Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2026 (44)** : 1–7.
ISSN-0037-8712
doi:10.5269/bspm.81298

# Certain Study of Lorentzian-Para Sasakian Manifolds

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ABSTRACT: The present paper deals with certain study of Lorentzian para-Sasakian manifolds endowed with extended quasi-conformal curvature tensor. Specifically, we have considered Lorentzian para-Sasakian manifolds admitting extended quasi conformally $\phi$-flat, extended quasi conformally $\phi$-semi-symmetric and $K_e(\xi, U) \cdot S = 0$ conditions and characterize some important results.

Keywords: Lorentzian para-Sasakian manifolds, extended quasi conformal curvature tensor, $\eta$-Einstein manifold, scalar curvature.

## Contents



## 1. Introduction

The idea of Lorentzian para-Sasakian manifolds was first introduced by Matsumoto [10] in 1989. Again the same notion was studied by Mihai and Rosca [11] and obtained many interesting results. The Lorentzian para-Sasakian manifolds have also been weakened by Shaikh et al. [17] and many others (See [5,12,18,35]).

On the other hand, in [36], Yano and Sawaki introduced the notion of quasi conformal curvature tensor of a $n$-dimensional Riemannian manifold and is given by

$$\begin{aligned}\tilde{K}(U,Y)Z &= aR(U,Y)Z + b[S(Y,Z)U - S(U,Z)Y + g(Y,Z)QU - g(U,Z)QY] \\ &\quad -\frac{r}{n}[\frac{a}{n-1} + 2b]\{g(Y,Z)U - g(U,Z)Y\}, \qquad (1)\end{aligned}$$

where $U, Y, Z \in T_pM$ and $a$, $b$ are constants and $r$ is a scalar curvature. In particular, if $a = 1$ and $b = -\frac{1}{n-2}$, then the quasi conformal curvature tensor reduces to conformal curvature tensor. Later, the extended form of quasi conformal curvature tensor were developed by the authors in [7] and is given by:

$$K_e(U,Y)Z = \tilde{K}(U,Y)Z - \eta(U)\tilde{K}(\xi,Y)Z - \eta(Y)\tilde{K}(U,\xi)Z - \eta(Z)\tilde{K}(U,Y)\xi, \qquad (2)$$

for any $U, Y, Z \in T_pM$. In [37], the authors studied the extended quasi conformal curvature tensor on $N(k)$ contact metric manifold. Some related works can be found in [1-4,13-16,20-34].

The present paper is organized in the following; After preliminaries in the Section 2, we have studied extended quasi-conformally $\phi$-flat Lorentzian para-Sasakian manifold and proved that the manifold is $\eta$-Einstein with constant scalar curvatue and always admits an $\eta$-parallel Ricci tenor. Later in Section 4, we proved that an extended quasi conformally $\phi$-semi-symmetric Lorentzian para-Sasakian manifold is of constant scalar curvature and the parameters $a$ and $b$ are linearly dependent to each other. Finally, Section 5 is devoted to study of extended quasi conformal curvature tensor on Lorentzian para-Sasakian manifold satisfying $K_e(\xi, U) \cdot S = 0$ and obtained the Ricci tensor.

2020 *Mathematics Subject Classification:* 53C15, 53C25.
Submitted January 11, 2026. Published March 28, 2026

## 2. Preliminaries

An $n$-dimensional differentiable manifold $M$ is said to be an Lorentzian para-Sasakian manifold [10] if it admits a $(1,1)$-tensor field $\phi$, a unit time like vector field $\xi$, a 1-form $\eta$ and a Lorentzian metric $g$ such that:

$$\phi^2 U = U + \eta(U)\xi, \quad \eta(\xi) = -1 \tag{3}$$

$$g(\phi U, \phi Y) = g(U,Y) + \eta(U)\eta(Y), \tag{4}$$

$$(\nabla_U \phi)(Y) = g(U,Y)\xi + \eta(Y)U + 2\eta(U)\eta(Y)\xi, \tag{5}$$

for any $U, Y \in T_pM$, where $\nabla$ denotes the operator of covariant differentiation with respect to the Lorentzian metric $g$. It can be easily seen that in an Lorentzian para-Sasakian manifold, the following relations hold (See [10]):

$$\begin{aligned} \phi\xi &= 0, \\ \eta(\phi U) &= 0, \\ rank\phi &= n-1, \\ g(U,\xi) &= \eta(U). \end{aligned} \tag{6}$$

Again, an $n$-dimensional Lorentzian para-Sasakian manifold satisfies following conditions:

$$\begin{aligned} \Omega(U,Y) &= g(U,\phi Y), \\ \nabla_U \xi &= \phi U, \end{aligned}$$

for any $U, Y \in T_pM$, then the tensor field $\Omega(U,Y)$ is a symmetric $(0,2)$ tensor field (See [10]).
Also, in an n-dimensional Lorentzian para-Sasakian manifold, since the vector field $\eta$ is closed [10,11], then we have:

$$\begin{aligned} (\nabla_U \eta)(Y) &= \Omega(U,Y), \\ \Omega(U,\xi) &= 0, \end{aligned}$$

for any $U, Y \in T_pM$. Let $M$ be an $n$-dimensional Lorentzian para-Sasakian manifold with the structure $(\phi, \xi, \eta, g)$. Then the following relations hold (See [10]):

$$R(U,Y)Z = g(Y,Z)U - g(U,Z)Y, \tag{7}$$

$$S(U,\xi) = (n-1)\eta(U), \tag{8}$$

$$S(\phi U, \phi Y) = S(U,Y) + (n-1)\eta(U)\eta(Y), \tag{9}$$

for any $U, Y, Z \in T_pM$, where R is the Riemannian curvature tensor and S is the Ricci tensor of the manifold.
The Lorentzian para-Sasakian manifolds endowed with extended quasi-conformal curvature satisfies the following:

$$\begin{aligned} K_e(U,Y)Z = {} & [a - \frac{r}{n}(\frac{a}{n-1} + 2b)]\{g(Y,Z)U - g(U,Z)Y + g(U,Z)\eta(Y)\xi \\ & -g(Y,Z)\eta(U)\xi + 2\eta(U)\eta(Z)Y - 2\eta(Y)\eta(Z)U\} \\ & +b\{S(Y,Z)U - S(U,Z)Y + \eta(Y)S(U,Z)\xi - \eta(U)S(Y,Z)\xi \\ & +g(Y,Z)QU - g(U,Z)QY + (n-1)\eta(Y)g(U,Z)\xi - (n-1)\eta(U)g(Y,Z)\xi \\ & +2\eta(U)\eta(Z)QY - 2\eta(Y)\eta(Z)QU + 2(n-1)\eta(U)\eta(Z)Y - 2\eta(Y)\eta(Z)U\}, \end{aligned} \tag{10}$$

$$\begin{aligned} K_e(\xi,Y)Z = {} & [a - \frac{r}{n}(\frac{a}{n-1} + 2b)]\{2g(Y,Z)\xi - 3\eta(Z)Y - \eta(Y)\eta(Z)\xi\} \\ & +b[2S(Y,Z)\xi - 3(n-1)\eta(Z)Y + 2(n-1)g(Y,Z)\xi - 3\eta(Z)QY \\ & -2(n-1)\eta(Y)\eta(Z)\xi] = -K_e(Y,\xi)Z, \end{aligned} \tag{11}$$

$$\begin{aligned} K_e(U,Y)\xi = {} & [a - \frac{r}{n}(\frac{a}{n-1} + 2b)][3\eta(Y)U - 3\eta(U)Y] \\ & +b[3(n-1)\eta(Y)U - 3(n-1)\eta(U)Y + 3\eta(Y)QU - 3\eta(U)QY], \end{aligned} \tag{12}$$

$$K_e(\xi,\xi)Z = 0. \tag{13}$$

**Definition 2.1** *An $n$-dimensional Lorentzian para-Sasakian manifold is said to be $\eta$-Einstein, if its Ricci tensor $S$ is of the form*

$$S(U,Y) = Ag(U,Y) + B\eta(U)\eta(Y),$$

*for all $U, Y \in T_pM$, where $A$ and $B$ are constants. If $B = 0$, then the manifold reduces to Einstein.*

**Definition 2.2** *In an $n$-dimensional Lorentzian para-Sasakian manifold, if Ricci tensor $S$ satisfies*

$$(\nabla_W S)(\phi U, \phi Y) = 0,$$

*for all $U, Y \in T_pM$, then the Ricci tensor is said to be $\eta$-parallel.*

## 3. Extended Quasi-Conformally $\phi$-Flat Lorentzian Para-Sasakian Manifold

**Definition 3.1** *A $n$-dimensional Lorentzian para-Sasakian manifold is said to be extended quasi-conformally $\phi$-flat if*

$$g(K_e(\phi U, \phi Y)\phi Z, \phi W) \quad = \quad 0, \tag{14}$$

*for all $U, Y, Z, W \in T_pM$.*

It follows from (14) that

$$\begin{aligned}
&[a - \frac{r}{n}(\frac{a}{n-1} + 2b)][g(\phi Y, \phi Z)g(\phi U, \phi W) - g(\phi U, \phi Z)g(\phi Y, \phi W)] \\
&+b[S(\phi Y, \phi Z)g(\phi U, \phi W) - S(\phi U, \phi Z)g(\phi Y, \phi W) + g(\phi Y, \phi Z)S(\phi U, \phi W) \\
&-g(\phi U, \phi Z)S(\phi Y, \phi W)] = 0.
\end{aligned} \tag{15}$$

On plugging $U = W = e_i$, where $e_i$ is an orthonormal basis for the tangent space at each point of the manifold and taking the summation over $i$, $i = 1, 2, \cdots, n$, we get:

$$\begin{aligned}
&[a - \frac{r}{n}(\frac{a}{n-1} + 2b)][(n-2)g(\phi Y, \phi Z)] \\
&+b[(n-3)S(\phi Y, \phi Z) + rg(\phi Y, \phi Z)] = 0.
\end{aligned} \tag{16}$$

Now it follows from the above equation that:

$$\begin{aligned}
S(Y,Z) \quad = \quad & \frac{a(n-2)[r - n(n-1)] + br(n-1)(n-4)}{bn(n-1)(n-3)} g(Y,Z) \\
& + \frac{a(n-2)[r - n(n-1)] + b(n-1)[r(n-4) - n(n-1)(n-3)]}{bn(n-1)(n-3)} \eta(Y)\eta(Z).
\end{aligned} \tag{17}$$

Hence we can state the following:

**Theorem 3.1** *An extended quasi conformally $\phi$-flat Lorentzian para-Sasakian manifold is an $\eta$-Einstein manifold.*

Again putting $Y = Z = e_i$ in (17), yields

$$r = \frac{an(n-1)(n-2)^2 + bn(n-1)^2(n-3)}{a(n-2)^2 + b(n-1)(-3n+8)}. \tag{18}$$

Hence this leads us to the following:

**Corollary 3.1** *In an extended quasi conformally $\phi$-flat Lorentzian para-Sasakian manifold, the scalar curvature is constant.*

Now replacing $Y$ by $\phi Y$ and $Z$ by $\phi Z$ in (17) and then taking an account of (6), we obtain

$$S(\phi Y, \phi Z) = \frac{a(n-2)[r-n(n-1)] + br(n-1)(n-4)}{bn(n-1)(n-3)} g(\phi Y, \phi Z). \tag{19}$$

Differentiating (19) covariantly with respect $X$, we get

$$(\nabla_X S)(\phi Y, \phi Z) = dr(X) \frac{a(n-2) + b2(n-1)(n-2)}{bn(n-1)(n-3)} g(\phi Y, \phi Z). \tag{20}$$

Since extended quasi conformally $\phi$-flat Lorentzian para-Sasakian manifold is of constant scalar curvature, then the above equation reduces to

$$(\nabla_X S)(\phi Y, \phi Z) = 0. \tag{21}$$

Hence this leads us to the following:

**Corollary 3.2** *An extended quasi conformally $\phi$-flat Lorentzian para-Sasakian manifold with constant scalar curvature admits a $\eta$-parallel Ricci tensor.*

## 4. Extended Quasi-Conformally $\phi$-Semi-Symmetric Lorentzian Para-Sasakian Manifold

**Definition 4.1** *An $n$-dimensional Lorentzian para-Sasakian manifold is said to be extended quasi conformally $\phi$-semi-symmetric if $K_e(U,Y) \cdot \phi = 0$.*

The above condition turns into

$$K_e((U,Y) \cdot \phi)Z = K_e(U,Y)\phi Z - \phi K_e(U,Y)Z = 0, \tag{22}$$

for any $U, Y, Z \in T_pM$.
In view of (1), (2), (11) and (12), it follows from (22) that

$$\begin{aligned}
&[a - \frac{r}{n}(\frac{a}{n-1} + 2b)]\{g(Y,\phi Z)\phi U - g(U,Z)\phi Y + 2\eta(U)\eta(Z)\phi Y - 2\eta(Y)\eta(Z)\phi U\} \\
&+b\{S(Y,Z)\phi U - S(U,Z)\phi Y + g(Y,Z)\phi(QU) - g(U,Z)\phi(QY) + 2\eta(U)\eta(Z)\phi(QY) \\
&-2\eta(Y)\eta(Z)\phi(QU) + 2(n-1)\eta(U)\eta(Z)\phi Y - 2(n-1)\eta(Y)\eta(Z)\phi U\} = 0.
\end{aligned} \tag{23}$$

Now taking inner product of (23) with respect to $W$, yields:

$$\begin{aligned}
&[a - \frac{r}{n}(\frac{a}{n-1} + 2b)]\{g(Y,\phi Z)g(\phi U, W) - g(U,Z)g(\phi Y, W) + 2\eta(U)\eta(Z)g(\phi Y, W) \\
&-2\eta(Y)\eta(Z)g(\phi U, W)\} + b\{S(Y,Z)g(\phi U, W) - S(U,Z)g(\phi Y, W) + g(Y,Z)S(\phi U, W) \\
&-g(U,Z)S(\phi Y, W) + 2\eta(U)\eta(Z)S(\phi Y, W) - 2\eta(Y)\eta(Z)S(\phi U, W) \\
&+2(n-1)\eta(U)\eta(Z)g(\phi Y, W) - 2(n-1)\eta(Y)\eta(Z)g(\phi U, W)\} = 0.
\end{aligned} \tag{24}$$

Putting $U = W = e_i$, where $e_i$ is an orthonormal basis for the tangent space at each point of the manifold and taking summation over $i$, $i = 1, 2, \cdots, n$, we get:

$$\begin{aligned}
&[a - \frac{r}{n}(\frac{a}{n-1} + 2b)]\{g(Y,Z)\alpha - g(\phi Y, Z) - 2\eta(Y)\eta(Z)\alpha\} \\
&+b\{S(Y,Z)\alpha - S(\phi Y, Z) + g(Y,Z)\beta - S(\phi Y, Z) \\
&-2\eta(Y)\eta(Z)\beta - 2(n-1)\eta(Y)\eta(Z)\alpha\} = 0.
\end{aligned} \tag{25}$$

From which it follows that

$$a[n(n-1)\alpha - r] + b[(n(n-1)^2 - 2r(n-1))\alpha + n(n-1)\beta] = 0. \tag{26}$$

Hence we can state the following:

**Theorem 4.1** *In an $n$-dimensional extended quasi conformally $\phi$-semi-symmetric Lorentzian para-Sasakian manifold, $a$ and $b$ are linearly dependent to each other.*

Again from (26), it follows that:

$$r = \frac{n(n-1)\alpha a + (2n(n-1)^2\alpha + n(n-1)\beta)b}{a + 2(n-1)\alpha b}. \tag{27}$$

Hence this leads us to the following:

**Corollary 4.1** *In an extended quasi conformally $\phi$-semi-symmetric Lorentzian para-Sasakian manifold, the scalar curvature is constant which is given by (27).*

## 5. Extended Quasi-Conformal Curvature Tensor on Lorentzian Para-Sasakian Manifold Satisfying $K_e(\xi, U) \cdot S = 0$

Let us consider a Lorentzian para-Sasakian manifold satisfying $K_e(\xi, U) \cdot S = 0$.

Now it follows from the above condition that

$$S(K_e(\xi, U)Y, \xi) + S(Y, K_e(\xi, U)\xi) = 0. \tag{28}$$

In view of (11), it follows that

$$\begin{aligned}&[a - \frac{r}{n}(\frac{a}{n-1} + 2b)]\{3S(Y,U) - 2(n-1)g(Y,U) + (n-1)\eta(Y)\eta(U)\} \\ &+b\{3S(Y,QU) + (n-1)S(Y,U) - 2(n-1)^2 g(Y,U) - 2(n-1)2\eta(Y)\eta(U)\} = 0.\end{aligned} \tag{29}$$

By using above equation (29), we get

$$\begin{aligned}3bS(Y,QU) &= (-3[a - \frac{r}{n}(\frac{a}{n-1} + 2b)] - b(n-1))S(Y,U) \\ &\quad +\{2(n-1)[a - \frac{r}{n}(\frac{a}{n-1} + 2b)] + 2b(n-1)^2\}g(Y,U) \\ &\quad +\{(n-1)[a - \frac{r}{n}(\frac{a}{n-1} + 2b)] + 2b(n-1)^2\}\eta(Y)\eta(U).\end{aligned} \tag{30}$$

Hence we can state the following:

**Theorem 5.1** *If an extended quasi conformal curvature tensor on $n$-dimensional Lorentzian para-Sasakian manifold satisfies $K_e(\xi, U) \cdot S = 0$, then the Ricci tensor is given by (30).*

Plugging $Y = U = e_i$, where $e_i$ is an orthonormal basis for the tangent space at each point of the manifold and taking summation over $i$, $i = 1, 2, \cdots, n$, we get:

$$\begin{aligned}3b\,\|Q\|^2 &= \frac{a[3r^2 - r(n-1)(5n-1) + n(n-1)2(2n-1)]}{n(n-1)} \\ &\quad + \frac{b[r^2 - r(n-1)(5n-2) + 2n(n-1)2(n-2)]}{n}\end{aligned} \tag{31}$$

Hence this leads us to the following:

**Corollary 5.1** *In an $n$-dimensional Lorentzian para-Sasakian manifold satisfying $K_e(\xi, U) \cdot S = 0$, then the norm of a Ricci operator is given by (31).*

**Acknowledgments**

The authors thank the anonymous reviewers for their careful reading of our manuscript and their many insightful comments and suggestions.

## References


1. Alloush, K. A. A., Rajendra, R., Siva Kota Reddy, P., Pavani, N., Somashekhara, G. and Shivaprasanna, G. S., *Geometry of $\eta$-Ricci Yamabe Soliton on Nearly Sasakian Manifold*, Bol. Soc. Parana. Mat. (3), 43, Article Id: 70582, 8 Pages, (2025).
2. Angadi, P. G., Shivaprasanna, G. S., Somashekhara, G. and Siva Kota Reddy, P., *Ricci-Yamabe solitons on submanifolds of some indefinite almost contact manifolds*, Adv. Math., Sci. J., 9(11), 10067–10080, (2020).
3. Angadi, P. G., Shivaprasanna, G. S., Somashekhara, G. and Siva Kota Reddy, P., *Ricci Solitons on $(LCS)$-Manifolds under D-Homothetic Deformation*, Italian Journal of Pure & Applied Mathematics, 46, 672–683, (2021).
4. Angadi, P. G., Siva Kota Reddy, P., Shivaprasanna, G. S., and Somashekhara, G., *On Weakly Symmetric Generalized $(k, \mu)$-Space Forms*, Proc. Jangjeon Math. Soc., 25(2), 133-144, (2022).
5. Bagewadi, C. S., Venkatesha and Basavarajappa, N. S., *On LP-Sasakian manifolds*, Sci., Ser. A, Math. Sci. (N.S.), 16. 1–8, (2008).
6. De, U. C. and Guha, N., *On Generalized Recurrent Manifolds*, J. Natl. Acad. Math. India, 9, 85–92, (1991).
7. De, U. C. and Sarkar, A., *On the quasi-conformal curvature tensor of a $(k, \mu)$-contact metric manifold*, Math. Rep., Buchar., 14(2), 115–129, (2012).
8. Hamilton, R. S., *The Ricci flow on surfaces*, Contemp. Math., 71, 237-262 (1988).
9. Kobayashi, S. and Nomizu, K., *Foundations of differential geometry I*, Intersci. Tracts Pure Appl. Math., 15, Interscience Publishers, a division of John Wiley & Sons., xi, 329 pages, (1963).
10. Matsumoto, K., *On Lorentzian paracontact manifolds*, Bull. Yamagata Univ., Nat. Sci., 12(2), 151–156, (1989).
11. Mihai, I. and Rosca, R., *On Lorentzian P-Sasakian Manifolds*, Classical Analysis, World Scientific Publ., Signapore, 155–169, (1992).
12. Mihai, I., Shaikh, A. A., De, U. C., *On Lorentzian Para-Sasakian Manifolds*, Korean J. Math. Sci., 6, 1–13, (1999).
13. Nagaraja, H. G., Dipansha Kumari and Siva Kota Reddy, P., *Submanifolds of $(k, \mu)$- Contact Metric Manifold as Ricci Solitons*, Proc. Jangjeon Math. Soc., 24(1), 11-19, (2021).
14. Naveen Kumar, R. T., Siva Kota Reddy, P., Venkatesha and Sangeetha, M., *Certain Results on $(k, \mu)$-Contact Metric Manifold endowed with Concircular Curvature Tensor*, Commun. Math. Appl., 14(1), 215-225, (2023).
15. Naveen Kumar, R. T., Siva Kota Reddy, P. and Venkatesha, *Certain Results of $(LCS)_n$-Manifolds Endowed with E-Bochner Curvature Tensor*, Bol. Soc. Parana. Mat. (3), 42, Article Id: 65813, 8 Pages, (2024).
16. Naveen Kumar, R. T., Siddesha, M. S., Swapna Sangeetha, P., Sowmyashree, B. M. and Siva Kota Reddy, P., *Characterization of Generalized Ricci-Type Solitons on Lorentzian Para-Kenmotsu Manifolds*, *Dyn. Contin. Discrete Impuls. Syst., Ser. A, Math. Anal.*, 32(4), 271–281, (2025).
17. Shaikh, A. A. and Baishya, K. K., *Some results on LP-Sasakian manifolds*, Bull. Math. Soc. Sci. Math. Roum., Nouv. Sér., 49(2), 193–205, (2006).
18. Shaikh, A. A., Baishya, K. K. and Eyasmin, S., *On the existence of some types of LP-Sasakian manifolds*, Commun. Korean Math. Soc., 23(1), 95-110, (2008).
19. Shaikh, A. A. and Biswas, S., *On LP-Sasakian manifolds*, Bull. Malays. Math. Sci. Soc. (2), 27(1), 17–26, (2004).
20. Shivaprasanna, G. S., Rajendra, R., Somashekhara, G. and Siva Kota Reddy, P., *On Submanifolds of a Sasakian Manifold*, Bol. Soc. Parana. Mat. (3), 42, Article Id: 66247, 8 Pages, (2024).
21. Shivaprasanna, G. S., Rajendra, R., Siva Kota Reddy, P., Somashekhara, G. and Pavithra, M., *Almost Ricci-Yamabe Solitons in $f$-Kenmotsu Manifolds*, Bol. Soc. Parana. Mat. (3), 43, Article Id: 69758, 8 Pages, (2025).
22. Somashekhara, G., Pavani, N. and Siva Kota Reddy, P., *Invariant Sub-manifolds of LP-Sasakian Manifolds with Semi-Symmetric Connection*, Bull. Math. Anal. Appl., 12(2), 35–44, (2020).
23. Somashekhara, G., Girish Babu, S. and Siva Kota Reddy, P., *Indefinite Sasakian Manifold with Quarter-Symmetric Metric Connection*, Proc. Jangjeon Math. Soc., 24(1), 91–98, (2021).
24. Somashekhara, G., Girish Babu, S. and Siva Kota Reddy, P., *Conformal Ricci Soliton in an Indefinite Trans-Sasakian manifold*, Vladikavkaz Math. J, 23(3), 43–49, (2021).
25. Somashekhara, G., Girish Babu, S. and Siva Kota Reddy, P., *Ricci Solitons and Generalized Weak Symmetries under D-Homothetically Deformed LP-Sasakian Manifolds*, Ital. J. Pure Appl. Math., 46, 684–695, (2021).
26. Somashekhara, G., Girish Babu, S. and Siva Kota Reddy, P., *Conformal $\eta$-Ricci Solitons in Lorentzian Para-Sasakian Manifold Admitting Semi-Symmetric Metric Connection*, Ital. J. Pure Appl. Math., 46, 1008–1019, (2021).
27. Somashekhara, G., Siva Kota Reddy, P., Shivashankara, K. and Pavani, N., *Slant Sub-manifolds of Generalized Sasakian-Space-Forms*, Proc. Jangjeon Math. Soc., 25(1), 83–88, (2022).
28. Somashekhara, G., Girish Babu, S., Siva Kota Reddy, P. and Shivashankara, K., *On LP-Sasakian Manifolds admitting Generalized Symmetric Metric Connection*, Proc. Jangjeon Math. Soc., 25(3), 287–296, (2022).

29. Somashekhara, G., Girish Babu, S. and Siva Kota Reddy, P., *$\eta$-Ricci soliton in an indefinite trans-Sasakian manifold admitting semi-symmetric metric connection*, Bol. Soc. Parana. Mat. (3), 41, Article Id: 51358, 9 Pages, (2023).

30. Somashekhara, G., Girish Babu, S. and Siva Kota Reddy, P., *$\eta$-Ricci soliton in an indefinite trans-Sasakian manifold admitting semi-symmetric metric connection*, Bol. Soc. Parana. Mat. (3), 41, 1–9, (2023).

31. Somashekhara, G., Rajendra, R., Shivaprasanna, G. S. and Siva Kota Reddy, P., *Pseudo Parallel and Generalized Ricci Pseudo Parallel Invariant Submanifolds of a Generalized Sasakian Space Form*, Proc. Jangjeon Math. Soc., 26(1), 69–78, (2023).

32. Somashekhara, P., Naveen Kumar, R. T., Siva Kota Reddy, P., Venkatesha and Alloush, K. A. A., *Pseudo Projective Curvature Tensor on Generalized Sasakian Space Forms*, Proc. Jangjeon Math. Soc., 26(3), 243–251, (2023).

33. Sowmyashree, B. M., Siva Kota Reddy, P., Pavithra, M., Shivaprasanna, G. S. and Somashekhara, G., *$\eta$-Ricci Yamabe Solitons on Pseudo-Projective Flat LP-Sasakian Space-Time Manifold*, *Dyn. Contin. Discrete Impuls. Syst., Ser. B, Appl. Algorithms*, 32(3), 163–176, (2025).

34. Sowmyashree, B. M., Aruna Kumara, H. and Siva Kota Reddy, P., *Geometry of $(\kappa,\mu)'$-almost Kenmotsu manifolds with divergence free Cotton tensor and vanishing Bach tensor*, Bol. Soc. Parana. Mat. (3), 43, Article Id: 76100, 9 Pages, (2025).

35. Yano, K. and Kon, M., *Structures on manifolds*, Ser. Pure Math., 3, World Scientific, Hackensack, NJ, (1984).

36. Yano, K. and Sawaki, S., *Riemannian manifolds admitting a conformal transformation group*, J. Differ. Geom., 2, 161–184, (1968).

37. Venkatesha, kumar, H. A. and Naveen Kumar, R. T., *Extended quasi conformal curvature on $N(k)$-contact metric manifold*, International J.Math. Combin., 1, 41-50, (2018).

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