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About M-Bounded Topological Groups

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ABSTRACT: Although the theory of uniform spaces is independent, it is closely related to the theory of topological groups and there is a profound analogy between them. Therefore, research into the theory of topological groups using uniform structures is relevant. In this paper some important properties of the M-bounded topological group using uniform structures are investigated. In particular, it is established that under precompact continuous homomorphism, the M-bounded property is preserved both in the image and the preimage direction.

Keywords: Topological group, uniform space, cover, M-boundedness, ω-boundedness, totally boundedness, homomorphism, uniformly continuous mapping.

Contents

1. Introduction

For covers α and β of a set X , we have: $\alpha \wedge \beta = \{A \cap B : A \in \alpha, B \in \beta$. Let M subset of X. Then $\alpha(M) = \cup St(\alpha, M)$, $St(\alpha, M) = \{A \in \alpha : A \cap M \neq \emptyset$. Let $x \in X$ be an arbitrary point. Then $\alpha(x) = \cup St(\alpha, x)$, $St(\alpha, x) = \{A \in \alpha : A \ni x\}$.

The symbol $\alpha \succ \beta$ means that the cover α is a refinement of the cover β, i.e., for any $A \in \alpha$ there exists $B \in \beta$ such that $A \subset B$. The symbol $\alpha \rhd \beta$ means that the cover α is a star refinement of the cover β, i.e., for any $x \in X$ there exists $B \in \beta$ such that $\alpha(x) \subset B$ and, the symbol $\alpha * \succ \beta$ means that the cover α is a strongly star refinement of the cover β, i.e., for any $A \in \alpha$ there exists $B \in \beta$ such that $\alpha(A) \subset B$.

A uniformity on a set $X \neq \emptyset$ is a family U of covers of X which satisfies the following axioms:

(U1) If $\alpha \in U$ and $\alpha \succ \beta$, then $\beta \in U$;

(U2) If $\alpha, \beta \in U$, then there exists $\gamma \in U$ such that $\gamma \succ \alpha$ and $\gamma \succ \beta$;

(U3) If $\alpha \in U$, then there exists $\beta \in U$ such that $\beta^* \succ \alpha$;

(U4) For any two points $x, y \in X$, $x \neq y$ there is an $\alpha \in U$ such that no member of α contains both x and y, [1], [4], [6], [7].

The pair (X, U) is called a uniform space.

Recall that a uniform space (X, U) is called:

(1) precompact if the uniformity U has a base consisting of finite covers [1];

(2) totally bounded, if each $\alpha \in U$ has a finite set $M \subset X$ such that $\alpha(M) = X$ [1], [7], [22];

(3) ω -bounded, if the uniformity U has a base consisting of countable cover [1], [7];

(4) has the uniform Menger property, if for each sequence $(\alpha_n | n \in N) \subset U$ there is a sequence $(\beta_n | n \in N)$ such that for each $n \in N$, β_n is a finite subset of α_n and $\underset{n \in N}{\cup} \beta_n$ is a cover of X [7], [21].

A mapping $f : (X, U) \to (Y, V)$ of the uniform space (X, U) to the uniform space (Y, V) is called a uniformly continuous mapping, if every $\beta \in U$ there is $\alpha \in U$ such that $f\alpha \succ \beta$ [7], [12].

The mapping f is called precompact, if for each $\alpha \in U$ there exist a uniform cover $\beta \in U$ and a finite uniform cove $\gamma \in U$ such that $f^{-1}\beta \wedge \gamma \succ \alpha$ [22].

A topological group G is said to be:

(a) totally bounded if for each neighborhood D of neutral element e there exists a finite set $A \subset G$ such that $A \cdot D = G$, [1], [12];

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(b) ω-bounded if for each neighborhood D of neutral element e there exists a countable set $A \subset G$ such that $A \cdot D = G$, [1], [12];

(c) M-bounded, if each sequence $(U_n | n \in N)$ of neighborhoods of neutral element e in G there exists a sequence $(A_n | n \in N)$ of finite subsets of G such that $\underset{n\in N}{\cup} A_n \cdot U_n = G$, [9];

(d) σ-totally bounded, if it can be represented as the union of countable many totally bounded subgroups, [1], [12].

A filter F in a uniform space (X, U) is said to be a Cauchy filter, if $\alpha \cap \neq \varnothing$ for all $\alpha \in U$, [15].

A point $x \in X$ is said to be an adherent point of a filter F in (X, U), if $x \in \cap\{[M] : M \in F\}$ or equivalently, $x \in X$ is an adherent point of each element $M \in F$ in (X, τ_U) [15].

A point $x \in X$ is said to be a limit of a filter F , if a filter F is finer than the filter F_x of neighborhoods of the point $x \in X$, [1], [15].

Let $\widetilde{X}$ be the set of all minimal Cauchy filters in the uniform space (X, U). For each uniform cover $\alpha \in U$ we denote $\widetilde{\alpha} = \{\widetilde{A} : A \in \alpha\}$ where $\widetilde{A} = \{F \in \widetilde{X} : A \in F\}$. The family of covers $\widetilde{B} = \{\widetilde{\alpha} : \alpha \in U\}$ forms a base of some uniformity $\widetilde{U}$ on $\widetilde{X}$,[12], [15], [22].

For any uniform space (X, U) there exists a unique (with respect to uniform isomorphism) complete uniform space $(\widetilde{X}, \widetilde{U})$ such that for some everywhere-dense subset $\widetilde{Y} \subset \widetilde{X}$ the uniform space (X, U) is uniformly isomorphic to the space $(\widetilde{X}, \widetilde{U}_{\widetilde{Y}})$. And $w(\widetilde{U}_{\widetilde{Y}}) = w(\widetilde{U})$ [1].

The uniform space $(\widetilde{X}, \widetilde{U})$ is called a completion of the uniform space (X, U) [1].

Many terminology and known results are used by the authors from the works [1]-[36].

2. About M-Bounded Topological Groups

Let G be a topological group.

Proposition 1. A topological group G is M-bounded, if and only if the uniform space (G, U_l) is uniformly Menger, where U_l is the left uniformity on G.

Proof: Let G be a M-bounded topological group and $(\alpha^l_{H_n} | n \in N) \subset U$ be an arbitrary sequence of uniform covers, $\alpha^l_{H_n} = \{x \cdot H_n : \ x \in G\}$, $H_n \in B(e)$, $B(e)$ is the base of neighborhoods of the neutral element e. Since G is M-bounded, then for a countable system $(H_n | n \in N)$ of neighborhoods of a neutral element $e \in G$ there exists a sequence $(A_n | \ n \in N)$ of finite subsets G such that $G = \cup A_n \cdot H_n$. Let $\{x \cdot H_n : \ x \in A_n\} = \beta^l_{H_n}$. Since A_n is finite for any $n \in N$, then $\beta^l_{H_n}$ is a finite subsystem of the cover $\alpha^l_{H_n}$. It is easy to see that $\underset{n\in N}{\cup} \beta^l_{H_n} = G$. Thus, (G, U_l) is uniformly Menger space.

Conversely, let (G, U_l) be a uniformly Menger space. We will show that topological group G is a M-bounded. Let $(H_n | n \in N)$ be an arbitrary sequence of neighborhoods of the neutral element e. Put $\alpha^l_{H_n} = \{x \cdot H_n : \ x \in A_n\}$, $H_n \in B(e)$. It is clear that $\alpha^l_{H_n} \in U_l$. Then $\{\alpha^l_{H_n} : \ n \in N\} \subset U$ is a sequence of uniform covers. Since (G, U_l) is a uniformly Menger space, there exists a sequence $(\gamma^l_{H_n} | n \in N)$ of finite subfamilies such that $\gamma^l_{H_n} \subset \alpha^l_{H_n}$, $\underset{n\in N}{\cup} \gamma^l_{H_n} = G$, $\gamma^l_{H_n} = \{x_1 \cdot H_1, ..., x_n \cdot H_n\}$. For each $i \leq n$ from $x_i \cdot H_i$, we select one element y_i and put $A_{\gamma^l_{H_n}} = \{y_i : i = 1, 2, ..., n\}$. It is easy to see that $\underset{n\in N}{\cup} A_{\gamma^l_{H_n}} \cdot H_n = G$. Hence, G is a M-bounded.

Proposition 2. The topological group G is ω-bounded if and only if the uniform space (G, U_l) is a ω-bounded, where U_l is the left uniformity on G.

Proof: Let G be a ω-bounded topological group. We will show the uniform space (G, U_l) is ω-bounded. Let $\alpha^l_H \in U_l$ be an arbitrary uniform cover. Choose a neighborhood $M \in B(e)$ of the neutral element e such that $M \subset H$. Since G is ω-bounded, there exists a countable set $A \subset G$ such that $A \cdot M = G$. Put $\alpha^l_M = \{x_i \cdot M : x_i \in A\}$. Since $A \cdot M = G$, then $\cup \alpha^l_M = G$. It is easy to see that $\alpha^l_M \succ \alpha^l_H$ and α^l_M-countable uniform cover. Hence, (G, U_l) is ω-bounded uniform space.

Conversely, let be $H \in B(e)$ an arbitrary neighborhood of the neutral element e. Then there exists $M, L \in B(e)$, such that $M \cdot M^{-1} \cdot M \subset L$ and $L \cdot L^{-1} \cdot L \subset H$. Since (G, U_l) is ω-bounded uniform space, then a uniform cover α^l_M contains a countable subcover $\hat{\alpha}^l_M = \{x_i \cdot M : i \in N\}$. For each $x_i \cdot M \in \hat{\alpha}^l_M$ choose one element $x_{x_i \cdot M} \in x_i \cdot M$. Denote $S = \{x_{x_i \cdot M} : i \in N\}$ is ω-bounded uniform space, then a uniform cover α^l_M contains a countable subcover $\hat{\alpha}^l_M = \{x_i \cdot M : i \in N\}$. Take an arbitrary element $x_i \cdot M \in \hat{\alpha}^l_M$ and $y \in x_i \cdot M$. Then there is $x_{x_i \cdot M} \in S$ such that $y \in \alpha^l_M(x_i \cdot M)$. There is $x \cdot L \in \alpha^l_L$ such that $x_i \cdot M \in \alpha^l_M(y) \subset \alpha^l_M(x_i \cdot M) \subset x \cdot L$. It follows from this, that $\alpha^l_M(x_i \cdot M) \subset a^l_L(x_i \cdot M)$. For $x_{x_i \cdot M}$ choose one $x_i \cdot H \in \alpha^l_H$ such that $\alpha^l_L(x_{x_i \cdot M}) \subset (x \cdot H)_{x_{x_i \cdot M}}$.

Put $\hat{\alpha}^l_H = \{(x \cdot H)_{x_{x_i \cdot M}} : i \in N\}$. Then $\alpha^l_M * \succ \hat{\alpha}^l_H$ i.e. $\hat{\alpha}^l_H \in U_l$. Hence, (G, U_l) is a ω-bounded.

Proposition 3. Any precompact topological group G is a M-bounded.

Proof: Let G be a precompact topological group. Then (G, U_l) is precompact. Take an arbitrary sequence $(\alpha^l_{U_n} | n \in N) \subset U_i$.Then, for any $n \in N$ the cover $\alpha^l_{U_n}$ contains a finite subcover $\hat{\alpha}^l_{U_n} \subset \alpha^l_{U_n}$, where U_n is a neighborhood of neutral element e in G. It is easy to see that $\bigcup_{n \in N} \hat{\alpha}^l_{U_n} = G$. Hence, G is M-bounded.

Proposition 4. Any M-bounded topological group G is ω-bounded.

Proof: Let $U \in B(e)$ be an arbitrary neighborhood of neutral element e in G.

Put $U_n = U$ for any $n \in N$.Then, for the sequence $(\alpha^l_{U_n} | n \in N) \subset U_l$, where $U_n = U$, $n \in N$, there is a sequence $(\beta^l_{U_n} | n \in N)$ of finite subfamilies such that for any $n \in N$, $\beta^l_{U_n}$ is a subfamily of the cover $\alpha^l_{U_n}$ and $\bigcup_{n \in N} \beta^l_{U_n} = G$. For each $B_{\beta^l_{U_n}(m)} \in \beta_n$, $m = 1, 2, ..., k$, $n \in N$, by selecting one element $A_{B_{\beta^l_{U_n}(m)}}$ from α_n, we obtain $\hat{\alpha}^l_{U_n} \subset \alpha^l_{U_n}$ and $\hat{\alpha}^l_{U_n}$ is a finite. Then $\bigcup_{n \in N} \hat{\alpha}^l_{U_n}$ is a countable subfamily of the cover $\alpha = \alpha_n$. Since $\bigcup_{n \in N} \beta_{U_n} = G$, then $\bigcup_{n \in N} \hat{\alpha}^l_{U_n} = G$. Hence, (G, U_l) is ω-bounded i.e. topological group G is ω-bounded.

Corollary 1. Any compact topological group G is M-bounded.

Proposition 5. Any σ-precompact topological group is M-bounded.

Proof: Let $(U_n | n \in N)$ be an arbitrary sequence of neighborhoods of neutral element e and $G = \bigcup_{n \in N} G_n$, where G_n is precompact. For each $n \in N$, the cover $\alpha^l_{U_n} \wedge \{G_n\} = \alpha^l_{U_n^{G_n}}$ of G_n contains a finite subcover $\hat{\alpha}^l_{U_n^{G_n}}$. Put $\beta^l_{U_n^{G_n}} = \hat{\alpha}^l_{U_n^{G_n}}$.Then, for each $n \in N$, $\beta^l_{U_n^{G_n}}$ is a finite subset of $\alpha^l_{U_n^{G_n}}$ and $\bigcup_{n \in N} \beta^l_{U_n^{G_n}} = G$. Thus, (G, U_l) is a uniformly Menger space i.e. G is M-bounded.

Theorem 1. If a topological group G' is a continuous homomorphism image of a M-bounded topological group G, then G' is also M-bounded.

Proof: Let $f : G \to G'$ be a continuous homomorphism of the M-bounded topological group G to the topological group G'. Since every continuous homomorphism is uniformly continuous, then homomorphism $f : (G, U_l) \to (G', V_l)$ is uniformly continuous. Let $(\gamma^l_{V_n} | n \in N)$ be an arbitrary sequence of V_l, where V_n is neighborhoods of neutral element e' in G'. Put $f^{-1}\gamma^l_{V_n} = \alpha_{f^{-1}V_n}$, $n \in N$. It is easy to see that of V_n is a neighborhood of neutral element e' in G' then $f^{-1}V_n$ is a neighborhood of neutral element e in G. Then there exists a sequence of finite subfamilies $(\hat{\alpha}^l_{f^{-1}V_n} | n \in N)$ such that for any $n \in N$ $\hat{\alpha}_{f^{-1}V_n} \subset \alpha_{f^{-1}V_n}$ and $\bigcup_{n \in N} \hat{\alpha}^l_{f^{-1}V_n} = G$. Denote $f\hat{\alpha}^l_{f^{-1}V_n} = \hat{\beta}^l_{V_n}$, $n \in N$. This implies $f(\bigcup_{n \in N} \hat{\alpha}^l_{f^{-1}V_n}) = \bigcup_{n \in N} \hat{\beta}^l_{V_n} = G'$, $n \in N$. Hence, (G', V_l) is uniformly Menger. Therefore, G' is M-bounded.

Continuous homomorphism $f : G \to G'$ of a topological group G onto a topological group G' said to be precompact, if the mapping $f : (G, U_l) \to (G', V_l)$ of a uniform space (G, U_l) to uniform space (G', V_l) is precompact.

Theorem 2. Let $f : G \to G'$ be a precompact homomorphism. If topological group G' is M-bounded, then a topological group G is also M-bounded.

Proof: Let $f : G \to G'$ be a precompact homomorphism of a topological group G onto a topological group G'. Take an arbitrary sequence $(\alpha^l_{U_n} | n \in N) \subset U_l$ of covers. Since f is a precompact, then there exists $\gamma^l_{U_n} \in U$ and $\beta^l_{V_n} \in V_l$, such that $f^{-1}\beta^l_{V_n} \wedge \gamma^l_{U_n} \succ \alpha^l_{U_n}$. The space (G', V_l) has a uniformly Menger property, then there exists a sequence $(\hat{\beta}^l_{V_n} | n \in N)$ of finite subfamilies such that $\bigcup_{n \in N} \hat{\beta}^l_{V_n} = G'$. For each $n \in N$ the family $f^{-1}\beta^l_{V_n} \wedge \gamma^l_{U_n}$ is finite and $\cup\{f^{-1}\beta^l_{V_n} \wedge \gamma^l_{U_n}\} = \cup f^{-1}\beta^l_{V_n}$. For any $f^{-1}(\{V_{n,m} \cdot y : y \in G'\}) \cap \{U_{n,m} \cdot x : x \in G\} \in f^{-1}\hat{\beta}^l_{V_n} \wedge \gamma^l_{U^l_n}$ choose $\{x \cdot U_{n,m} : x \in G\}$ such that $f^{-1}(\{y \cdot V_{n,m} : y \in G') \cap \{x \cdot U_{n,m} : x \in G\} \subset \{x \cdot U_{n,m} : x \in G\}$. Put $\hat{\alpha}^l_{U_n} = \{x \cdot U_{n,1}, x \cdot U_{n,2}, ..., x \cdot U_{n,k}\}$, where k is the cardinality of $f^{-1}\hat{\beta}^l_{V_n} \wedge \gamma^l_{U_n}$. It is easy to see that $\bigcup_{n \in N} \hat{\alpha}^l_{U_n} = G$. Thus, G is a M-bounded.

Proposition 6. The additive group of real numbers $(R, +, \tau)$ is a M-bounded.

Proof: Let $(U_n | n \in N)$ be an arbitrary sequence of neighborhoods of neutral element e in G. Then

$(\alpha^l_{U_n}|n \in N) \subset U_l$ is a sequence of covers. Let $B(0) = \{(-n, n) : n \in N\}$ be a base of 0-neighborhoods. Consider the following construction. For $n = 1$, from the cover α_1 select a finite subsystem $\hat{\alpha}^l_{U_1} \subset \alpha_{U_1}$ such that $(-1, 1) \subset [-1, 1] \subset \cup\, \hat{\alpha}^l_{U_1}$, for $n = 2$, from the cover α_2 select a finite subsystem $\hat{\alpha}^l_{U_2} \subset \alpha_{U_2}$ such that $(-2, 2) \subset [-2, 2] \subset \cup\, \hat{\alpha}^l_{U_2}$, for $m \leq n$, from the cover α_m select a finite subsystem $\hat{\alpha}^l_{U_m} \subset \alpha_{U_m}$ such that $(-m, m) \subset [-m, m] \subset \cup \hat{\alpha}^l_{U_m}$, etc. Continuing process, we have a sequence $(\hat{\alpha}^l_{U_n}|n \in N)$ of finite subsystems. Since $\beta^l_{U_n}$, $U_n \in B(0)$ is a cover of the group $(R, +, \tau)$, the system $\underset{n \in N}{\cup} \hat{\alpha}^l_{U_n}$ is also a cover of the group G. Hence, G is M-bounded.

Theorem 3. A locally compact topological group G is M-bounded if and only if it is a ω-bounded.

Proof: Necessity. Let G be an M-bounded topological group. Then from the Proposition 2, G is a ω-bounded.

Sufficiency. Let G be a locally compact and ω-bounded topological group. Let $(\alpha^l_{U_n}|n \in N) \subset U_l$ be an arbitrary sequence of covers. Then there exists a countable uniform cover $\beta^l_W = \{x_n \cdot W : n \in N\}$, where W is a compact 0-neighborhood, consisting of compact subsets. For each $n \in N$, due to the compactness of $x_n \cdot W$, there exists a finite subsystem $\hat{\alpha}^l_{U_n} \subset \alpha_{U_n}$ such that $x_n \cdot W \subset \hat{\alpha}^l_{U_n}$.

Since $\cup \beta^l_W = G$, then $\underset{n \in N}{\cup} \hat{\alpha}^l_{U_n} = G$. Consequently, G is M-bounded.

Theorem 4. The completion of an M-bounded topological group is M-bounded.

Proof: Let G be a M-bounded topological group, then (G, U_t) is uniformly Menger space. Let $(\tilde{G}, \tilde{U}_t)$ be the completion of (G, U_t).Take an arbitrary sequence $(\tilde{\alpha}^l_{U_n}|n \in N) \subset \tilde{U}_t$ of covers. Put $\alpha^l_{U_n} = \tilde{\alpha}^l_{U_n} \wedge \{G\}$, $\tilde{\alpha}^l_{U_n} = \{\tilde{x} \cdot \tilde{U}_n : x \in \tilde{G}\}$, $\tilde{U}_n = \{F : U_n \in F\}$, where F is a Cauchy filter in $(\tilde{G}, \tilde{U}_t)$. From the definition of completion of uniform spaces we have $(\alpha^l_{U_n}|n \in N) \subset U_t$. Then there exists a sequence $(\beta^l_{U_n}|n \in N)$ of finite subsystems, such that for each $n \in N$, $\beta^l_{U_n} \subset \alpha^l_{U_n}$ and $\underset{n \in N}{\cup} \beta^l_{U_n} = G$. Then, there is a sequence $(\tilde{\beta}^l_{U_n}|n \in N)$ of finite subsystems, such that for each $n \in N$, $\tilde{\beta}^l_{U_n} \subset \tilde{\alpha}^l_{U_n}$ and $\tilde{\beta}^l_{U_n} \wedge \{G\} = \beta^l_{U_n}$ for any $n \in N$. It is easy to see that $\underset{n \in N}{\cup} \tilde{\beta}^l_{U_n} = \tilde{G}$, $\tilde{\beta}^l_{U_n} = \{\tilde{x} \cdot \tilde{U}_n : \tilde{x} \in \tilde{G}\}$, $\tilde{U}_n = \{F : U_n \in F\}$. $\tilde{x}$ is minimal Cauchy filter and $f^{-1}(\tilde{x}) = F_x$, F_x is the filter neighborhood of the point x, $f : G \to \tilde{G}$ - homomorphism. Hence, G is M-bounded.

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