Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2026 (44) 7** : 1–11.
ISSN-0037-8712
doi:10.5269/bspm.81030

Stability Model of Robot Manipulators by Using Adaptive Fast Terminal WNN-Based Sliding Mode Control

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ABSTRACT: This paper investigates a control strategy for robot manipulators to address the stability of the system, in which the introduced controller is based on adaptive fast terminal sliding mode control (FTSMC) with wavelet neural networking. To handle uncertainties and unknown dynamics in the robotic system, a wavelet neural network (WNN) is introduced to compensate and approximate for the nonlinearity of the dynamic model through specifically designed the adaptive update laws. The adaptive tuning rules for the WNN parameters are determined through a Lyapunov-based stability analysis, which ensures convergence and stability of the closed-loop system. To enhance control performance, a FTSMC framework is designed to determine the system's tracking error to the sliding surface within finite time and subsequently improves convergence characteristics. The proposed control scheme demonstrates that the WNN can effectively learn and represent the robot dynamics while maintaining finite-time convergence and adaptive FTSMC performance.

Key Words: Wavelet neural network, adaptive bound, FTSMC, Lyapunov stability.

Contents

1. Introduction

Robotic manipulators are complex dynamical systems characterized by strong coupling, nonlinear behaviors, and time-varying properties. In practical applications, they are further influenced by parametric uncertainties and a wide range of external disturbances. As a result, designing effective stability strategies for such systems has become a topic of significant research interest. Over the years, numerous control methodologies have been proposed in the literature to address these challenges and ensure precise and robust operation of robot manipulators.

A wide range of control strategies have been proposed in the literature for the control of robotic manipulators. Traditional model-based controllers are one such approach [29]. Nevertheless they require complete knowledge of the system dynamics and often neglect unmodeled effects, which limits their practical applicability. So, model-free control methods have been explored to achieve more effective real-world performance. In practice, the dynamic model of a manipulator is subject to both structured and unstructured uncertainties, which can lead to tracking or positioning errors and even destabilize the system [18]. To address these challenges and enhance control performance, researchers have developed adaptive and robust control techniques. These methods are designed to ensure that the system states reach their desired values while simultaneously driving the tracking errors to zero [28,8]. In contrast, sliding mode control (SMC) has emerged as a robust alternative for controlling robotic manipulators. This approach

* Corresponding author.
2020 *Mathematics Subject Classification:* 70E60, 93D05.
Submitted December 25, 2025. Published April 11, 2026

has recently attracted significant interest for systems with uncertainties due to its fast response, improved transient behavior, and inherent robustness to disturbances [5,6,11,23]. Conventional SMC, which relies on a linear switching surface, can only guarantee asymptotic convergence [17]. For applications requiring high precision, this asymptotic behavior may not achieve sufficiently fast convergence without applying large control forces. To improve transient performance, a nonlinear switching surface—known as a terminal sliding mode—has been introduced [31]. Tang [32] and Mu *et al.* [24] proposed control schemes based on terminal sliding mode control (TSMC). By employing a nonlinear switching manifold during the transient phase, these approaches reduce the control effort and consequently lower the steady-state tracking error.

Although sliding mode controllers are known for their robustness, their implementation typically requires prior knowledge of the system dynamics. In practice, however, obtaining exact system information is often difficult due to inherent uncertainties. Recently, intelligent techniques such as neural networks and fuzzy systems—owing to their self-learning capability and nonlinear mapping features—have gained significant attention among control researchers. Numerous neural-network-based control strategies have been developed for rigid robot manipulators [19,15,16]. Nevertheless, comparatively few studies have combined the neural-networks strength with terminal sliding mode control (TSMC) for robotic applications. Leveraging this idea, Wang *et al.* [37] proposed a robust neural networks controller with integrating TSMC for robot manipulators. A neuro-SMC approach for mechanical systems was described in [22]. Supper-Twisting Control Law (STCL) is utilized with in [10] and SMC in [14] for finite-time convergence and to improve transient performance, and to minimize the effect of input saturation with saturation compensation of the system respectively. Fault tolerance control (FTC) mechanism [33] is combineded with finite-time commentator and promoted FTSMC to enhance the convergence rate, stability with high tracking accuracy and to estimate exterio disturbances, uncertainties. Yousuf *et al.* [40] proposed multi-agent tracking to achieve finite-time convergence of non-holonomic mobile robots by combining high-gain observers with FTSMC, these schemes handle unmeasured states, improve transient performance, and maintain stability and collision avoidance under uncertainties. Finite-time TSMC strategies are proposed the singularity-free neural network based robust control that simplify the implementation of tracking system for uncertain robots with high steady-state accuracy [26,42,34]. Derrouaoui *et al.* [9] presents a nonlinear robust FTSMC for a novel reconfigurable Unmanned Aerial Vehicle (UAV) whose arm lengths and angles vary, causing significant changes in its dynamics. The controller ensures finite-time convergence, high accuracy, and robustness to parameter uncertainties while reducing chattering, with stability verified via Lyapunov theory. Unlike previous studies focusing separately on extendable or rotating arms and mostly using linear control, this research applies FTSMC to a UAV combining both features, addressing a gap in existing literature.

In control technologies, Fixed-time TSMC associated with a radial basis function neural network to improve response speed, reduce steady-state error and chattering, and remove dependence on the robot's dynamic model. By employing pre-specified performance functions and transformation errors, the method ensures global fixed-time convergence of both control errors and adaptive neural rules [36,35]. Fuzzy fractional order PID approach [27] delivers superior tracking performance over conventional fuzzy, fuzzy-PID, and fuzzy-FOPID controllers under varying loads and model uncertainties. Fractional-order FTSMC [1,7] was proposed that ensures rapid finite-time convergence and provides an explicit estimate of stopping time by integrating RBFNN for nonlinear dynamics approximation. At present, studies have demonstrated that WNNs exhibit superior learning capabilities as compared to other neural networks, during controller design, this work employs a WNN to estimate the unknowable dynamics of the manipulator [20,2,25]. Ba *et al.* [4] combined the fuzzy TSMC with time-delay estimation (TDE) to enhance hydraulic drive unit (HDU) performance in legged robots and by adaptively tuning the reaching law parameters and estimating external disturbances, the method achieves improved position accuracy, strong disturbance rejection, and better adaptability under varying operating conditions compared to traditional controllers. Hosseinabadi *et al.* [3] introduced the observer-based SMC to achieve fixed-time convergence for uncertain nonlinear double-integrator systems with only partial state measurements, it estimates unmeasured states and disturbances while eliminating chattering through an integral signum design, guaranteeing convergence time and closed-loop stability. Yin *et al.* [39] introducesd an adaptive non-singular TSMC method combined with a non-singular disturbance observer to enhance trajectory tracking of multi-joint robotic arms under modeling uncertainties and time-varying disturbances. Su [30] presented a robust SMC approach for achieving precise trajectory tracking of robotic manipulators operating in demanding environments, and a Cartesian-coordinate dynamic model of the end-effector is developed. Ma *et al.* [21] proposed a dual fixed-time second-order SMC strategy to achieve high-precision, anti-saturation control for hybrid robots under uncertainties and input limits, and fixed-time disturbance observer compensates for modeling errors and external disturbances, while a fixed-time auxiliary system handles actuator saturation, and by integrating these elements into the sliding surface, the method guarantees fixed-time convergence of both tracking and observation errors with reduced chattering. A function approximation technique for continuum robots was introduced [38] the adaptive nonsingular FTSMC to estimate unknown dynamics and actuator faults with a disturbance observer to counter external disturbances, its Simulation and experimental results show that this combined approach achieves faster convergence and significantly improved trajectory tracking accuracy under uncertainties.

This paper introduces the novel form FTSMC has emerged as an effective nonlinear control strategy that offers finite-time convergence and stronger robustness. By designing a nonlinear sliding surface, FTSMC ensures that the tracking error not only reaches the surface in finite time but also converges to zero, proven in stability model. WNN are incorporated into the control scheme to effectively approximate unknown nonlinearities in real time, Adaptive control methods are designed to cope with parameter uncertainties by adjusting control parameters online, while robust control techniques such as sliding mode control enhance disturbance rejection and system stability under bounded uncertainties. The integration of WNN with adaptive FTSMC allows the controller to compensate for uncertain and complex dynamics without requiring exact prior knowledge of the system model. Stability of the overall closed-loop system is established using Lyapunov-based analysis, guaranteeing that all signals remain bounded and asymptotic convergence. This approach not only improves robustness against disturbances and uncertainties but also achieves high-precision motion control stability for robotic manipulators.

The remainder of this article is organized as follows: Section 2 introduces the dynamic model of an n-DOF manipulator and its properties. Section 3 is devoted to the design of the proposed controller. Section 4 presents the corresponding stability analysis. Lastly, the conclusion part is described in Section 5.

2. Dynamic Model for Robotic System

A dynamic equation is derived for n-link robotic manipulator, which applies Euler–Lagrange formulation, can be expressed as follows:

$$B(\psi)\ddot{\psi} + H(\psi, \dot{\psi}) + G(\psi) + F(\dot{\psi}) + \mu_d = \mu, \tag{2.1}$$

where $\psi = [\psi_1,\ \psi_2, ...\psi_n]^T$, $\dot{\psi} = [\dot{\psi}_1,\ \dot{\psi}_2, ..., \dot{\psi}_n]^T$, and $\ddot{\psi} = [\ddot{\psi}_1,\ \ddot{\psi}_2, ...\ddot{\psi}_n]^T \in \mathbb{R}^n$ represent the position, velocity, and acceleration of the joints, respectively. $B(\psi) \in \mathbb{R}^{n\times n}$ denotes the inertia effects, $H(\psi, \dot{\psi}) \in \mathbb{R}^{n\times n}$ signifies the centripetal-coriolis force matrix, $G(\psi) \in \mathbb{R}^n$ is the gravity vector acting on the manipulator, $F(\dot{\psi}) \in \mathbb{R}^n$ accounts for the frictional effects, external disturbance is denoted by $\mu_d \in \mathbb{R}^n$, and $\mu \in \mathbb{R}^n$ describes as the vector of control torque input. The robot dynamics of model (2.1) satisfies the following subsequent development properties.

The dynamic model stated above is considered under the following properties and assumptions:

Property 2.1 $B(\psi)$ is a mass matrix with symmetric, positive definite, invertible and bounded form, which possesses the following inequality:

$$\alpha_B \leqslant B(\psi) \leqslant \beta_B, \tag{2.2}$$

where α_B and β_B are positive constants.

Property 2.2 The matrix $(\dot{B}(\psi) - 2H(\psi, \dot{\psi}))$ is satisfy the property of skew-symmetric, i.e. $\varsigma^T(\dot{B}(\psi) - 2H(\psi, \dot{\psi}))\varsigma = 0$, for any $\varsigma \in \mathbb{R}^n$.

Assumption 2.1 The friction term of the manipulator dynamics is bounded such that $\|F(\dot{\psi})\| \leq \alpha_1 + \alpha_2\|\dot{\psi}\|$, where α_1 and α_2 are positive constants.

Assumptions 2.2 The external disturbances affecting the manipulator are bounded, i.e., $\|\mu_d\| \leq \alpha_3$, for some positive constant α_3.

3. Controller Design

The main novelty of this paper is introduced the controller strategy to enhance the stability of the system under the efficacy of disturbances and uncertainties. The introduced controller is designed by integration of Adaptive FTSMC with WNN to ensure strong robustness against these challenges while reducing chattering effects, and guaranteeing the stability of the closed-loop system.

3.1. FTSMC

The surface of FTSMC is formulated [41] in the under mentioned differential equation as:

$$\kappa(t) = \dot{e}(t) + \Upsilon_1 e(t) + \Upsilon_2 \text{sign}(e)^\eta, \tag{3.1}$$

where $e(t) = \psi_d(t) - \psi(t)$ denotes the tracking error with $\psi_d(t) \in \mathbb{R}^{n\times n}$ as the desired bounded trajectory, $\Upsilon_1 = \text{diag}(\upsilon_{11}, \upsilon_{12}, ...\upsilon_{1n}) \in \mathbb{R}^{n\times n}, \Upsilon_2 = \text{diag}(\upsilon_{21}, \upsilon_{22}, ...\upsilon_{2n}) \in \mathbb{R}^{n\times n}$, $\eta = \frac{s}{r}$ with $s,\ r > 0$ as integers satisfying $s < r < 2s$, and $\text{sign}(e)^\eta = [|e_1|^\eta \text{sign}(e_1), |e_2|^\eta \text{sign}(e_2), ...|e_n|^\eta \text{sign}(e_n)]$. The i-th component of the sliding surface is expressed as:

$$\kappa_i = \dot{e}_i + \upsilon_{1i} e_i + \upsilon_{2i} |e_i|^\eta \text{sign}(e_i), \tag{3.2}$$

To prove equilibrium point of (3.2) is globally finite-time stable, we apply the finite time stability definition [12]. For all phase $z_i(0) = z_{i0}$, the settling time of the method is:

$$T_i = \frac{1}{\upsilon_{1i}(1-\eta)} \ln\left(\frac{\upsilon_{1i}|z_{0i}|^{1-\eta} + \upsilon_{2i}}{\upsilon_{2i}}\right).$$

To support the stability analysis, the following results are utilized:

Lemma 3.1 For $p_1 > 0,\ p_2 > 0$ and $0 < q < 1$, the inequality holds [41]:

$$(p_1 + p_2)^q \leq p_1^q + p_2^q.$$

Lemma 3.2 Let $p = [p_1, p_2, ...p_n]^T$, with $|p| = |p_1| + |p_2| + ... + |p_n|$ denoting the sum norm and $\|p\| = (p_1^2 + p_2^2 + ... + p_n^2)^{1/2}$ denoting the Euclidean norm. Then, [13]

$$\|p\| \leq |p|.$$

Lemma 3.3 Let in a neighborhood of origin $\wp \subset \mathbb{R}^n$, $\exists$ a continuously differentiable function $C(z)$, and the real numbers $\ell_1 > 0,\ 0 < \ell_2 < 1$, such that

$$\dot{C}(z) + \ell_1 C^{\ell_2}(z) \leq 0;\ \forall\ z \in \wp.$$

Then, there exists a set $\wp_0 \subset \mathbb{R}^n$ such that whichever trajectory starting from $x_0 \in \wp_0$ will reach $C(z) = 0$ in finite time. The corresponding settling time satisfies

$$T_\kappa \leq \frac{C(z_0)^{1-\ell_2}}{\ell_1(1-\ell_2)}.$$

Differentiating (3.2), we obtain:

$$\dot{\kappa}_i = \ddot{e}_i + \upsilon_{1i}\dot{e}_i + \upsilon_{2i} e_{ji};$$

$$e_{ji} = \begin{cases} \eta|e_i|^{\eta-1}\dot{e}_i, & e_i \neq 0, \\ 0, & e_i = 0. \end{cases}$$

By assembling all terms in vector form, the sliding dynamics become, $\dot{\kappa} = \ddot{e} + \Upsilon_1\dot{e} + \Upsilon_2 e_j$. The manipulator dynamics expressed in terms of the FTSM surface are given as:

$$B\dot{\kappa} = -H\kappa - \mu + V(z) + \mu_d, \tag{3.3}$$

where the nonlinear function $V(z) = B(\psi)(\ddot{\psi}_d + \Upsilon_1\dot{e} + \Upsilon_2 e_j) + H(\psi, \dot{\psi})(\dot{q}_d + \Upsilon_1 e + \Upsilon_2 \text{sign}(e)^\eta) + G(\psi) + F(\dot{\psi})$, and the state vector is chosen as $z = [e^T,\ \dot{e}^T,\ \psi_d^T,\ \dot{\psi}_d^T,\ \ddot{\psi}_d^T]^T$. In general, the nonlinear function $V(z)$ is influenced by both structured and unstructured uncertainties. In this study, $V(z)$ is considered unknown, and a WNN is employed to estimate its value.

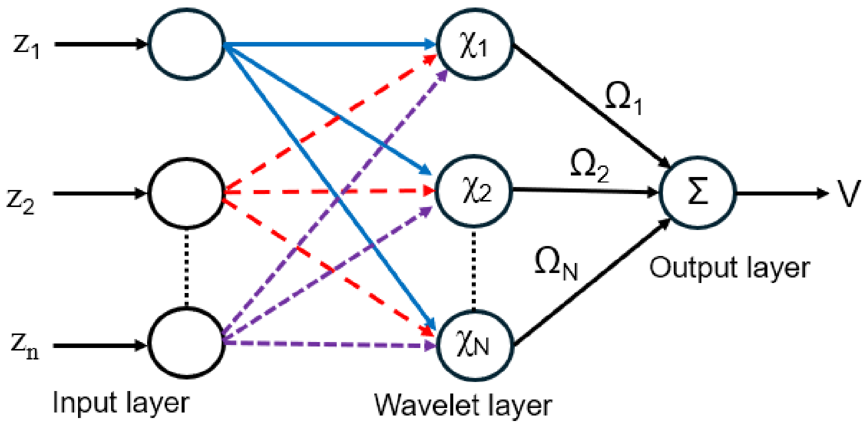

Figure 1: WNN structure

3.2. WNN

The architecture of the WNN is illustrated in Fig. 1. In general, a WNN with n-dimensional input signal and m-dimensional output signal can be expressed as a mapping $V : \mathbb{R}^n \to \mathbb{R}^m$ is described as:

$$V(z) = \Omega^T \chi(z, \delta, \zeta), \tag{3.4}$$

where the vector $z = [z_1, z_2, z_3, ..., z_n]^T \in \mathbb{R}^n$ stands for input signal of the network, $\Omega = [\Omega_{ij}] \in \mathbb{R}^{N\times m}$ is the output weighting matrix of the layer, and for $i = [1, 2..., N]$, within this network the hidden layer N stands for a number of neuron, $\delta_i = [\delta_{i1}, \delta_{i2}, ..., \delta_{in}]^T \in \mathbb{R}^n$ is the translation parameters, $\zeta_i = [\zeta_{i1}, \zeta_{i2}, ..., \zeta_{in}]^T \in \mathbb{R}^n$ is the dilation parameters. The translation layer output is denoted by $\chi(\cdot, \cdot, \cdot) = [\vartheta_1, \vartheta_2, \dots, \vartheta_N]^T \in \mathbb{R}^N$, with each component given as:

$$\vartheta = \chi_i(z) exp\left(-\frac{\sum_{j=1}^{n} \delta_{ij}(z_j - \zeta_{ij})^2}{2}\right), \tag{3.5}$$

where $\chi_i(z) = \prod_{j=1}^{n}(1 - \delta_{ij}^2 z_j^2)$ is a Mexican hat function. According to the global approximation hypothesis, there exists a WNN capable of approximating an unknown nonlinear function such that

$$V(z) = V^*(z) + \varepsilon(z) = \Omega^{*T}\chi(z, \delta^*, \zeta^*) + \varepsilon(z),$$

where $\varepsilon(z)$ is the approximation error, bounded as $||\varepsilon(z)|| \leqslant \varepsilon_N$ for some $\varepsilon_N > 0$. The parameters Ω^*, ζ^* and δ^* denote the optimal bounded values of the weights, dilation, and translation vectors, respectively. Since determining the exact optimal parameters is generally intractable, and estimated values are employed instead. The approximated model is then represented as:

$$\hat{V}(z) = \hat{\Omega}^T \chi(z, \hat{\delta}, \hat{\zeta}), \tag{3.6}$$

where $\hat{\Omega}$, $\hat{\delta}$ and $\hat{\zeta}$ are the estimated counterparts of the optimal parameters Ω^*, δ^* and ζ^* respectively. Assigning the annotations $\tilde{\Omega} = \Omega^* - \hat{\Omega}$, $\tilde{\delta} = \delta^* - \hat{\delta}$, $\tilde{\zeta} = \zeta^* - \hat{\zeta}$, $\hat{\chi} = \chi(z, \hat{\delta}, \hat{\zeta})$, and $\tilde{\chi} = \chi^* - \hat{\chi} = [\tilde{\vartheta}_1, \tilde{\vartheta}_2, ..., \tilde{\vartheta}_N]^T$, we identify the errors in estimation as:

$$\tilde{V} = V - \hat{V} = V^* - \hat{V} + \varepsilon = \Omega^{*T}\chi^* - \hat{\Omega}^T\hat{\chi} + \varepsilon = (\hat{\Omega}^T + \tilde{\Omega}^T)(\hat{\chi} + \tilde{\chi}) - \hat{\Omega}^T\hat{\chi} + \varepsilon = \tilde{\Omega}^T\hat{\chi} + \hat{\Omega}^T\tilde{\chi} + \tilde{\Omega}^T\tilde{\chi} + \varepsilon. \tag{3.7}$$

By applying a Taylor series expansion of $\tilde{v}_i$ around the point $(\hat{\delta}, \hat{\zeta})$, a semi-linear expression is obtained, which facilitates the origination of online adaptation laws for the WNN parameters:

$$\tilde{\vartheta}_i = \sum_{j=1}^{n} \frac{\partial \vartheta_i}{\partial \delta_{ij}}\bigg|_{(\hat{\delta}_{ij},\hat{\zeta}_{ij})} \tilde{\delta}_{ij} + \sum_{j=1}^{n} \frac{\partial \vartheta_i}{\partial \zeta_{ij}}\bigg|_{(\hat{\delta}_{ij},\hat{\zeta}_{ij})} \tilde{\zeta}_{ij} + \lambda_i;\ i = 1, 2, \dots, N$$
$$= Y_i\tilde{\delta}_i + Z_i\tilde{\zeta}_i + \lambda_i. \tag{3.8}$$

where $Y_i = [\frac{\partial\vartheta_i}{\partial\delta_{i1}}, \frac{\partial\vartheta_i}{\partial\delta_{i2}}, ..., \frac{\partial\vartheta_i}{\partial\delta_{in}}]$, $Z_i = [\frac{\partial\vartheta_i}{\partial\zeta_{i1}}, \frac{\partial\vartheta_i}{\partial\zeta_{i2}}, ..., \frac{\partial\vartheta_i}{\partial\zeta_{in}}]$ and λ_i represents the highest-order terms neglected in the Taylor elaboration. Equation (3.8) is expressed in a compact vector form, this relation becomes:

$$\tilde{\chi} = Y\tilde{\delta} + Z\tilde{\zeta} + \lambda, \tag{3.9}$$

where $Y = blockdiag[Y_1, Y_2, ..., Y_N] \in \mathbb{R}^{N\times N_n}$, $Z = blockdiag[Z_1, Z_2, ..., Z_N] \in \mathbb{R}^{N\times N_n}$, and $\lambda = [\lambda_1, \lambda_2, ..., \lambda_N]^T \in \mathbb{R}^N$. Substituting (3.9) in (3.7), the WNN approximation equation yields:

$$\tilde{V} = \tilde{\Omega}^T\hat{\chi} + \hat{\Omega}^T(Y\tilde{\delta} + Z\tilde{\zeta} + \lambda) + \tilde{\Omega}^T(Y\tilde{\delta} + Z\tilde{\zeta} + \lambda) + \varepsilon$$
$$= \tilde{\Omega}^T\hat{\chi} + \hat{\Omega}^T Y\tilde{\delta} + \hat{\Omega}^T Z\tilde{\zeta} + \hat{\Omega}^T\lambda + \tilde{\Omega}^T Y(\delta^* - \hat{\delta}) + \tilde{\Omega}^T Z(\zeta^* - \hat{\zeta}) + \tilde{\Omega}^T\lambda + \varepsilon$$
$$= \tilde{\Omega}^T(\hat{\chi} - Y\hat{\delta} - Z\hat{\zeta}) + \hat{\Omega}^T Y\tilde{\delta} + \hat{\Omega}^T Z\tilde{\zeta} + \mu_{d1} + \varepsilon.$$

where $\mu_{d1} = \hat{\Omega}^T\lambda + \tilde{\Omega}^T Y\delta^* + \tilde{\Omega}^T Z\zeta^* + \tilde{\Omega}^T\lambda$ represents the combined disturbance term, which accounts for approximation errors and bounded by some positive constant i.e. α_4; the manipulator dynamics are expressed as:

$$B\dot{\kappa} = -H\kappa - \mu + \hat{\Omega}^T\chi(z, \hat{\delta}, \hat{\zeta}) + \tilde{\Omega}^T(\hat{\chi} - Y\hat{\delta} - Z\hat{\zeta}) + \hat{\Omega}^T Y\tilde{\delta} + \hat{\Omega}^T Z\tilde{\zeta} + F(\dot{\psi}) + \lambda_d + \mu_{d1} + \varepsilon, \tag{3.10}$$

3.3. Adaptive bound

Considering the previously defined bound $\varepsilon(z)$ and bound of μ_{d1} for the neural network reconstruction error, together with Assumptions 2.1 and 2.2, such that:

$$\|F(\dot{\psi}) + \mu_d + \mu_{d1} + \varepsilon(z)\| = \alpha_1 + \alpha_2\|\dot{\psi}\| + \alpha_3 + \alpha_4 + \varepsilon_N. \tag{3.11}$$

Further, we define $\varphi = \alpha_1 + \alpha_2\|\dot{\psi}\| + \alpha_3 + \alpha_4 + \varepsilon_N$ as adaptive bound and which can also be written in a more compact form as:

$$\varphi = [1\ \|\dot{\psi}\|\ 1\ 1\ 1][\alpha_1\ \alpha_2\ \alpha_3\ \alpha_4\ \varepsilon_N]^T = U^T(\|\dot{\psi}\|)\nu, \tag{3.12}$$

where $U \in \mathbb{R}^b$ is a known function of the joint velocities, and $\nu \in \mathbb{R}^b$ is an unknown parameter vector, for a fixed positive number. To suppress the impact of disturbances, an adaptive compensator is designed as:

$$\Theta = \frac{(\hat{\varphi})^2\kappa}{\hat{\varphi}\|\kappa\| + \xi} \tag{3.13}$$

where the auxiliary signal ξ evolves according to $\dot{\xi} = -\beta\xi$, $\xi(0) =$designed constant > 0, $\beta > 0$, $\hat{\varphi} = U^T\hat{\nu}$ denotes the adaptive estimation of φ, and $\hat{\Omega}$ and $\hat{\nu}$ formulated by tuning algorithm. In order to achieve accurate trajectory tracking, the neural network-based control law is defined as:

$$\mu = \hat{\Omega}^T\hat{\chi} + Q_1\text{sign}(\kappa) + Q_2\text{sign}(\kappa)^\eta + \frac{(\hat{\varphi})^2\kappa}{\hat{\varphi}\|\kappa\| + \xi}, \tag{3.14}$$

where Q_1 and Q_2 are gain matrices, $\hat{\chi}$ is the activation vector, and $\hat{\Omega}^T$ represents the estimated neural network weight matrix. The values of $\hat{\Omega}^T$ are updated using online tuning laws. Substituting (3.14) in (3.10), the closed-loop dynamics of the system can be expressed as:

$$B\dot{\kappa} = -H\kappa - \hat{\Omega}^T\hat{\chi} - Q_1\text{sign}(\kappa) - Q_2\text{sign}(\kappa)^\eta + \hat{\Omega}^T\chi(z, \hat{\delta}, \hat{\zeta}) + \tilde{\Omega}^T(\hat{\chi} - Y\hat{\delta} - Z\hat{\zeta}) + \hat{\Omega}^T Y\tilde{\delta}$$
$$+ \hat{\Omega}^T Z\tilde{\zeta} + F(\dot{\psi}) + \mu_d + \mu_{d1} + \varepsilon(z) - \frac{(\hat{\varphi})^2\kappa}{\hat{\varphi}\|\kappa\| + \xi}. \tag{3.15}$$

4. Stability Analysis

Considering the system dynamics described in (2.1) and the control law defined in (3.14), the parameter adaptation rules are designed as follows:

$$\dot{\hat{\Omega}} = \Gamma_\Omega(\hat{\chi} - Y\hat{\delta} - Z\hat{\zeta})\kappa^T, \tag{4.1}$$

$$\dot{\hat{\delta}} = \Gamma_\delta Y^T \hat{\Omega}\kappa, \tag{4.2}$$

$$\dot{\hat{\zeta}} = \Gamma_\zeta Z^T \hat{\Omega}\kappa, \tag{4.3}$$

$$\dot{\hat{\nu}} = \Gamma_\nu U\|\kappa\|. \tag{4.4}$$

where $\Gamma_\Omega \in \mathbb{R}^{N\times N}$, $\Gamma_\delta \in \mathbb{R}^{Nn\times Nn}$, $\Gamma_\zeta \in \mathbb{R}^{Nn\times Nn}$ and $\Gamma_\nu \in \mathbb{R}^{b\times b}$ are positive definite gain matrices. Under these update laws, all closed-loop signals remain bounded, the tracking error is guaranteed to converge to zero in finite time and the system is completely asymptotically stable.
Proof: Select the Lyapunov function as

$$W = \frac{1}{2}\kappa^T B\kappa + \frac{1}{2}tr(\tilde{\Omega}^T\Gamma_\Omega^{-1}\tilde{\Omega}) + \frac{1}{2}\tilde{\delta}^T\Gamma_\delta^{-1}\tilde{\delta} + \frac{1}{2}\tilde{\zeta}^T\Gamma_\zeta^{-1}\tilde{\zeta} + \frac{1}{2}tr(\tilde{\nu}^T\Gamma_\nu^{-1}\tilde{\nu}) + \frac{\xi}{\beta}. \tag{4.5}$$

Taking the time derivative of (4.5) as follows:

$$\dot{W} = \kappa^T B\dot{\kappa} + \frac{1}{2}\kappa^T\dot{B}\kappa + tr(\tilde{\Omega}^T\Gamma_\Omega^{-1}\dot{\tilde{\Omega}}) + \tilde{\delta}^T\Gamma_\delta^{-1}\dot{\tilde{\delta}} + \tilde{\zeta}^T\Gamma_\zeta^{-1}\dot{\tilde{\zeta}} + tr(\tilde{\nu}^T\Gamma_\nu^{-1}\dot{\tilde{\nu}}) + \frac{\dot{\xi}}{\beta}. \tag{4.6}$$

After using the (3.15), then (4.6) is simplified as:

$$\begin{aligned}\dot{W} = {}& \frac{1}{2}\kappa^T(\dot{B}-2H)\kappa - \kappa^T Q_1\mathrm{sign}(\kappa) - \kappa^T Q_2\mathrm{sign}(\kappa)^\eta + \kappa^T\tilde{\Omega}^T(\hat{\chi}-Y\hat{\delta}-Z\hat{\zeta}) + \kappa^T\hat{\Omega}^T Y\tilde{\delta} + \kappa^T\hat{\Omega}^T Z\tilde{\zeta} + \kappa^T(F(\dot{\psi}) \\ & + \mu_d + \mu_{d1} + \varepsilon(z)) - \frac{(\hat{\varphi})^2\kappa\kappa^T}{\hat{\varphi}\|\kappa\| + \xi} + tr(\tilde{\Omega}^T\Gamma_\Omega^{-1}\dot{\tilde{\Omega}}) + \tilde{\delta}^T\Gamma_\delta^{-1}\dot{\tilde{\delta}} + \tilde{\zeta}^T\Gamma_\zeta^{-1}\dot{\tilde{\zeta}} + tr(\tilde{\nu}^T\Gamma_\nu^{-1}\dot{\tilde{\nu}}) + \frac{\dot{\xi}}{\beta}.\end{aligned} \tag{4.7}$$

Using property 2.2, $\kappa^T\kappa = ||\kappa||^2$ and update laws (4.1), (4.2), (4.3), (4.4) together with $\dot{\tilde{\Omega}} = -\dot{\hat{\Omega}}$, $\dot{\tilde{\delta}} = -\dot{\hat{\delta}}$, $\dot{\tilde{\zeta}} = -\dot{\hat{\zeta}}$, $\dot{\tilde{\nu}} = -\dot{\hat{\nu}}$ and $\dot{\xi} = -\beta\xi$, we get

$$\dot{W} = -\kappa^T Q_1\mathrm{sign}(\kappa) - \kappa^T Q_2\mathrm{sign}(\kappa)^\eta + \kappa^T(F(\dot{\psi}) + \mu_d + \mu_{d1} + \varepsilon(z)) - \tilde{\nu}^T U\|\kappa\| - \frac{(U^T\hat{\nu})^2\|\kappa\|^2}{(U^T\hat{\nu})\|\kappa\| + \xi} - \xi. \tag{4.8}$$

By using (3.11) and adaptive bound (3.12) form, we get

$$\kappa^T(F(\dot{\psi}) + \mu_d + \mu_{d1} + \varepsilon(z)) \le \|\kappa\|\|F(\dot{\psi}) + \mu_d + \mu_{d1} + \varepsilon(z)\| \le \varphi\|\kappa\| = U^T\nu\|\kappa\| = U^T(\hat{\nu} + \tilde{\nu})\|\kappa\|. \tag{4.9}$$

Now, we used the inequality (4.9) in (4.8), we get

$$\dot{W} \le -\kappa^T Q_1\mathrm{sign}(\kappa) - \kappa^T Q_2\mathrm{sign}(\kappa)^\eta + U^T(\hat{\nu} + \tilde{\nu})\|\kappa\| - \tilde{\nu}^T U\|\kappa\| - \frac{(U^T\hat{\nu})^2\|\kappa\|^2}{(U^T\hat{\nu})\|\kappa\| + \xi} - \xi, \tag{4.10}$$

$$\dot{W} \le -\kappa^T Q_1\mathrm{sign}(\kappa) + (U^T\hat{\nu})\|\kappa\| - \frac{(U^T\hat{\nu})^2\|\kappa\|^2}{(U^T\hat{\nu})\|\kappa\| + \xi} - \xi, \tag{4.11}$$

$$\dot{W} \le -\kappa^T Q_1\mathrm{sign}(\kappa) + \frac{\xi(U^T\hat{\nu})\|\kappa\|}{(U^T\hat{\nu})\|\kappa\| + \xi} - \xi, \tag{4.12}$$

$$\dot{W} \leq -\kappa^T Q_1 \text{sign}(\kappa) - \frac{\xi^2}{(U^T\hat{\nu})\|\kappa\| + \nu}. \tag{4.13}$$

$$\dot{W} \leq -\kappa^T Q_1 \text{sign}(\kappa) \tag{4.14}$$

$$\dot{W} \leq -\theta_{min}(Q_1)\|\kappa\|. \tag{4.15}$$

where θ_{min} represent the minimum eigenvalue of the gain matrix Q_1. Since $\dot{W}(\kappa(t), \tilde{\Omega}, \tilde{\delta}, \tilde{\zeta}, \tilde{\nu}) \leq 0$ this implies $W(\kappa(t), \tilde{\Omega}, \tilde{\delta}, \tilde{\zeta}, \tilde{\nu}) \leq W(\kappa(0), \tilde{\Omega}, \tilde{\delta}, \tilde{\zeta}, \tilde{\nu})$, it shows that $\kappa(t)$, $\tilde{\Omega}$, $\tilde{\delta}$, $\tilde{\zeta}$ and $\tilde{\nu}$ remain bounded. Define the auxiliary function $\phi(t)\theta_{min}(Q_1\|\kappa\|^2) \leq -\dot{W}$. By integrating $\phi(t)$ with respect to time, we obtain the inequality as:

$$\int_0^t \phi(t)dt \leq W(0) - W(t) = W(\kappa(0), \tilde{\Omega}, \tilde{\delta}, \tilde{\zeta}, \tilde{\nu}) - W(\kappa(t), \tilde{\Omega}, \tilde{\delta}, \tilde{\zeta}, \tilde{\nu}) \tag{4.16}$$

where $W(\kappa(0), \tilde{\Omega}, \tilde{\delta}, \tilde{\zeta}, \tilde{\nu})$ is bounded, while $W(\kappa(t), \tilde{\Omega}, \tilde{\delta}, \tilde{\zeta}, \tilde{\nu})$ is a non-increasing and bounded function. Therefore, the boundedness of κ, $\tilde{\Omega}$, $\tilde{\delta}$, $\tilde{\zeta}$, and $\tilde{\nu}$) is guaranteed. Since the sliding surface κ is defined in terms of the tracking error e and its derivative $\dot{e}$ the error signal is also bounded.

To further establish finite-time stability of the closed-loop system, the following Lyapunov function is introduced:

$$W = \frac{1}{2}\kappa^T B \kappa \tag{4.17}$$
$$\dot{W} = \frac{1}{2}\kappa^T \dot{B} \kappa + \kappa^T B \dot{\kappa}.$$

After substituting the value of (3.15), we get:

$$\dot{W} = \frac{1}{2}\kappa^T(\dot{B} - 2H)\kappa - \kappa^T Q_1 \text{sign}(\kappa) - \kappa^T Q_2 \text{sign}(\kappa)^\eta + \kappa^T \tilde{\Omega}^T(\hat{\chi} - Y\hat{\delta} - Z\hat{\zeta}) + \kappa^T \hat{\Omega}^T Y \tilde{\delta} + \kappa^T \hat{\Omega}^T Z \tilde{\zeta} + \kappa^T(F(\dot{\psi}) + \mu_d + \mu_{d1} + \varepsilon(z)) - \frac{(\hat{\varphi})^2 \kappa\kappa^T}{\hat{\varphi}\|\kappa\| + \xi}. \tag{4.18}$$

Using property (2.2) and $\kappa^T\kappa = \|\kappa\|^2$, we obtain:

$$\dot{W} = -\kappa^T Q_1 \text{sign}(\kappa) - \kappa^T Q_2 \text{sign}(\kappa)^\eta + \kappa^T \tilde{\Omega}^T(\hat{\chi} - Y\hat{\delta} - Z\hat{\zeta}) + \kappa^T \hat{\Omega}^T Y \tilde{\delta} + \kappa^T \hat{\Omega}^T Z \tilde{\zeta} + \kappa^T(F(\dot{\psi}) + \mu_d + \mu_{d1} + \varepsilon(z)) - \frac{(\hat{\varphi})^2 \|\kappa\|^2}{\hat{\varphi}\|\kappa\| + \xi}. \tag{4.19}$$

$$\dot{W} \leq -\theta_{min}(Q_1)\|\kappa\| - \kappa^T Q_2 \text{sign}(\kappa)^\eta + \|\kappa\|\|\tilde{\Omega}^T(\hat{\chi} - Y\hat{\delta} - Z\hat{\zeta}) + \hat{\Omega}^T Y \tilde{\delta} + \hat{\Omega}^T Z \tilde{\zeta}\| + \|\kappa\|\|F(\dot{\psi}) + \mu_d + \mu_{d1} + \varepsilon(z)\| - \frac{(\hat{\varphi})^2 \|\kappa\|^2}{\hat{\varphi}\|\kappa\| + \xi}. \tag{4.20}$$

where λ_{min} denote the smallest singular value of the matrix Q_1. Because any Gaussian basis function $\chi(z)$ satisfies $0 < \chi(z) \leq 1$, and so the WNN parameter estimates $\hat{\Omega}$, $\hat{\delta}$, $\hat{\zeta}$ as well as the input z are bounded, the boundedness of $\hat{\chi}$, the matrices Y and Z follows and also the boundedness of κ, adaptive bound term and disturbance terms are concluded. Then, the following bound is obtained:

$$\begin{aligned}
&\|\kappa\|\|\tilde{\Omega}^T(\hat{\chi} - Y\hat{\delta} - Z\hat{\zeta}) + \hat{\Omega}^T Y\tilde{\delta} + \hat{\Omega}^T Z\tilde{\zeta}\| + \|\kappa\|\|F(\dot{\psi}) + \mu_d + \mu_{d1} + \varepsilon(z)\| \\
&\leq \|\kappa\|\|\tilde{\Omega}^T(\hat{\chi} - Y\hat{\delta} - Z\hat{\zeta})\| + \|\kappa\|\|\hat{\Omega}^T Y\tilde{\delta}\| + \|\kappa\|\|\hat{\Omega}^T Z\tilde{\zeta}\| + \varphi\|\kappa\| \\
&\leq \|\kappa\|\|\tilde{\Omega}\|_F\|\hat{\chi} - Y\hat{\delta} - Z\hat{\zeta}\| + \|\kappa\|\|\hat{\Omega}\|_F\|Y\|_F\|\tilde{\delta}\| + \|\kappa\|\|\hat{\Omega}\|_F\|Z\|_F\|\tilde{\zeta}\| + U^T\nu\|\kappa\| \\
&\leq M_1 + M_2 + M_3 + M_4 = M_m,
\end{aligned}$$

where $\|.\|_F$ represents the Frobenius norm.

$$\dot{W} \leq -\theta_{min}(Q_1)\|\kappa\| - \kappa^T Q_2 \text{sign}(\kappa)^\eta + M_m, \tag{4.21}$$

Choosing the gain matrix Q_1 such a way that $\theta_{min} > M_m$, we get

$$\dot{W} \leq -\kappa^T Q_2 \text{sign}(\kappa)^\eta = \sum_{i=1}^{n} \kappa_i Q_{1i} |\kappa_i|^\eta \text{sign}(\kappa_i) = -\sum_{i=1}^{n} Q_{1i} |\kappa_i|^{\eta+1} \leq -\rho \left(\sum_{i=1}^{n} \frac{1}{2} \bar{b} \kappa_i^2 \right)^\gamma \leq -\rho W^\gamma.$$

This implies

$$\dot{W} + \rho W^\gamma \leq 0, \tag{4.22}$$

where $\gamma = (1+\eta)/2$, $\rho = Q_{min}(2/\bar{b})^\gamma$ and $Q_{min} = min Q_{1i}$. By Lemma 3.3, κ converges to zero in limited time. From the interpretation of the sliding surface, it then follows the convergence of tracking errors to zero level in limited time along the fast terminal sliding surface.

Hence, The chosen Lyapunov function guarantees that the tracking errors, the adaptive parameters, and the WNN weight estimates remain uniformly bounded during the entire control operation, and also it proves that approximation errors and external disturbances stay bounded, do not compromise system stability. The fast terminal sliding surface further ensures that the sliding variable and tracking errors converge to zero within a finite time rather than only asymptotically.

5. Conclusion

This paper introduces a novety of the stability model of robotic manipulators has been addressed through the integration of Adaptive FTSMC with WNN. The proposed control framework successfully combines the finite-time convergence property of FTSMC with the powerful approximation capability of WNN to handle nonlinearities, parametric uncertainties, and external disturbances. The adaptive mechanism incorporated in the controller removes the dependency on prior knowledge of system bounds, thereby enhancing adaptability and making the scheme more suitable for real-world robotic applications. The stability model of the closed-loop system has been rigorously verified using Lyapunov-based analysis, which guarantees finite-time convergence and overall system stability.

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