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Some Properties on Possibility Interval-Valued Intuitionistic Fuzzy Soft Sets

Arunadevi Sivaraj, Jayanthi Duraisamy and Priyadharshini Manoharan

ABSTRACT: This paper's main goal is to present and examine some fundamental characteristics of possibility interval-valued intuitionistic fuzzy soft sets (PIVIFSSs). Initially, the cardinality measure is concentrated on PIVIFSSs and its performance under the union and intersection are analyzed. Furthermore, the operators @,$ and * are formally defined and applied to PIVIFSSs. Their validity is established through rigorous mathematical proofs verifying the commutative, associative, and distributive properties. Additionally, the definitions and analyses of the necessary and sufficient operators are presented to justify their adequacy and relevance. Finally the cardinality measure is applied to a group decision making problem and obtained an optimal solution.

Keywords: Possibility sets, possibility soft sets, possibility fuzzy soft sets, interval-valued intuitionistic fuzzy soft sets, possibility interval-valued intuitionistic fuzzy soft sets.

Contents

1. Introduction

The advancement of theoretical mathematics plays a crucial role in the progress of applied mathematics. Although classical set theory provides precise results, its rigid decision-making nature can sometimes lead to unrealistic outcomes. To address this, Zadeh [24] introduced the concept of fuzzy sets, which are more flexible than traditional crisp sets by assigning a degree of membership to each element. However, even fuzzy sets can yield unsatisfactory results in certain cases, such as in decision-making scenarios like elections. To overcome these limitations, Atanassov [4] proposed intuitionistic fuzzy sets, incorporating both membership and non-membership functions which was further developed by Maji [17]. He later extended this concept further, introducing interval-valued intuitionistic fuzzy sets as a more generalized form.

Simultaneously, Zadeh [25] introduced possibility theory, which was later advanced by Dubois [9]. In addition, Molodtsov [16] introduced soft set theory, which significantly influenced the development of set theory and was subsequently explored in greater depth by Maji [13]. Building on these foundations, Maji et al. [11], Salleh [20], and Jiang [10] introduced and analyzed fuzzy soft sets, intuitionistic fuzzy soft sets, and interval-valued intuitionistic fuzzy soft sets, respectively. The continuous drive for research in this area further led to the development of possibility fuzzy soft sets, possibility intuitionistic fuzzy soft sets, and possibility interval-valued fuzzy soft sets, proposed by Alkhazaleh [2], Maruah et al. [18], and Alkhazaleh [1], respectively.

The concept of relative cardinality for interval-valued fuzzy sets, along with its definition, properties, and computational aspects, was first addressed by Zywica [26] in 2006. In the same year, Yager [23]

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explored the use of a fuzzy subset of non-negative integers to represent the cardinality of a fuzzy set. His work established fundamental relationships between the degree of fuzziness in a set A and the accuracy of the fuzzy subset used to express its cardinality.

In 2013, Deschrijvers and Pavol [7] presented an axiomatic formulation of scalar cardinality for interval-valued fuzzy sets. During the same period, Dhar [14] introduced a notion of fuzzy set cardinality that is closely tied to the concept of fuzzy set complementation. Chamorro-Martinez [6] further expanded the discussion by examining multiple representations of fuzzy set cardinality and their relevance in fuzzy quantification. The study specifically focused on well-known approaches such as sigma-count, fuzzy numbers, and gradual numbers. Later, in 2015, Tripathy et al. [21,22] proposed a bag-theoretic framework for determining the cardinality of an intuitionistic fuzzy set, contributing a novel perspective to the ongoing development in this area of research.

Decision making is crucial when data are imprecise and uncertain. To overcome the difficulties, soft sets have been applied by Maji et al. [12], fuzzy soft sets by Roy and Maji [19], generalized intuitionistic fuzzy soft sets by Dinda et al. [8] in decision making.

Building upon these advancements, a new concept called the possibility interval-valued intuitionistic fuzzy soft set (PIVIFSS) has been introduced, and its mathematical properties have been thoroughly analyzed. Initially, the study focuses on defining the cardinality measure of PIVIFSSs under union and intersection operations. Furthermore, the operators @, and * are formally defined and applied to PIVIFSSs. Their validity is established through mathematical proofs of the commutative, associative, and distributive properties. Finally, the necessity and possibility operators are defined and examined in detail and the cardinality measure is applied to a group decision making problem.

2. Preliminaries

This section provides a thorough summary of the core ideas and theoretical underpinnings necessary to comprehend the approaches and frameworks covered in this research study.

Definition 2.1 [13] Let U be the universe set and E be the set of parameters. A pair (F,E) is called a soft set over U if and only if F is a mapping of E into the set of all subsets of the set U.

Definition 2.2 [4] An intuitionistic fuzzy set (IFS) A in a non-empty set U is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in U\}$ where the functions $\mu_A : U \to [0,1]\, and\, \nu_A : U \to [0,1]$ denote the degree of membership and the degree of non-membership of each element $x \in U$ to the set A,respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in U$.

Definition 2.3 [5] An interval-valued intuitionistic fuzzy set (IVIFS) A in a non-empty set U is an object having the form

$$A = \{\langle x, [\mu_A^-(x), \mu_A^+(x)], [\nu_A^-(x), \nu_A^+(x)]\rangle : x \in U\}$$

where the functions $\mu_A^-, \mu_A^+ : U \to [0,1]\, and\, \nu_A^-, \nu_A^+ : U \to [0,1]$ denote the degree of membership and the degree of non-membership of each element $x \in U$ to the set A, respectively and $0 \leq \mu_A^-(x) \leq \mu_A^+(x) \leq 1$, $0 \leq \nu_A^-(x) \leq \nu_A^+(x) \leq 1$, $0 \leq \mu_A^+(x) + \nu_A^+(x) \leq 1\, for\, all\, x \in U$.

Definition 2.4 [20] Let U be the universe set and E be a set of parameters. Let P(U) denote the power set of U and $A \subseteq E$.A pair (F,E) is an intuitionistic fuzzy soft set over U, where F is a mapping given by $F : A \to P(U)$.

Definition 2.5 [2] Let $U = \{x_1, x_2, ..., x_n\}$ be the universal set of elements and $E = \{e_1, e_2, ..., e_n\}$ be the universal set of parameters.The pair (U,E) will be the soft universe. Let $F : E \to (I \times I)^U \times I^U$ where $(I \times I)^U$ is the collection of all intuitionistic fuzzy subsets of U and I^U is the collection of all fuzzy subsets of U. Let $P : E \to I^U$ be a fuzzy subset of E and let $F_p : E \to (I \times I)^U \times I^U$ be a function defined as follows:

$$F_p(e) = (F(e)(x), p(e)(x)), \forall x \in U$$

where

$$F(e)(x) = (\mu(x), \nu(x)).$$

Then F_p is called a possibility intuitionistic fuzzy soft set (PIFSS) over (U,E). For each parameter $e_i, F_p(e_i) = (F(e_i)(x), p(e_i)(x)$, indicates not only the degree of belongingness of the elements of U in $F(e_i)$ but also the degree of possibility of belongingness of the elements of U in $F(e_i)$, which is represented by $p(e_i)$. So we write $F_p(e_i)$ as follows:

$F(e_i) = \{(\frac{x_1}{F(e_i)(x_1)}, p(e_i)(x_1)), (\frac{x_2}{F(e_i)(x_2)}, p(e_i)(x_2)), ..., (\frac{x_n}{F(e_i)(x_n)}, p(e_i)(x_n))\}$

Sometime we write F_p as (F_p, E). If $A \subseteq E$, then we have a PIFSS (F_p, A).

Definition 2.6 [3] Let $U = \{x_1, x_2, ..., x_n\}$ be the universal set of elements and $E = \{e_1, e_2, ..., e_n\}$ be the universal set of parameters.The pair (U,E) will be the soft universe. Let $F : E \to (I \times I)^U \times I^U where (I \times I)^U$ is the collection of all intuitionistic fuzzy subsets of U and I^U is the collection of all fuzzy subsets of U. Let $P : E \to I^U$ be a fuzzy subset of E and let $F_p : E \to (I \times I)^U \times I^U$ be a function defined as follows: $F_p(e) = (F(e)(x), p(e)(x)), \forall x \in U$ where $F(e)(x) = ([\mu^-(x), \mu^+(x)], [\nu^-(x), \nu^+(x)]$ Then F_p is called a possibility intuitionistic fuzzy soft set (PIFSS) over (U,E). For each parameter $e_i, F_p(e_i) = (F(e_i)(x), p(e_i)(x)$, indicates not only the degree of belongingness of the elements of U in $F(e_i)$ but also the degree of possibility of belongingness of the elements of U in $F(e_i)$, which is represented by $P(e_i)$.So we write $F_p(e_i)$ as follows:

$\{(\frac{x_1}{F(e_i)(x_1)}, p(e_i)(x_1)), (\frac{x_2}{F(e_i)(x_2)}, p(e_i)(x_2)), \ldots, (\frac{x_n}{F(e_i)(x_n)}, p(e_i)(x_n))\}$

3. Measures and Operators on Possibility Interval-Valued Intuitionistic Fuzzy Soft Set

This section focuses on the introduction and analysis of the cardinality measure, as well as the union and intersection of PIVIFSSs with respect to this measure. Additionally, the @ and * operators on PIVIFSSs and their properties are examined. The necessity and possibility operators are also defined and explored. All the concepts and operations presented are mathematically verified and illustrated with appropriate numerical examples.

Definition 3.1 The cardinality of a PIVIFSS F_P is defined as the interval,

$card(F_p) = [min \sum count(F_p), max \sum count(F_p)]$

where

$$\min \sum count(F_p) = \sum_{x \in X} \left[\frac{[\mu^-_{F_p}(x) + \mu^+_{F_p}(x)]}{2} \cdot p(x) \right]$$

and

$$\max \sum count(F_p) = \sum_{x \in X} \left[\frac{2 - \nu^-_{F_p}(x) - \nu^+_{F_p}(x)}{2} \right]$$

Definition 3.2 The average possible cardinality of a PIVIFSS A is given by,

$$A(F_p) = \frac{1}{2} \sum_{x \in X} \left(\frac{\mu^-_{F_p}(x) + \mu^+_{F_p}(x)}{2} \cdot p(x) + \frac{2 - \nu^-_{F_p}(x) - \nu^+_{F_p}(x)}{2} \right)$$

Example 3.3 Let $U = \{u_1, u_2\}$ be the universe set, $E = \{e_1, e_2\}$ be the parameter set, and let $p : E \to I^U$. Then the function $F_p : E \to (I \times I)^U \times I^U$ is defined as

$$F_p(e_1) = \left\{ \left(\frac{u_1}{([0.5, 0.6], [0.4, 0.4])}, 0.6 \right), \left(\frac{u_2}{([0.7, 0.8], [0.1, 0.2])}, 0.7 \right) \right\}$$
$$F_p(e_2) = \left\{ \left(\frac{u_1}{([0.6, 0.6], [0.3, 0.4])}, 0.5 \right), \left(\frac{u_2}{([0.4, 0.5], [0.3, 0.5])}, 0.3 \right) \right\}$$

Then F_p is a PIVIFSS over (U, E). Now,

$$\min \sum count(F_p(e_1)) = 0.855; \qquad \max \sum count(F_p(e_1)) = 1.45$$
$$\therefore card(F_p(e_1)) = [0.855, 1.450] \qquad A(F_p(e_1)) = 1.1525$$

Example 3.4 Let $\bar{F}_p$ be the complement of a PIVIFSS F_p. Then the cardinality of $\bar{F}_p$ is given by

$$Card\,\bar{F}_p = \left[\min \sum count\bar{F}_p, \max \sum count\bar{F}_p\right]$$

where,

$$\min \sum count\,\bar{F}_p = \sum_{x \in X} \left[\frac{\nu^-_{F_p}(x) + \nu^+_{F_p}(x)}{2}\right] \cdot (1 - p(x))$$
$$\max \sum count\,\bar{F}_p = \sum_{x \in X} \left[\frac{2 - \mu^-_{F_p}(x) - \mu^+_{F_p}(x)}{2}\right]$$

Example 3.5 From Example 3.3, we have

$$\overline{F_p(e_1)} = \left\{\left(\frac{u_1}{([0.4, 0.4], [0.5, 0.6])}, 0.4\right), \left(\frac{u_2}{([0.1, 0.2], [0.7, 0.8])}, 0.3\right)\right\}$$

Then,

$$\min \sum count(\bar{F}_p(e_1)) = 0.205; \qquad \max \sum count(\bar{F}_p(e_1)) = 0.7$$

Therefore

$$Card\,\bar{F}_p(e_1) = [0.205, 0.700]$$

Proposition 3.6 For any two PIVIFSSs F_p,and G_q, the following are true:

(a) $Card(F_p \cup G_q) + Card(F_p \cap G_q) = Card(F_p) + Card(G_q)$

(b) $Card(\overline{(F_p \cup G_q)} + Card(\overline{F_p \cap G_q}) = Card(\bar{F}_p) + Card(\bar{G}_q)$

Proof: Let F_p and G_p be any two PIVIFSSs over (U, E) such that $F_p \leq G_q$, where for every $x \in X$

$$F_p(e)(x) = \langle([\mu^-_F(x), \mu^+_F(x)], [\nu^-_F(x), \nu^+_F(x)]), p(e)(x)\rangle$$
$$G_q(e)(x) = \langle([\mu^-_G(x), \mu^+_G(x)], [\nu^-_G(x), \nu^+_G(x)]), q(e)(x)\rangle$$

(a)Then

$$\begin{aligned} F_p \cup G_q &= \langle([\max(\mu^-_F, \mu^-_G), \max(\mu^+_F, \mu^+_G)], [\min(\nu^-_F, \nu^-_G), \min(\nu^+_F, \nu^+_G)]), \max(p(e), q(e))\rangle \\ &= \langle([\mu^-_G, \mu^+_G], [\nu^-_G, \nu^+_G]), q(e)\rangle \end{aligned} \tag{3.1}$$

and

$$\begin{aligned} F_p \cap G_q &= \langle([\min(\mu^-_F, \mu^-_G), \min(\mu^+_F, \mu^+_G)], [\max(\nu^-_F, \nu^-_G), \max(\nu^+_F, \nu^+_G)]), \min(p(e), q(e))\rangle \\ &= \langle([\mu^-_F, \mu^+_F], [\nu^-_F, \nu^+_F]), p(e)\rangle \end{aligned} \tag{3.2}$$

Now,

$$Card(F_p \cup G_q) = [\min \sum count(F_p \cup G_q), \max \sum count(F_p \cup G_q)]$$

where,

$$\min \sum count(F_p \cup G_q) = \left[\frac{\mu_G^- + \mu_G^+}{2}\right] \cdot q(e)$$
$$\max \sum count(F_p \cup G_q) = \left[\frac{2 - \nu_G^- - \nu_G^+}{2}\right]$$

and $Card(F_p \cap G_q) = [\min \sum count(F_p \cap G_q), \max \sum count(F_p \cap G_q)]$ Where,

$$\min \sum count(F_p \cap G_q) = \left[\frac{\mu_F^- + \mu_F^+}{2}\right] \cdot p(e)$$
$$\max \sum count(F_p \cap G_q) = \left[\frac{2 - \nu_F^- - \nu_F^+}{2}\right]$$

Now,

$$\begin{aligned} Card(F_p \cup G_q) + Card(F_p \cap G_q) &= \left[\frac{\mu_G^- + \mu_G^+}{2} \cdot q(e), \frac{2 - \nu_G^- - \nu_G^+}{2}\right] + \left[\frac{\mu_F^- + \mu_F^+}{2} \cdot p(e), \frac{2 - \nu_F^- - \nu_F^+}{2}\right] \\ &= \left[\frac{\mu_G^- + \mu_G^+}{2} \cdot q(e) + \frac{\mu_F^- + \mu_F^+}{2} \cdot p(e), \frac{4 - \nu_G^- - \nu_G^+ - \nu_F^- - \nu_F^+}{2}\right] \end{aligned} \tag{3.3}$$

Consider,

$$Card(F_p) = \left[\frac{\mu_F^- + \mu_F^+}{2} \cdot p(e), \frac{2 - \nu_F^- - \nu_F^+}{2}\right] \tag{3.4}$$
$$Card(G_q) = \left[\frac{\mu_G^- + \mu_G^+}{2} \cdot q(e), \frac{2 - \nu_G^- - \nu_G^+}{2}\right] \tag{3.5}$$

Adding (3.4) & (3.5),

$$\begin{aligned} Card(F_p) + Card(G_q) &= \left[\frac{\mu_F^- + \mu_F^+}{2} \cdot p(e), \frac{2 - \nu_F^- - \nu_F^+}{2}\right] + \left[\frac{\mu_G^- + \mu_G^+}{2} \cdot q(e), \frac{2 - \nu_G^- - \nu_G^+}{2}\right] \\ &= \left[\frac{\mu_F^- + \mu_F^+}{2} \cdot p(e) + \frac{\mu_G^- + \mu_G^+}{2} \cdot q(e), \frac{4 - \nu_G^- - \nu_G^+ - \nu_F^- - \nu_F^+}{2}\right] \end{aligned} \tag{3.6}$$

From (3.3) and (3.6), it is clear that

$$Card(F_p \cup G_q) + Card(F_p \cap G_q) = Card(F_p) + Card(G_q).$$

Similarly (b) can also be proved.
It is to be noted that we can prove the above results similarly for the case $F_p \geq G_q$.

Definition 3.7 Let F_p and G_q be any two PIVIFSSs over (U, E), where for every $x \in X$:

$$F_p(e)(x) = \langle([\mu_F^-(x), \mu_F^+(x)], [\nu_F^-(x), \nu_F^+(x)]), p(e)(x)\rangle$$
$$G_q(e)(x) = \langle([\mu_G^-(x), \mu_G^+(x)], [\nu_G^-(x), \nu_G^+(x)]), q(e)(x)\rangle$$

Then

$$F_p \,@\, G_q = \left\langle\left(\left[\frac{\mu_F^- + \mu_G^-}{2}, \frac{\mu_F^+ + \mu_G^+}{2}\right], \left[\frac{\nu_F^- + \nu_G^-}{2}, \frac{\nu_F^+ + \nu_G^+}{2}\right]\right), \frac{p(e) + q(e)}{2}\right\rangle$$
$$F_p \,\$\, G_q = \left\langle\left(\left[\sqrt{\mu_F^- \cdot \mu_G^-}, \sqrt{\mu_F^+ \cdot \mu_G^+}\right], \left[\sqrt{\nu_F^- \cdot \nu_G^-}, \sqrt{\nu_F^+ \cdot \nu_G^+}\right]\right), \sqrt{p(e) \cdot q(e)}\right\rangle$$
$$F_p * G_q = \left\langle\left(\left[\frac{\mu_F^- + \mu_G^-}{2(\mu_F^- \cdot \mu_G^- + 1)}\right], \left[\frac{\mu_F^+ + \mu_G^+}{2(\mu_F^+ \cdot \mu_G^+ + 1)}\right], \left[\frac{\nu_F^- + \nu_G^-}{2(\nu_F^- \cdot \nu_G^- + 1)}\right], \left[\frac{\nu_F^+ + \nu_G^+}{2(\nu_F^+ \cdot \nu_G^+ + 1)}\right]\right), \left[\frac{p(e) + q(e)}{2(p(e) \cdot q(e) + 1)}\right]\right\rangle$$

Proposition 3.8 The operators @, $, and * are commutative.

Proof: Consider any two PIVIFSSs F_p and G_q over (U, E), where for each $x \in X$:

$$F_p(e)(x) = \langle([\mu_F^-(x), \mu_F^+(x)], [\nu_F^-(x), \nu_F^+(x)]), p(e)(x)\rangle$$
$$G_q(e)(x) = \langle([\mu_G^-(x), \mu_G^+(x)], [\nu_G^-(x), \nu_G^+(x)]), q(e)(x)\rangle$$

To prove @ is commutative:
We have,

$$F_p \,@\, G_q = \left\langle\left(\left[\frac{\mu_F^- + \mu_G^-}{2}, \frac{\mu_F^+ + \mu_G^+}{2}\right], \left[\frac{\nu_F^- + \nu_G^-}{2}, \frac{\nu_F^+ + \nu_G^+}{2}\right]\right), \frac{p(e) + q(e)}{2}\right\rangle \tag{3.1}$$

Then

$$G_q \,@\, F_p = \left\langle\left(\left[\frac{\mu_G^- + \mu_F^-}{2}, \frac{\mu_G^+ + \mu_F^+}{2}\right], \left[\frac{\nu_G^- + \nu_F^-}{2}, \frac{\nu_G^+ + \nu_F^+}{2}\right]\right), \frac{q(e) + p(e)}{2}\right\rangle \tag{3.2}$$

From (3.1) and (3.2) it is clear that

$$F_p \,@\, G_q = G_q \,@\, F_p$$

Hence @ is commutative. Similarly we can prove $ and * are commutative.

Remark 3.9 The operators @, $, and * are not associative.

Proof: Consider any three PIVIFSSs F_p, G_q, and H_r over (U, E), where for each $x \in X$:

$$F_p(e)(x) = \langle([\mu_F^-(x), \mu_F^+(x)], [\nu_F^-(x), \nu_F^+(x)]), p(e)(x)\rangle$$
$$G_q(e)(x) = \langle([\mu_G^-(x), \mu_G^+(x)], [\nu_G^-(x), \nu_G^+(x)]), q(e)(x)\rangle$$
$$H_r(e)(x) = \langle([\mu_H^-(x), \mu_H^+(x)], [\nu_H^-(x), \nu_H^+(x)]), r(e)(x)\rangle$$

To prove @ is not associative: We have

$$F_p \,@\, G_q = \left\langle\left(\left[\frac{\mu_F^- + \mu_G^-}{2}, \frac{\mu_F^+ + \mu_G^+}{2}\right], \left[\frac{\nu_F^- + \nu_G^-}{2}, \frac{\nu_F^+ + \nu_G^+}{2}\right]\right), \frac{p(e) + q(e)}{2}\right\rangle \tag{3.1}$$

Now,

$$\begin{aligned}(F_p \,@\, G_q) \,@\, H_r &= \left\langle\left(\left[\frac{(\frac{\mu_F^- + \mu_G^-}{2}) + \mu_H^-}{2}, \frac{(\frac{\mu_F^+ + \mu_G^+}{2}) + \mu_H^+}{2}\right],\right.\right. \\ &\qquad \left.\left.\left[\frac{(\frac{\nu_F^- + \nu_G^-}{2}) + \nu_H^-}{2}, \frac{(\frac{\nu_F^+ + \nu_G^+}{2}) + \nu_H^+}{2}\right]\right), \frac{(\frac{p(e)+q(e)}{2}) + r(e)}{2}\right\rangle \\ &= \left\langle\left(\left[\frac{\mu_F^- + \mu_G^- + 2\mu_H^-}{4}, \frac{\mu_F^+ + \mu_G^+ + 2\mu_H^+}{4}\right],\right.\right. \\ &\qquad \left.\left.\left[\frac{\nu_F^- + \nu_G^- + 2\nu_H^-}{4}, \frac{\nu_F^+ + \nu_G^+ + 2\nu_H^+}{4}\right]\right), \frac{p(e) + q(e) + 2r(e)}{4}\right\rangle\end{aligned} \tag{3.2}$$

Similarly,

$$G_q \,@\, H_r = \left\langle\left(\left[\frac{\mu_G^- + \mu_H^-}{2}, \frac{\mu_G^+ + \mu_H^+}{2}\right], \left[\frac{\nu_G^- + \nu_H^-}{2}, \frac{\nu_G^+ + \nu_H^+}{2}\right]\right), \frac{q(e) + r(e)}{2}\right\rangle \tag{3.3}$$

Now,

$$F_p \;@\; (G_q \;@\; H_r) = \left\langle \left(\left[\frac{\mu_F^- + (\mu_G^- + \mu_H^-)/2}{2}, \frac{\mu_F^+ + (\mu_G^+ + \mu_H^+)/2}{2} \right], \left[\frac{\nu_F^- + (\nu_G^- + \nu_H^-)/2}{2}, \frac{\nu_F^+ + (\nu_G^+ + \nu_H^+)/2}{2} \right] \right), \frac{p(e) + (q(e) + r(e))/2}{2} \right\rangle$$
$$= \left\langle \left(\left[\frac{2\mu_F^- + \mu_G^- + \mu_H^-}{4}, \frac{2\mu_F^+ + \mu_G^+ + \mu_H^+}{4} \right], \left[\frac{2\nu_F^- + \nu_G^- + \nu_H^-}{4}, \frac{2\nu_F^+ + \nu_G^+ + \nu_H^+}{4} \right] \right), \frac{2p(e) + q(e) + r(e)}{4} \right\rangle \quad (4)$$

From (2) and (4), it is evident that

$$(F_p \;@\; G_q) \;@\; H_r \neq F_p \;@\; (G_q \;@\; H_r)$$

Hence @ is not associative. Similarly we can prove that $ and * are not associative.

Proposition 3.10 The operation @ on PIVIFSSs is distributive over union and intersection.

Proof: Consider any three PIVIFSSs F_p, G_q, and H_r over (U, E) where for each $x \in X$:

$$F_p(e)(x) = \langle([\mu_F^-(x), \mu_F^+(x)], [\nu_F^-(x), \nu_F^+(x)]), p(e)(x)\rangle$$
$$G_q(e)(x) = \langle([\mu_G^-(x), \mu_G^+(x)], [\nu_G^-(x), \nu_G^+(x)]), q(e)(x)\rangle$$
$$H_r(e)(x) = \langle([\mu_H^-(x), \mu_H^+(x)], [\nu_H^-(x), \nu_H^+(x)]), r(e)(x)\rangle$$

To prove @ is distributive over union: That is to prove

$$(F_p \cup G_q) \;@\; H_r = (F_p \;@\; H_r) \cup (G_q \;@\; H_r)$$

Consider,

$$F_p \cup G_q = \langle([\max(\mu_F^-, \mu_G^-), \max(\mu_F^+, \mu_G^+)], [\min(\nu_F^-, \nu_G^-), \min(\nu_F^+, \nu_G^+)]), \max(p(e), q(e))\rangle$$

Now,

$$(F_p \cup G_q) \;@\; H_r = \left\langle \left(\left[\frac{\max(\mu_F^-, \mu_G^-) + \mu_H^-}{2}, \frac{\max(\mu_F^+, \mu_G^+) + \mu_H^+}{2} \right], \left[\frac{\min(\nu_F^-, \nu_G^-) + \nu_H^-}{2}, \frac{\min(\nu_F^+, \nu_G^+) + \nu_H^+}{2} \right] \right), \frac{\max(p(e), q(e)) + r(e)}{2} \right\rangle \quad (3.1)$$

We have,

$$F_p \;@\; H_r = \left\langle \left(\left[\frac{\mu_F^- + \mu_H^-}{2}, \frac{\mu_F^+ + \mu_H^+}{2} \right], \left[\frac{\nu_F^- + \nu_H^-}{2}, \frac{\nu_F^+ + \nu_H^+}{2} \right] \right), \frac{p(e) + r(e)}{2} \right\rangle$$

And

$$G_q \;@\; H_r = \left\langle \left(\left[\frac{\mu_G^- + \mu_H^-}{2}, \frac{\mu_G^+ + \mu_H^+}{2} \right], \left[\frac{\nu_G^- + \nu_H^-}{2}, \frac{\nu_G^+ + \nu_H^+}{2} \right] \right), \frac{q(e) + r(e)}{2} \right\rangle$$

Then

$$(F_p \;@\; H_r) \cup (G_q \;@\; H_r) = \left\langle \left(\left[\max\left(\frac{\mu_F^- + \mu_H^-}{2}, \frac{\mu_G^- + \mu_H^-}{2} \right), \max\left(\frac{\mu_F^+ + \mu_H^+}{2}, \frac{\mu_G^+ + \mu_H^+}{2} \right) \right], \left[\min\left(\frac{\nu_F^- + \nu_H^-}{2}, \frac{\nu_G^- + \nu_H^-}{2} \right), \min\left(\frac{\nu_F^+ + \nu_H^+}{2}, \frac{\nu_G^+ + \nu_H^+}{2} \right) \right] \right), \max\left(\frac{p(e) + r(e)}{2}, \frac{q(e) + r(e)}{2} \right) \right\rangle \quad (3.2)$$

From (3.1) and (3.2) it is evident that

$$(F_p \cup G_q) \ @ \ H_r = (F_p \ @ \ H_r) \cup (G_q \ @ \ H_r)$$

Hence @ is distributive over union.

To prove @ is distributive over intersection: That is to prove

$$(F_p \cap G_q) \ @ \ H_r = (F_p \ @ \ H_r) \cap (G_q \ @ \ H_r)$$

Consider,

$$F_p \cap G_q = \langle([\min(\mu_F^-, \mu_G^-), \min(\mu_F^+, \mu_G^+)], [\max(\nu_F^-, \nu_G^-), \max(\nu_F^+, \nu_G^+)]), \min(p(e), q(e))\rangle$$

Now,

$$(F_p \cap G_q) \ @ \ H_r = \left\langle \left(\left[\frac{\min(\mu_F^-, \mu_G^-) + \mu_H^-}{2}, \frac{\min(\mu_F^+, \mu_G^+) + \mu_H^+}{2} \right], \left[\frac{\max(\nu_F^-, \nu_G^-) + \nu_H^-}{2}, \frac{\max(\nu_F^+, \nu_G^+) + \nu_H^+}{2} \right] \right), \frac{\min(p(e), q(e)) + r(e)}{2} \right\rangle \tag{3.3}$$

We have,

$$F_p \ @ \ H_r = \left\langle \left(\left[\frac{\mu_F^- + \mu_H^-}{2}, \frac{\mu_F^+ + \mu_H^+}{2} \right], \left[\frac{\nu_F^- + \nu_H^-}{2}, \frac{\nu_F^+ + \nu_H^+}{2} \right] \right), \frac{p(e) + r(e)}{2} \right\rangle$$

And

$$G_q \ @ \ H_r = \left\langle \left(\left[\frac{\mu_G^- + \mu_H^-}{2}, \frac{\mu_G^+ + \mu_H^+}{2} \right], \left[\frac{\nu_G^- + \nu_H^-}{2}, \frac{\nu_G^+ + \nu_H^+}{2} \right] \right), \frac{q(e) + r(e)}{2} \right\rangle$$

Then

$$\begin{aligned}(F_p \ @ \ H_r) \cap (G_q \ @ \ H_r) = \Bigg\langle \Bigg(\Bigg[&\min\left(\frac{\mu_F^- + \mu_H^-}{2}, \frac{\mu_G^- + \mu_H^-}{2} \right), \min\left(\frac{\mu_F^+ + \mu_H^+}{2}, \frac{\mu_G^+ + \mu_H^+}{2} \right) \Bigg], \\ &\Bigg[\max\left(\frac{\nu_F^- + \nu_H^-}{2}, \frac{\nu_G^- + \nu_H^-}{2} \right), \\ &\max\left(\frac{\nu_F^+ + \nu_H^+}{2}, \frac{\nu_G^+ + \nu_H^+}{2} \right) \Bigg] \Bigg), \\ &\min\left(\frac{p(e) + r(e)}{2}, \frac{q(e) + r(e)}{2} \right) \Bigg\rangle\end{aligned}$$

$$\begin{aligned} = \langle &\left[\frac{\min(\mu_F^-, \mu_G^-) + \mu_H^-}{2}, \frac{\min(\mu_F^+, \mu_G^+) + \mu_H^+}{2} \right], \\ &\left[\frac{\max(v_F^-, v_G^-) + v_H^-}{2}, \frac{\max(v_F^+, v_G^+) + v_H^+}{2} \right], \\ &\frac{\min(p(e), q(e)) + r(e)}{2} \Big\rangle \end{aligned} \tag{3.4}$$

From (3.3) and (3.4) it is evident that

$$(F_p \cap G_q) \ @ \ H_r = (F_p \ @ \ H_r) \cap (G_q \ @ \ H_r)$$

Hence @ is distributive over intersection.

Similarly we can prove that $ and * are distributive over union and intersection.

Definition 3.11 The necessity $\square$ and $\diamond$ possibility operators on a PIVIFSS

$$F_p(e)(x) = \langle([\mu_F^-(x), \mu_F^+(x)], [\nu_F^-(x), \nu_F^+(x)]), p(e)(x)\rangle$$

are defined as follows:

$$\square F_p(e)(x) = \langle([\mu_F^-(x), \mu_F^+(x)], [\nu_F^-(x), 1 - \mu_F^+(x)]), p(e)(x)\rangle$$
$$\diamond F_p(e)(x) = \langle([\mu_F^-(x), 1 - \nu_F^+(x)], [\nu_F^-(x), \nu_F^+(x)]), p(e)(x)\rangle$$

Proposition 3.12 For any two PIVIFSSs F_p and G_q, the following are true:

1. $\square(F_p \cup G_q) = \square F_p \cup \square G_q$
2. $\square(F_p \cap G_q) = \square F_p \cap \square G_q$
3. $\diamond(F_p \cup G_q) = \diamond F_p \cup \diamond G_q$
4. $\diamond(F_p \cap G_q) = \diamond F_p \cap \diamond G_q$

Proof: Consider any two PIVIFSSs F_p, G_q over (U, E) where for each $x \in X$:

$$F_p(e)(x) = \langle([\mu_F^-(x), \mu_F^+(x)], [\nu_F^-(x), \nu_F^+(x)]), p(e)(x)\rangle$$
$$G_q(e)(x) = \langle([\mu_G^-(x), \mu_G^+(x)], [\nu_G^-(x), \nu_G^+(x)]), q(e)(x)\rangle$$

Then

$$F_p \cup G_q = \langle([\max(\mu_F^-, \mu_G^-), \max(\mu_F^+, \mu_G^+)], [\min(\nu_F^-, \nu_G^-), \min(\nu_F^+, \nu_G^+)]), \max(p(e), q(e))\rangle$$

And

$$F_p \cap G_q = \langle([\min(\mu_F^-, \mu_G^-), \min(\mu_F^+, \mu_G^+)], [\max(\nu_F^-, \nu_G^-), \max(\nu_F^+, \nu_G^+)]), \min(p(e), q(e))\rangle$$

1. Consider,

$$\square(F_p \cup G_q) = \langle([\max(\mu_F^-, \mu_G^-), \max(\mu_F^+, \mu_G^+)], [\min(\nu_F^-, \nu_G^-), 1 - \max(\mu_F^+, \mu_G^+)]), \max(p(e), q(e))\rangle \tag{3.1}$$

Now

$$\square F_p(e) = \langle([\mu_F^-, \mu_F^+], [\nu_F^-, 1 - \mu_F^+]), p(e)\rangle$$

and

$$\square G_q(e) = \langle([\mu_G^-, \mu_G^+], [\nu_G^-, 1 - \mu_G^+]), q(e)\rangle$$

Then their union is:

$$\begin{aligned}\square F_p \cup \square G_q &= \langle([\max(\mu_F^-, \mu_G^-), \max(\mu_F^+, \mu_G^+)], [\min(\nu_F^-, \nu_G^-), \min(1 - \mu_F^+, 1 - \mu_G^+)]), \max(p(e), q(e))\rangle \\ &= \langle([\max(\mu_F^-, \mu_G^-), \max(\mu_F^+, \mu_G^+)], [\min(\nu_F^-, \nu_G^-), 1 - \max(\mu_F^+, \mu_G^+)]), \max(p(e), q(e))\rangle \end{aligned} \tag{3.2}$$

From (3.1) and (3.2),

$$\square(F_p \cup G_q) = \square F_p \cup \square G_q.$$

2. Consider,

$$\square(F_p \cap G_q) = \langle([\min(\mu_F^-, \mu_G^-), \min(\mu_F^+, \mu_G^+)], [\max(\nu_F^-, \nu_G^-), 1 - \min(\mu_F^+, \mu_G^+)]), \min(p(e), q(e))\rangle \tag{3.3}$$

And

$$\begin{aligned}\square F_p \cap \square G_q &= \langle([\min(\mu_F^-, \mu_G^-), \min(\mu_F^+, \mu_G^+)], [\max(\nu_F^-, \nu_G^-), \max(1 - \mu_F^+, 1 - \mu_G^+)]), \min(p(e), q(e))\rangle \\ &= \langle([\min(\mu_F^-, \mu_G^-), \min(\mu_F^+, \mu_G^+)], [\max(\nu_F^-, \nu_G^-), 1 - \min(\mu_F^+, \mu_G^+)]), \min(p(e), q(e))\rangle \end{aligned} \tag{3.4}$$

From (3.3) and (3.4),

$$\Box(F_p \cap G_q) = \Box F_p \cap \Box G_q.$$

3. Consider,

$$\Diamond(F_p \cup G_q) = \langle([\max(\mu_F^-, \mu_G^-), 1 - \min(\nu_F^+, \nu_G^+)], [\min(\nu_F^-, \nu_G^-), \min(\nu_F^+, \nu_G^+)]), \max(p(e), q(e))\rangle \tag{3.5}$$

Now,

$$\Diamond F_p(e) = \langle([\mu_F^-, 1 - \nu_F^+], [\nu_F^-, \nu_F^+]), p(e)\rangle$$
$$\Diamond G_q(e) = \langle([\mu_G^-, 1 - \nu_G^+], [\nu_G^-, \nu_G^+]), q(e)\rangle$$

Then,

$$\begin{aligned}\Diamond F_p \cup \Diamond G_q &= \langle([\max(\mu_F^-, \mu_G^-), \max(1 - \nu_F^+, 1 - \nu_G^+)], [\min(\nu_F^-, \nu_G^-), \min(\nu_F^+, \nu_G^+)]), \max(p(e), q(e))\rangle \\ &= \langle([\max(\mu_F^-, \mu_G^-), 1 - \min(\nu_F^+, \nu_G^+)], [\min(\nu_F^-, \nu_G^-), \min(\nu_F^+, \nu_G^+)]), \max(p(e), q(e))\rangle \end{aligned} \tag{3.6}$$

From (3.5) and (3.6),

$$\Diamond(F_p \cup G_q) = \Diamond F_p \cup \Diamond G_q.$$

4. Consider,

$$\Diamond(F_p \cap G_q) = \langle([\min(\mu_F^-, \mu_G^-), 1 - \max(\nu_F^+, \nu_G^+)], [\max(\nu_F^-, \nu_G^-), \max(\nu_F^+, \nu_G^+)]), \min(p(e), q(e))\rangle \tag{3.7}$$

Now,

$$\begin{aligned}\Diamond F_p \cap \Diamond G_q &= \langle([\min(\mu_F^-, \mu_G^-), \min(1 - \nu_F^+, 1 - \nu_G^+)], [\max(\nu_F^-, \nu_G^-), \max(\nu_F^+, \nu_G^+)]), \min(p(e), q(e))\rangle \\ &= \langle([\min(\mu_F^-, \mu_G^-), 1 - \max(\nu_F^+, \nu_G^+)], [\max(\nu_F^-, \nu_G^-), \max(\nu_F^+, \nu_G^+)]), \min(p(e), q(e))\rangle \end{aligned} \tag{3.8}$$

From (3.7) and (3.8),

$$\Diamond(F_p \cap G_q) = \Diamond F_p \cap \Diamond G_q.$$

4. Multi-Criteria Decision-Making Problem for the Cardinality Measure on Possibility Interval-Valued Intuitionistic Fuzzy Soft Sets

In this Section, we introduced a method to solve a multi-criteria decision-making problem using cardinality measure on PIVIFSSs.
Let $F_p = \{F_{p_1}, F_{p_2}, \ldots, F_{p_m}\}$ be a set of options, and $C = \{C_1, C_2, \ldots, C_n\}$ be a set of criteria.

The proposed decision-making approach is carried out through the following steps:

Step 1: Construct the decision matrix D in the form of PIVIFSSs.

Step 2: Compute the minimum and maximum sigma counts according to Definition 3.1.

Step 3: Determine the midpoint of the sigma count values for each option and identify the option(s) with the highest midpoint value.

Example 4.1 Consider the supplier selection problem discussed by Mohamed, S. S., et al. (2019) [15]. A company evaluates potential suppliers based on six criteria: price (C_1), deadline (C_2), quality (C_3), the level of technology (C_4), service (C_5), and the future cooperation (C_6). Five suppliers, denoted by F_{p_j}, $(j = 1, 2, 3, 4, 5)$, are assessed by experts.

Step 1: The collected evaluations are presented as PIVIFSSs in Table 1.

Step 2: The minimum and maximum sigma count values are obtained as:

$$\begin{aligned}
\min \sum count(F_{p_1}) &= 2.435, & \max \sum count(F_{p_1}) &= 4.800\\
\min \sum count(F_{p_2}) &= 1.460, & \max \sum count(F_{p_2}) &= 4.350\\
\min \sum count(F_{p_3}) &= 2.005, & \max \sum count(F_{p_3}) &= 4.350\\
\min \sum count(F_{p_4}) &= 1.820, & \max \sum count(F_{p_4}) &= 4.150\\
\min \sum count(F_{p_5}) &= 3.245, & \max \sum count(F_{p_5}) &= 5.100
\end{aligned}$$

Step 3: Finally, the midpoint values for $\sum count$ of PIVIFSSs are calculated as:

$$\begin{aligned}
card(F_{p_1}) &= 3.6175; & card(F_{p_2}) &= 2.9050;\\
card(F_{p_3}) &= 3.1775; & card(F_{p_4}) &= 2.9850\\
card(F_{p_5}) &= 4.1725
\end{aligned}$$

Step 4: Based on the midpoint cardinality values, the suppliers are ranked as follows:

$$F_{p_5} > F_{p_1} > F_{p_3} > F_{p_4} > F_{p_2}$$

Hence, F_{p_5} is the proper supplier among the considered suppliers.
This ranking aligns with the results reported in the original study [15].

Table 1: The alternatives in terms of possibility interval-valued intuitionistic fuzzy soft numbers

$\langle([0.4,0.5],[0.2,0.3]),0.5\rangle$	$\langle([0.6,0.8],[0.1,0.2]),0.8\rangle$	$\langle([0.4,0.5],[0.2,0.4]),0.5\rangle$	$\langle([0.8,0.9],[0.1,0.1]),0.9\rangle$	$\langle([0.2,0.6],[0.2,0.3]),0.6\rangle$	$\langle([0.5,0.7],[0.1,0.2]),0.7\rangle$
$\langle([0.5,0.7],[0.1,0.2]),0.4\rangle$	$\langle([0.6,0.8],[0.1,0.2]),0.6\rangle$	$\langle([0.3,0.4],[0.4,0.6]),0.3\rangle$	$\langle([0.8,0.9],[0.0,0.1]),0.7\rangle$	$\langle([0.2,0.5],[0.3,0.4]),0.2\rangle$	$\langle([0.1,0.2],[0.4,0.5]),0.2\rangle$
$\langle([0.2,0.3],[0.6,0.7]),0.3\rangle$	$\langle([0.4,0.5],[0.3,0.4]),0.5\rangle$	$\langle([0.7,0.8],[0.1,0.2]),0.8\rangle$	$\langle([0.2,0.5],[0.1,0.2]),0.5\rangle$	$\langle([0.7,0.8],[0.0,0.1]),0.8\rangle$	$\langle([0.5,0.6],[0.2,0.4]),0.6\rangle$
$\langle([0.5,0.6],[0.1,0.2]),0.7\rangle$	$\langle([0.3,0.4],[0.2,0.4]),0.5\rangle$	$\langle([0.5,0.8],[0.1,0.2]),0.9\rangle$	$\langle([0.6,0.7],[0.1,0.2]),0.7\rangle$	$\langle([0.3,0.4],[0.3,0.4]),0.5\rangle$	$\langle([0.1,0.2],[0.7,0.8]),0.3\rangle$
$\langle([0.4,0.5],[0.3,0.4]),0.5\rangle$	$\langle([0.8,0.9],[0.0,0.1]),0.9\rangle$	$\langle([0.6,0.8],[0.1,0.2]),0.8\rangle$	$\langle([0.8,0.9],[0.0,0.1]),0.9\rangle$	$\langle([0.7,0.8],[0.1,0.2]),0.8\rangle$	$\langle([0.5,0.6],[0.1,0.2]),0.6\rangle$

Note: The table values are adapted from (Mohamed, S. S., et al., 2019 [15].)

5. Conclusion

In this research, we introduced the concepts of cardinality and average PIVIFSSs and established their fundamental properties. In addition, we proposed a set of basic operators, namely @, $, and *, defined on PIVIFSSs and analyzed their mathematical characteristics. The study also incorporated necessity and possibility operators, providing deeper insight into the structure and flexibility of PIVIFSSs. Furthermore, the developed cardinality measure is effectively applied to a multi-criteria decision-making problem, demonstrating its practical utility and ability to handle uncertainty in real-world decision settings. Overall, the proposed framework enriches the theoretical foundation of PIVIFSSs and expands their applicability in complex decision-making environments.

6. Future Scope

In future work, we intend to further explore the use of cardinality-based measures to evaluate the effectiveness of PIVIFSSs in various image processing applications, such as segmentation, and feature extraction. Beyond cardinality, several essential measures including entropy, similarity, and subsethood remain to be examined in detail. These measures have the potential to enhance the analytical capability of PIVIFSSs and contribute to more robust modeling of uncertainty. Future studies may also focus on developing hybrid operators, optimizing computational efficiency, and extending the methodology to group decision-making and classification problems.

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Arunadevi Sivaraj,

Department of Mathematics,

School of Physical Sciences and Computational Sciences,

Avinashilingam Institute for Home Science and Higher Education for Women,

Coimbatore, Tamilnadu

India.

E-mail address: 18phmap005@avinuty.ac.in

and

Jayanthi Duraisamy,
Department of Mathematics,
School of Physical Sciences and Computational Sciences,
Avinashilingam Institute for Home Science and Higher Education for Women,
Coimbatore, Tamilnadu
India.
E-mail address: jayanthimaths2006@gmail.com

and

Priyadharshini Manoharan,
Department of Mathematics,
School of Physical Sciences and Computational Sciences,
Avinashilingam Institute for Home Science and Higher Education for Women,
Coimbatore, Tamilnadu
India.
E-mail address: 120phmaf001@avinuty.ac.in