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Reduced Second Stress Index

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ABSTRACT: The stress of a vertex is a node centrality index, which has been introduced by Shimbel (1953). The stress of a vertex in a graph is the number of geodesics (shortest paths) passing through it. A topological index of a chemical structure (graph) is a number that correlates the chemical structure with chemical reactivity or physical properties. In this paper, we introduce a new topological index for graphs called reduced second stress index using stresses of vertices. Further, we establish some inequalities, prove some results and compute reduced second stress index for some standard graphs.

Key Words: Graph, geodesic, topological index, stress of a vertex.

Contents

1. Introduction

For standard terminology and notation in graph theory, we follow the textbook by Harary [7]. Any non-standard terminology required in this work will be introduced at the appropriate points.

Let $G = (V, E)$ be a finite, undirected graph. The distance between two vertices u and v in G, denoted by $d(u, v)$, is the number of edges in a shortest path a geodesic joining them. A geodesic P is said to pass through a vertex v if v appears as an internal vertex of P; that is, v lies on P but is not one of its endpoints. The degree of a vertex v is denoted by $d(v)$.

The notion of stress of a vertex as a centrality measure was introduced by Shimbel in 1953 [27]. This measure has since found applications in biology, sociology, psychology, and other disciplines (see [25,11]). The stress of a vertex v in a graph G, denoted $str_G(v)$ or simply $str(v)$, is defined as the number of geodesics that pass through v. The maximum and minimum stress among all vertices of G are denoted by Θ_G and θ_G, respectively. The concepts of the stress number and stress-regular graphs were introduced by Bhargava, Dattatreya, and Rajendra [3]. A graph G is said to be k-stress regular if $str(v) = k$ for every vertex $v \in V(G)$.

The well-known Zagreb indices, defined in terms of vertex degrees, have been widely used to study molecular structures of chemical compounds [5,6]. For a simple graph G, the first and second Zagreb indices are defined as

$$M_1(G) = \sum_{v \in V(G)} d(v)^2, \tag{1.1}$$

$$M_2(G) = \sum_{uv \in E(G)} d(u)d(v). \tag{1.2}$$

Inspired by these classical indices, Rajendra et al. [16] introduced two analogous invariants based on vertex stress, called the first stress index and the second stress index. For a simple graph G, they are

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defined as

$$S_1(G) = \sum_{v \in V(G)} str(v)^2, \tag{1.3}$$

$$S_2(G) = \sum_{uv \in E(G)} str(u)\, str(v). \tag{1.4}$$

It is well known that the first Zagreb index satisfies the identity

$$M_1(G) = \sum_{uv \in E(G)} \big(d(u) + d(v)\big). \tag{1.5}$$

However, an analogous identity does not hold for the first stress index. To illustrate this, consider the path P_3 on three vertices, shown in Figure 1.

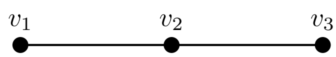

Figure 1: The path P_3.

For P_3, the vertex stresses are $str(v_1) = str(v_3) = 0$ and $str(v_2) = 1$. Hence,

$$S_1(P_3) = 0^2 + 1^2 + 0^2 = 1,$$

while

$$\sum_{uv \in E(P_3)} str(u) + str(v) = 0 + 1 + 1 + 0 = 2.$$

The reduced second Zagreb index, introduced in [4], is defined for a graph G by

$$R_2M(G) = \sum_{v_i v_j \in E(G)} (d(v_i) - 1)(d(v_j) - 1). \tag{1.6}$$

Motivated by this identity, we introduce in the present work a new topological index based on vertex stress, called the reduced second stress index. We derive several inequalities involving this index and compute its values for various standard classes of graphs. Many stress related concepts in graphs and topological indices have been defined and studied by several authors [1,2,8-10,13-24,26,28-30].

2. Reduced Second Stress Index

Definition 2.1 The reduced second stress sum index $RS_2(G)$ of a simple graph G is defined as

$$RS_2(G) = \sum_{uv \in E(G)} (\mathrm{str}(u) - 1)(\mathrm{str}(v) - 1). \tag{2.1}$$

Theorem 2.1 *For a graph G, $RS_2(G) = |E|$ if and only if neighbours of every vertex induce a complete subgraph of G.*

Proof: Suppose that $RS_2(G) = |E|$. By the definition of the reduced second stress index,

$$RS_2(G) = \sum_{uv \in E(G)} (\mathrm{str}(u) - 1)(\mathrm{str}(v) - 1).$$

Since the sum equals the number of edges, every summand must be equal to 1. Hence,

$$(\mathrm{str}(u) - 1)(\mathrm{str}(v) - 1) = 1 \qquad \text{for all } uv \in E(G),$$

which implies that

$$\mathrm{str}(v) = 0 \qquad \text{for every } v \in V(G).$$

Let $v \in V(G)$. We show that the neighbours of v induce a complete subgraph. If v is a pendant vertex, the statement is trivial. Suppose that v is not pendant. Assume, for contradiction, that there exist neighbours $u, w \in N(v)$ such that $uw \notin E(G)$. Then the path uvw is a geodesic of length 2 passing through v, which implies $\mathrm{str}(v) \geq 1$, a contradiction. Therefore, any two neighbours of v are adjacent, so the subgraph induced by $N(v)$ is complete. Since v is arbitrary, this holds for all vertices.

Conversely, suppose that the neighbours of every vertex in G induce a complete subgraph. Let $v \in V(G)$. Since every two neighbours of v are adjacent, there exists no geodesic of length at least 2 passing through v. Hence,

$$\mathrm{str}(v) = 0 \qquad \text{for all } v \in V(G).$$

It follows that for every edge $uv \in E(G)$,

$$(\mathrm{str}(u) - 1)(\mathrm{str}(v) - 1) = (-1)(-1) = 1.$$

Summing over all edges, we obtain

$$RS_2(G) = \sum_{uv \in E(G)} 1 = |E|.$$

□

Proposition 2.1 *For the complete bipartite $K_{m,n}$,*

$$RS_2(K_{m,n}) = \frac{mn}{4}\left[(n^2 - n - 2)(m^2 - m - 2)\right].$$

Proof: Let $V_1 = \{v_1, \ldots, v_m\}$ and $V_2 = \{u_1, \ldots, u_n\}$ be the partite sets of $K_{m,n}$. We have,

$$\mathrm{str}(v_i) = \frac{n(n-1)}{2} \text{ for } 1 \leq i \leq m \tag{2.2}$$

and

$$\mathrm{str}(u_j) = \frac{m(m-1)}{2} \text{ for } 1 \leq j \leq n. \tag{2.3}$$

Using (2.2) and (2.3) in the Definition 2.1, we have

$$\begin{aligned} RS_2(K_{m,n}) &= \sum_{uv \in E(G)} (\mathrm{str}(u) - 1)(\mathrm{str}(v) - 1) \\ &= \sum_{1 \leq i \leq m,\ 1 \leq j \leq m} (str(v_i) - 1)(str(u_j) - 1) \\ &= \sum_{1 \leq i \leq m,\ 1 \leq j \leq n} \left[\frac{n(n-1)}{2} - 1\right]\left[\frac{m(m-1)}{2} - 1\right] \\ &= \frac{mn}{4}\left[(n^2 - n - 2)(m^2 - m - 2)\right]. \end{aligned}$$

□

Proposition 2.2 *If $G = (V, E)$ is a k-stress regular graph, then*

$$RS_2(G) = 2(k-1)^2|E|.$$

Proof: Suppose that G is a k-stress regular graph. Then

$$\mathrm{str}(v) = k \text{ for all } v \in V(G).$$

By the Definition 2.1, we have

$$\begin{aligned} RS_2(G) &= \sum_{uv\in E(G)} (\mathrm{str}(u)-1)(\mathrm{str}(v)-1) \\ &= \sum_{uv\in E(G)} (k-1)(k-1) \\ &= 2(k-1)^2|E|. \end{aligned}$$

□

Corollary 2.1 *For a cycle C_n,*

$$RS_2(C_n) = \begin{cases} \dfrac{n(n^2-4n-5)^2}{32}, & \text{if } n \text{ is odd} \\ \dfrac{n(n^2-2n-8)^2}{32}, & \text{if } n \text{ is even.} \end{cases}$$

Proof: For any vertex v in C_n, we have,

$$\mathrm{str}(v) = \begin{cases} \dfrac{(n-1)(n-3)}{8}, & \text{if } n \text{ is odd} \\ \dfrac{n(n-2)}{8}, & \text{if } n \text{ is even.} \end{cases}$$

Hence C_n is

$$\begin{cases} \dfrac{(n-1)(n-3)}{8}\text{-stress regular}, & \text{if } n \text{ is odd} \\ \dfrac{n(n-2)}{8}\text{-stress regular}, & \text{if } n \text{ is even.} \end{cases}$$

Since C_n has n vertices and n edges, by the Proposition 2.2, we have

$$\begin{aligned} RS_2(C_n) &= 2n \times \begin{cases} \left[\dfrac{(n-1)(n-3)}{8} - 1\right]^2, & \text{if } n \text{ is odd} \\ \left[\dfrac{n(n-2)}{8} - 1\right]^2, & \text{if } n \text{ is even.} \end{cases} \\ &= \begin{cases} \dfrac{n(n^2-4n-5)^2}{32}, & \text{if } n \text{ is odd} \\ \dfrac{n(n^2-2n-8)^2}{32}, & \text{if } n \text{ is even.} \end{cases} \end{aligned}$$

□

Proposition 2.3 *Let T be a tree on n vertices. Then*

$$\begin{aligned} RS_2(T) = \sum_{uv\in J} &\left[\left(\sum_{1\le i<j\le m(u)} |C_i^u||C_j^u| - 1\right)\left(\sum_{1\le i<j\le m(v)} |C_i^v||C_j^v| - 1\right)\right] \\ &+ \sum_{w\in Q}\left(\sum_{1\le i<j\le m(w)} |C_i^w||C_j^w| - 1\right). \end{aligned}$$

where J is the set of internal(non-pendant) edges in T, Q denotes the set of all vertices adjacent to pendent vertices in T, and the sets $C_1^v, \ldots, C_m^v$ denotes the vertex sets of the components of $T - v$ for an internal vertex v of degree $m = m(v)$.

Proof:

We know that a pendant vertex in T has zero stress. Let v be an internal vertex of T of degree $m = m(v)$. Let $C_1^v, \ldots, C_m^v$ be the components of $T - v$. Since there is only one path between any two vertices in a tree, it follows that,

$$\mathrm{str}(v) = \sum_{1 \le i < j \le m} |C_i^v||C_j^v| \tag{2.4}$$

Let J denotes the set of internal(non-pendant) edges, and P denotes pendant edges and Q denotes the set of all vertices adjacent to pendent vertices in T. Then using (2.4) in the Definition (2.1), we have

$$\begin{aligned} RS_2(T) &= \sum_{uv \in J} (\mathrm{str}(u) - 1)(\mathrm{str}(v) - 1) + \sum_{uv \in P} (\mathrm{str}(u) - 1)(\mathrm{str}(v) - 1) \\ &= \sum_{uv \in J} (\mathrm{str}(u) - 1)(\mathrm{str}(v) - 1) + \sum_{w \in Q} \mathrm{str}(w) - 1 \\ &= \sum_{uv \in J} \left[\left(\sum_{1 \le i < j \le m(u)} |C_i^u||C_j^u| - 1 \right) \left(\sum_{1 \le i < j \le m(v)} |C_i^v||C_j^v| - 1 \right) \right] \\ &\quad + \sum_{w \in Q} \left(\sum_{1 \le i < j \le m(w)} |C_i^w||C_j^w| - 1 \right). \end{aligned}$$

□

Corollary 2.2 *For the path P_n on n vertices*

$$RS_2(P_n) = \sum_{i=1}^{n-1} \left[i^4 - 2ni^3 + (n^2 + n + 1)i^2 - 2ni + (n+1) \right].$$

Proof: Let P_n be the path with vertex sequence $v_1, v_2, \ldots, v_n$ (shown in Figure 2).

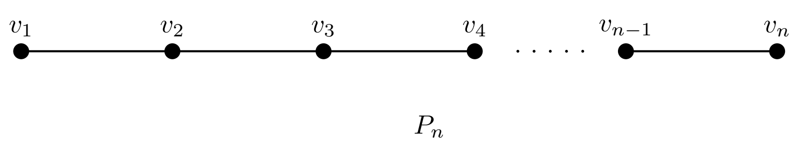

Figure 2: The path P_n on n vertices.

We have,

$$\mathrm{str}(v_i) = (i-1)(n-i), \qquad 1 \le i \le n.$$

Then

$$\begin{aligned} RS_2(P_n) &= \sum_{uv \in E(P_n)} (\mathrm{str}(u) - 1)(\mathrm{str}(v) - 1) \\ &= \sum_{i=1}^{n-1} (str(v_i) - 1)(str(v_{i+1}) - 1) \\ &= \sum_{i=1}^{n-1} ((i-1)(n-i) - 1)((i)(n-i-1) - 1) \\ &= \sum_{i=1}^{n-1} \left[i^4 - 2ni^3 + (n^2 + n + 1)i^2 - 2ni + (n+1) \right]. \end{aligned}$$

□

Proposition 2.4 *Let $G_1, G_2, \ldots, G_m$ be the components of a disconnected graph H. Then The reduced second stress index of H is given by*

$$RS_2(H) = RS_2(G_1) + RS_2(G_2) + \cdots + RS_2(G_m).$$

Proof: We have $H = \bigcup_{i=1}^{m} G_i$. Note that an edge $uv \in E(H)$ if and only if uv belongs to the same component. Hence,

$$RS_2(H) = RS_2\left(\bigcup_{i=1}^{m} G_i\right)$$

$$\begin{aligned} &= \sum_{u_{1i}v_{1i} \in E(G_1)} (str(u_{1i}) - 1)(str(v_{1i}) - 1) + \cdots \\ &\quad + \sum_{u_{mi}v_{mi} \in E(G_m)} (str(u_{mi}) - 1)(str(v_{mi}) - 1) \\ &= RS_2(G_1) + RS_2(G_2) + RS_2(G_3) + \cdots + RS_2(G_m). \end{aligned}$$

□

Proposition 2.5 *Let $Wd(n, m)$ denotes the windmill graph constructed for $n \geq 2$ and $m \geq 2$ by joining m copies of the complete graph K_n at a shared universal vertex v. Then*

$$RS_2(Wd(n, m)) = \frac{m(n-1)\left[m(m-1)(n-1)^2 - 2\right]}{2}.$$

Hence, for the friendship graph F_k on $2k + 1$ vertices,

$$RS_2(F_k) = 4k^3 - 4k^2 - 2k.$$

Proof: Clearly the stress of any vertex other than universal vertex is zero in $Wd(n, m)$, because neighbors of that vertex induces a complete subgraph of $Wd(n, m)$. Also, since there are m copies of K_n in $Wd(n, m)$ and their vertices are adjacent to v, it follows that, the only geodesics passing through v are of length 2 only. So, $\mathrm{str}(v) = \frac{m(m-1)(n-1)^2}{2}$. Note that there are $m(n - 1)$ edges incident on v and the edges that are not incident on v have end vertices of stress zero. Hence by the Definition 2.1, we have

$$\begin{aligned} RS_2(Wd(n, m)) &= m(n-1)\left(\mathrm{str}(v) - 1\right) \\ &= m(n-1)\left(\frac{m(m-1)(n-1)^2}{2} - 1\right) \\ &= \frac{m(n-1)\left[m(m-1)(n-1)^2 - 2\right]}{2}. \end{aligned}$$

Since the friendship graph F_k on $2k + 1$ vertices is nothing but $Wd(3, k)$, it follows that

$$RS_2(F_k) = 4k^3 - 4k^2 - 2k.$$

□

Proposition 2.6 *Let W_n denotes the wheel graph constructed on $n \geq 4$ vertices. Then*

$$RS_2(W_n) = 0.$$

Proof: In W_n with $n \geq 4$, there are $(n-1)$ peripheral vertices and one central vertex, say v. It is easy to see that

$$\text{str}(v) = \frac{(n-1)(n-4)}{2} \tag{2.5}$$

Let p be a peripheral vertex. Since v is adjacent to all the peripheral vertices in W_n, there is no geodesic passing through p and containing v. Hence contributing vertices for str(p) are the rest peripheral vertices. So, by denoting the cycle $W_n - p$ (on $n-1$ vertices) by C_{n-1}, we have

$$\begin{aligned} \text{str}_{W_n}(p) &= \text{str}_{W_n - v}(p) \\ &= \text{str}_{C_{n-1}}(p) \\ &= 1 \end{aligned} \tag{2.6}$$

Let us denote the set of all radial edges in W_n by R, and the set of all peripheral edges by Q. Note that there are $(n-1)$ radial edges and $(n-1)$ peripheral edges in W_n. By using Definition 2.1, we have

$$\begin{aligned} RS_2(W_n) = &\sum_{xy \in R} [(str(x)-1)(str(y)-1)] \\ &+ \sum_{xy \in Q} [(str(x)-1)(str(y)-1)] \\ =&(n-1)[[(str(v)-1)(str(p)-1)]] + (n-1)\left(str(p)-1\right)^2 \\ =&0 \end{aligned}$$

□

Proposition 2.7 *For the complement of a cycle C_n $(n \geq 5)$, the reduced second stress index is given by*

$$RS_2(\overline{C_n}) = \frac{n(n-3)(n-5)^2}{2}.$$

Proof: Let $G = \overline{C_n}$ and $V(G) = \{v_1, v_2, \ldots, v_n\}$ (from [12] of Theorem 2.5)

$$st(v_i) = n-4, \quad v_i \in V(G)$$

and the total number of edges in G is

$$|E(G)| = \frac{n(n-3)}{2}$$

By the Definition 2.1, we have

$$RS_2(G) = \sum_{uv \in E(G)} [(st(u)-1)(st(v)-1)] .$$

we have $st(u) = st(v) = n-4$ for all $u, v \in V(G)$

$$RS_2(\overline{C_n}) = \sum_{uv \in E(G)} [(n-5)(n-5)] .$$

$$RS_2(\overline{C_n}) = \frac{n(n-3)}{2}(n-5)^2 = \frac{n(n-3)(n-5)^2}{2}.$$

□

Definition 2.2 The line graph of G, denoted by $L(G)$, is defined such that for every edge of G there is a corresponding vertex in $L(G)$, and two vertices in $L(G)$ are adjacent if and only if the corresponding edges of G have a common vertex.

Proposition 2.8 *For the line graph of the complete graph K_n,*

$$RS_2(L(K_n)) = n(n-1)(n-2)\left(n^2 - 5n + 5\right)^2.$$

Proof: For $G = L(K_n)$, each vertex satisfies (from [12] of Theorem 2.12)

$$st(v) = (n-2)(n-3), \quad v \in V(G),$$

and the total number of edges in G is

$$|E(G)| = n(n-1)(n-2).$$

By Definition 2.1,

$$RS_2(L(K_n)) = \sum_{uv \in E(G)} \left[(st(u)-1)(st(v)-1)\right].$$

Now, we have: $st(u) = st(v) = (n-2)(n-3)$ for all $u, v \in V(G)$.

$$RS_2(L(K_n)) = \sum_{uv \in E(G)} \left[((n-2)(n-3)-1)((n-2)(n-3)-1)\right]$$

$$RS_2(L(K_n)) = n(n-1)(n-2)\left(n^2 - 5n + 5\right)^2.$$

□

Proposition 2.9 *For the line graph of $K_{m,n}$,*

$$RS_2(L(K_{m,n})) = \frac{mn(m+n-2)(mn-m-n)^2}{2}.$$

Proof: For $G = L(K_{m,n})$, each vertex satisfies (from [12] of Theorem 2.14)

$$st(v) = mn - m - n + 1, \quad v \in V(G),$$

and the total number of edges in G is

$$|E(G)| = \frac{mn(m+n-2)}{2}.$$

By Definition 2.1,

$$RS_2(L(K_{m,n})) = \sum_{uv \in E(G)} \left[(st(u)-1)(st(v)-1)\right]$$

we have, $st(u) = st(v) = mn - m - n + 1$ for all $u, v \in V(G)$

$$RS_2(L(K_{m,n})) = \sum_{uv \in E(G)} \left[(mn-m-n)(mn-m-n)\right]$$

$$RS_2(L(K_{m,n})) = \frac{mn(m+n-2)(mn-m-n)^2}{2}.$$

□

Proposition 2.10 *Let K_n and K_m be complete graphs on n and m vertices, respectively. Then for the Kronecker product of K_n and K_m, denoted by $K_n \otimes K_m$,*

$$RS_2(K_n \otimes K_m) = nm(n-1)(m-1)\left[\frac{(n-1)(m-1)(m+n-4)-2}{2}\right]^2.$$

Proof: For $G = K_n \otimes K_m$ each vertex satisfies (from [12] of Theorem 2.14)

$$str(v) = \frac{(n-1)(m-1)(m+n-4)}{2} \quad v \in V(G)$$

$$\begin{aligned} RS_2(K_n \otimes K_m) &= \sum_{uv \in E(G)} [(st(u)-1)(st(v)-1)] \\ &= \sum_{uv \in E(G)} \left[\left(\frac{(n-1)(m-1)(m+n-4)}{2} - 1 \right) \left(\frac{(n-1)(m-1)(m+n-4)}{2} - 1 \right) \right] \\ &= nm(n-1)(m-1) \left[\frac{(n-1)(m-1)(m+n-4)-2}{2} \right]^2 . \end{aligned}$$

□

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