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Tracing Chāyā and Bimba: Mathematical Harmony of Indian Eclipse Computations and Modern Astronomy

Vanaja V., Shailaja M., Sangayya G.

ABSTRACT: Eclipses are the natural phenomena occurring periodically. In modern astronomy i.e., after Kepler and Newton, an eclipse is a syzygy that is a straight-line configuration of three heavenly bodies in a gravitational system. In this phenomenon the participating three heavenly bodies are the Sun, the Earth and the Moon. By definition, the times of New Moon, First Quarter, Full Moon, and the Last Quarter are excess of the apparent geocentric longitude of the Moon over the apparent geocentric longitude of the Sun is 0^0, 90^0, 180^0, and 270^0 respectively. To calculate timings of the lunar eclipse, here we used the algorithms based on the phases of the Moon. In Indian classical astronomy texts, the computation of lunar eclipse is given based on the positions of the Sun and the Moon and with their true daily motions. In modern astronomy the gravitational parameters are also involved for calculations. The algorithms of both Indian and modern methods are discussed using mathematical software Scilab and compared these results with NASA data. In this paper we discussed Improved Siddhantic Procedure of Indian method and modern procedure using number of periodic terms and the number of lunations from the epoch 2000 C.E which are comparable to those of modern data.

Keywords: Large eclipse, ISP, JDE, the sun, the moon, SDM, MDM, SDIA, MDIA.

Contents

1. Introduction

The lunar eclipse occurs on a full-moon day. In this particular day the Sun and the Moon are on the opposite sides of the earth. The light rays of the Sun falls on the earth facing the Sun, and a shadow will be cast on the other side. When the Moon enters the shadow of the earth, a lunar eclipse occurs. This happens when the Sun and the Moon are in opposition i.e., their longitudinal difference is 180^0. However, a lunar eclipse does not occur on every full-moon day, because the plane of the Moon's orbit is inclined at about $5^0 8'$ to the ecliptic (the apparent path traced by the Sun). Generally, on a full-moon day, the Moon will be either far above or far below the plane of the ecliptic and so does not pass through the shadow of the earth. But, on that full-moon day, when the Moon does pass through the earth's shadow, a lunar eclipse occurs.

A necessary condition for a lunar eclipse to occur, the Moon must come close to the ecliptic that means the Moon must be close to one of the nodes. In Figure 1, roughly speaking, the orbit of the Moon intersects with the ecliptic at two points N and N'. These two points are referred to as the ascending and the descending nodes of the Moon. They are called Rāhu and Ketu in Indian astronomy.

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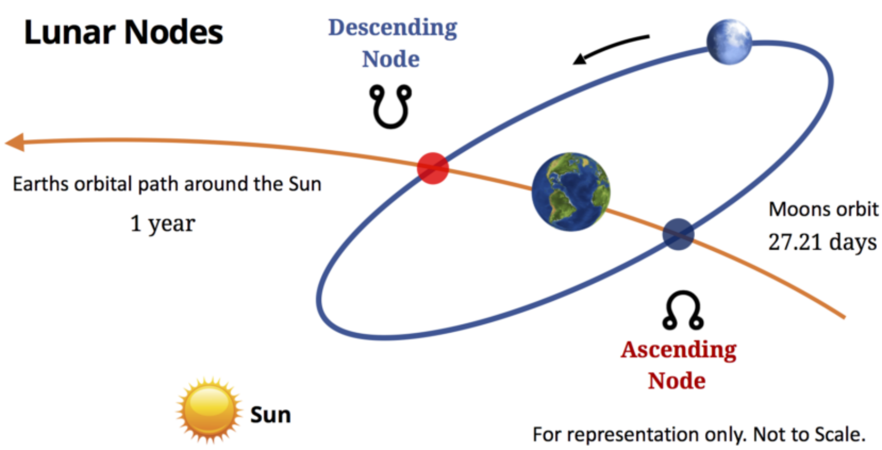

Figure 1: Nodes of the Moon.

2. Geometrical Interpretation of Lunar Eclipse

In Figure 2, S and E represent the centres of the Sun and the Earth, respectively. Draw a pair of direct tangents AB and CD to the surface of the Sun, the earth, meeting SE in V. If these lines are imagined to revolve round SE as axis, they will generate a cone. There is, thus, a conical shadow BVD, with V as its vertex, across which no direct ray from the Sun can fall. This conical shadow is called umbra.

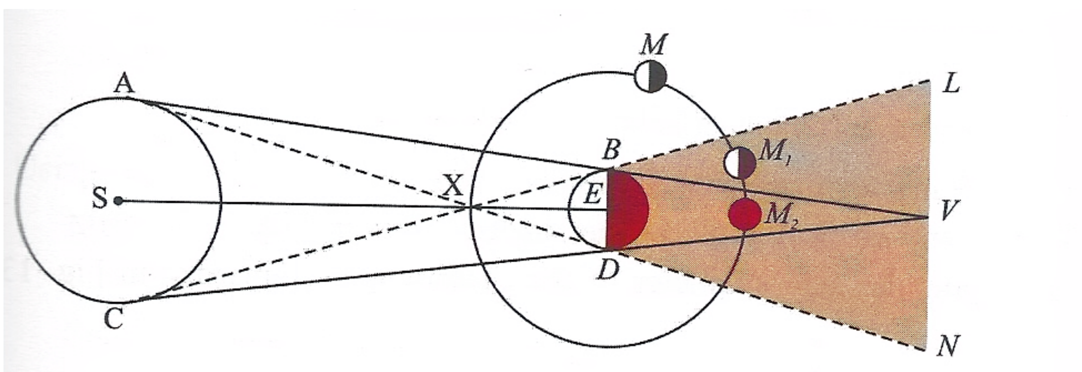

Figure 2: Earth's shadow cone and the lunar eclipse.

The spaces around the umbra, represented by VBL and VDN, form what is called penumbra, from which only a part of the Sun's light is excluded. It is to be noted that the passage of the Moon through the penumbra does not prompt an eclipse. It results only in diminution of the Moon's brightness. In Indian classical astronomy a penumbral eclipse is not considered as an eclipse. In Fig 2, the Moon is at M1 it receives light from portions of the Sun next to A, but rays from the parts near C will not reach the Moon at M1. Therefore, the brightness is diminished, the diminution growing greater as the Moon approaches the edge of the umbra. An umbral eclipse is considered as just commencing when the Moon enters the umbra or the shadow-cone. M2 represents the Moon which is completely immersed in the shadow-cone or umbra.

The lunar eclipse is said to be total when the whole of the Moon passes through the shadow. The eclipse is partial when only a part of the Moon enters the shadow.

3. Half-Durations of Eclipse and of Totality

The next important step is to determine the instants of the beginning and the end of a lunar eclipse as also of the totality. For this, we need to find the durations of the first half and the second half of the total duration of the eclipse. The configuration is shown in Figure 3.

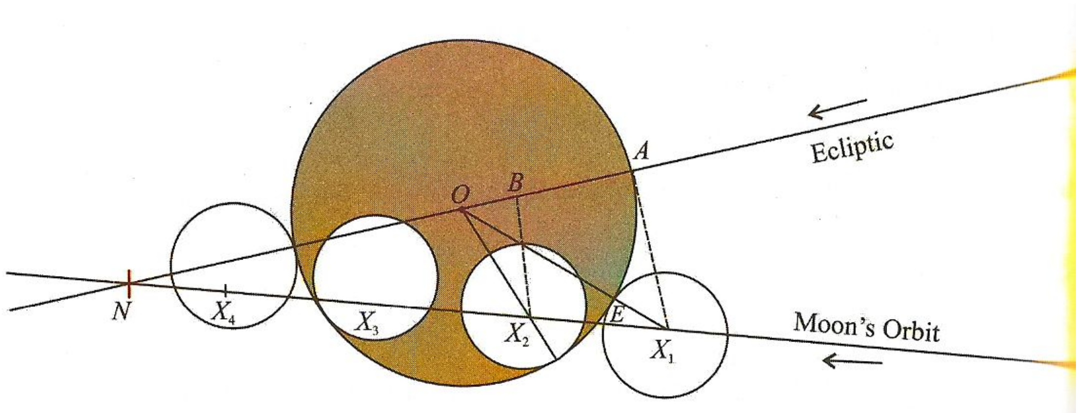

Figure 3: Half-duration of lunar eclipse.

A half-duration (Sthiti) is the time taken by the Moon, relative to the Sun, so that the point A in Fig.3 moves through OA. We have

$$OA^2 = OX_1^2 - AX_1^2 = (OE + EX_1)^2 - AX_1^2 = (d_1 + d_2)^2 - \beta^2 \tag{3.1}$$

where

$$\begin{aligned} OE = d_1 &= \text{semi-diameter of the shadow,} \\ EX_1 = d_2 &= \text{semi-diameter of the Moon,} \\ \beta = AX_1 &= \text{latitude of the Moon } (\textit{vikṣepa} \text{ or } \textit{śara}). \end{aligned}$$

When the Moon's centre is at X_1,

$$\text{Half-duration} = \frac{\sqrt{(d_1 + d_2)^2 - \beta^2}}{(\text{Moon's daily motion} - \text{Sun's daily motion})} \tag{3.2}$$

Since the actual instant of the beginning (*sparśa*) of the eclipse, and hence the Moon's latitude then, are not known, *śara* in the above formula is used iteratively.

By a similar analysis, the half-duration of maximum obscuration (totality or annularity as the case may be) is given by

$$\text{Half-duration of maximum obscuration} = \frac{\sqrt{(d_1 - d_2)^2 - \beta^2}}{(\text{Moon's daily motion - Sun's daily motion})} \tag{3.3}$$

4. Improved Siddhāntic Procedure (ISP)

Indian Classical Astronomical texts such as Āryabhaṭīyam, Sūryasiddhānta, Karṇakutūhalam, Grahalāghavam etc., and some of the Kerala astronomers also discussed in detail about the computation of eclipses in that particular period.

In this section we applied the algorithm of Improved Siddhāntic Procedure (ISP), based on Indian classical siddhāntic texts with updated parameters. This procedure is inspired by the works of the great savant in the field of Indian astronomy, the late Prof.T.S.Kuppaṇṇa Śāstrī. The Ecliptic system is considered to calculate the timings of eclipses, i.e., the entire computation of lunar eclipse is based on the longitudes of the Sun and the Moon in terms of degree, minute and second. Here we have considered one example to explain the computations of lunar eclipse according to ISP with explanatory notes.

Example: Lunar eclipse on 4^{th} April 2015, Saturday.
Instant of opposition is $17^{\text{h}}\,36^{\text{m}}$ (IST).
At the instant of opposition
True Sun (S): $350°20'$; True Moon (M): $170°20'$; *Rāhu* (R): $165°56'$
Sun's daily motion, SDM: $59'08''$
Moon's daily motion, MDM: $714'54''$.

(i) Moon's latitude (*Candra śara*) $= \beta$:

$$\beta = 308° \times \sin(M - R) \tag{4.1}$$

$$= 0°23'37''.$$

(ii) Moon's angular diameter (*Candra bimba*):

$$MDIA = 2\left[\frac{939.6 + (61.1)\cos GM}{60}\right] \quad \text{in minutes of arc} \tag{4.2}$$

where GM is the Moon's anomaly (*mandakendra*) measured from its perigee and it is given by

$$GM = 134°.9633964 + 13°.06499295\,T + \cdots$$

where T is the number of days completed since the epoch 2000 January 1, noon (GMT), i.e., $17^{\text{h}}30^{\text{m}}$ (IST). Julian days (JD) for this particular date i.e., 4^{th} April 2015 is 2457117.

$$JD \text{ for } 6^{\text{m}} = \frac{(17^{\text{h}}30^{\text{m}} - 17^{\text{h}}36^{\text{m}})}{24^{\text{h}}} = \frac{6^{\text{m}}}{24^{\text{h}}} = 0.00416 \text{ days} \tag{4.3}$$

Therefore, the days from the epoch (2000 Jan 1, noon GM) is

$$T = JD \text{ for } 4^{\text{th}} \text{ April 2015} - JD \text{ for } 1^{\text{st}} \text{ Jan 2000}$$

$$T = 2457117.00416 - 2451545 = 5572.00416$$

Using the value of T in GM, it obtains the value

$$GM = 134°.9633964 + 13°.06499295\,T + \cdots$$

$$= 213°.051858$$

$\therefore MDIA = 29'.614662$.

$GS =$ The Sun's mean anomaly from its perigee

$$= 357°.529092 + 0°.985600231\,T$$

$$= 89°.297679$$

(iii) Diameters of the earth's shadow (*chāyā bimba*):

$$SHDIA = 2\left[\frac{2545.4 + 228.9\cos GM - (16.4)\cos GS}{60}\right] \quad \text{in minutes of arc} \tag{4.4}$$

$SHDIA = 78'.451230$.

(iv) True daily motions of the Sun and the Moon:

Sun's daily motion, SDM: $59'08''$; Moon's daily motion, MDM: $714'54''$.

Vyarkendu sphuṭa nāḍi gati, $VRKSN = (MDM - SDM)$ per nāḍi.

$$VRKSN = \frac{(MDM - SDM)}{60} = 10'.929444$$

Note: One day = 60 *nāḍīs*; 1 *nāḍī* = 60 *vināḍīs* = 24 minutes.

(v) *Bimba yogārdham* = D:

$$D = \frac{(MDIA + SHDIA)}{2} = 54'.032946$$

(vi) *Bimba viyogārdham* = D':

$$D' = \frac{(SHDIA - MDIA)}{2} = 24'.418284$$

(vii) *Sphuṭa śara* = β':

$$\beta' = \beta \times \left(1 - \frac{1}{205}\right) = \frac{23'.61666 \times 204}{205} = 23'.514195$$

where β is the Moon's latitude from step (i) above.

(viii) $MDOT = \mathring{m}$:

$$\mathring{m} = VRKSN \times \left(1 + \frac{1}{205}\right) = 10'.9327 \times \frac{206}{205} = 10'.982759$$

(ix) If $|\beta'| < D$, then lunar eclipse occurs. If $|\beta'| < D'$, then the eclipse is total.

In this case $|\beta'| < D'$, i.e., 23'.5141 ¡ 24'.4182. Hence eclipse is total.

(x) *ViRāhu Candra*=$VRCH$

$$VRCH = (\text{True Moon} - \text{True Rāhu}) = 4°24'$$

(xi) Calculate *Correction*

$$COR = \frac{|\beta'| \times 59}{10 \times m} \quad \textit{vināḍīs}$$

If $VRCH$ is in an odd quadrant (i.e., I or III), then subtract the above value COR from the instant of opposition to get the instant of the middle of the eclipse.

If $VRCH$ is in an even quadrant (i.e., II or IV), then add the above value COR to the instant of opposition to get the instant of the middle of the eclipse.

In the current example,

$$COR = \frac{|\beta'| \times 59}{10 \times m} = \frac{23'.5141 \times 59}{10 \times 10'.982759} \tag{4.5}$$

$$= 12'.63190 \textit{ vināḍīs}$$

$$COR = 12'.63190 \times \frac{2}{5} \approx 0^{\text{h}}5^{\text{m}}3^{\text{s}}$$

Now, $VRCH = 4°24'$. Since $VRCH < 90°$, i.e., $VRCH$ is in 1 quadrant (odd), the above value is subtractive from the instant of opposition.

∴ Middle of the eclipse

$$= \text{Instant of opposition} - COR$$
$$= 17^{\text{h}}36^{\text{m}} - 0^{\text{h}}5^{\text{m}}3^{\text{s}}$$
$$= 17^{\text{h}}30^{\text{m}}57^{\text{s}}$$

(xii) Half-duration of the eclipse (*Sthiti*)

$$HDUR = \frac{\sqrt{D^2 - (\beta')^2}}{m} = \frac{\sqrt{(54'.032946)^2 - (23'.514195)^2}}{10'.982759} = 4.429501 \ \textit{nāḍīs} \quad (4.6)$$

$$= 4.429501 \times \frac{2}{5} = 1^{\text{h}}46^{\text{m}}18^{\text{s}}$$

(xiii) Half-duration of totality (*marda*)

$$THDUR = \frac{\sqrt{(D')^2 - (\beta')^2}}{m} = \frac{\sqrt{(24'.418284)^2 - (23'.514195)^2}}{10'.982759} = 0.599389 \ \textit{nāḍīs} \quad (4.7)$$

$$= 0.599389 \times \frac{2}{5} = 0^{\text{h}}14^{\text{m}}23^{\text{s}}$$

(xiv) *Pramāṇam* (Magnitude)

$$= \frac{D - |\beta'|}{MDIA} = \frac{54'.032946 - 23'.514195}{29'.614662} = 1.030528 \quad (4.8)$$

Summary of the eclipse **IST**

1. Beginning of the eclipse (*Sparśa*)

$$= \text{Middle} - HDUR = 17^{\text{h}}30^{\text{m}}57^{\text{s}} - 1^{\text{h}}46^{\text{m}}18^{\text{s}} = 15^{\text{h}}44^{\text{m}}39^{\text{s}}$$

2. Beginning of totality (*Sammilana*)

$$= \text{Middle} - THDUR = 17^{\text{h}}30^{\text{m}}57^{\text{s}} - 0^{\text{h}}14^{\text{m}}23^{\text{s}} = 17^{\text{h}}21^{\text{m}}37^{\text{s}}$$

3. Middle (*Madhya*)

$$= \text{Instant of full moon} - COR = 17^{\text{h}}36^{\text{m}} - 0^{\text{h}}5^{\text{m}}3^{\text{s}} = 17^{\text{h}}30^{\text{m}}57^{\text{s}}$$

4. End of totality (*Unmilana*)

$$= \text{Middle} + THDUR = 17^{\text{h}}30^{\text{m}}57^{\text{s}} + 0^{\text{h}}14^{\text{m}}23^{\text{s}} = 17^{\text{h}}45^{\text{m}}20^{\text{s}}$$

5. End of the eclipse (*Mokṣa*)

$$= \text{Middle} + HDUR = 17^{\text{h}}30^{\text{m}}57^{\text{s}} + 1^{\text{h}}46^{\text{m}}18^{\text{s}} = 19^{\text{h}}17^{\text{m}}15^{\text{s}}$$

Based on the above procedure we computed the circumstances of total lunar eclipses from 2000 to 2025 C.E. and the magnitude and middle timings of the eclipses in IST are given in the following Table. Comparing these computed values with those of the NASA values, they are equivalent up to the minutes.

Table 1: Total lunar Eclipses : 2000-2025 C.E

Sl. No.	Dates	ISP		NASA	
		Magnitude	Middle of Eclipse	Magnitude	Middle of Eclipse
1	2000 January 21	1.335	10^h14^m	1.3246	$10^h13^m30.6^s$
2	2000 July 16	1.77	19^h26^m	1.7684	$19^h25^m34.8^s$
3	2001 January 09	1.189	25^h50^m	1.1889	$25^h50^m35.4^s$
4	2003 May 16	1.136	09^h12^m	1.1276	$09^h10^m08.7^s$
5	2003 November 09	1.0204	06^h49^m	1.0178	$06^h48^m33.5^s$
6	2004 May 04	1.0788	26^h02^m	1.3035	$26^h00^m12.6^s$
7	2004 October 28	1.3195	08^h34^m	1.3081	$08^h34^m06.7^s$
8	2007 March 03	1.2367	28^h51^m	1.2328	$28^h50^m53.2^s$
9	2007 August 28	1.4789	16^h07^m	1.4758	$16^h07^m21^s$
10	2008 February 21	1.118	08^h55^m	1.1062	$08^h56^m03^s$
11	2010 December 21	1.2648	13^h47^m	1.2561	$13^h46^m57.1^s$
12	2011 June 15	1.710	25^h42^m	1.6999	$25^h42^m36.2^s$
13	2011 December 10	1.111	20^h02^m	1.1061	$20^h01^m49^s$
14	2014 April 15	1.306	13^h15^m	1.2907	$13^h15^m38.9^s$
15	2014 October 08	1.116	16^h24^m	1.1659	$16^h24^m35.1^s$
16	**2015 April 04**	**1.030**	$\mathbf{17^h30^m}$	**1.0008**	$\mathbf{17^h30^m14.5^s}$
17	2015 September 28	1.288	08^h16^m	1.2764	$08^h17^m07.5^s$
18	**2018 January 31**	**1.321**	$\mathbf{19^h00^m}$	**1.3155**	$\mathbf{18^h59^m49.6^s}$
19	2018 July 27	1.612	25^h51^m	1.6087	$25^h51^m43.5^s$
20	2019 January 21	1.200	10^h42^m	1.1953	$10^h42^m16^s$
21	2021 May 26	1.012	16^h50^m	1.0095	$16^h48^m40.3^s$
22	2022 May 16	1.423	09^h41^m	1.4137	$09^h41^m28.8^s$
23	2022 November 08	1.372	16^h28^m	1.3589	$16^h29^m08.8^s$
24	2025 March 14	1.180	12^h28^m	1.1784	$12^h28^m41.7^s$
25	2025 September 07	1.359	23^h40^m	1.3619	$23^h41^m43.1^s$

5. Conclusion

We have discussed the computation of eclipses based on Indian procedure including modern astronomical terms such as GS (the Sun's anomaly), GM (the Moon's anomaly). The results are compared and which are comparable to the prestigious modern NASA data and the middle of the eclipse timings is correct to their minutes, which shows the accuracy of ISP method, which is very easy to compute with minimum number of steps.

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Vanaja V.,
Department of Mathematics,
Government First Grade College,
Yelahanka, Bengaluru 560064,
Karnataka, India.
E-mail address: vanajavenkatachalaiah@gmail.com

and

Shailaja M.,
Department of Mathematics,
School of Applied Sciences,
Vijayanagara, Bengaluru 560104,
Karnataka, India.
E-mail address: shaila.ac55@yahoo.com

and

Sangayya G.,
Department of Computer Science,
Government First Grade College and PG centre,
Chintamani 563125,
Karnataka, India.
E-mail address: gsswamy@gmail.com