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About R-Bounded Topological Groups

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ABSTRACT: Selection principles in topological groups have been developing intensively in recent years. In topological group theory, there are so-called left, right, two-sided, and universal uniform structures. In this article, we use these uniform structures to study R-bounded topological groups.

Keywords: Uniform space, topological group, cover, M-boundedness, ω-boundedness, R-boundedness, homomorphism, uniformly continuous mapping.

Contents

1. Introduction

A mapping $f : X \cdot X \to X$ is said to be a binary operation on a set X. Such operation maps any ordered pair (x, y) of elements of the set X to an element $z = f(x, y)$ of the same set. As usually, a binary operation is denoted by a certain sign, for example " $\cdot$ ", and such sign is used in infix form, for example $z = x \cdot y$. If such operation is associative on the set X, i.e. $(x \cdot y) \cdot z = x \cdot (y \cdot z)$ for all $x, y, z \in X$ then the pair $(X, \cdot)$ is said to be a semigroup. A semigroup $(G, \cdot)$ is said to be a group, if the following conditions are relized. There exists such neutral element $e \in G$ that $x \cdot e = e \cdot x = x$ for all $x \in G$ and each element $x \in G$ has the inverse one, i.e. such element $x^{-1} \in G$ that $x \cdot x^{-1} = x^{-1} \cdot x = x$. The main binary operation in a group is frequently denoted with the symbol " $+$ " and is said to be addition. A group equipped with addition is said to be additive.

For covers α and β of a set X , we have: $\alpha \wedge \beta = \{A \cap B : A \in \alpha, B \in \beta$. Let M subset of X. Then $\alpha(M) = \cup St(\alpha, M)$, $St(\alpha, M) = \{A \in \alpha : A \cap M \neq \emptyset\}$. Let $x \in X$ be an arbitrary point. Then $\alpha(x) = \cup St(\alpha, x)$, $St(\alpha, x) = \{A \in \alpha : A \ni x\}$.

The symbol $\alpha \succ \beta$ means that the cover α is a refinement of the cover β, i.e., for any $A \in \alpha$ there exists $B \in \beta$ such that $A \subset B$. The symbol $\alpha \rhd \beta$ means that the cover α is a star refinement of the cover β, i.e., for any $x \in X$ there exists $B \in \beta$ such that $\alpha(x) \subset B$ and, the symbol $\alpha * \succ \beta$ means that the cover α is a strongly star refinement of the cover β, i.e., for any $A \in \alpha$ there exists $B \in \beta$ such that $\alpha(A) \subset B$.

A uniformity on a set $X \neq \emptyset$ is a family U of covers of X which satisfies the following axioms:

(U1) If $\alpha \in U$ and $\alpha \succ \beta$, then $\beta \in U$;

(U2) If $\alpha, \beta \in U$, then there exists $\gamma \in U$ such that $\gamma \succ \alpha$ and $\gamma \succ \beta$;

(U3) If $\alpha \in U$, then there exists $\beta \in U$ such that $\beta * \succ \alpha$;

(U4) For any two points $x, y \in X$, $x \neq y$ there is an $\alpha \in U$ such that no member of α contains both x and y, [1], [4], [6], [7], [12].

The pair (X, U) is called a uniform space.

A uniform space (X, U) is called:

(1) ω -bounded, if the uniformity U has a base consisting of countable cover [1], [7], [12];

(2) has the uniform Menger property, if for each sequence $(\alpha_n | n \in N) \subset U$ there is a sequence $(\beta_n | n \in N)$ such that for each $n \in N$, β_n is a finite subset of α_n and $\underset{n \in N}{\cup} \beta_n$ is a cover of X [7], [21], [23].

(3) has the uniform Rothberger property, if for each sequence $(\alpha_n | n \in N) \subset U$ there is a sequence $(A_n | n \in N)$ such that for each $n \in N$, $A_n \in \alpha_n$ and $\underset{n \in N}{\cup} A_n = X$, [7], [31].

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A mapping $f : (X, U) \to (Y, V)$ of the uniform space (X, U) to the uniform space (Y, V) is called a uniformly continuous mapping, if every $\beta \in V$ there is $\alpha \in U$ such that $f\alpha \succ \beta$ [1], [12], [23], [36].

The mapping f is called precompact, if for each $\alpha \in U$ there exist a uniform cover $\beta \in U$ and a finite uniform cover $\gamma \in U$ such that $f^{-1}\beta \wedge \gamma \succ \alpha$ [1], [12], [17], [22].

A set G is said to be a topological group if it is equipped with structures of a group and a topology realizing the following two axioms:

(TG1) The mapping $(x, y) \to x \cdot y$ of the product $G \times G$ to G is continuous;

(TG2) The mapping $x \to x^{-1}$ of group G to itself is continuous, [1].

A topological group G is said to be:

(a) ω-bounded if for each neighborhood D of neutral element e there exists a countable set $A \subset G$ such that $A \cdot D = G$, [1], [12], [22];

(b) M-bounded, if each sequence $(U_n | n \in N)$ of neighborhoods of neutral element e in G there exists a sequence $(A_n | n \in N)$ of finite subsets of G such that $\underset{n\in N}{\cup} A_n \cdot U_n = G$, [7];

(d) R-bounded, if for each sequence $(U_n | n \in N)$ of neighborhoods of neutral element e in G there exists a sequence $(x_n | n \in N)$ of elements of G such that $\underset{n\in N}{\cup} x_n \cdot U_n = G$, [7].

A filter F in a uniform space (X, U) is said to be a Cauchy filter, if $\alpha \cap F \neq \varnothing$ for all $\alpha \in U$, [15]; a point $x \in X$ is said to be an adherent point of a filter F in (X, U), if $x \in \cap\{[M] : M \in F\}$ or equivalently, $x \in X$ is an adherent point of each element $M \in F$ in (X, τ_U) [15]; a point $x \in X$ is said to be a limit of a filter F , if a filter F is finer than the filter F_x of neighborhoods of the point $x \in X$, [1], [15].

Let $\widetilde{X}$ be the set of all minimal Cauchy filters in the uniform space (X, U). For each uniform cover $\alpha \in U$ we denote $\widetilde{\alpha} = \{\widetilde{A} : A \in \alpha\}$, where $\widetilde{A} = \{F \in \widetilde{X} : A \in F\}$. The family of covers $\widetilde{B} = \{\widetilde{\alpha} : \alpha \in U\}$ forms a base of some uniformity $\widetilde{U}$ on $\widetilde{X}$, [12], [15], [22].

For any uniform space (X, U) there exists a unique (with respect to uniform isomorphism) complete uniform space $(\widetilde{X}, \widetilde{U})$ such that for some everywhere-dense subset $\widetilde{Y} \subset \widetilde{X}$ the uniform space (X, U) is uniformly isomorphic to the space $(\widetilde{X}, \widetilde{U}_{\widetilde{Y}})$. And $w(\widetilde{U}_{\widetilde{Y}}) = w(\widetilde{U})$. The uniform space $(\widetilde{X}, \widetilde{U})$ is called a completion of the uniform space (X, U) [1].

A continuous mapping $f : G \to H$ being a homomorphism (in algebraic terms) of a group G to group H is said to be a homomorphism of a topological group G to a topological group H; a filter F is called $\aleph_0$-centered, if $\cap F' \neq \emptyset$ for any subfamily $F' \subset F$ of cardinality $|F'| \in \aleph_0$. In other words, any countable subfamily of filter F has non-empty intersection; a uniform space in which every $\aleph_0$-centered Cauchy filter converges is called $\aleph_0$-complete; a topological group G is called $\aleph_0$-complete in the sense of Raikov if the uniform space (G, U_t) is $\aleph_0$-complete.In other words, the two-sided uniform structure U_t of the group G is an $\aleph_0$-complete uniformity, [1].

Any $\aleph_0$-complete topological group $G^{\aleph_0}$ containing the group G as an everywhere dense subgroup is called an $\aleph_0$-completion of the group G. A topological group G is called complete in the sense of Raikov if the uniform space (G, U_t) is complete. In other words, the two-sided uniform structure U_t of the group G is a $\aleph_0$-complete uniformity, [1].

Any complete topological group $\widetilde{G}$ containing the group G as an everywhere dense subgroup is called a completion of the group G.

A topological group μG is said to be a D-completion of a topological group G, if the following conditions are realized:

1) μG is Dieudonne complete;

2) G is a subgroup of the group μG;

3) G is everywhere dense in μG, [1].

Important properties of selection principles are investigated in the works [3], [5], [7]-[10], [21], [28]-[35].

Many terminology and known results are used by the authors from the works [1], [2], [7], [11]-[27], [34], [36].

2. About *R*-Bounded Topological Groups

Let G be a topological group.

Proposition 1. A topological group G is R-bounded if and only if the uniform space (G, U_l) is uniformly Rothberger, where U_l is the left uniformity on G.

Proof: Let G be a R-bounded topological group and take an arbitrary sequence $(\alpha^l_{H_n}|n \in N)\subset U_l$ of uniform covers, where $\alpha^l_{H_n} = \{x \cdot H_n : x \in G\}$, $H_n \in B(e)$, $B(e)$ – base of neighborhoods of the neutral element e. Since G is R-bounded, then for a countable family $(H_n|n \in N)$ of neighborhoods of a neutral element e in G there exists a sequence $(x_n|n \in N)$ elements of G such that $G = \underset{n\in N}{\cup} x_n \cdot H_n$. Put $\beta^l_{H_n} = \{B_n : n\in N\}$, where $B_n = x_n \cdot H_n$ for each $n\in N$. It is easy to see that $B_n \in \alpha^l_{H_n}$ for each $n\in N$. Since $\underset{n\in N}{\cup} x_n \cdot H_n = G$, then $\underset{n\in N}{\cup} \beta^l_{H_n} = G$. Hence, (G, U_l) is uniformly Rothberger.

Conversely, let (G, U_l) be a uniformly Rothberger space and $(H_n|n\in N)$ be an arbitrary sequence of neighborhoods of the neutral element e in G. Denote $\alpha^l_{H_n} = \{x \cdot H_n : x\in G\}$, $H_n\in B(e)$. Then $(\alpha^l_{H_n}|n\in N)\subset U$ is a sequence of uniform covers. Since the uniform space (G, U_l) has a uniformly Rothberger property, then there exists a sequence $(A_n|n\in N)$ such that $A_n\in\alpha^l_{H_n}$ and $\underset{n\in N}{\cup} A_n = G$. For each i, from $A_n\in\alpha^l_{H_n}$ we select one element x_i. Then $\{x_1, x_2, ..., x_n, ...\}\subset G$ is a countable subset of G. It is easy to see that $\underset{n\in N}{\cup} x_n \cdot H_n = G$. Thus, G is R-bounded.

Proposition 2. A topological group G is R-bounded, if and only if the uniform space (G, U_l) is uniformly Rothberger, where U_r is the right uniformity on G.

Proof: The proof, with minor changes, is similar to the proof of Proposition 1.

Proposition 3. A topological group G is R-bounded, if and only if the uniform space (G, U_t) is uniformly Rothberger, where U_t is the two-sided uniformity on G.

Proof: The proof, with minor changes, is similar to the proof of Proposition 1.

Proposition 4. Any R-bounded topological group G is M-bounded.

Proof: Let G be an R-bounded topological group. We will proof that it is M-bounded. It suffices to show that the uniform space (G, U_l), where U_l is the left uniformity on G, is a uniformly Menger space. Let $(\alpha^l_{U_n}|n\in N)\subset U_i$ be an arbitrary uniform cover, $\alpha^l_{U_n} = \{x \cdot U_n : x\in G\}$, U_n is a neighborhood of the neutral element e in G. Then there exists a sequence $(x \cdot U_n|n\in G)$ such that for each n, $x \cdot U_n \in \alpha^l_{U_n}$ and $\underset{n\in N}{\cup} x \cdot U_n = G$. Put $\{x \cdot U_n : n \in G\}=\beta^l_{U_n}$. Then it is easy to see that $\beta^l_{U_n}$ is a finite subfamily of $\alpha^l_{U_n}$. Hence, G is M-bounded.

Corollary 1. Any R-bounded topological group is ω-bounded.

Theorem 1. Any countable discrete topological group G is a R-bounded.

Proof: Let G be a countable discrete topological group. We will proof that G is R-bounded. It suffices to proof that the uniform space (G, U_l) is a uniformly Rothberger space. Let $(\alpha^l_{W_n}|n\in N)\subset U_l$ be a sequence of uniform covers. Since the topological group G is countable discrete, there exists a base $B = (\alpha^l_W)$ consisting of countable covers $\alpha^l_W = \{x_n \cdot W : x_n\in G\}$. It is easy to see that $\alpha^l_W \succ \alpha^l_{W_n}$. Take $x_n \cdot W\in\alpha^l_{W_n}$, $W = \{e\}$, $e\in G$ – neutral element, such that $x_n \cdot W\subset x_n \cdot W_n$ and $\underset{n\in N}{\cup} x_n \cdot W = G$. Thus, G is a R-bounded.

Theorem 2. If a topological group G^* is a continuous homomorphic image of a R-bounded topological group G, then G^* is also R-bounded topological group.

Proof: Let G be a R-bounded topological group and $f : G \to G^*$ be an arbitrary continuous homomorphism of the R-bounded topological group G to the topological group $G^\star$. It is easy to see that the homomorphism $f : (G, U_l) \to (G^*, V_l)$ is a uniformly continuous mapping. We proof that the uniform space (G^*, V_l) is a uniformly Rothberger space. Let $(\beta^l_{V_n}|n\in N)\subset V_l$ be an arbitrary sequence of uniform covers. Since f is uniformly continuous, the cover $f^{-1}\beta^l_{V_n}$ is uniformly cover i.e. $f^{-1}\beta^l_{V_n}\in U_l$. Put $f^{-1}\beta^l_{V_n} = \alpha^l_{f^{-1}V_n}$, $n\in N$. It is easily seen that we have: if V_n is a neighborhood of neutral element e in topological group G^*, then $f^{-1}V_n$ is a neighborhood of neutral element e in the topological group G. Then there exists a sequence $(x \cdot U_n|n\in N)$, such that for each $n\in N$, $x \cdot U_n\in\alpha^l_{U_n}$ and $\underset{n\in N}{\cup} x \cdot U_n = G$, where $U_n = f^{-1}V_n$. For each $n\in N$ put $f(x \cdot U_n) = y \cdot V_n$, $y = f(x)$. Then, $\underset{n\in N}{\cup} f(x \cdot U_n) = \underset{n\in N}{\cup} y \cdot V_n = G^*$, $n\in N$. Hence, G^* is R-bounded.

Theorem 3. Let $f : G \to G^*$ be a precompact continuous homomorphism. If the topological group G^* is R-bounded, then a topological group G is M-bounded.

Proof: Let $f : G \to G^*$ be a precompact continuous homomorphism of a topological group G onto a topological group G^* and $(\alpha^l_{U_n}|n\in N)\subset U_i$ be an arbitrary sequence of uniform covers. For each $n\in N$ there exists a finite cover $\gamma^l_{V_n}\in U_l$ and $\beta^l_{W_n}\in V_l$, such that $f^{-1}\beta^l_{W_n}\wedge\gamma^l_{V_n}\succ\alpha^l_{U_n}$. Apply to the sequence $(\beta^l_{W_n}|n\in N)\subset V_l$ the fact that the space (G^*,V_l) has a uniformly Rothberger property we find a sequence $(y\cdot W_n|n\in N)$ such that $y\cdot W_n\in\beta^l_{W_n}$ and $\underset{n\in N}{\cup} y\cdot W_n = G^*$, $y\in G^*$. We have for each $n\in N$, $f^{-1}(y\cdot W_n)\cap(x\cdot V_{n,i})\neq\emptyset$, $i=1,2,...,k$ and $f^{-1}(y\cdot W_n)\cap(x\cdot V_{n,i})\subset x\cdot U_{n,i}$, $U_{n,i}\in\alpha^l_{U_n}$, $x\in G$, $y\in G^*$. Put $\hat{\alpha}^l_{U_n}=\{x\cdot U_{n,i} : i=1,2,...,k\}$. Since $f^{-1}(y\cdot W_n)\subset\hat{\alpha}^l_{U_n}$ and $\underset{n\in N}{\cup} f^{-1}(y\cdot W_n)\in G$, $y\in G^*$ for each $n\in N$, then $\underset{n\in N}{\cup}\hat{\alpha}^l_{U_n}=G$. Hence, G is M-bounded.

Theorem 4. The completion $\widetilde{G}$ of an R-bounded topological group G is R-bounded.

Proof: Let $(\widetilde{G},\widetilde{U}_t)$ be the completion of the uniformly Rothberger space (G,U_t) and $(\widetilde{\alpha}^t_n|n\in N)\subset\widetilde{U}_t$ be an arbitrary sequence. Put $\alpha^t_n=\widetilde{\alpha}^t_n\wedge\{G\}$, $\widetilde{\alpha}^t_n=\{\widetilde{A}_n : A_n\in\alpha_n\}$, $\widetilde{A}_n=\{F : A_n\in F\}$, F – minimal Cauchy filter. Then from the construction [1] of completion of uniform spaces $(\alpha^t_n|n\in N)\subset U_t$. Since (G,U_t) is a uniformly Rothberger space, then there exists a sequence $(A_n|n\in N)$ such that for any $n\in N$, $A_n\in\alpha^t_n$ and $\underset{n\in N}{\cup} A_n = G$. It is easy to see that $\underset{n\in N}{\cup}\widetilde{A}_n=\widetilde{G}$, $\widetilde{A}_n=\{F : A_n\in F\}$, F - minimal Cauchy filter. Hence, $(\widetilde{G},\widetilde{U}_t)$ is a uniformly Rothberger space, i.e. $\widetilde{G}$ is R-bounded.

Theorem 5. The $\aleph_0$-completion $G^{\aleph_0}$ of a R-bounded topological group G is a R-bounded.

Proof: It is known that any topological group G has a unique, up to a topological isomorphism, $\aleph_0$-completion $G^{\aleph_0}_t$ in the sense of Raikov [1]. For the uniform space (G,U_t), $G^{\aleph_0}_t$ denotes the set $G^{\aleph_0}_t=G'\cup G''$, where G' is the set of all neighborhood filters of points of the group G, and G'' is the set of all free $\aleph_0$-centered Cauchy filters of the uniform space (G,U_t). Without loss of generality, we can assume that each element of the set G'' is a minimal Cauchy filter, which is also $\aleph_0$-centered. Let $(\widetilde{G},\widetilde{U}_t)$ be the Raikov completion of the topological group G. Then, according to the construction of completions of uniform spaces [1], the inclusion $G^{\aleph_0}_t\subset\widetilde{G}$ holds. Put $U^{\aleph_0}_t=\widetilde{U}_t\wedge\{G^{\aleph_0}_t\}$. It is known [1] the topological group G is everywhere dense in $\aleph_0$-complete topological group $G^{\aleph_0}_t$. Let $(G^{\aleph_0}_t,U^{\aleph_0}_t)$ be the $\aleph_0$ -completion of the uniformly Rothberger space (G,U_t) and $(\alpha^{\aleph_0,t}_n|n\in N)\subset U^{\aleph_0}_t$ be an arbitrary sequence, where $\alpha^{\aleph_0,t}_n=\{\widetilde{A}_n : A_n\in\alpha^t_n\}$, $\widetilde{A}_n=\{F\in G^{\aleph_0}_t : A\in F\}$. Put $\alpha^t_n=\alpha^{\aleph_0,t}_n\wedge\{G\}$. Then from the construction $(G^{\aleph_0}_t,U^{\aleph_0}_t)$, $(\alpha^t_n|n\in N)\subset U_t$. Since (G,U_t) is a uniformly Rothberger, there exists $(A_n|n\in N)$ such that for any $n\in N$, $A_n\in\alpha^t_n$ and $\underset{n\in N}{\cup} A_n$ is a cover of the space (G,U_t). Then from the construction $\aleph_0$-completion of uniform space (G,U_t), $\underset{n\in N}{\cup}\widetilde{A}_n$ is a cover of the $(G^{\aleph_0},U^{\aleph_0}_t)$. Consequently, $(G^{\aleph_0},U^{\aleph_0}_t)$ is a uniformly Rothberger space, i.e. a topological group $G^{\aleph_0}$ is a R-bounded.

Lemma 1. A Tychonoff space G is a Rothberger space if and only if the uniform space (G,U_G), where U_G is the universal uniformity, is a uniformly Rothberger space.

Proof: Let G be a Rothberger space and $(\alpha_n|n\in N)\subset U_G$ be an arbitrary sequence of uniform covers. Since the interior $\langle\alpha_n\rangle$ of each uniform cover α_n is an open cover, then $(\langle\alpha_n\rangle|n\in N)$ is a sequence of open covers of the space G, $\langle\alpha_n\rangle=\{\langle A\rangle : A\in\alpha_n\}$, where $\langle A\rangle$ is the interior of the set A. Then, there is a sequence $(A_n|n\in N)$ of finite open subfamilies such that for any $n\in N$, $A_n\in\alpha_n$ and $\underset{n\in N}{\cup} A_n$ is an open cover of X. Consequently, (X,U_G) is a uniformly Rothberger space.

Conversely, let $(\alpha_n|n\in N)$ be an arbitrary sequence of open covers of the space G. Then $(\alpha_n|n\in N)\subset U_G$. Therefore, there is a sequence $(A_n|n\in N)$ such that for any $n\in N$, $A_n\in\alpha_n$ and $\underset{n\in N}{\cup}\beta_n$ is a cover of the space (G,U_G). Assume $(\langle A_n\rangle|n\in N)$ where $\langle A_n\rangle$ is the interior of the set A_n. Note that $\underset{n\in N}{\cup}\langle A_n\rangle$ is an open cover of the space G. Consequently, G is a Rothberger space.

It is known that for the universal uniformity U_X of a Tychonoff space X the $\aleph_0$-completion $(X^{\aleph_0},U^{\aleph_0}_X)$ is homeomorphically embedded into every $\aleph_0$-completion $(X^{\aleph_0},U^{\aleph_0})$ of the uniform space (X,U), where U is any uniformity on X congruent with the topology of the space X [1].

For a Tychonoff space X we denote by $\mathbf{U}(X)$ the set of all uniform structures congruent with the topology of the space X. For each uniformity $U\in\mathbf{U}(X)$ we have the uniformly continuous mapping $1 : (X,U_X)\to(X,U)$ and its natural uniformly continuous extension $\widetilde{1} : (X^{\aleph_0},U^{\aleph_0}_X)\to(X^{\aleph_0},U^{\aleph_0})$ over the $\aleph_0$-completions. As we noted above, the $\aleph_0$-completion $(X^{\aleph_0},U^{\aleph_0}_X)$ of the uniform space (X,U_X)

coincides with the Dieudonne completion μX of the Tychonoff space X, i.e. it is identical with the uniform space $(\widetilde{X}, \widetilde{U}_X)$.

Then the following theorem holds.

Theorem 6. The D-completion μG of an R-bounded topological group G is R-bounded.

The proof follows from the fact [1] that for the universal uniformity U_G on a group G the $\aleph_0$-completion $(G^{\aleph_0}, U_G^{\aleph_0})$ is homeomorphically embedded into every $\aleph_0$-completion $(G^{\aleph_0}, U_G^{\aleph_0})$, where U is an arbitrary uniformity on the group G, and from Lemma 1.

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References

1. A.A. Borubaev, *Uniform Topology and its Applications*, Bishkek: Ilim, 2021.
2. A.A. Borubaev, B.E. Kanetov, A.M. Baidzhuranova, T.Z. Zhumaliev, and A. Bekbolsunova, *On the R-compactification of uniform spaces*, AIP Conference Proceedings Sixth International Conference of Mathematical Sciences, 2879 (2023), 020001.
3. L. Babinkostova, Lj. Kocinac, M. Scheepers, *Combinatorics of open covers (XI): Menger- and Rothberger-bounded groups*, Topology Appl., 154 (2007), 1269-1280.
4. R. Engelking, *General Topology*, Berlin: Heldermann, 1989.
5. W. Hurewicz, *UbereineVerallgemeinerung des Borelschen Theorems*, Math. Z. 24 (1925), 401421.
6. J. Isbell, *Uniform space*, Providence, 1964.
7. Lj.D.R. Kocinac, *Selection principles in uniform spaces*, Note di Mathematica, 22 (2003), 127139.
8. Lj.D.R. Kocinac, M. Scheepers, *Function spaces and a property of Reznichenko, Function spaces and a property of Reznichenko*, Topology Appl., 123 (2002), 135-143.
9. Lj.D.R. Kocinac, *Some covering properties in topological and uniform spaces*, Proceedings of the Steklov Institute of Mathematics, 252 (2006), 122-137.
10. H.-P.A. Kunzi, M. Mrsevic, I.L. Reilly, M.K. Vamanamurthy, *Pre-Lindelof quasi-pseudo-metric, quasi-uniform spaces*, Mat. Vesnik, 46 (1994), 131-135.
11. B.E. Kanetov, D.E. Kanetova and M.K. Beknazarova, *About uniform analogues of strongly paracompact and Lindelof spaces*, Transactions issue mathematics National academy of sciences of Azerbaijan. Series of physical-technical and mathematics science, 44 (2024), 117-127.
12. B.E. Kanetov, *Some classes of uniform spaces an uniformly continuous mappings*, Bishkek, KNU named after J. Balasagyn, 2013.
13. B.E. Kanetov, A.M. Baidzhuranova, B.A. Almazbekova, *About weakly uniformly paracompact spaces*, AIP Conference Proc., 2483 (2022), 020004-020009.
14. B.E. Kanetov, U.A. Saktanov, D.E. Kanetova, *Some remainders properties of uniform spaces and uniformly continuous mappings*, AIP Conference Proc., 2183 (2019), 030011-030015.
15. B.E. Kanetov, D.E. Kanetova, M.O. Zhanakunova, *On some completeness properties of uniform spaces*, AIP Conference Proc., 2183 (2019), 030010-030014.
16. B.E. Kanetov, N.A. Baigazieva, N.I. Altybaev, *About uniformly μ-paracompactspaces*, International J. of Appl. Math., 34 (2021), 353-362.
17. B.E. Kanetov, A.M. Baidzhuranova, *Paracompact-type mappings*, Bull. of the Karaganda Univ., 2 (2021), 62-66.
18. B. Kanetov, N. Baigazieva, *Strong uniform paracompactness*, AIP Conference Proc., 1997 (2018), 020085-020089.
19. B.E. Kanetov, A.M. Baidzhuranova, *On a uniform analogue of paracompact spaces*, AIP Conference Proc., 2183 (2019), 030009-030013.
20. B.E. Kanetov, D.E. Kanetova, N.I. Altybaev, *On countably uniformly paracompact spaces*, AIP Conference Proc., 2334 (2020), 020011-020015.
21. B.E. Kanetov, D.E. Kanetova, A.M. Baidzhuranova, *About uniformly Menger Spaces*, Mathematica Moravika, 28 (2024), 53-61.
22. B.E. Kanetov, U.A. Saktanov, A.M. Baidzhuranova, *Totally bounded remainders of uniform spaces and samuelcompactification of uniformly continuous mappings*, AIP Conference Proc., 2334 (2021), 020013-020016.
23. B.E. Kanetov, U.A. Saktanov, E.N. Zhusupbekova, Altybaev N. A. *About u-uniformly plume spaces*, AIP Conference Proc., 3431 (2025), 020010-020014.

24. B.E. Kanetov and M.O. Zhanakunova, *On uniformly Lindelof spaces*, AIP Conference Proceedings International Conference on Analysis and Applied Mathematics, 2325 (2021), 020055.

25. B.E. Kanetov, D.E. Kanetova and N.A. Baigazieva, *Uniformly locally compact and close to them spaces*, AIP Conference Proceedings Fourth International Conference of Mathematical Sciences, 2334 (2021), 020012.

26. B.E. Kanetov and D.E. Kanetova, *Characterization of Some Types of Compactness and a Construction of Index Compactness Extensions by Means of Uniform Structures*, AIP Conference Proceedings International Conference on Analysis and Applied Mathematics, 1997 (2018), 020023.

27. B.E. Kanetov and N.N. Baigazieva, *On one property of uniform spaces*, AIP Conference Proceedings International Conference Functional Analysis in Interdisciplinary Applications, 1880 (2017), 030016.

28. M. Menger, *EinigeUberdeckungssatze der Punktmengenlehre*, Sitzungsberischte Abt. 2a, Mathematik, Astronomie, Physik, Meteorolgie und Mechanik (vienerAkademie, Wien), 133 (1924), 421-444.

29. M. Machura, S. Shelah, B. Tsaban, *Squares of Menger-bounded groups*, Trans. Amer. Math. Soc., 362 (2010), 1751-1764.

30. A.V. Osipov, *The functional characterizations of the Rothberger and Menger properties*, Topology Appl., 243 (2018), 146-152.

31. F. Rothberger, *EineVerscharfung der Eigenscafts*, Fund. Math., 30 (1938), 50-55.

32. M. Sakai, *Property* C'' *and function spaces*, Proc. Amer. Math. Soc., 104 (1988), 917-919.

33. M. Scheepers, *Combinatorics of open covers I: Ramsey theory*, Topology Appl., 69 (1996), 31-62.

34. U.A. Saktanov, E.N. Zhusupbekova, B.E. Kanetov, *Selection principles in topological vector spaces*, Kragujevac Journal of Mathematics., 50(10) (2026), 1621-1626.

35. B. Tsaban, *Selection principles and special sets of reals*, In: E. Pearl (ed.), Open Problems in Topology II, Elsevier Science, (2007), 91-108.

36. M.O. Zhanakunova and B.E. Kanetov, *On strongly uniformly paracompact spaces and mappings*, in AIP Conference Proceedings International Conference on Analysis and Applied Mathematics, 2325 (2021), 020030.

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