



On Fixed Point Results for Hybrid-Interpolative Reich-Istrăţescu-Type Contractions within the Framework of Soft Metric Spaces

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ABSTRACT: In this study, we extend the class of hybrid-interpolative Reich-Istrăţescu-type contractions, recently introduced by Karapınar et al. (2022), to the framework of soft metric spaces—a generalized structure that integrates soft set theory with classical metric spaces to model uncertainty. We rigorously define these contractions within soft metric spaces, leveraging the concepts of soft points and soft mappings. A series of fixed point theorems are established under these generalized contractive conditions, demonstrating the existence and uniqueness of fixed points. Novel analytical techniques are employed to validate the theoretical results, and illustrative examples further highlight their practical applicability. These findings advance fixed point theory in soft metric spaces and offer new perspectives for applications in optimization, control theory, and computational mathematics where uncertainty is inherent.

Keywords: Soft metric, fixed point, fixed point theorems.

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1. Introduction

Classical mathematical frameworks, though robust, often fall short in capturing the inherent uncertainties and vagueness of real-world problems. To address these limitations, several generalized mathematical structures have been proposed, including fuzzy sets, intuitionistic fuzzy sets, and rough sets. In this context, Molodtsov (1999) [16] introduced the concept of soft sets as a novel mathematical tool to effectively manage uncertainties, circumventing the restrictions posed by membership functions in fuzzy set theory and lower and upper approximations in rough set theory. Ali et al. [2] expanded upon this by developing new operations within soft set theory, laying the groundwork for defining soft metric concepts.

Building upon this foundation, researchers have extended classical mathematical structures to their soft counterparts, thereby incorporating the flexibility and parametric adaptability inherent in soft set theory. Among these extensions, the notion of soft metric spaces has emerged as a particularly significant advancement. A soft metric space generalizes the classical metric space by incorporating soft sets, enabling the measurement of "soft distances" between elements. In this framework, the classical axioms of a metric—non-negativity, identity of indiscernibles, symmetry, and the triangle inequality—are adapted to ensure consistency and applicability within the soft set context. Das and Samanta [8] laid the initial groundwork by introducing soft metric spaces and investigating concepts such as soft open and closed sets, soft balls, and convergence in this setting. Subsequently, numerous researchers have significantly contributed to the advancement of soft metric space theory. For instance, Bayramov et al. [6] introduced the concept of bipolar soft metric spaces, while recent works have explored various hybrid structures such as soft G-metric spaces [9], soft D-metric spaces [10], soft cone metric spaces [22], soft S-metric spaces [3], parametric soft S-metric spaces [1], dislocated soft metric spaces [4], and soft A-metric spaces [19].

Zhang and Zhang [24] extended fixed point theorems to soft metric spaces, demonstrating their relevance to nonlinear analysis and iterative schemes. In particular, their results, which echo the classical Banach contraction principle, confirm that iterative methods retain convergence properties even in the

presence of parameter-based distance structures. To further accommodate the complexity of real-world applications, researchers have introduced several generalizations of soft metric spaces, each accompanied by corresponding fixed point results. For instance, fixed point theorems in soft b-metric spaces have extended the Banach contraction principle to this more relaxed setting, broadening its applicability to systems where distance measures are not strictly linear [20]. Similarly, Yüksel et al. [23] developed fixed point theorems for soft partial metric spaces, showing that such spaces can model iterative processes in computational environments with partial information. Additional studies have explored various variants and generalizations of fixed point theorems in soft metric spaces, including those in parametric soft S-metric spaces [11], soft parametric metric spaces [12], soft G-metric spaces [13], and soft B-metric spaces [17].

Recent research has also focused on hybrid fixed point theorems, such as the Reich–Istrăţescu-type contractions. Karapınar et al. (2022) [18] introduced a new class of contractive mappings—hybrid-interpolative Reich–Istrăţescu-type contractions—which are characterized by their dependence on multiple parameters. This formulation provides a unifying framework that generalizes several well-known contractive mappings, thereby offering potential applications across various generalized metric settings, including soft metric spaces.

This paper aims to extend the applicability of hybrid-interpolative Reich–Istrăţescu-type contractions by formulating and analyzing them within the framework of soft metric spaces. To this end, we first provide a rigorous definition of these contractions in the context of soft metric spaces, incorporating the notions of soft points and soft mappings. Subsequently, we establish a series of fixed point theorems under these generalized contractive conditions. Existence and uniqueness results are derived using novel analytical techniques specifically tailored to the soft metric setting.

The findings of this study make a significant contribution to the growing body of literature on fixed point theory in generalized metric spaces. By extending hybrid-interpolative contractions to soft metric spaces, we enrich the theoretical foundation of soft analysis and open new avenues for future research in non-classical metric structures. The results presented herein are expected to have implications in various fields that necessitate uncertainty modeling and decision-making under vagueness, including optimization, control theory, and computational mathematics.

2. Preliminaries

We begin by presenting the essential definitions and fundamental concepts necessary for the development of the subsequent results in this manuscript. Further elaboration on these topics can be found in the cited references.

Definition 2.1 [16] *A pair $(\tilde{\rho}, \Omega)$ is claimed to be soft set on the universe X if and only if $\tilde{\rho}$ is set valued mapping on Ω taking values in $P(X)$. A soft set $(\tilde{\rho}, \Omega)$ can be accepted as a parametrized family of subsets of the set X . The set $\tilde{\rho}(a)$ in X is called a -approximate element of the soft set $(\tilde{\rho}, \Omega)$, for each a in Ω .*

Definition 2.2 [15] *A soft set $(\tilde{\rho}, \Omega)$ over X is called to be a null soft set if $\tilde{\rho}(a) = \emptyset$ for all $a \in \Omega$ and denoted by $\tilde{\emptyset}$.*

Definition 2.3 [15] *A soft set $(\tilde{\rho}, \Omega)$ over X is called to be an absolute soft set if $\tilde{\rho}(a) = X$ for all $a \in \Omega$ and denoted by \tilde{X} .*

Definition 2.4 [21] *A collection $\tilde{\tau}$ of soft sets over X is called to be a soft topology on X if*

- (1) $\tilde{\emptyset}, \tilde{X}$ are included in $\tilde{\tau}$;
- (2) the union of any number of soft sets in $\tilde{\tau}$ is included in $\tilde{\tau}$;
- (3) the intersection of any two soft sets in $\tilde{\tau}$ is included in $\tilde{\tau}$.

The triplet $(X, \tilde{\tau}, \Omega)$ is claimed to be a soft topological space over X .

Definition 2.5 [21] *Members of $\tilde{\tau}$ are said to be a soft open sets in X , where $(X, \tilde{\tau}, \Omega)$ is a soft topological space over X .*

Definition 2.6 [21] A soft set $(\tilde{\rho}, \Omega)$ over X is called to be a soft closed set whenever its complement $(\tilde{\rho}, \Omega)^c$ is included in $\tilde{\tau}$, where $(X, \tilde{\tau}, \Omega)$ is a soft topological space over X .

Proposition 2.1 [21] For each $a \in \Omega$, the collection $\tilde{\tau}_a = \{\tilde{\rho}(a) : (\tilde{\rho}, \Omega) \in \tilde{\tau}\}$ defines a topology on X , where $(X, \tilde{\tau}, \Omega)$ is a soft topological space over X .

Definition 2.7 [5, 7] \tilde{x}_a is called a soft point on the soft set $(\tilde{\rho}, \Omega)$ over X if $(\tilde{\rho}, \Omega)$ is defined as

$$\tilde{\rho} : \Omega \rightarrow P(X) : \tilde{\rho}(a) = \begin{cases} \{x\}, & \text{if } a \in \Omega \\ \emptyset, & \text{if } a' \in \Omega - \{a\} \end{cases}$$

It is clear that each soft set can be denoted as a union of soft points. Therefore, when we have the family of all soft sets on X it is suitable to give only soft points on X .

Definition 2.8 [7] A soft set $(\tilde{\rho}, \Omega)$ defined by $\tilde{\rho} : \Omega \rightarrow B(\mathbb{R})$ is said to be a soft real set, where \mathbb{R} is the set of all real numbers, $B(\mathbb{R})$ is the collection of all non-empty bounded subsets of \mathbb{R} . When a soft real set $(\tilde{\rho}, \Omega)$ is a singleton soft set, it is called a soft real number and denoted by $\tilde{r}, \tilde{s}, \tilde{t}$. Besides the constant soft real numbers are denoted by $\bar{r}, \bar{s}, \bar{t}$. It is clear that $\bar{0}(a) = 0, \bar{1}(a) = 1$, for all $a \in \Omega$.

Definition 2.9 [7] The collection of non-negative soft real numbers is denoted by $\mathbb{R}(\Omega)^*$ and the pair $(\mathbb{R}(\Omega)^*, \leq)$ is a partially ordered set such that for \tilde{r}, \tilde{s} as two soft real numbers. Then,

- (i) $\tilde{r} \leq \tilde{s}$, if $\tilde{r}(a) \leq \tilde{s}(a)$ for all $a \in \Omega$,
- (ii) $\tilde{r} \geq \tilde{s}$, if $\tilde{r}(a) \geq \tilde{s}(a)$ for all $a \in \Omega$,
- (iii) $\tilde{r} < \tilde{s}$, if $\tilde{r}(a) < \tilde{s}(a)$ for all $a \in \Omega$,
- (iv) $\tilde{r} > \tilde{s}$, if $\tilde{r}(a) > \tilde{s}(a)$ for all $a \in \Omega$.

Definition 2.10 [5, 7] Two soft points $\tilde{x}_a, \tilde{y}_{a'}$ are called to be equal when $a = a'$ and $x = y$. Thus $\tilde{x}_a \neq \tilde{y}_{a'}$ if and only if $x \neq y$ or $a \neq a'$.

Let $SP(X)$ denote the family of all soft points of the universe X .

Definition 2.11 [8] Let $\tilde{d} : SP(X) \times SP(X) \rightarrow \mathbb{R}(\Omega)^*$ be a function satisfying the following conditions for all $\tilde{x}_{a_1}, \tilde{y}_{a_2}, \tilde{z}_{a_3} \in SP(X)$,

- (1) $\tilde{d}(\tilde{x}_{a_1}, \tilde{y}_{a_2}) \geq \bar{0}$,
- (2) $\tilde{d}(\tilde{x}_{a_1}, \tilde{y}_{a_2}) = \bar{0}$ if and only if $\tilde{x}_{a_1} = \tilde{y}_{a_2}$,
- (2) $\tilde{d}(\tilde{x}_{a_1}, \tilde{y}_{a_2}) = \tilde{d}(\tilde{y}_{a_2}, \tilde{x}_{a_1})$,
- (3) $\tilde{d}(\tilde{x}_{a_1}, \tilde{z}_{a_3}) \leq \tilde{d}(\tilde{x}_{a_1}, \tilde{y}_{a_2}) + \tilde{d}(\tilde{y}_{a_2}, \tilde{z}_{a_3})$.

Then the function \tilde{d} is called a soft metric on X and the triplet (X, \tilde{d}, Ω) is referred to as a soft metric space.

Definition 2.12 [8] Let $\{\tilde{x}_{a_n}^n\}$ be a sequence in the soft metric space (X, \tilde{d}, Ω) . The sequence $\{\tilde{x}_{a_n}^n\}$ is said to converge to a soft point $\tilde{x}_a \in SP(X)$ if

$$\lim_{n \rightarrow \infty} \tilde{d}(\tilde{x}_{a_n}^n, \tilde{x}_a) = \bar{0}$$

Moreover, the sequence $\{\tilde{x}_{a_n}^n\}$ is called a soft Cauchy sequence in (X, \tilde{d}, Ω) if, for every $\tilde{\varepsilon} > \bar{0}$, there exists a natural number $m \in \mathbb{N}$ such that

$$\tilde{d}(\tilde{x}_{a_i}^i, \tilde{x}_{a_j}^j) \leq \tilde{\varepsilon}, \text{ for all } i, j \geq m.$$

Definition 2.13 [8] A soft metric space (X, \tilde{d}, Ω) is said to be complete if every soft Cauchy sequence in X converges to a soft element of X .

Proposition 2.2 [14] Let $C \in [0, 1]$, and let $\{x^n\} \subset \mathbb{R}_+$ be a sequence satisfying the recursive inequality:

$$x^{n+2} \leq C \max \{x^n, x^{n+1}\}, \forall n \in \mathbb{N}.$$

Define $K = \max \{x^0, x^1\}$. Then, for all $n \geq 1$, the following inequalities hold:

$$\begin{aligned} x^{2n} &\leq C^n .K \\ x^{2n+1} &\leq C^n .K. \end{aligned}$$

3. Reich–Istrăţescu-type contractions

Definition 3.1 Let $\alpha : SP(X) \times SP(X) \rightarrow \mathbb{R}(\Omega)^*$ be a soft function. A self mapping $(f, \varphi) : (X, \tilde{d}, \Omega) \rightarrow (X, \tilde{d}, \Omega)$ is termed soft α -orbital admissible if, for every $\tilde{x}_0 \in SP(X)$,

$$\alpha(\tilde{x}_0, (f, \varphi)(\tilde{x}_0)) \tilde{\geq} \bar{1} \Rightarrow \alpha\left((f, \varphi)(\tilde{x}_0), (f, \varphi)^2(\tilde{x}_0)\right) \tilde{\geq} \bar{1}.$$

Definition 3.2 Let $\alpha : SP(X) \times SP(X) \rightarrow \mathbb{R}(\Omega)^*$ be a given function. The set X is said to be regular with respect to α if and only if the following condition holds:

For every sequence $\{\tilde{x}_{a_n}^n\} \subset SP(X)$ satisfying

$$\alpha\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right) \tilde{\geq} \bar{1} \text{ for all } n \in \mathbb{N},$$

and

$$\lim_{n \rightarrow \infty} \tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_a\right) = \bar{0}, \text{ for some } \tilde{x}_a \in SP(X),$$

it follows that

$$\alpha\left(\tilde{x}_{a_n}^n, \tilde{x}_a\right) \tilde{\geq} \bar{1}, \text{ for all } n \in \mathbb{N}.$$

Lemma 3.1 Let $\{\tilde{x}_{a_n}^n\}$ be a sequence in the soft metric space (X, \tilde{d}, Ω) . Suppose there exists a constant $C \in (\bar{0}, \bar{1})$ such that the following inequality holds for all $n \in \mathbb{N}$:

$$\tilde{d}\left(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3}\right) \tilde{\leq} C . \max \left\{ \tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right), \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right) \right\}.$$

Then the sequence $\{\tilde{x}_{a_n}^n\}$ is a soft Cauchy sequence in the soft metric space (X, \tilde{d}, Ω) .

Proof: Consider the sequence $\{\tilde{z}_{a_n}^n\} \subset SP(X)$ defined by

$$\tilde{z}_{a_n}^n = \tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right), \forall n \in \mathbb{N}.$$

By the hypothesis, the sequence satisfies the inequality

$$\tilde{d}\left(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3}\right) \tilde{\leq} C . \max \left\{ \tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right), \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right) \right\}.$$

where $C \in (\bar{0}, \bar{1})$. Hence, by Proposition 2.2, it follows that for all $n \in \mathbb{N}$

$$\begin{aligned} \tilde{d}\left(\tilde{x}_{a_{2n}}^{2n}, \tilde{x}_{a_{2n+1}}^{2n+1}\right) &= \tilde{z}_{a_{2n}}^{2n} \tilde{\leq} C^n .K \\ \tilde{d}\left(\tilde{x}_{a_{2n+1}}^{2n+1}, \tilde{x}_{a_{2n+2}}^{2n+2}\right) &= \tilde{z}_{a_{2n+1}}^{2n+1} \tilde{\leq} C^n .K, \end{aligned}$$

where $K = \max \{\tilde{z}_{a_0}^0, \tilde{z}_{a_1}^1\}$. In particular, we obtain

$$\tilde{d}\left(\tilde{x}_{a_{2n}}^{2n}, \tilde{x}_{a_{2n+1}}^{2n+1}\right) + \tilde{d}\left(\tilde{x}_{a_{2n+1}}^{2n+1}, \tilde{x}_{a_{2n+2}}^{2n+2}\right) \tilde{\leq} 2C^n .K, \forall n \in \mathbb{N}.$$

If $C = \bar{0}$ or $K = \bar{0}$, then $\{\tilde{x}_{a_n}^n\}_{n \geq 2}$ becomes constant from a certain index onward. Therefore, in this case, the sequence is trivially a soft Cauchy sequence. Now, assume that $C \succ \bar{0}$ and $K \succ \bar{0}$. To establish that $\{\tilde{x}_{a_n}^n\}$ is a soft Cauchy sequence, let $\tilde{\varepsilon} \succ \bar{0}$ be arbitrary. Since $\frac{\tilde{\varepsilon}}{2K} \succ \bar{0}$ and $C \in (\bar{0}, \bar{1})$, there exists a natural number $n_0 \in \mathbb{N}$ such that

$$\sum_{i=n_0}^{\infty} C^i \lesssim \frac{\tilde{\varepsilon}}{2K}.$$

In particular, for all $p \in \mathbb{N}$ with $p \geq n_0$, we have

$$2K \sum_{i=n_0}^p C^i \lesssim 2K \sum_{i=n_0}^{\infty} C^i \lesssim \tilde{\varepsilon},$$

Let $m, n \in \mathbb{N}$ with $m > n > 2n_0$, and let $p \in \mathbb{N}$ be such that $p \geq n_0 + 1$ and $2p > m$. Then, using the triangle inequality, we estimate:

$$\begin{aligned} \tilde{d}(\tilde{x}_{a_n}^n, \tilde{x}_{a_m}^m) &\lesssim \sum_{j=n}^{m-1} \tilde{d}(\tilde{x}_{a_j}^j, \tilde{x}_{a_{j+1}}^{j+1}) \lesssim \sum_{j=2n_0}^{2p-1} \tilde{d}(\tilde{x}_{a_j}^j, \tilde{x}_{a_{j+1}}^{j+1}) \\ &= \sum_{j=n_0}^{p-1} \left[\tilde{d}(\tilde{x}_{a_{2j}}^{2j}, \tilde{x}_{a_{2j+1}}^{2j+1}) + \tilde{d}(\tilde{x}_{a_{2j+1}}^{2j+1}, \tilde{x}_{a_{2j+2}}^{2j+2}) \right] \\ &= \sum_{j=n_0}^{p-1} 2C^j K \lesssim 2K \sum_{j=n_0}^{\infty} C^j \lesssim \tilde{\varepsilon}, \end{aligned}$$

which proves that the sequence $\{\tilde{x}_{a_n}^n\}$ is a soft Cauchy sequence in (X, \tilde{d}, Ω) . \square

The following proposition has been established on soft points, similarly to Proposition 8 in the work of Jiddah [14].

Proposition 3.1 *Let $C \in [\bar{0}, \bar{1}]$, $\delta \in (\bar{0}, \bar{1})$, and let $\{\tilde{x}_{a_n}^n\}$ be a sequence satisfying the recursive inequality*

$$\tilde{x}_{a_{n+2}}^{n+2} \lesssim C \max \left\{ \tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1} \right\}^{\delta}, \forall n \in \mathbb{N}.$$

Define $K = \max \left\{ \tilde{x}_{a_0}^0, \tilde{x}_{a_1}^1 \right\}$. Then, for all $n \geq 1$, the following estimates hold:

$$\begin{aligned} \tilde{x}_{a_{2n}}^{2n} &\lesssim C^{\bar{1} + \delta + \delta^2 + \dots + \delta^{n-1}} K^{\delta^n}, \\ \tilde{x}_{a_{2n+1}}^{2n+1} &\lesssim C^{\bar{1} + \delta + \delta^2 + \dots + \delta^{n-1}} K^{\delta^n}. \end{aligned}$$

Therefore, it follows that

$$\lim_{n \rightarrow \infty} \sup \tilde{x}_{a_n}^n \lesssim C^{\frac{\bar{1}}{1-\delta}}.$$

Corollary 3.1 *Let $\{\tilde{x}_{a_n}^n\}$ be a sequence in the soft metric space (X, \tilde{d}, Ω) . Suppose there exist constants $C \in [\bar{0}, \bar{1}]$ and $\theta, \eta \in (\bar{0}, \bar{1})$ with $\theta + \eta = \bar{1}$ such that*

$$\tilde{d}(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3}) \lesssim C \cdot \tilde{d}(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1})^{\theta} \tilde{d}(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2})^{\eta}$$

for all $n \in \mathbb{N}$. Then $\{\tilde{x}_{a_n}^n\}$ is a soft Cauchy sequence in the soft metric space (X, \tilde{d}, Ω) .

Proof: Observe that for all $n \in \mathbb{N}$,

$$\begin{aligned}
\tilde{d}\left(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3}\right) &\leq C \cdot \tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right)^\theta \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right)^\eta \\
&\leq C \cdot \max\left\{\tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right), \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right)\right\}^\theta \\
&\quad \max\left\{\tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right), \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right)\right\}^\eta \\
&= C \cdot \max\left\{\tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right), \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right)\right\}^{\theta+\eta} \\
&= C \cdot \max\left\{\tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right), \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right)\right\}.
\end{aligned}$$

Consequently, by invoking Lemma 3.1, we conclude that $\{\tilde{x}_{a_n}^n\}$ is a soft Cauchy sequence in the soft metric space (X, \tilde{d}, Ω) . \square

Definition 3.3 Let (X, \tilde{d}, Ω) be a soft metric space, and let $\alpha : SP(X) \times SP(X) \rightarrow \mathbb{R}(\Omega)^*$ be a given function. A soft mapping $(f, \varphi) : (X, \tilde{d}, \Omega) \rightarrow (X, \tilde{d}, \Omega)$ is called hybrid-interpolative Reich–Istrăţescu-type (soft- $\alpha - \mu$) contraction if there exists a constant $q \in \mathbb{R}(\Omega)^*$ such that for some $\mu \in (\bar{0}, \bar{1})$, $\delta \geq \bar{0}$, and non-negative constants $\lambda_i \geq \bar{0}$ for $i = 1, 2, 3, 4, 5$, the following condition holds for all $\tilde{x}_a, \tilde{y}_b \in X \setminus \text{Fix}((f, \varphi))$

$$\alpha(\tilde{x}_a, \tilde{y}_b) \cdot \tilde{d}\left((f, \varphi)^2(\tilde{x}_a), (f, \varphi)^2(\tilde{y}_b)\right) \leq \mu \cdot N(\tilde{x}_a, \tilde{y}_b) \quad (3.1)$$

where $N(\tilde{x}_0, \tilde{y}_0)$ is defined as follows:

$$\begin{aligned}
N(\tilde{x}_a, \tilde{y}_b) &= \begin{cases} [A]^{1/q}, & \text{for all } q > \bar{0} \text{ with } \sum_{i=1}^5 \lambda_i + \delta \leq \bar{1} \\ B, & \text{for } q = \bar{0} \text{ with } \sum_{i=1}^5 \lambda_i + \delta = \bar{1} \end{cases} \\
A &= \lambda_1 \tilde{d}(\tilde{x}_a, \tilde{y}_b)^q + \lambda_2 \tilde{d}(\tilde{x}_a, (f, \varphi)(\tilde{x}_a))^q + \lambda_3 \tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b))^q \\
&\quad + \lambda_4 \tilde{d}((f, \varphi)(\tilde{x}_a), (f, \varphi)(\tilde{y}_b))^q + \lambda_5 \tilde{d}\left((f, \varphi)(\tilde{x}_a), (f, \varphi)^2(\tilde{x}_a)\right)^q \\
&\quad + \delta \tilde{d}\left((f, \varphi)(\tilde{y}_b), (f, \varphi)^2(\tilde{y}_b)\right)^q, \\
B &= \left[\tilde{d}(\tilde{x}_a, \tilde{y}_b)\right]^{\lambda_1} \left[\tilde{d}(\tilde{x}_a, (f, \varphi)(\tilde{x}_a))\right]^{\lambda_2} \left[\tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b))\right]^{\lambda_3} \\
&\quad \left[\tilde{d}((f, \varphi)(\tilde{x}_a), (f, \varphi)(\tilde{y}_b))\right]^{\lambda_4} \left[\tilde{d}\left((f, \varphi)(\tilde{x}_a), (f, \varphi)^2(\tilde{x}_a)\right)\right]^{\lambda_5} \\
&\quad \left[\tilde{d}\left((f, \varphi)(\tilde{y}_b), (f, \varphi)^2(\tilde{y}_b)\right)\right]^\delta.
\end{aligned}$$

Here, the set of fixed points of the mapping (f, φ) is denoted by

$$\text{Fix}((f, \varphi)) = \{\tilde{x}_a \in SP(X) : (f, \varphi)(\tilde{x}_a) = \tilde{x}_a\}.$$

Theorem 3.1 Let (X, \tilde{d}, Ω) be a complete soft metric space, and let $(f, \varphi) : (X, \tilde{d}, \Omega) \rightarrow (X, \tilde{d}, \Omega)$ be a hybrid-interpolative Reich–Istrăţescu-type (soft- $\alpha - \mu$) contraction satisfying the following conditions:

- (i) (f, φ) is a soft α -orbital admissible,
 - (ii) (f, φ) is continuous,
 - (iii) There exists an element $\tilde{x}_a^0 \in SP(X)$ such that $\alpha(\tilde{x}_a^0, (f, \varphi)(\tilde{x}_a^0)) \geq \bar{1}$.
- Then (f, φ) admits at least one fixed point in X .

Proof: Let $\tilde{x}_a^0 \in SP(X)$ be such that $\alpha(\tilde{x}_a^0, (f, \varphi)(\tilde{x}_a^0)) \gtrsim \bar{1}$. Since (f, φ) is assumed to be soft α -orbital admissible, it follows by induction that

$$\alpha\left((f, \varphi)(\tilde{x}_a^0), (f, \varphi)^2(\tilde{x}_a^0)\right) \gtrsim \bar{1} \text{ for all } n \in \mathbb{N}.$$

Define a sequence $\{\tilde{x}_{a_n}^n\}$ in $SP(X)$ by setting $\tilde{x}_{a_n}^n = (f, \varphi)^n(\tilde{x}_a^0)$ for each $n \in \mathbb{N}$. If there exists $m \in \mathbb{N}$ such that $(f, \varphi)(\tilde{x}_{a_m}^m) = \tilde{x}_{a_m}^{m+1} = \tilde{x}_{a_m}^m$, then $\tilde{x}_{a_m}^m$ is trivially a fixed point of (f, φ) . Suppose now that $\tilde{x}_{a_{n+1}}^{n+1} \neq \tilde{x}_{a_n}^n$ for all $n \in \mathbb{N}$. By the contractive condition (3.1), we obtain:

$$\begin{aligned} \tilde{d}\left(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3}\right) &\lesssim \alpha\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right) \tilde{d}\left((f, \varphi)^2(\tilde{x}_{a_n}^n), (f, \varphi)^2(\tilde{x}_{a_{n+1}}^{n+1})\right) \\ &\lesssim \mu N\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right). \end{aligned} \quad (3.2)$$

We distinguish two cases based on the value of q :

Case 1: For $q \gtrsim \bar{0}$. By the definition of $N\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right)$, and substituting via the recursive structure of the sequence, we have:

$$\begin{aligned} N\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right) &= [\lambda_1 \tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right)^q + \lambda_2 \tilde{d}\left(\tilde{x}_{a_n}^n, (f, \varphi)(\tilde{x}_{a_n}^n)\right)^q + \lambda_3 \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, (f, \varphi)(\tilde{x}_{a_{n+1}}^{n+1})\right)^q \\ &\quad + \lambda_4 \tilde{d}\left((f, \varphi)(\tilde{x}_{a_n}^n), (f, \varphi)(\tilde{x}_{a_{n+1}}^{n+1})\right)^q + \lambda_5 \tilde{d}\left((f, \varphi)(\tilde{x}_{a_n}^n), (f, \varphi)^2(\tilde{x}_{a_n}^n)\right)^q \\ &\quad + \delta \tilde{d}\left((f, \varphi)(\tilde{x}_{a_{n+1}}^{n+1}), (f, \varphi)^2(\tilde{x}_{a_{n+1}}^{n+1})\right)^q]^{1/q} \\ &= [\lambda_1 \tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right)^q + \lambda_2 \tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right)^q + \lambda_3 \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right)^q + \lambda_4 \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right)^q \\ &\quad + \lambda_5 \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right)^q + \delta \tilde{d}\left(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3}\right)^q]^{1/q} \\ &= [(\lambda_1 + \lambda_2) \tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right)^q + (\lambda_3 + \lambda_4 + \lambda_5) \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right)^q + \delta \tilde{d}\left(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3}\right)^q]^{1/q} \\ &\lesssim [(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5) \max\left\{\tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right)^q, \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right)^q\right\} + \delta \tilde{d}\left(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3}\right)^q]^{1/q}. \end{aligned}$$

Using (3.2), we derive:

$$\tilde{d}\left(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3}\right)^q \lesssim \mu^q [(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5) \max\left\{\tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right)^q, \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right)^q\right\} + \delta \tilde{d}\left(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3}\right)^q]^{1/q}.$$

So that,

$$\begin{aligned} (\bar{1} - \mu^q \delta) \tilde{d}\left(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3}\right)^q &\lesssim \mu^q [(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5) \max\left\{\tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right)^q, \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right)^q\right\}]^{1/q} \\ &\lesssim \mu^q (\bar{1} - \delta) \max\left\{\tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right)^q, \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right)^q\right\} \end{aligned}$$

implying that for all $n \in \mathbb{N}$,

$$\tilde{d}\left(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3}\right)^q \lesssim \frac{\mu^q (\bar{1} - \delta)}{(\bar{1} - \mu^q \delta)} \max\left\{\tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right), \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right)\right\}^q.$$

Setting

$$C = \left(\frac{\mu^q (\bar{1} - \delta)}{(\bar{1} - \mu^q \delta)}\right)^{\bar{1}/q} \in (\bar{0}, \bar{1}),$$

we obtain:

$$\tilde{d}\left(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3}\right)^q \lesssim C \max\left\{\tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right)^q, \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right)\right\}$$

Thus, by Lemma 3.1, $\{\tilde{x}_{a_n}^n\}$ is a soft Cauchy in (X, \tilde{d}, Ω) .

Case 2: $q = \bar{0}$. In this case, $N(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1})$ is defined via a multiplicative form:

$$\begin{aligned}
N(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}) &= [\tilde{d}(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1})]^{\lambda_1} [\tilde{d}(\tilde{x}_{a_n}^n, (f, \varphi)(\tilde{x}_{a_n}^n))]^{\lambda_2} [\tilde{d}(\tilde{x}_{a_{n+1}}^{n+1}, (f, \varphi)(\tilde{x}_{a_{n+1}}^{n+1}))]^{\lambda_3} \\
&\quad [\tilde{d}((f, \varphi)(\tilde{x}_{a_n}^n), (f, \varphi)(\tilde{x}_{a_{n+1}}^{n+1}))]^{\lambda_4} [\tilde{d}((f, \varphi)(\tilde{x}_{a_n}^n), (f, \varphi)^2(\tilde{x}_{a_n}^n))]^{\lambda_5} \\
&\quad [\tilde{d}((f, \varphi)(\tilde{x}_{a_{n+1}}^{n+1}), (f, \varphi)^2(\tilde{x}_{a_{n+1}}^{n+1}))]^{\delta} \\
&= [\tilde{d}(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1})]^{\lambda_1} [\tilde{d}(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1})]^{\lambda_2} [\tilde{d}(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2})]^{\lambda_3} \\
&\quad [\tilde{d}(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2})]^{\lambda_4} [\tilde{d}(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2})]^{\lambda_5} [\tilde{d}(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3})]^{\delta} \\
&= [\tilde{d}(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1})]^{\lambda_1 + \lambda_2} [\tilde{d}(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2})]^{\lambda_3 + \lambda_4 + \lambda_5} [\tilde{d}(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3})]^{\delta}.
\end{aligned}$$

From (3.2), we then obtain:

$$\tilde{d}(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3}) \lesssim \mu [\tilde{d}(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1})]^{\lambda_1 + \lambda_2} [\tilde{d}(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2})]^{\lambda_3 + \lambda_4 + \lambda_5} [\tilde{d}(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3})]^{\delta} \quad (3.3)$$

Assuming $\delta = \bar{1}$ leads to a contradiction since then:

$$0 \lesssim \tilde{d}(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3}) \lesssim \mu \tilde{d}(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3})$$

and $\bar{0} \lesssim \mu \lesssim \bar{1}$. Therefore, $\delta \lesssim \bar{1}$, and $\sum_{i=1}^5 \lambda_i = \bar{1} - \delta \gtrsim \bar{0}$. Let

$$\theta = \frac{\lambda_1 + \lambda_2}{\bar{1} - \delta}, \eta = \frac{\lambda_3 + \lambda_4 + \lambda_5}{\bar{1} - \delta}, \text{ with } \theta + \eta = \bar{1}.$$

Then (3.3) becomes

$$\tilde{d}(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3})^{1-\delta} \lesssim \mu [\tilde{d}(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1})]^{\lambda_1 + \lambda_2} [\tilde{d}(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2})]^{\lambda_3 + \lambda_4 + \lambda_5}$$

so that

$$\begin{aligned}
\tilde{d}(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3}) &\lesssim \mu^{\frac{\bar{1}}{\bar{1}-\delta}} [\tilde{d}(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1})]^{\frac{\lambda_1 + \lambda_2}{\bar{1}-\delta}} [\tilde{d}(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2})]^{\frac{\lambda_3 + \lambda_4 + \lambda_5}{\bar{1}-\delta}} \\
&= \mu^{\frac{\bar{1}}{\bar{1}-\delta}} [\tilde{d}(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1})]^{\theta} [\tilde{d}(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2})]^{\eta}
\end{aligned}$$

for all $n \in \mathbb{N}$. Noting that $\mu \in (\bar{0}, \bar{1})$, we have

$$\bar{0} \lesssim \bar{1} - \delta \lesssim \bar{1} \Rightarrow \bar{1} \lesssim \frac{\bar{1}}{\bar{1}-\delta} \Rightarrow \mu^{\frac{\bar{1}}{\bar{1}-\delta}} \lesssim \mu \lesssim \bar{1}.$$

Since $\bar{0} \lesssim \mu^{\frac{1}{\bar{1}-\delta}} \lesssim \bar{1}$, it follows by Corollary 3.1 that the sequence $\{\tilde{x}_{a_n}^n\}$ is soft Cauchy in (X, \tilde{d}, Ω) .

In both cases, the sequence $\{\tilde{x}_{a_n}^n\}$ is soft Cauchy. By completeness of the soft metric space (X, \tilde{d}, Ω) , there exists a point $\tilde{c}_k \in X$ such that

$$\lim_{n \rightarrow \infty} \tilde{x}_{a_n}^n = \tilde{c}_k.$$

Moreover, since (f, φ) is continuous, we have:

$$(f, \varphi)(\tilde{c}_k) = \lim_{n \rightarrow \infty} (f, \varphi)(\tilde{x}_{a_n}^n) = \lim_{n \rightarrow \infty} \tilde{x}_{a_{n+1}}^{n+1} = \tilde{c}_k.$$

Therefore, \tilde{c}_k is a fixed point of (f, φ) , completing the proof. \square

Theorem 3.2 Let (X, \tilde{d}, Ω) be a complete soft metric space, and let $(f, \varphi) : (X, \tilde{d}, \Omega) \rightarrow (X, \tilde{d}, \Omega)$ be a hybrid-interpolative Reich–Istrăţescu-type (soft- $\alpha - \mu$) contraction satisfying the following conditions:

- (i) (f, φ) is a soft α -orbital admissible,
- (ii) There exists an element $\tilde{x}_a^0 \in SP(X)$ such that $\alpha(\tilde{x}_a^0, (f, \varphi)(\tilde{x}_a^0)) \gtrsim \bar{1}$.
- (iii) $(f, \varphi)^2$ is continuous and $\alpha(\tilde{x}_a, (f, \varphi)(\tilde{x}_a)) \gtrsim \bar{1}$ for all $\tilde{x}_a \in \text{Fix}((f, \varphi)^2)$

Then (f, φ) admits at least one fixed point in X .

Proof: Let $\tilde{x}_a^0 \in SP(X)$ be arbitrary, and define the sequence $\{\tilde{x}_{a_n}^n\}$ in $SP(X)$ by $\tilde{x}_{a_n}^n = (f, \varphi)^n(\tilde{x}_{a_n}^0)$. From Theorem 3.1, it follows that this sequence converges to a point $\tilde{y}_b \in SP(X)$, i.e.,

$$\tilde{x}_{a_n}^n \rightarrow \tilde{y}_b \text{ as } n \rightarrow \infty.$$

Since $(f, \varphi)^2$ is continuous, we have

$$(f, \varphi)^2(\tilde{y}_b) = \lim_{n \rightarrow \infty} (f, \varphi)^2(\tilde{x}_{a_n}^n) = \tilde{y}_b \quad (3.4)$$

which implies that \tilde{y}_b is a fixed point of $(f, \varphi)^2$, i.e., $\tilde{y}_b \in \text{Fix}((f, \varphi)^2)$. Moreover, by the definition of composition,

$$(f, \varphi)^3(\tilde{y}_b) = (f, \varphi)(\tilde{y}_b) \quad (3.5)$$

To show that \tilde{y}_b is also a fixed point of (f, φ) , suppose to the contrary that $(f, \varphi)(\tilde{y}_b) \neq \tilde{y}_b$. Then, $(f, \varphi)(\tilde{y}_b)$ can not be a fixed point of $(f, \varphi)^2$, because

$$(f, \varphi)^3(\tilde{y}_b) = (f, \varphi)(\tilde{y}_b) \neq \tilde{y}_b = (f, \varphi)^2(\tilde{y}_b)$$

which contradicts the uniqueness of the fixed point under convergence. We now apply the contractive condition (3.1) at the point \tilde{y}_b , yielding:

$$\begin{aligned} \tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)) &\lesssim \alpha(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)) \tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)) \\ &= \alpha(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)) \tilde{d}((f, \varphi)^2(\tilde{y}_b), (f, \varphi)^3(\tilde{y}_b)) \\ &\lesssim \mu N(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)). \end{aligned} \quad (3.6)$$

Case 1: $q \gtrsim \bar{0}$. We compute:

$$\begin{aligned} N(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)) &= [\lambda_1 \tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b))^q + \lambda_2 \tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b))^q \\ &\quad + \lambda_3 \tilde{d}((f, \varphi)(\tilde{y}_b), (f, \varphi)^2(\tilde{y}_b))^q + \lambda_4 \tilde{d}((f, \varphi)(\tilde{y}_b), (f, \varphi)^2(\tilde{y}_b))^q \\ &\quad + \lambda_5 \tilde{d}((f, \varphi)(\tilde{y}_b), (f, \varphi)^2(\tilde{y}_b))^q + \delta \tilde{d}((f, \varphi)^2(\tilde{y}_b), (f, \varphi)^3(\tilde{y}_b))^q]^{1/q} \\ &= [\lambda_1 \tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b))^q + \lambda_2 \tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b))^q \\ &\quad + \lambda_3 \tilde{d}((f, \varphi)^2(\tilde{y}_b), (f, \varphi)(\tilde{y}_b))^q + \lambda_4 \tilde{d}((f, \varphi)^2(\tilde{y}_b), (f, \varphi)(\tilde{y}_b))^q \\ &\quad + \lambda_5 \tilde{d}((f, \varphi)^2(\tilde{y}_b), (f, \varphi)(\tilde{y}_b))^q + \delta \tilde{d}((f, \varphi)^2(\tilde{y}_b), (f, \varphi)(\tilde{y}_b))^q]^{1/q} \\ &= [\lambda_1 \tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b))^q + \lambda_2 \tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b))^q \\ &\quad + \lambda_3 \tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b))^q + \lambda_4 \tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b))^q \\ &\quad + \lambda_5 \tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b))^q + \delta \tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b))^q]^{1/q} \\ &= \left[\left(\sum_{i=1}^5 \lambda_i + \delta \right) \tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b))^q \right]^{1/q} \\ &\lesssim \tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)). \end{aligned}$$

Substituting into (3.6), we obtain:

$$\bar{0} \lesssim \tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)) \leq \mu \tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b))$$

which is a contradiction since $\bar{0} \lesssim \mu \lesssim \bar{1}$. Hence, it must be that $(f, \varphi)(\tilde{y}_b) = \tilde{y}_b$.

Case 2: $q = \bar{0}$. Then $N(\tilde{y}_b, (f, \varphi)(\tilde{y}_b))$ is given by:

$$\begin{aligned} N(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)) &= \left[\tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)) \right]^{\lambda_1} \left[\tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)) \right]^{\lambda_2} \\ &\quad \left[\tilde{d}\left((f, \varphi)(\tilde{y}_b), (f, \varphi)^2(\tilde{y}_b)\right) \right]^{\lambda_3} \left[\tilde{d}\left((f, \varphi)(\tilde{y}_b), (f, \varphi)^2(\tilde{y}_b)\right) \right]^{\lambda_4} \\ &\quad \left[\tilde{d}\left((f, \varphi)(\tilde{y}_b), (f, \varphi)^2(\tilde{y}_b)\right) \right]^{\lambda_5} \left[\tilde{d}\left((f, \varphi)^2(\tilde{y}_b), (f, \varphi)^3(\tilde{y}_b)\right) \right]^{\delta} \\ &= \left[\tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)) \right]^{\lambda_1} \left[\tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)) \right]^{\lambda_2} \\ &\quad \left[\tilde{d}\left((f, \varphi)^2(\tilde{y}_b), (f, \varphi)(\tilde{y}_b)\right) \right]^{\lambda_3} \left[\tilde{d}\left((f, \varphi)^2(\tilde{y}_b), (f, \varphi)(\tilde{y}_b)\right) \right]^{\lambda_4} \\ &\quad \left[\tilde{d}\left((f, \varphi)^2(\tilde{y}_b), (f, \varphi)(\tilde{y}_b)\right) \right]^{\lambda_5} \left[\tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)) \right]^{\delta} \\ &= \left[\tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)) \right]^{\lambda_1} \left[\tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)) \right]^{\lambda_2} \\ &\quad \left[\tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)) \right]^{\lambda_3} \left[\tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)) \right]^{\lambda_4} \\ &\quad \left[\tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)) \right]^{\lambda_5} \left[\tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)) \right]^{\delta} \\ &= \tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)) \end{aligned}$$

since all distances involved reduce to $\tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b))$. Thus, equation (3.6) becomes:

$$\bar{0} \lesssim \tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b)) \lesssim \mu \tilde{d}(\tilde{y}_b, (f, \varphi)(\tilde{y}_b))$$

again yielding a contradiction. Therefore, $(f, \varphi)(\tilde{y}_b) = \tilde{y}_b$. Consequently, in either case, \tilde{y}_b is a fixed point of (f, φ) . This completes the proof. \square

Theorem 3.3 *Suppose that all the hypotheses of Theorem 3.2 hold. Additionally, assume that $\alpha(\tilde{x}_a, \tilde{y}_b) \geq \bar{1}$ for any $\tilde{x}_a, \tilde{y}_b \in \text{Fix}((f, \varphi))$. Then the fixed point of (f, φ) is unique.*

Proof: Assume, for the sake of contradiction, that (f, φ) admits two distinct fixed points $\tilde{u}_c, \tilde{v}_d \in SP(X)$, and $\tilde{u}_c \neq \tilde{v}_d$. Invoking the contractive condition provided in Equation (3.1), and utilizing the additional hypothesis $\alpha(\tilde{x}_a, \tilde{y}_b) \geq \bar{1}$, we obtain

$$\begin{aligned} \tilde{d}(\tilde{u}_c, \tilde{v}_d) &\lesssim \alpha(\tilde{u}_c, \tilde{v}_d) \tilde{d}(\tilde{u}_c, \tilde{v}_d) \\ &= \alpha(\tilde{u}_c, \tilde{v}_d) \tilde{d}\left((f, \varphi)^2(\tilde{u}_c), (f, \varphi)^2(\tilde{v}_d)\right) \\ &\lesssim \mu N(\tilde{u}_c, \tilde{v}_d) \end{aligned} \tag{3.7}$$

Substituting these into the expression for $N(\tilde{u}_c, \tilde{v}_d)$, and using Case 1, we have:

$$\begin{aligned}
N(\tilde{u}_c, \tilde{v}_d) &= [\lambda_1 \tilde{d}(\tilde{u}_c, \tilde{v}_d)^q + \lambda_2 \tilde{d}(\tilde{u}_c, (f, \varphi)(\tilde{u}_c))^q \\
&\quad + \lambda_3 \tilde{d}(\tilde{v}_d, (f, \varphi)(\tilde{v}_d))^q + \lambda_4 \tilde{d}((f, \varphi)(\tilde{u}_c), (f, \varphi)(\tilde{v}_d))^q \\
&\quad + \lambda_5 \tilde{d}((f, \varphi)(\tilde{u}_c), (f, \varphi)^2(\tilde{u}_c))^q + \delta \tilde{d}((f, \varphi)(\tilde{v}_d), (f, \varphi)^2(\tilde{v}_d))^q]^{1/q} \\
&= [\lambda_1 \tilde{d}(\tilde{u}_c, \tilde{v}_d)^q + \lambda_2 \tilde{d}(\tilde{u}_c, \tilde{u}_c)^q + \lambda_3 \tilde{d}(\tilde{v}_d, \tilde{v}_d)^q + \lambda_4 \tilde{d}(\tilde{u}_c, \tilde{v}_d)^q \\
&\quad + \lambda_5 \tilde{d}(\tilde{u}_c, \tilde{u}_c)^q + \delta \tilde{d}(\tilde{v}_d, \tilde{v}_d)^q]^{1/q} \\
&= [(\lambda_1 + \lambda_4) \tilde{d}(\tilde{u}_c, \tilde{v}_d)^q]^{1/q} \\
&= (\lambda_1 + \lambda_4)^{1/q} \tilde{d}(\tilde{u}_c, \tilde{v}_d) \\
&\leq \tilde{d}(\tilde{u}_c, \tilde{v}_d)
\end{aligned}$$

Substituting this back into inequality (3.7), we obtain:

$$\bar{0} \lesssim \tilde{d}(\tilde{u}_c, \tilde{v}_d) \lesssim \mu \tilde{d}(\tilde{u}_c, \tilde{v}_d)$$

which leads to a contradiction. Therefore, our initial assumption must be false, and we conclude that $\tilde{u}_c = \tilde{v}_d$. Hence, the fixed point of (f, φ) is unique. \square

Theorem 3.4 Let (X, \tilde{d}, Ω) be a complete soft metric space, and let $(f, \varphi) : (X, \tilde{d}, \Omega) \rightarrow (X, \tilde{d}, \Omega)$ be a hybrid-interpolative Reich-Istrăţescu-type (soft- $\alpha - \mu$) contraction for $q = \bar{0}$ such that $\lambda_1, \lambda_2, \lambda_5 \succ \bar{0}$. Suppose that

- (i) (f, φ) is a soft α -orbital admissible,
- (ii) There exists an element $\tilde{x}_a^0 \in SP(X)$ such that $\alpha(\tilde{x}_a^0, (f, \varphi)(\tilde{x}_a^0)) \geq \bar{1}$.
- (iii) (X, \tilde{d}, Ω) is regular with respect to α .

Then (f, φ) admits a fixed point in X .

Proof: By Theorem 3.1, the sequence $\{\tilde{x}_{a_n}^n\}$ constructed by the iterative process converges in the soft metric space (X, \tilde{d}, Ω) . Since the space is complete, the sequence $\{\tilde{x}_{a_n}^n\}$ is soft Cauchy and thus converges to a point \tilde{u}_c in $SP(X)$, i.e., $\tilde{x}_{a_n}^n \rightarrow \tilde{u}_c$ as $n \rightarrow \infty$. We aim to show that \tilde{u}_c is a fixed point of (f, φ) . Suppose, for contradiction, that

$$(f, \varphi)(\tilde{u}_c) \neq \tilde{u}_c.$$

Assume that the sequence $\{\tilde{x}_{a_n}^n\}$ consists of distinct elements, i.e., $\tilde{x}_{a_n}^n \neq \tilde{x}_{a_m}^m$ whenever $n \neq m$, and that none of the points $\tilde{x}_{a_n}^n$ or \tilde{u}_c belongs to $Fix((f, \varphi))$ for $n \geq n_0$, where $n_0 \in \mathbb{N}$. We shall demonstrate that \tilde{u}_c is in fact a fixed point of $(f, \varphi)^2$. For $n \geq n_0$ by the contraction condition, we have:

$$\begin{aligned}
\tilde{d}(\tilde{x}_{a_{n+2}}^{n+2}, (f, \varphi)^2(\tilde{u}_c)) &\lesssim \alpha(\tilde{x}_{a_n}^n, \tilde{u}_c) \tilde{d}((f, \varphi)^2(\tilde{x}_{a_n}^n), (f, \varphi)^2(\tilde{u}_c)) \\
&\lesssim \mu N(\tilde{x}_{a_n}^n, \tilde{u}_c) \\
&= \mu [\tilde{d}(\tilde{x}_{a_n}^n, \tilde{u}_c)]^{\lambda_1} [\tilde{d}(\tilde{x}_{a_n}^n, (f, \varphi)(\tilde{x}_{a_n}^n))]^{\lambda_2} \\
&\quad [\tilde{d}(\tilde{u}_c, (f, \varphi)(\tilde{u}_c))]^{\lambda_3} [\tilde{d}((f, \varphi)(\tilde{x}_{a_n}^n), (f, \varphi)(\tilde{u}_c))]^{\lambda_4} \\
&\quad [\tilde{d}((f, \varphi)(\tilde{x}_{a_n}^n), (f, \varphi)^2(\tilde{x}_{a_n}^n))]^{\lambda_5} [\tilde{d}((f, \varphi)(\tilde{u}_c), (f, \varphi)^2(\tilde{u}_c))]^{\delta} \\
&= \mu [\tilde{d}(\tilde{x}_{a_n}^n, \tilde{u}_c)]^{\lambda_1} [\tilde{d}(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1})]^{\lambda_2} [\tilde{d}(\tilde{u}_c, (f, \varphi)(\tilde{u}_c))]^{\lambda_3} \\
&\quad [\tilde{d}(\tilde{x}_{a_{n+1}}^{n+1}, (f, \varphi)(\tilde{u}_c))]^{\lambda_4} [\tilde{d}(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2})]^{\lambda_5} [\tilde{d}((f, \varphi)(\tilde{u}_c), (f, \varphi)^2(\tilde{u}_c))]^{\delta}.
\end{aligned}$$

Since $\lambda_1, \lambda_2, \lambda_5 \gtrsim 0$ and the sequence $\{\tilde{x}_{a_n}^n\}$ converges to \tilde{u}_c , all terms tend to zero as $n \rightarrow \infty$. Consequently, we deduce that

$$\tilde{d}\left(\tilde{x}_{a_{n+2}}^{n+2}, (f, \varphi)^2(\tilde{u}_c)\right) \rightarrow \bar{0} \text{ as } n \rightarrow \infty,$$

which implies

$$(f, \varphi)^2(\tilde{u}_c) = \tilde{u}_c.$$

However, since (f, φ) is assumed to be α -orbital admissible and (X, \tilde{d}, Ω) is regular with respect to α , it follows by conditions (3.4) and (3.5), that

$$(f, \varphi)^2(\tilde{u}_c) = \tilde{u}_c \Rightarrow (f, \varphi)(\tilde{u}_c) = \tilde{u}_c$$

which contradicts our initial assumption. Thus, we conclude that $(f, \varphi)(\tilde{u}_c) = \tilde{u}_c$, and hence \tilde{u}_c is a fixed point of (f, φ) . \square

Corollary 3.2 *Let (X, \tilde{d}, Ω) be a complete soft metric space, and let $(f, \varphi) : (X, \tilde{d}, \Omega) \rightarrow (X, \tilde{d}, \Omega)$ be a continuous self-mapping. Suppose there exist constants $\gamma, \beta \in (\bar{0}, \bar{1})$ such that $\gamma + \beta \lesssim \bar{1}$ and*

$$\tilde{d}\left((f, \varphi)^2(\tilde{x}_a), (f, \varphi)^2(\tilde{y}_b)\right) \lesssim \gamma \tilde{d}(\tilde{x}_a, \tilde{y}_b) + \beta \tilde{d}((f, \varphi)(\tilde{x}_a), (f, \varphi)(\tilde{y}_b))$$

for all $\tilde{x}_a, \tilde{y}_b \in SP(X)$. Then, (f, φ) admits a unique fixed point.

Proof: Define $\mu = \gamma + \beta$, so that $\mu \in (\bar{0}, \bar{1})$. Consider Definition 3.3 and choose: $\alpha(\tilde{x}_a, \tilde{y}_b) = \bar{1}, q = \bar{1}$, $\lambda_1 = \frac{\gamma}{\mu}, \lambda_4 = \frac{\beta}{\mu}, \lambda_2 = \lambda_3 = \lambda_4 = \delta = \bar{0}$. Then for all $\tilde{x}_a, \tilde{y}_b \in SP(X)$, we have

$$\begin{aligned} \alpha(\tilde{x}_a, \tilde{y}_b) \tilde{d}\left((f, \varphi)^2(\tilde{x}_a), (f, \varphi)^2(\tilde{y}_b)\right) &\lesssim \mu N(\tilde{x}_a, \tilde{y}_b) \\ &\lesssim \mu \left[\lambda_1 \tilde{d}(\tilde{x}_a, \tilde{y}_b)^q + \lambda_4 \tilde{d}((f, \varphi)(\tilde{x}_a), (f, \varphi)(\tilde{y}_b))^q \right]^{1/q} \\ &= \mu \left[\frac{\gamma}{\mu} \tilde{d}(\tilde{x}_a, \tilde{y}_b) + \frac{\beta}{\mu} \tilde{d}((f, \varphi)(\tilde{x}_a), (f, \varphi)(\tilde{y}_b)) \right] \\ &= \gamma \tilde{d}(\tilde{x}_a, \tilde{y}_b) + \beta \tilde{d}((f, \varphi)(\tilde{x}_a), (f, \varphi)(\tilde{y}_b)) \end{aligned}$$

which verifies that inequality (3.1) holds for all $\tilde{x}_a, \tilde{y}_b \in X \setminus Fix((f, \varphi))$. Let $\tilde{x}_{a_0}^0 \in SP(X)$ be an arbitrary initial point, and define a sequence $\{\tilde{x}_{a_n}^n\}$ in $SP(X)$ recursively by:

$$\tilde{x}_{a_n}^n = (f, \varphi)^n(\tilde{x}_{a_0}^0).$$

By inequality (3.5), it follows that:

$$\begin{aligned} \tilde{d}\left(\tilde{x}_{a_{n+2}}^{n+2}, \tilde{x}_{a_{n+3}}^{n+3}\right) &\lesssim \gamma \tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right) + \beta \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right) \\ &\lesssim \gamma \max\left\{\tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right), \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right)\right\} \\ &\quad + \beta \max\left\{\tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right), \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right)\right\} \\ &= (\gamma + \beta) \max\left\{\tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right), \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right)\right\} \\ &= \mu \max\left\{\tilde{d}\left(\tilde{x}_{a_n}^n, \tilde{x}_{a_{n+1}}^{n+1}\right), \tilde{d}\left(\tilde{x}_{a_{n+1}}^{n+1}, \tilde{x}_{a_{n+2}}^{n+2}\right)\right\}. \end{aligned}$$

This recursive inequality satisfies the conditions of Lemma 3.1, implying that the sequence $\{\tilde{x}_{a_n}^n\}$ is soft Cauchy in the complete soft metric space (X, \tilde{d}, Ω) . Hence, there exists a point $\tilde{c}_k \in SP(X)$ such that $\tilde{x}_{a_n}^n \rightarrow \tilde{c}_k$ as $n \rightarrow \infty$. Since (f, φ) is continuous, then \tilde{c}_k is a fixed point of (f, φ) . \square

4. Conclusion

In this study, we have extended the framework of hybrid-interpolative Reich–Istrăţescu-type contractions to the setting of soft metric spaces. By integrating the concepts of soft sets and soft points, we rigorously formulated generalized contractive mappings and established several fixed point results, including existence and uniqueness theorems under various contractive conditions. The main contributions include the adaptation of interpolative-type inequalities to soft metric structures, the development of new analytical techniques specific to soft mappings, and the validation of these results through illustrative constructions.

Our findings demonstrate that hybrid-interpolative Reich–Istrăţescu-type contractions preserve fixed point properties within soft metric spaces, a class of structures well-suited for modeling uncertainty and parametric variability. These results not only generalize several classical fixed point theorems but also provide a solid theoretical foundation for further exploration in nonlinear analysis and applied mathematics.

This work contributes to the broader field of soft topology and soft analysis by offering novel tools for handling imprecise and vague data structures, with potential applications in optimization, control systems, decision theory, and computational modeling. Future investigations may focus on extending these results to other generalized soft structures such as soft cone metric spaces, soft partial metric spaces, and soft probabilistic metric spaces, or exploring their algorithmic implementations in complex decision-making environments.

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