Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2026 (44) 11** : 1–9.
ISSN-0037-8712
doi:10.5269/bspm.81200

A Note on Eigenvalues Significance in Digital Image Processing

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ABSTRACT: This paper discusses the importance of eigenvalues in the area of digital image processing (DIP). A digital image may be in the form of a matrix data. The properties of eigenvalues, in the case of standard matrices, have been discussed. We started by converting a standard color image a grayscale image to make it easier to understand how the image is represented in a matrix, and then applied the properties and theorems using the MATLAB software. We also give a comparative analysis of different image noise reduction methods that make use of the eigenvalues of the image data. To measure the quality of image compression using various filters, we used mean square error (MSE) and peak signal to noise ratio (PSNR). Mean square error (MSE) and PSNR have also been used to conduct the experiments and to further prove the validity of the proposed theories and methodologies.

Keywords: Eigenvalues, trace, determinant, digital image, filter.

Contents

1. Introduction

DIP is the digital image manipulation od digital images through computer algorithms to improve their quality, derive valuable information, transmit digitally, communicate, or prepare digital images to be further processed. History of DIP started in the mid 20^{th} century, with the invention of computers [1]. The field began to emerge as a result of the need to process satellite images, medical images, and televisions [2]. The development of image enhancement, restoration, and compression algorithms made possible advances in medical imaging. The introduction of digital cameras and smart phones started in the 1990s and 2000s, and DIP started to be used in electronic devices [3]. The computer vision area has made great strides in this period and mostly applies the DIP techniques to achieve functions such as recognition of objects, face recognition and video surveillance [4]. Image enhancement is the process of improving the clarity of an image by adjusting its sharpness contrast ot brightness [5]. DIP involves a lot

2020 *Mathematics Subject Classification:* 62H35, 65F15, 05C50, 68U10, 94A08.
Submitted January 03, 2026. Published March 13, 2026

of eigenvalues, which is a fundamental term in linear algebra. It is revant to numerous others techniques, including feature extraction, picture and compression [6]. The two most important uses of eigenvalues in DIP are PCA and SVD [7]. According to [8], PCA is frequently employed for face recognition, while SVD is typically used for image compression [9]. A matrix's trace, which is the sum of its diagonal elements, is also its eigenvalues. Trace has several applications image denoising and filtering, variation and energy minimization, tensor and multispectral image analysis, etc., in DIP, including corner detection, particularly using the Harris corner detector, which is one of the most significant applications for trace [10]. The determinant is the product of matrix eigenvalues and has a variety of uses, such as PCA, corner detection, covariance, and geometric transformations etc. [11]. Image denoising, image super-resolution, video compression, and medical image processing are among the DIP fields in which MSE and PSNR are used in picture denoising to evaluate the degree of noise removal [12]. More efficient noise reduction is indicated by lower MSE and higher PSNR values [13].

Notations: IM: Input image, GN: Gaussian noise, PN: Poisson noise, SN: Speckle noise, SPN: Salt and pepper noise, MSE: Mean square error, AMF: Arithmetic mean filter, GMF: Geometric mean filter, HMF: Harmonic mean filter, PSNR: Peak signal-to-noise ratio, DIP: Digital image processing, PCA: Principal component analysis, SVD: Singular value decomposition.

2. Definitions and Properties

Definition 2.1 *If $A = [a_{ij}]_{(n\times n)}$ square matrix of order n. $|A - \lambda I|$ is a characteristic matrix of A. Let $\lambda_1, \lambda_2 ..., \lambda_n$ be roots of a polynomial equation, then $\lambda_1, \lambda_2 ..., \lambda_n$ are called eigenvalues of A [14].*

Proposition 2.1 *Let $A = [a_{ij}]_{(n\times n)}$, n^{th} order square matrix, $a_{11}, a_{22}, a_{33} ..., a_{nn}$ are diagonal entries of A and $\lambda_1, \lambda_2 ..., \lambda_n$ are eigenvalues of A, then the trace of matrix $A = \sum_{i=1}^{n} a_{ii} = \sum_{i=1}^{n} \lambda_i$ [15].*

Proposition 2.2 *Let $A = [a_{ij}]_{(n\times n)}$ n^{th} order square matrix and $\lambda_1, \lambda_2 ..., \lambda_n$ are eigenvalues of A, then the determinant of matrix $A = \prod_{i=1}^{n} \lambda_i$ [15].*

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2.1. Analytical relationship PSNR/MSE

Given a reference image A and a test image B, both of size $m \times n$, the PSNR between A and B is defined by:

$$\mathrm{SVD}(A, B) = 10 \log_{10}\left(\frac{255}{\mathrm{MSE}(A, B)}\right), \tag{2.1}$$

where

$$\mathrm{MSE}(A, B) = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} (a_{ij} - b_{ij})^2 . \tag{2.2}$$

Here, a_{ij} is the pixel intensity value of the reference image at position (i, j), b_{ij} is the pixel intensity value of the test image at position (i, j). As the MSE approaches zero, the PSNR value approaches infinity, indicating that a greater PSNR value corresponds to a higher-quality image. On the other hand, a low PSNR score indicates significant numerical disparities between images [16].

2.2. Basic noise and filter types in DIP

Image noise refers to the random variation of brightness or color in images generated by a scanner or digital camera's sensor and electronics. Unwanted fluctuations, such as dithering, have been referred to as "noise" due to their inaudibility [17]. Common types of noise observed in image processing are the following:

2.2.1. Salt and pepper noise. A form of impulsive noise, errors in data transfer are the primary cause of this. It only has two possible values a and b. Each has a probability of less than 0.1. Corrupted pixels are set to either the minimum or maximum value, resulting in a "salt and pepper" appearance. Unaffected pixels stay unmodified. In an 8-bit image, the normal value for pepper noise is 0, while salt noise is 255. SPN is typically caused by failing pixel elements in camera sensors, faulty memory locations, or digitization timing issues [18].

2.2.2. Speckle noise. SN is a multiplicative noise. This form of noise exists in practically all coherent imaging technologies, including laser, acoustics, and SAR (Synthetic Aperture Radar) images. The noise is caused by random interference between coherent replies. Fully grown SN resembles multiplicative noise. SN has a gamma distribution and is defined as:

$$F(g) = \frac{g^{\alpha-1}}{(\alpha-1)!\, a^{\alpha}}\, e^{\frac{g}{a}}, \tag{2.3}$$

where variance $a^2\alpha$ and g is the gray level [18].

2.2.3. Gaussian noise. GN has a uniform distribution across the signal. In noisy images, each pixel is the sum of the true pixel value and a random GN value. This sort of noise follows a Gaussian distribution, characterized by a bell-shaped probability distribution function,

$$F(g) = \frac{1}{\sqrt{2\,\pi\,\sigma^2}}\, e^{-\frac{(g-m)^2}{\sigma^2}} \tag{2.4}$$

Here, g represents the gray level, m is the function's mean or average, and σ is the noise standard deviation [18].

2.2.4. Poisson noise. A type of electronic noise that occurs when the finite number of particles carrying energy, such as electrons in an electrical circuit or photons in an optical device, is tiny enough to cause visible statistical fluctuations in measurement [18]. Various application-oriented mean filtering techniques exist to remove noise from images. According to the noise category, some filtering techniques perform better than others. The mean filtering approaches are given below.

2.2.5. Arithmetic mean filter. Let S_{xy} be the set of coordinates in a rectangular sub-image window with a center at (x, y) and a size of $m \times n$. The average value of the corrupted image $g(x, y)$ in the region delineated by S_{xy} is computed using the AMF approach. The arithmetic mean, which is determined by utilizing the pixels within the region delineated by S_{xy}, is the value of the restored image f at any given position (x, y) [19].

$$F(x, y) = \frac{1}{mn} \sum_{(r,c)\in\mathcal{S}_{xy}} g(r, c) \tag{2.5}$$

2.2.6. Geometric mean filter. Every restored pixel in the GMF approach is equal to the product of the sub-image window's pixels, raised to the power of $\frac{1}{mn}$. Comparable to an AMF, a GMF produces smooth images but often loses less image quality in the process [19].

$$F(x, y) = \prod_{(r,c)\in\mathcal{S}_{xy}} [g(r, c)]^{\frac{1}{mn}} \tag{2.6}$$

2.2.7. Harmonic mean filter. For SN, the HMF performs well, but not well for PN. It functions well even with different kinds of noise, such as GN [19].

$$F(x, y) = \frac{mn}{\sum_{(r,c)\in\mathcal{S}_{xy}} \frac{1}{g(r,c)}} \tag{2.7}$$

3. Analytical and computational analysis of digital image noises concerning AMF, GMF, and HMF filters

None of the authors use matrix trace and determinant to assess the structural behavior of filters in image de-noising, even though these metrices have received significant research in both pure and applied mathematics. In this article, authors work on traces and determinant of digital image data matrix based on analyzing eigenvalues. Also compares different types of filters based on MSE and PSNR values, which shows some interesting results and relationship between trace, determinant, MSE and PSNR representing using tables and figures. The advancement of this work attached below in author's contribution table;

Authors	**Matrices Used**	**Trace and Determinant used**
Al-Amri, et al., (2010) [20]	MSE, PSNR	Not used
Huang, H., et al. (2016) [21]	Not used	Used theoretically
Alkan, E., Yörük, E. S. (2019) [22]	Not used	Used theoretically
Bharati, S., et al. (2020) [23]	MSE, SNR, SSIM	Not used
Salamat, N., et al. (2024) [24]	MSE, PSNR, SSIM	Not used
Our work in this article	MSE, PSNR	Used based on image eigenvalues data

Based on matrix trace, determinant, MSE, and PSNR values, we applied different filters (AMF, GMF, HMF) on various types of noisy (SPN, SN, GN, PN) images and analyzed the filters to determine which one performs the best.

3.1. Analysis of SPN for DIP

For a basic understanding and study of SPN in DIP, one can compute key factors of digital images after converting input digital images into a two-dimensional tensor or matrix. Properties of the matrix, such as eigenvalues, trace, determinant, MSE, and PSNR, reveal many characteristics of digital image noises and particular image noise filters.

Table 1: Computational analysis of SPN using three different filters

	IM	SPN	AMF	GMF	HMF
Trace	3.9373	3.1706	4.3528	4.1693	3.8186
Determinant	$-2.29 \times 10^{-5} + 9.681 \times 10^{-6}$	-2.1×10^{-3}	7.47×10^{-14}	-2.31×10^{-10}	$1.92 \times 10^{-10} - 8.331 \times 10^{-23}$
MSE	0.000	2378.5	165.0422	107.35	108.92
PSNR(dB)	0.000	14.3678	25.9549	7.8227	7.7597

Our analysis states that the trace of the input image and trace after using an HMF for the noise image are approximately nearest,(For instance see Table 1 and Figure 1) i.e., HMF works better than AMF and GMF based on trace data, but according to the determinant, MSE, and PSNR data values, respectively, GMF and AMF give the best results.

3.2. Analysis of SN for DIP

Analyzing Table 2 and Figure 2, based on the trace and determinant of the matrix, GMF produces better results compared to the AMF and HMF. But on MSE and PSNR values, the AMF yielded the best results.

Table 2: Computational analysis of SN using three different filters

	IM	SN	AMF	GMF	HMF
Trace	3.9373	5.0393	6.1373	3.4688	3.4688
Determinant	$-2.29 \times 10^{-5} + 9.68i \times 10^{-6}$	1.45×10^{-2}	$8.9173 - 1.72i \times 10^{-14}$	$-8.71 \times 10^{-8} + 4.81i \times 10^{-26}$	4.66×10^{-10}
MSE	0.000	1792.4	622.0422	686.856	932.189
PSNR(dB)	0.000	15.5964	21.8709	19.7621	18.4358

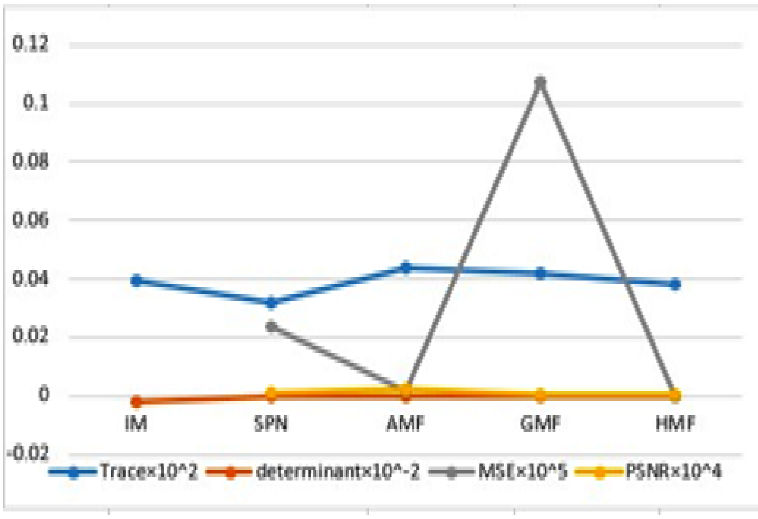

Figure 1: Comparison of SPN using three different filters

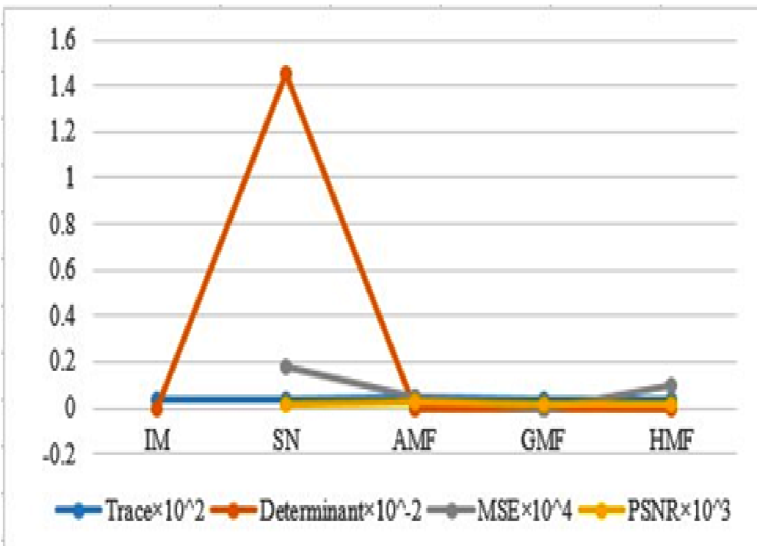

Figure 2: Comparison of SN using three different filters

3.3. Analysis of GN for DIP

In Table 3 and Figure 3, show that, according to the trace, MSE, and PSNR values, the HMF yields the best competitive results compared to AMF and GMF. However, GMF produces the best results based on determinant comparison.

Table 3: Computational analysis of GN using three different filters

	IM	GN	AMF	GMF	HMF
Trace	3.9373	4.4551	4.6157	3.6776	3.8173
Determinant	$-2.29 \times 10^{-5} + 9.681 \times 10^{-6}$	-4.6×10^{-3}	2.94×10^{-13}	$-3.50 \times 10^{-9} - 1.881 \times 10^{-25}$	8.38×10^{-8}
MSE	0.000	871.7842	475.1194	396.6120	326.9666
PSNR(dB)	0.000	18.7267	21.3628	22.1471	22.9858

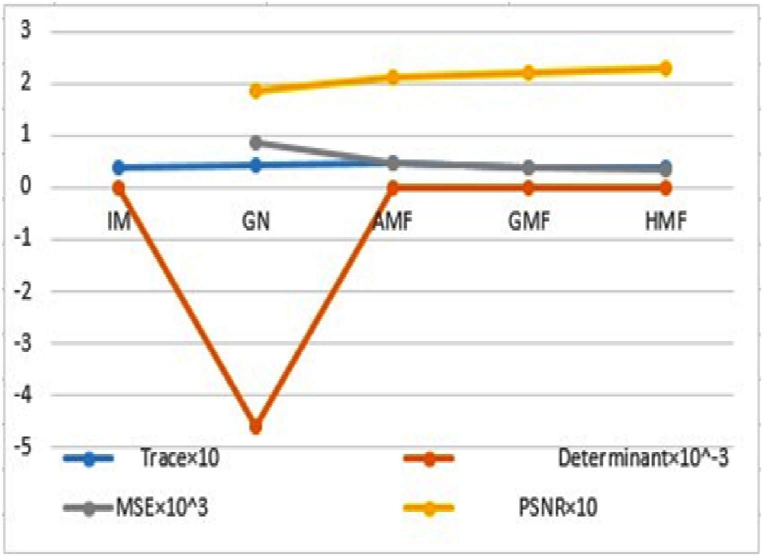

Figure 3: Comparison of GN using three different filters

3.4. Analysis of PN for DIP

As per Figure 4 and Table 4, if we apply different types of filters (AMF, GMF, HMF) on PN, we find that GMF gives the best outcomes based on trace, MSE, and PSNR values. However, based on the determinant data, the AMF gives the best outcomes.

Table 4: Computational analysis of PN using three different filters

	IM	PN	AMF	GMF	HMF
Trace	3.9373	3.8353	3.7331	3.9984	3.7428
Determinant	$-2.29 \times 10^{-5} + 9.681 \times 10^{-6}$	2.138×10^{-5}	$-8.04 \times 10^{-10} - 73.20i \times 10^{-23}$	6.93×10^{-12}	$-2.3 + 1.41 \times 10^{-12}$
hline MSE	0.000	113.2388	47.6131	26.8714	59.2589
PSNR(dB)	0.000	27.5909	31.3535	33.8379	33.3838

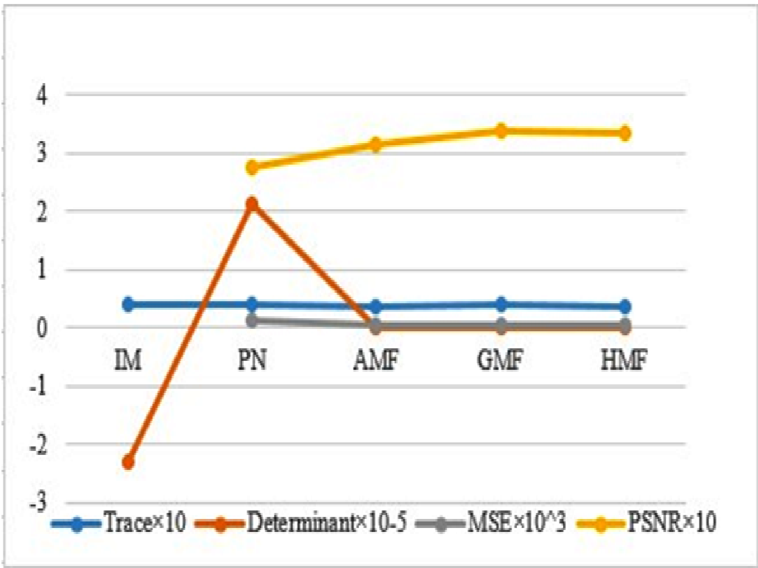

Figure 4: Comparison of PN using three different filters

4. Conclusion

Based on our comprehensive analysis, the evaluation criteria impact the filters' performance. Strong performance can be observed by the HMF for preserving the trace of noisy images, especially when SPN noise is present. On the other hand, the GMF regularly shows better results in terms of determinant and also does well in MSE and PSNR for different kinds of noise. AMF, on the other hand, performs highly effectively in descending MSE and maximizing PSNR, particularly when PN is present. In some situations, it also produces the greatest results in determinant. The fact that no single filter dominates across every measure highlights the significance of choosing filters according to particular matrix-based properties and quality of image measures that are important to the noise characteristic.

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