{-2}[1] P^0 a_{r-1} R^{-2}[17] P^0 a_{20} I^{-2}[16] P^0 a_0 a_{15} a_{12} a_{14} a_{11} a_{13} a_{10} I^{-2}[2] P^0 a_r R^{-2}[22]$$

is the Hamiltonian path.

**Subcase (ii):** For  $nm$  odd,

$$P : a_9 I^2[1] P^0 a_{r-1} R^{-2}[28] P^0 a_0 a_{12} a_{15} I^2[21] P^0 a_{24} a_{26} a_{23} I^2[r] P^0 a_2 I^2[r] P^0 a_2 I^2[10] P^0 a_{13} a_{11} a_{14} I^2[22]$$

is the Hamiltonian path.

### 3. Conclusion

In this paper, the Hamiltonian- $t^*$ -laceability properties of the distance graph of the Jahangir graph  $D_{jr}(n, m)$  for  $2 \leq n \leq 13$  and  $m \geq 3$  are explored. The Hamiltonian laceability properties of distance graphs and product graphs can be used to design network topologies. Hamiltonian laceability in a single connected graph can be used to overcome network failures when the nodes are connected in the form of a hybrid topology.

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