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$\mathrm{C_G}$-Class Function Approach to Random Coincidence Point Results for Weakly Increasing Contraction Mappings

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ABSTRACT: This paper aims to develop random coincidence point theorems for weakly increasing random operators within the framework of ordered metric spaces, utilizing generalized altering distance functions in conjunction with $\mathrm{C_G}$-**class** functions. The findings offer stochastic extensions and generalizations of several established results from the existing literature.

Keywords: Fixed point, $\mathrm{C_G}$-class function, random coincidence point, weakly increasing contraction.

Contents

1. Introduction

The theory of random metric fixed points serves as a significant stochastic extension of classical fixed point theory. It plays a critical role in the study of random differential and integral equations and finds applications in various fields such as optimization, variational inequalities, and approximation theory. Particularly, random fixed point results for contractive mappings in Polish spaces are foundational. Initial contributions in this area were made by Ŝpaĉek [37] and Hans [16,17], with further attention drawn to the topic following a comprehensive survey by Bharucha-Reid [11].

Several random fixed point results have proven essential in analyzing random integral and differential equations [5,8,20,23,24,25,27]. Notably, Sehgal and Singh [36] established stochastic analogs of Schauder fixed point theorem. Additionally, random coincidence point results, which generalize classical coincidence theorems in a stochastic context, have garnered attention. Recent advancements include coupled random fixed and coincidence point theorems in partially ordered complete metric spaces, as demonstrated by Ćirić and Lakshmikantham [15], and Hussain et al. [19].

Khan et al. [22] introduced the concept of an altering distance function, which was effectively utilized to address fixed point problems in metric spaces. This approach has since been extended by several authors [6,7,13] to establish new fixed point results. Additional insights can be found in [14,32,34]. In 2005, Choudhury [12] proposed a generalized three-variable distance function and derived a common fixed point theorem for a pair of self-mappings in a complete metric space. Later, Nashine and Aydi [31] broadened the work of Nashine et al. [30] by introducing a four-variable framework, obtaining coincidence and common fixed point theorems in complete ordered metric spaces under contractive conditions involving two generalized altering distance functions.

Building on these developments, the present work establishes random coincidence point results for weakly contractive mappings in ordered metric spaces. These results are derived using generalized altering distance functions and a newly introduced class of functions developed recently by the first author.

Definition 1.1 *[22] A function $\phi : [0, +\infty) \to [0, +\infty)$ is called an altering distance function if and only if*

(i) ϕ is continuous,
(ii) ϕ is nondecreasing,
(iii) $\phi(t) = 0 \Leftrightarrow t = 0$.

In 1997, Alber and Guerre-Delabriere [4] introduced the concept of weak contractions in Hilbert spaces. This concept was extended to metric spaces by Rhoades in [34].

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Definition 1.2 *A mapping $T : X \to X$, where (X, d) is a metric space, is said to be weakly contractive if and only if for all $x, y \in X$:*

$$d(Tx, Ty) \leq d(x, y) - \phi(d(x, y)),$$

where $\phi : [0, +\infty) \to [0, +\infty)$ is an altering distance function.

In [12], Choudhury introduced the concept of a generalized distance function for three variables.

Definition 1.3 *[12] A function $\phi : [0, +\infty)^3 \to [0, +\infty)$ is said to be a generalized altering distance function if and only if*

(i) ϕ is continuous,
(ii) ϕ is nondecreasing in all three variables,
(iii) $\phi(x, y, z) = 0 \Leftrightarrow x = y = z = 0$.
Define $\psi(x) = \phi(x, x, x)$ for $x \in [0, \infty)$. Clearly, $\psi(x) = 0$ if and only if $x = 0$. Examples of ϕ are
$\phi(a, b, c) = k \max\{a, b, c\}$, for $k > 0$,
$\phi(a, b, c) = a^p + b^q + c^r, p, q, r \geq 1$,
$\phi(a, b, c) = (a + \alpha b^q) r + c^s$, where $p, q, r, s \geq 1$ and $\alpha > 0$.
Rao et al. [33] generalized the above definition for four variables.

Definition 1.4 *[33] A function $\phi : [0, +\infty)^4 \to [0, +\infty)$ is said to be a generalized altering distance function if and only if*

(i) ϕ is continuous,
(ii) ϕ is nondecreasing in all four variables,
(iii) $\phi(t_1, t_2, t_3, t_4) = 0 \Leftrightarrow t_1 = t_2 = t_3 = t_4 = 0$.

Definition 1.5 *[21] Let (X, d) be a metric space and $f, g : X \to X$. If $\gamma = fx = gx$, for some $x \in X$, then x is called a coincidence point of f and g, and γ is called a point of coincidence of f and g. The pair $\{f, g\}$ is said to be compatible if and only if $\lim_{n\to\infty} d(fgx_n, gfx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some $t \in X$.*

Let X be a nonempty set and $R : X \to X$ be a given mapping. For every $x \in X$, we denote by $R^{-1}(x)$ the subset of X defined by

$$R^{-1}(x) = \{u \in X \mid Ru = x\}.$$

Definition 1.6 *[29] Let $(X, \preceq)$ be a partially ordered set and $T, S, R : X \to X$ be given mappings such that $TX \subseteq RX$ and $SX \subseteq RX$. We say that S and T are weakly increasing with respect to R if and only if for all $x \in X$, we have*

$$Tx \preceq Sy \ \text{ for all } \ y \in R^{-1}(Tx) \ \text{ and } Sx \preceq Ty \ \text{ for all } y \in R^{-1}(Sx).$$

Theorem 1.1 *Let $(X, d, \preceq)$ be an ordered complete metric space. Let $T, S, R : X \to X$ be given mappings satisfying the following inequality for every pair $(x, y) \in X \times X$ with Rx and Ry comparable,*

$$\begin{aligned}\phi_1(d(Sx, Ty)) \leq\ & \psi_1(d(Rx, Ry), d(Rx, Sx), d(Ry, Ty), \\ & \frac{1}{2}[d(Rx, Ty) + d(Ry, Sx)]) \\ & -\psi_2(d(Rx, Ry), d(Rx, Sx), d(Ry, Ty), \\ & \frac{1}{2}[d(Rx, Ty) + d(Ry, Sx)])\end{aligned}$$

where ψ_1 and ψ_2 are generalized altering distance functions and $\phi_1(x) = \psi_1(x, x, x, x)$. One assumes the following hypotheses:

(i) T, S and R are continuous,

(ii) $TX \subseteq RX$, $SX \subseteq RX$,

(iii) T and S are weakly increasing with respect to R,

(iv) the pairs $\{T, R\}$ and $\{S, R\}$ are compatible.

Then, T, S and R have a coincidence point, that is, there exists $u \in X$ such that $Ru = Tu = Su$.

Definition 1.7 *Let $(X, d, \preceq)$ be a partially ordered metric space. We say that X is regular if and only if the following hypothesis holds: if $\{z_n\}$ is a nondecreasing sequence in X with respect to $\preceq$ such that $z_n \to z \in X$ as $n \to +\infty$, then $z_n \preceq z$ for all $n \in N$.*

In the next theorem, Nashine and Aydi [31] omitted the continuity hypothesis satisfied by T, S and R in Theorem 1.

Theorem 1.2 *Let $(X, d, \preceq)$ be an ordered complete metric space. Let $T, S, R : X \to X$ be given mappings satisfying the following inequality for every pair $(x, y) \in X \times X$ with Rx and Ry comparable,*

$$\begin{aligned} \phi_1(d(Sx, Ty)) \leq\ & \psi_1(d(Rx, Ry), d(Rx, Sx), d(Ry, Ty), \\ & \frac{1}{2}[d(Rx, Ty) + d(Ry, Sx)]) \\ & -\psi_2(d(Rx, Ry), d(Rx, Sx), d(Ry, Ty), \\ & \frac{1}{2}[d(Rx, Ty) + d(Ry, Sx)]) \end{aligned}$$

where ψ_1 and ψ_2 are generalized altering distance functions and $\phi_1(x) = \psi_1(x, x, x, x)$. We assume the following hypotheses:

(i) X is regular,

(ii) T and S are weakly increasing with respect to R,

(iii) RX is a complete subspace of (X, d),

(iv) $TX \subseteq RX, SX \subseteq RX$.

Then, T, S and R have a coincidence point.

In 2014, the concept of C-class functions (see Definition 1.8) was introduced by A.H. Ansari in [2]. C-class functions play a crucial role in extending and generalizing fixed point theorems, particularly in more complex or abstract settings like ordered metric spaces or random fixed point theory.

Definition 1.8 *A mapping $F : [0, \infty)^2 \to \mathbb{R}$ is called C-class function if it is continuous and satisfies following axioms:*

(1) $F(s, t) \leq s$;

(2) $F(s, t) = s$ implies that either $s = 0$ or $t = 0$; for all $s, t \in [0, \infty)$.

Note that for some F we have that $F(0, 0) = 0$. The set of C-class functions is denoted $\mathcal{C}$.

Example 1.1 The following are elements of $\mathcal{C}$:

(1) $F(s, t) = s - t$, $F(s, t) = s \Rightarrow t = 0$;

(2) $F(s, t) = ms$, $0<m<1$, $F(s, t) = s \Rightarrow s = 0$;

(3) $F(s, t) = \frac{s}{(1+t)^r}$; $r \in (0, \infty)$, $F(s, t) = s \Rightarrow s = 0$ or $t = 0$;

(4) $F(s, t) = \log(t + a^s)/(1 + t)$, $a > 1$, $F(s, t) = s \Rightarrow s = 0$ or $t = 0$;

(5) $F(s, t) = \ln(1 + a^s)/2$, $a > e$, $F(s, 1) = s \Rightarrow s = 0$;

(6) $F(s, t) = (s + l)^{(1/(1+t)^r)} - l$, $l > 1, r \in (0, \infty)$, $F(s, t) = s \Rightarrow t = 0$;

(7) $F(s, t) = s \log_{t+a} a$, $a > 1$, $F(s, t) = s \Rightarrow s = 0$ or $t = 0$;

(8) $F(s, t) = s - (\frac{1+s}{2+s})(\frac{t}{1+t})$, $F(s, t) = s \Rightarrow t = 0$;

(9) $F(s, t) = s\beta(s)$, $\beta : [0, \infty) \to [0, 1)$, $F(s, t) = s \Rightarrow s = 0$;

(10) $F(s, t) = s - \frac{t}{k+t}$, $F(s, t) = s \Rightarrow t = 0$;

(11) $F(s,t) = s - \varphi(s), F(s,t) = s \Rightarrow s = 0$, here $\varphi : [0,\infty) \to [0,\infty)$ is a continuous function such that $\varphi(t) = 0 \Leftrightarrow t = 0$;

(12) $F(s,t) = sh(s,t), F(s,t) = s \Rightarrow s = 0$, here $h : [0,\infty) \times [0,\infty) \to [0,\infty)$ is a continuous function such that $h(t,s) < 1$ for all $t, s > 0$;

(13) $F(s,t) = s - (\frac{2+t}{1+t})t$, $F(s,t) = s \Rightarrow t = 0$.

(14) $F(s,t) = \sqrt[n]{\ln(1+s^n)}$, $F(s,t) = s \Rightarrow s = 0$.

(15) $F(s,t) = \phi(s), F(s,t) = s \Rightarrow s = 0$, here $\phi : [0,\infty) \to [0,\infty)$ is a upper semicontinuous function such that $\phi(0) = 0$, and $\phi(t) < t$ for $t > 0$,

(16) $F(s,t) = \frac{s}{(1+s)^r}$; $r \in (0,\infty)$, $F(s,t) = s \Rightarrow s = 0$;

Definition 1.9 *An ultra altering distance function is a continuous, nondecreasing mapping* $\varphi : [0,\infty) \to [0,\infty)$ *such that* $\varphi(t) > 0$ *for all* $t > 0$.

Definition 1.10 *A function* $\phi : [0,+\infty)^4 \to [0,+\infty)$ *is said to be a generalized ultra altering distance function if and only if*

(i) ϕ is continuous,
(ii) ϕ is nondecreasing in all four variables,
(iii) $\phi(t_1,t_2,t_3,t_4) > 0$ for all $(t_1,t_2,t_3,t_4) \neq (0,0,0,0)$,.

In 2022, the concept of generalized C_G-class functions (see Definition 1.11) was introduced by Kumssa in [10].

Definition 1.11 *[10]A mapping* $\mathcal{I}_G$*:* $\mathbb{R}_+^3 \to \mathbb{R}$ *is called a generalized* C_G*-class function if for all* $s,t,r \in \mathbb{R}_+$*,*

1. *$\mathcal{I}_G$ is continuous;*
2. *$\mathcal{I}_G(s,t,r) \leq \max\{s,t\}$;*
3. *$\mathcal{I}_G(s,t,r) = s$ or t implies that either of s,t or r is zero.*

Example 1.2 In the following, we give some members of generalized C_G-class functions, where $\mathcal{I}_G$: $\mathbb{R}_+^3 \to \mathbb{R}$ is given as:

1. $\mathcal{I}_G(s,t,r) = F(s,t)$, where F is a C-class function, $\mathcal{I}_G(s,t,r) = s$ implies $s = 0$ or $t = 0$.
2. $\mathcal{I}_G(s,t,r) = \frac{s}{(1+t)(1+r)}$, $\mathcal{I}_G(s,t,r) = s$ implies $s = 0$ or $t = r = 0$.
3. $\mathcal{I}_G(s,t,r) = h(r)s$, where $h : [0,\infty) \to [0,1), h(0) = 0$,is a continuous function such that,$h(0) = 0$,$h(t) \neq 0$ for all $t \neq 0$, $\mathcal{I}_G(s,t,r) = s$ implies $s = 0$ or $t = 0$.
4. $\mathcal{I}_G(s,t,r) = e^{-r}s$, $\mathcal{I}_G(s,t,r) = s$ implies $s = 0$ or $t = 0$.
5. $\mathcal{I}_G(s,t,r) = F(s,t+r)$, where F is a C-class function, $\mathcal{I}_G(s,t,r) = s$ implies $s = 0$ or $t = r = 0$.
6. $\mathcal{I}_G(s,t,r) = \frac{s}{(1+t+r)}$, $\mathcal{I}_G(s,t,r) = s$ implies $s = 0$ or $t = r = 0$.

Also see [10].

2. Main Results

Let (Ω,Σ) be a measurable space with a Σ sigma algebra of subsets of Ω and let (X,d) be a metric space. A mapping $\zeta : \Omega \to X$ is called $\Sigma -$ measurable if for any open subset O of X, $\zeta^{-1}(O) \in \Sigma$. A mapping $S : \Omega \times X \to X$ is said to be a random map if and only if for each fixed $x \in X$, the mapping $S(.,x) : \Omega \to X$ is measurable. A random map $S : \Omega \times X \to X$ is continuous if for each $\omega \in \Omega$, the mapping $S(\omega,.) : X \to X$ is continuous. A measurable mapping $\zeta : \Omega \to X$ is a random fixed point of the random map $S : \Omega \times X \to X$ if and only if $S(\omega,\zeta(\omega)) = \zeta(\omega)$ for each $\omega \in \Omega$.

Definition 2.1 *A measurable mapping $\zeta : \Omega \to K$, where K is a Polish subspace of X, is said to be*

(i) a random fixed point of $R : \Omega \times K \to K$, if for each $\omega \in \Omega$,

$$\zeta(\omega) = R(\omega, \zeta(\omega)).$$

(ii) a random coincidence point of $R : \Omega \times K \to K$, $S : \Omega \times K \to K$ and $T : \Omega \times K \to K$ if for each $\omega \in \Omega$,

$$R(\omega, \zeta(\omega)) = S(\omega, \zeta(\omega)) = T(\omega, \zeta(\omega)).$$

Definition 2.2 *Let (X, d) be a separable metric space and (Σ, Ω) be a measurable space. The pair $\{f, g\}$ is said to be compatible random operator pair if and only if*

$$\lim_{n \to +\infty} d(f(\omega, g(\omega, x_n)), g(\omega, f(\omega, x_n))) = 0$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to+\infty} f(\omega, x_n) = \lim_{n\to+\infty} g(\omega, x_n) = t$ for some $t \in X$ and for each $\omega \in \Omega$.

Now, we state our main result.

Theorem 2.1 *Let $(X, d, \preceq)$ be a separable ordered metric space and K be a nonempty Polish subspace of X. Let $T, S, R : \Omega \times K \to K$ be random operators satisfying the following inequality, for every pair $(x, y) \in K \times K$ with $R(\omega, x)$ and $R(\omega, y)$ comparable,*

$$\begin{aligned} \phi_1(d(S(\omega, x), T(\omega, y))) \leq\ & \mathcal{I}_G(\psi_1(d(R(\omega, x), R(\omega, y)), d(R(\omega, x), S(\omega, x)), \\ & d(R(\omega, y), T(\omega, y)), \frac{1}{2}[d(R(\omega, x), T(\omega, y)) + \\ & d(R(\omega, y), S(\omega, x))]) \\ & , \psi_2(d(R(\omega, x), R(\omega, y)), d(R(\omega, x), S(\omega, x)), \\ & d(R(\omega, y), T(\omega, y)), \frac{1}{2}[d(R(\omega, x), T(\omega, y)) + \\ & d(R(\omega, y), S(\omega, x))])), \end{aligned} \tag{2.1}$$

$$\begin{aligned} & \psi_3(d(R(\omega, x), R(\omega, y)), d(R(\omega, x), S(\omega, x)), \\ & d(R(\omega, y), T(\omega, y)), \frac{1}{2}[d(R(\omega, x), T(\omega, y)) + \\ & d(R(\omega, y), S(\omega, x))]))). \end{aligned}$$

where $\mathcal{I}_G$ is a generalized C_G-class function, ψ_1 is a generalized altering distance function and $\phi_1(x) = \psi_1(x, x, x, x)$ and ψ_2, ψ_3 are generalized ultra altering distance functions. Assume that

(i)T, S and R are continuous random operators,
(ii) $T(\omega, X) \subseteq R(\omega, X)$ and $S(\omega, X) \subseteq R(\omega, X)$ for each $\omega \in \Omega$,
(iii) $T(\omega, .)$ and $S(\omega, .)$ are weakly increasing with respect to $R(\omega, .)$ for each $\omega \in \Omega$,
(iv) the pairs $\{T, R\}$ and $\{S, R\}$ are compatible random operators.
Then there exists a measurable mapping $\zeta : \Omega \to K$ such that

$$R(\omega, \zeta(\omega)) = T(\omega, \zeta(\omega)) = S(\omega, \zeta(\omega))$$

for each $\omega \in \Omega$.

Proof. Let $\zeta_0 : \Omega \to K$ be a given measurable map. Since $T(\omega, X) \subseteq R(\omega, X)$, there exists $\zeta_1 : \Omega \to K$ such that $R(\omega, \zeta_1(\omega)) = T(\omega, \zeta_0(\omega))$. Since $S(\omega, X) \subseteq R(\omega, X)$, there is $\zeta_2(\omega) \in \Omega$ such that $R(\omega, \zeta_2(\omega)) = T(\omega, \zeta_1(\omega))$. Inductively, we construct a sequence of maps $\{\zeta_n(\omega)\}$ from Ω to K such that

$$R(\omega, \zeta_{2n+1}(\omega)) = T(\omega, \zeta_{2n}(\omega)) \text{ and } R(\omega, \zeta_{2n+2}(\omega)) = S(\omega, \zeta_{2n+1}(\omega)). \tag{2.2}$$

Since R, S and T are continuous random operators, by a result of Himmelberg [18], $\{\zeta_n(\omega)\}$ is a measurable sequence. Now, we claim that

$$R(\omega, \zeta_n(\omega)) \preceq R(\omega, \zeta_{n+1}(\omega)) \text{ for all } n \in \mathbb{N}. \tag{2.3}$$

Since $R(\omega, \zeta_1(\omega)) = T(\omega, \zeta_0(\omega))$, one has $(\omega, \zeta_1(\omega)) \in R^{-1}(T(\omega, \zeta_0(\omega)))$. By the increasing property of the mappings $S(\omega, .)$ and $T(\omega, .)$ with respect to $R(\omega.,)$, we get

$$R(\omega, \zeta_1(\omega)) = T(\omega, \zeta_0(\omega)) \preceq S(\omega, \zeta_1(\omega)) = R(\omega, \zeta_2(\omega)) \tag{2.4}$$

and

$$R(\omega, \zeta_2(\omega)) = S(\omega, \zeta_1(\omega)) \leq T(\omega, \zeta_2(\omega)) = R(\omega, \zeta_3(\omega)) \quad \text{for each } \omega \in \Omega. \tag{2.5}$$

Hence, by induction, (2.3) holds. Without the loss of generality, we can assume that

$$R(\omega, \zeta_n(\omega)) \neq R(\omega, \zeta_{n+1}(\omega)) \text{ for all } n \in N \text{ and for each } \omega \in \Omega. \tag{2.6}$$

Now, we prove that

$$\lim{}_{n\to\infty} d(R(\omega, \zeta_{n+1}(\omega)), R(\omega, \zeta_{n+2}(\omega))) = 0. \tag{2.7}$$

From (2.1), we have

$$\begin{aligned}
&\phi_1(d(R(\omega, \zeta_{2n+2}(\omega)), R(\omega, \zeta_{2n+1}(\omega)))) = \phi_1(d(S(\omega, \zeta_{2n+1}(\omega)), T(\omega, \zeta_{2n}(\omega))))\\
&\leq \mathcal{I}_{\mathrm{G}}(\psi_1(d(R(\omega, \zeta_{2n+1}(\omega)), R(\omega, \zeta_{2n}(\omega))), d(R(\omega, \zeta_{2n+1}(\omega)), S(\omega, \zeta_{2n+1}(\omega))),\\
&\quad d(R(\omega, \zeta_{2n}(\omega)), T(\omega, \zeta_{2n}(\omega))), \frac{1}{2}[d(R(\omega, \zeta_{2n+1}(\omega)), T(\omega, \zeta_{2n}(\omega)) +\\
&\quad d(R(\omega, \zeta_{2n}(\omega)), S(\omega, \zeta_{2n+1}(\omega)))]), \psi_2(d(R(\omega, \zeta_{2n+1}(\omega)), R(\omega, \zeta_{2n}(\omega))),\\
&\quad d(R(\omega, \zeta_{2n+1}(\omega)), S(\omega, \zeta_{2n+1}(\omega))), d(R(\omega, \zeta_{2n}(\omega)), T(\omega, \zeta_{2n}(\omega))),\\
&\quad \frac{1}{2}[d(R(\omega, \zeta_{2n+1}(\omega)), T(\omega, \zeta_{2n}(\omega)) + d(R(\omega, \zeta_{2n}(\omega)), S(\omega, \zeta_{2n+1}(\omega)))])\\
&\quad , \psi_3(d(R(\omega, \zeta_{2n+1}(\omega)), R(\omega, \zeta_{2n}(\omega))),\\
&\quad d(R(\omega, \zeta_{2n+1}(\omega)), S(\omega, \zeta_{2n+1}(\omega))), d(R(\omega, \zeta_{2n}(\omega)), T(\omega, \zeta_{2n}(\omega))),\\
&\quad \frac{1}{2}[d(R(\omega, \zeta_{2n+1}(\omega)), T(\omega, \zeta_{2n}(\omega)) + d(R(\omega, \zeta_{2n}(\omega)), S(\omega, \zeta_{2n+1}(\omega)))])).\\
&= \mathcal{I}_{\mathrm{G}}(\psi_1(d(R(\omega, \zeta_{2n+1}(\omega)), R(\omega, \zeta_{2n}(\omega))), d(R(\omega, \zeta_{2n+1}(\omega)), R(\omega, \zeta_{2n+2}(\omega))),\\
&\quad d(R(\omega, \zeta_{2n}(\omega)), R(\omega, \zeta_{2n+1}(\omega))), \frac{1}{2}[d(R(\omega, \zeta_{2n}(\omega)), R(\omega, \zeta_{2n+2}(\omega))])\\
&\quad , \psi_2(d(R(\omega, \zeta_{2n+1}(\omega)), R(\omega, \zeta_{2n}(\omega))), d(R(\omega, \zeta_{2n+1}(\omega)), R(\omega, \zeta_{2n+2}(\omega))),\\
&\quad d(R(\omega, \zeta_{2n}(\omega)), R(\omega, \zeta_{2n+1}(\omega))), \frac{1}{2}[d(R(\omega, \zeta_{2n}(\omega)), R(\omega, \zeta_{2n+2}(\omega))])\\
&\quad , \psi_3(d(R(\omega, \zeta_{2n+1}(\omega)), R(\omega, \zeta_{2n}(\omega))), d(R(\omega, \zeta_{2n+1}(\omega)), R(\omega, \zeta_{2n+2}(\omega))),\\
&\quad d(R(\omega, \zeta_{2n}(\omega)), R(\omega, \zeta_{2n+1}(\omega))), \frac{1}{2}[d(R(\omega, \zeta_{2n}(\omega)), R(\omega, \zeta_{2n+2}(\omega))]]).
\end{aligned} \tag{2.8}$$

Suppose, for some $n \in \mathbb{N}$, that

$$d(R(\omega, \zeta_{2n}(\omega)), R(\omega, \zeta_{2n+1}(\omega))) < d(R(\omega, \zeta_{2n+1}(\omega)), R(\omega, \zeta_{2n+2}(\omega))). \tag{2.9}$$

Using (2.9) and triangle inequality, we have

$$\frac{1}{2}d\left(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right)\leq\frac{1}{2}(d\left(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right)$$
$$+d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right))<d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right). \quad (2.10)$$

Using (2.9) and (2.10) together with a property of the generalized altering distance function ψ_1, we get

$$\begin{aligned}&\psi_1(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),\\&\quad d\left(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right),\frac{1}{2}[d(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right))])\\&\leq\ \phi_1\left(d(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right).\end{aligned} \quad (2.11)$$

Hence, we obtain

$$\begin{aligned}&\phi_1\left(d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right)\right)\leq\mathcal{I}_{\mathrm{G}}(\phi_1\left(d(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right)\\&,\psi_2(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),\\&d\left(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right),\frac{1}{2}[d(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right))])\\&,\psi_3(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),\\&d\left(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right),\frac{1}{2}[d(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right))]))\end{aligned} \quad (2.12)$$

which implies that

$$\phi_1\left(d(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right)=0,$$

or

$$\begin{aligned}&\psi_2(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),\\&d\left(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right),\frac{1}{2}[d(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right))]),\\=&\ 0.\end{aligned}$$

or

$$\begin{aligned}&\psi_3(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),\\&d\left(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right),\frac{1}{2}[d(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right))])\\=&\ 0.\end{aligned} \quad (2.13)$$

Thus, we have

$$d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n}\left(\omega\right)\right)\right)=0 \quad (2.14)$$

which contradicts (2.6). Hence, we deduce that

$$d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right)\leq d\left(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right) \quad (2.15)$$

for all $n\in\mathbb{N}$ and for each $\omega\in\Omega$. Again, from (2.1) and (2.3), we have

$$\begin{aligned}&\phi_1\left(d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)\right)\right)=\phi_1\left(d\left(S\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),T\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right)\right)\\&\leq\mathcal{I}_{\mathrm{G}}(\psi_1(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),S\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right),\end{aligned}$$

$$d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),T\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),\frac{1}{2}[d(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),T\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)+$$
$$d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),S\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right)]),\psi_2(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),$$
$$d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),S\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),T\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),$$
$$\frac{1}{2}[d(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),T\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)+d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),S\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right)])$$
$$,\psi_3(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),$$
$$d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),S\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),T\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),$$
$$\frac{1}{2}[d(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),T\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)+d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),S\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right)]))$$
$$=\mathcal{I}_{\mathrm{G}}(\psi_1(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),$$
$$d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)\right),\frac{1}{2}[d(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)+$$
$$d(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)]),\psi_2(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),$$
$$d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)\right),$$
$$\frac{1}{2}[d(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)+d(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)])$$

$$\psi_3(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),$$
$$d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)\right),$$
$$\frac{1}{2}[d(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)+d(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)])).$$
$$\phi_1\left(d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)\right)\right)$$
$$\leq\mathcal{I}_{\mathrm{G}}(\psi_1(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),$$
$$d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)\right),\frac{1}{2}d(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)$$
$$,\psi_2(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),$$
$$d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)\right),\frac{1}{2}d(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)$$
$$,\psi_3(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),$$
$$d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)\right),\frac{1}{2}d(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right))). \tag{2.16}$$

Suppose, for some $n\in\mathbb{N}$, that

$$d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right)<d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)\right). \tag{2.17}$$

Then, by triangle inequality, we have

$$\frac{1}{2}d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)\right)\leq\frac{1}{2}(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right)$$
$$+d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)\right))<d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)\right). \tag{2.18}$$

Hence, by (2.16) , (2.17) and (2.18) together with a property of the generalized altering function ψ_1, we obtain

$$\phi_1\left(d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)\right)\right)\leq\mathcal{I}_{\mathrm{G}}(\phi_1\left(d(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)\right)$$
$$,\psi_2(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),$$

$$d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)\right),\frac{1}{2}d(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right))$$
$$,\psi_3(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),$$
$$d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)\right),\frac{1}{2}d(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right))) \tag{2.19}$$

which implies that

$$\phi_1\left(d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)\right)\right)=0,$$

or

$$\begin{aligned}&\psi_2(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),\\&d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)\right),\frac{1}{2}d(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right))\\=\;&0,\end{aligned}$$

or

$$\begin{aligned}&\psi_3(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),\\&d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)\right),\frac{1}{2}d(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right))\\=\;&0.\end{aligned} \tag{2.20}$$

That is,

$$d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right)=0. \tag{2.21}$$

Hence, we obtain a contradiction to (2.6). We deduce that

$$d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+3}\left(\omega\right)\right)\right)\leq d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right) \tag{2.22}$$

for all $n\in\mathbb{N}$ and for each $\omega\in\Omega$. Combining (2.15) and (2.22), we obtain

$$d\left(R\left(\omega,\zeta_{n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{n+3}\left(\omega\right)\right)\right)\leq d\left(R\left(\omega,\zeta_{n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{n+2}\left(\omega\right)\right)\right) \tag{2.23}$$

for all n
$in\mathbb{N}$ and for each $\omega\in\Omega$. Thus, $\{d\left(R\left(\omega,\zeta_{n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{n+2}\left(\omega\right)\right)\right)\}$ is a nonincreasing sequence of positive real numbers for each $\omega\in\Omega$. This implies that there exists $r\geq 0$ such that

$$\lim_{n\to\infty}d\left(R\left(\omega,\zeta_{n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{n+2}\left(\omega\right)\right)\right)=r. \tag{2.24}$$

By (2.8), we have

$$\begin{aligned}&\phi_1\left(d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right)\right)\\&\leq\mathcal{I}_{\mathrm{G}}(\psi_1(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),\\&\quad d\left(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right),\frac{1}{2}d(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right))\\&\quad,\psi_2(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),\\&\quad d\left(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right),\frac{1}{2}d(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right))\\&\quad,\psi_3(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),\\&\quad d\left(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right),\frac{1}{2}d(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)))\\&\leq\mathcal{I}_{\mathrm{G}}(\psi_1(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right),\end{aligned}$$

$$d(R(\omega,\zeta_{2n}(\omega)),R(\omega,\zeta_{2n+1}(\omega))),d(R(\omega,\zeta_{2n}(\omega)),R(\omega,\zeta_{2n+1}(\omega))))$$
$$,\psi_2(d(R(\omega,\zeta_{2n+1}(\omega)),R(\omega,\zeta_{2n}(\omega))),d(R(\omega,\zeta_{2n+1}(\omega)),R(\omega,\zeta_{2n+2}(\omega))),$$
$$d(R(\omega,\zeta_{2n}(\omega)),R(\omega,\zeta_{2n+1}(\omega))),0)$$
$$,\psi_3(d(R(\omega,\zeta_{2n+1}(\omega)),R(\omega,\zeta_{2n}(\omega))),d(R(\omega,\zeta_{2n+1}(\omega)),R(\omega,\zeta_{2n+2}(\omega))),$$
$$d(R(\omega,\zeta_{2n}(\omega)),R(\omega,\zeta_{2n+1}(\omega))),0))$$

$$= \mathcal{I}_{\mathrm{G}}(\phi_1(d(R(\omega,\zeta_{2n}(\omega)),R(\omega,\zeta_{2n+1}(\omega)))),\psi_2(d(R(\omega,\zeta_{2n+1}(\omega)),R(\omega,\zeta_{2n}(\omega))),$$
$$d(R(\omega,\zeta_{2n+1}(\omega)),R(\omega,\zeta_{2n+2}(\omega))),d(R(\omega,\zeta_{2n}(\omega)),R(\omega,\zeta_{2n+1}(\omega))),0)). \tag{2.25}$$

Letting $n \to +\infty$ in (2.25) and using the continuity of ϕ_1 and ψ_2, we obtain

$$\phi_1(r) \leq \mathcal{I}_{\mathrm{G}}(\phi_1(r),\psi_2(r,r,r,0),\psi_3(r,r,r,0)), \tag{2.26}$$

which implies that $\phi_1(r) = 0$,or $\psi_2(r,r,r,0) = 0$, or $\psi_3(r,r,r,0) = 0$, so $r = 0$. Hence

$$\lim_{n\to+\infty} d(R(\omega,\zeta_{n+1}(\omega)),R(\omega,\zeta_{n+2}(\omega))) = 0 \text{ for each } \omega \in \Omega.$$

Thus (2.7) holds. Now, we claim that for $\omega \in \Omega$, $\{R(\omega,\zeta_n(\omega))\}$ is a Cauchy sequence. From (2.7), it will be sufficient to prove that $\{R(\omega,\zeta_{2n}(\omega))\}$ is a Cauchy sequence. We proceed by negation and suppose that $\{R(\omega,\zeta_{2n}(\omega))\}$ is not a Cauchy sequence. Then, there exists $\epsilon > 0$ for which we can find two sequences of positive integers $\{m_i\}$ and $\{n_i\}$ such that, for all positive integers i,

$$n(i) > m(i), \quad d\left(R\left(w,\zeta_{2m(i)}(\omega)\right),R\left(w,\zeta_{2n(i)}(\omega)\right)\right) \geq \epsilon \quad \text{and}$$
$$d\left(R\left(w,\zeta_{2m(i)}(\omega)\right),R\left(w,\zeta_{2n(i)-2}(\omega)\right)\right) < \epsilon. \tag{2.27}$$

From (2.27) and using the triangle inequality, we get

$$\begin{aligned}
\epsilon &\leq d\left(R\left(w,\zeta_{2m(i)}(\omega)\right),R\left(w,\zeta_{2n(i)}(\omega)\right)\right)\\
&\leq d\left(R\left(w,\zeta_{2m(i)}(\omega)\right),R\left(w,\zeta_{2n(i)-2}(\omega)\right)\right)+d\left(R\left(w,\zeta_{2n(i)-2}(\omega)\right),R\left(w,\zeta_{2n(i)-1}(\omega)\right)\right)\\
&\quad +d\left(R\left(w,\zeta_{2n(i)-1}(\omega)\right),R\left(w,\zeta_{2n(i)}(\omega)\right)\right)\\
&< \epsilon + d\left(R\left(w,\zeta_{2n(i)-2}(\omega)\right),R\left(w,\zeta_{2n(i)-1}(\omega)\right)\right)+d\left(R\left(w,\zeta_{2n(i)-1}(\omega)\right),R\left(w,\zeta_{2n(i)}(\omega)\right)\right).
\end{aligned}$$

Letting $i \to +\infty$ in the above inequality and using (2.7), we obtain

$$\lim_{i\to+\infty} d\left(R\left(w,\zeta_{2m(i)}(\omega)\right),R\left(w,\zeta_{2n(i)}(\omega)\right)\right) = \epsilon. \tag{2.28}$$

Again, triangle inequality gives

$$\begin{aligned}
&\left|d\left(R\left(w,\zeta_{2n(i)}(\omega)\right),R\left(w,\zeta_{2m(i)-1}(\omega)\right)\right)-d\left(R\left(w,\zeta_{2n(i)}(\omega)\right),R\left(w,\zeta_{2m(i)}(\omega)\right)\right)\right|\\
\leq\ & d\left(R\left(w,\zeta_{2m(i)-1}(\omega)\right),R\left(w,\zeta_{2m(i)}(\omega)\right)\right).
\end{aligned}$$

Letting $i \to +\infty$ in the above inequality and using (2.7)and (2.28), we get

$$\lim_{i\to+\infty} d\left(R\left(w,\zeta_{2n(i)}(\omega)\right),R\left(w,\zeta_{2m(i)-1}(\omega)\right)\right) = \epsilon. \tag{2.29}$$

On the other hand, we have

$$\begin{aligned}
\phi_1\left(d\left(R\left(w,\zeta_{2n(i)}(\omega)\right),R\left(w,\zeta_{2m(i)}(\omega)\right)\right)\right) &\leq \phi_1(d\left(R\left(w,\zeta_{2n(i)}(\omega)\right),R\left(w,\zeta_{2n(i)+1}(\omega)\right)\right)\\
&\quad +d\left(R\left(w,\zeta_{2n(i)+1}(\omega)\right),R\left(w,\zeta_{2m(i)}(\omega)\right)\right))\\
&= \phi_1(d\left(R\left(w,\zeta_{2n(i)}(\omega)\right),R\left(w,\zeta_{2n(i)+1}(\omega)\right)\right)\\
&\quad +d\left(T\left(w,\zeta_{2n(i)}(\omega)\right),S\left(w,\zeta_{2m(i)-1}(\omega)\right)\right)).
\end{aligned}$$

Then, from (2.7), (2.28) and the continuity of ϕ_1, we get by letting $i\to\infty$ in the above inequality

$$\phi_1(\epsilon)\le \lim_{i\to+\infty}\phi_1\left(d\left(T\left(w,\zeta_{2n(i)}(\omega)\right),S\left(w,\zeta_{2m(i)-1}(\omega)\right)\right)\right). \tag{2.30}$$

Now, using the contractive condition (2.1), we have

$$\begin{aligned}
&\phi_1\left(d\left(S\left(w,\zeta_{2m(i)-1}(\omega)\right),T\left(w,\zeta_{2n(i)}(\omega)\right)\right)\right)\\
&\le \mathcal{I}_G(\psi_1(d\left(R\left(w,\zeta_{2m(i)-1}(\omega)\right),T\left(w,\zeta_{2n(i)}(\omega)\right)\right),d\left(R\left(w,\zeta_{2m(i)-1}(\omega)\right),R\left(w,\zeta_{2m(i)}(\omega)\right)\right)\\
&d\left(R\left(w,\zeta_{2n(i)}(\omega)\right),R\left(w,\zeta_{2n(i)+1}(\omega)\right)\right),\frac{1}{2}[d\left(R\left(w,\zeta_{2m(i)-1}(\omega)\right),R\left(w,\zeta_{2n(i)+1}(\omega)\right)\right)+\\
&d\left(R\left(w,\zeta_{2n(i)}(\omega)\right),T\left(w,\zeta_{2m(i)}(\omega)\right)\right)]),\psi_2(d\left(R\left(w,\zeta_{2m(i)-1}(\omega)\right),R\left(w,\zeta_{2n(i)}(\omega)\right)\right),\\
&d\left(R\left(w,\zeta_{2m(i)-1}(\omega)\right),R\left(w,\zeta_{2m(i)}(\omega)\right)\right),d\left(R\left(w,\zeta_{2n(i)}(\omega)\right),R\left(w,\zeta_{2n(i)+1}(\omega)\right)\right),\\
&\frac{1}{2}[d\left(R\left(w,\zeta_{2m(i)-1}(\omega)\right),R\left(w,\zeta_{2n(i)+1}(\omega)\right)\right)+d\left(R\left(w,\zeta_{2n(i)}(\omega)\right),R\left(w,\zeta_{2m(i)}(\omega)\right)\right)])\\
&\psi_3(d\left(R\left(w,\zeta_{2m(i)-1}(\omega)\right),R\left(w,\zeta_{2n(i)}(\omega)\right)\right),\\
&d\left(R\left(w,\zeta_{2m(i)-1}(\omega)\right),R\left(w,\zeta_{2m(i)}(\omega)\right)\right),d\left(R\left(w,\zeta_{2n(i)}(\omega)\right),R\left(w,\zeta_{2n(i)+1}(\omega)\right)\right),\\
&\frac{1}{2}[d\left(R\left(w,\zeta_{2m(i)-1}(\omega)\right),R\left(w,\zeta_{2n(i)+1}(\omega)\right)\right)+d\left(R\left(w,\zeta_{2n(i)}(\omega)\right),R\left(w,\zeta_{2m(i)}(\omega)\right)\right)])).
\end{aligned} \tag{2.31}$$

From (2.7), (2.29) and the continuity of ψ_1 and ψ_2, we get by letting $i\to+\infty$ in the above inequality

$$\begin{aligned}
&\lim_{i\to+\infty}\phi_1\left(d\left(S\left(w,\zeta_{2m(i)-1}(\omega)\right),T\left(w,\zeta_{2n(i)}(\omega)\right)\right)\right)\\
\le\;& \mathcal{I}_G(\psi_1(\epsilon,0,0,\epsilon),\psi_2(\epsilon,0,0,\epsilon),\psi_3(\epsilon,0,0,\epsilon))\\
\le\;& \mathcal{I}_G(\phi_1(\epsilon),\psi_2(\epsilon,0,0,\epsilon),\psi_3(\epsilon,0,0,\epsilon)).
\end{aligned} \tag{2.32}$$

Now, combining (2.30) with (2.32), we get

$$\phi_1(\epsilon)\le\mathcal{I}_G(\phi_1(\epsilon),\psi_2(\epsilon,0,0,\epsilon),\psi_3(\epsilon,0,0,\epsilon)). \tag{2.33}$$

which implies that $\phi_1(\epsilon)=0$,or $\psi_2(\epsilon,0,0,\epsilon)=0$,or $\psi_3(\epsilon,0,0,\epsilon)=0$, so $\epsilon=0$, which contradict $\epsilon>0$. We deduce that for $\omega\in\Omega$, $\{R(\omega,\zeta_n(\omega))\}$ is a Cauchy sequence. Since $\{R(\omega,\zeta_n(\omega))\}$ is a Cauchy sequence in the complete metric space K, therefore there exists $\zeta:\Omega\to K$ such that

$$\lim_{n\to+\infty}R(\omega,\zeta_n(\omega))=\zeta(\omega). \tag{2.34}$$

From (2.34) and the continuity of R, we get

$$\lim_{n\to+\infty}R(\omega,R(\omega,\zeta_n(\omega)))=R(\omega,\zeta(\omega)) \tag{2.35}$$

By the triangle inequality, we have

$$\begin{aligned}
&d(R(\omega,\zeta(\omega)),T(\omega,\zeta(\omega)))\le d(R(\omega,\zeta(\omega)),R(\omega,T(\omega,\zeta_{2n}(\omega))))+\\
&d(R(\omega,T(\omega,\zeta_{2n}(\omega))),T(\omega,R(\omega,\zeta_{2n}(\omega))))+d(T(\omega,R(\omega,\zeta_{2n}(\omega))),T(\omega,\zeta(\omega))).
\end{aligned} \tag{2.36}$$

On the other hand, we have $R(\omega,\zeta_{2n}(\omega))\to\zeta(\omega)$, $T(\omega,\zeta_{2n}(\omega))\to\zeta(\omega)$ as $n\to\infty$. As R and T are compatible mappings, so we have

$$\lim_{n\to+\infty}d(R(\omega,T(\omega,\zeta_{2n}(\omega))),T(\omega,R(\omega,\zeta_{2n}(\omega))))=0. \tag{2.37}$$

Now, from the continuity of T and (2.34), we have

$$\lim_{n\to+\infty} d\left(T\left(\omega,R\left(\omega,\zeta_{2n}\left(\omega\right)\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right)=0. \tag{2.38}$$

Combining (2.35), (2.37) and (2.38) and letting $n\to+\infty$ in (2.36), we have

$$d\left(R\left(\omega,\zeta\left(\omega\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right)\leq 0. \tag{2.39}$$

That is,

$$R\left(\omega,\zeta\left(\omega\right)\right)=T\left(\omega,\zeta\left(\omega\right)\right). \tag{2.40}$$

Again, by triangle inequality, we have

$$d\left(R\left(\omega,\zeta\left(\omega\right)\right),S\left(\omega,\zeta\left(\omega\right)\right)\right)\leq d\left(R\left(\omega,\zeta\left(\omega\right)\right),R\left(\omega,R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right)\right)$$
$$+d\left(R\left(\omega,S\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right),S\left(\omega,R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right)\right)+d\left(S\left(\omega,R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right),S\left(\omega,\zeta\left(\omega\right)\right)\right). \tag{2.41}$$

On the other hand, we have $R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\to\zeta\left(\omega\right)$, $S\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\to\zeta\left(\omega\right)$ as $n\to\infty$. Since R and S are compatible mappings, therefore we get

$$\lim_{n\to+\infty} d\left(R\left(\omega,S\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right),S\left(\omega,R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right)\right)=0. \tag{2.42}$$

Now, from the continuity of S and (2.34), we have

$$\lim_{n\to+\infty} d\left(S\left(\omega,R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right),S\left(\omega,\zeta\left(\omega\right)\right)\right)=0. \tag{2.43}$$

Combining (2.35), (2.42) and (2.43) and letting $n\to\infty$ in (2.41), we obtain

$$d\left(R\left(\omega,\zeta\left(\omega\right)\right),S\left(\omega,\zeta\left(\omega\right)\right)\right)\leq 0. \tag{2.44}$$

That is,

$$R\left(\omega,\zeta\left(\omega\right)\right)=S\left(\omega,\zeta\left(\omega\right)\right). \tag{2.45}$$

Finally, from (2.40) and (2.45), we have

$$T\left(\omega,\zeta\left(\omega\right)\right)=R\left(\omega,\zeta\left(\omega\right)\right)=S\left(\omega,\zeta\left(\omega\right)\right)$$

that is, $\zeta\left(\omega\right)$ is a random coincidence point of T, S and R.

With choice $\mathcal{I}_G(s,t,r)=s-t$ in Theorem 2.1, we have the following:

Corollary 2.1 *[26] Let $(X,d,\preceq)$ be a separable ordered metric space and K be a nonempty Polish subspace of X. Let T, S, $R:\Omega\times K\to K$ be random operators satisfying the following inequality, for every pair $(x,y)\in K\times K$ with $R\left(\omega,x\right)$ and $R\left(\omega,y\right)$ comparable,*

$$\begin{aligned}\phi_1\left(d\left(S\left(\omega,x\right),T\left(\omega,y\right)\right)\right) \quad \leq \quad & \psi_1(d\left(R\left(\omega,x\right),R\left(\omega,y\right)\right),d\left(R\left(\omega,x\right),S\left(\omega,x\right)\right),\\ & d\left(R\left(\omega,y\right),T\left(\omega,y\right)\right),\frac{1}{2}[d\left(R\left(\omega,x\right),T\left(\omega,y\right)\right)+\\ & d\left(R\left(\omega,y\right),S\left(\omega,x\right)\right)])\\ & -\psi_2(d\left(R\left(\omega,x\right),R\left(\omega,y\right)\right),d\left(R\left(\omega,x\right),S\left(\omega,x\right)\right),\\ & d\left(R\left(\omega,y\right),T\left(\omega,y\right)\right),\frac{1}{2}[d\left(R\left(\omega,x\right),T\left(\omega,y\right)\right)+\\ & d\left(R\left(\omega,y\right),S\left(\omega,x\right)\right)])\end{aligned}$$

where , ψ_1 and. ψ_2 are generalized altering distance function and $\phi_1\left(x\right)=\psi_1\left(x,x,x,x\right)$. Assume that

(i)T,S and R are continuous random operators,

(ii) $T(\omega,X)\subseteq R(\omega,X)$ and $S(\omega,X)\subseteq R(\omega,X)$ for each $\omega\in\Omega$,

(iii) $T(\omega,.)$ and $S(\omega,.)$ are weakly increasing with respect to $R(\omega,.)$ for each $\omega\in\Omega$,

(iv) the pairs $\{T,R\}$ and $\{S,R\}$ are compatible random operators.

Then there exists a measurable mapping $\zeta:\Omega\to K$ such that

$$R\left(\omega,\zeta\left(\omega\right)\right)=T\left(\omega,\zeta\left(\omega\right)\right)=S\left(\omega,\zeta\left(\omega\right)\right)$$

for each $\omega\in\Omega$.

With choice $\mathcal{I}_{\mathrm{G}}(s,t,r)=\frac{s}{(1+t+r)}$ in Theorem 2.1, we have:

Corollary 2.2 *Let $(X,d,\preceq)$ be a separable ordered metric space and K be a nonempty Polish subspace of X. Let $T,S:\Omega\times K\to K$ be random operators satisfying the following inequality, for every pair $(x,y)\in K\times K$ with x and y comparable,*

$$\begin{aligned}\phi_1\left(d\left(S\left(\omega,x\right),T\left(\omega,y\right)\right)\right) \leq\; & \psi_1(d\left(R\left(\omega,x\right),R\left(\omega,y\right)\right),d\left(R\left(\omega,x\right),S\left(\omega,x\right)\right),\\ & d\left(R\left(\omega,y\right),T\left(\omega,y\right)\right),\frac{1}{2}[d\left(R\left(\omega,x\right),T\left(\omega,y\right)\right)+d\left(R\left(\omega,y\right),S\left(\omega,x\right)\right)])\\ & /(1+\psi_2(d\left(R\left(\omega,x\right),R\left(\omega,y\right)\right),d\left(R\left(\omega,x\right),S\left(\omega,x\right)\right),\\ & d\left(R\left(\omega,y\right),T\left(\omega,y\right)\right),\frac{1}{2}[d\left(R\left(\omega,x\right),T\left(\omega,y\right)\right)+d\left(R\left(\omega,y\right),S\left(\omega,x\right)\right)]))\\ & +\psi_3(d\left(R\left(\omega,x\right),R\left(\omega,y\right)\right),d\left(R\left(\omega,x\right),S\left(\omega,x\right)\right),\\ & d\left(R\left(\omega,y\right),T\left(\omega,y\right)\right),\frac{1}{2}[d\left(R\left(\omega,x\right),T\left(\omega,y\right)\right)+d\left(R\left(\omega,y\right),S\left(\omega,x\right)\right)])).\end{aligned}$$

where ψ_1 is a generalized altering distance function and $\phi_1(x)=\psi_1(x,x,x,x)$ and ψ_2, ψ_3 are generalized ultra altering distance functions. Assume that

(i)T,S and R are continuous random operators,

(ii) $T(\omega,X)\subseteq R(\omega,X)$ and $S(\omega,X)\subseteq R(\omega,X)$ for each $\omega\in\Omega$,

(iii) $T(\omega,.)$ and $S(\omega,.)$ are weakly increasing with respect to $R(\omega,.)$ for each $\omega\in\Omega$,

(iv) the pairs $\{T,R\}$ and $\{S,R\}$ are compatible random operators.

Then there exists a measurable mapping $\zeta:\Omega\to K$ such that

$$R\left(\omega,\zeta\left(\omega\right)\right)=T\left(\omega,\zeta\left(\omega\right)\right)=S\left(\omega,\zeta\left(\omega\right)\right)$$

for each $\omega\in\Omega$.

With choice $\mathcal{I}_{\mathrm{G}}(s,t,r)=e^{-r}s$ in Theorem 2.1, we have:

Corollary 2.3 *Let $(X,d,\preceq)$ be a separable ordered metric space and K be a nonempty Polish subspace of X. Let $T,S:\Omega\times K\to K$ be random operators satisfying the following inequality, for every pair $(x,y)\in K\times K$ with x and y comparable,*

$$\begin{aligned}\phi_1\left(d\left(S\left(\omega,x\right),T\left(\omega,y\right)\right)\right) \leq\; & e^{-\psi_3(d(R(\omega,x),R(\omega,y)),d(R(\omega,x),S(\omega,x)),d(R(\omega,y),T(\omega,y)),\frac{1}{2}[d(R(\omega,x),T(\omega,y))+d(R(\omega,y),S(\omega,x))])}\\ & \times\psi_1(d\left(R\left(\omega,x\right),R\left(\omega,y\right)\right),d\left(R\left(\omega,x\right),S\left(\omega,x\right)\right),\\ & d\left(R\left(\omega,y\right),T\left(\omega,y\right)\right),\frac{1}{2}[d\left(R\left(\omega,x\right),T\left(\omega,y\right)\right)+d\left(R\left(\omega,y\right),S\left(\omega,x\right)\right)])\end{aligned}$$

where ψ_1 is a generalized altering distance function and $\phi_1(x)=\psi_1(x,x,x,x)$ and ψ_3 is generalized ultra altering distance functions. . Assume that

(i)T,S and R are continuous random operators,

(ii) $T(\omega,X)\subseteq R(\omega,X)$ and $S(\omega,X)\subseteq R(\omega,X)$ for each $\omega\in\Omega$,

(iii) $T(\omega,.)$ and $S(\omega,.)$ are weakly increasing with respect to $R(\omega,.)$ for each $\omega\in\Omega$,

(iv) the pairs $\{T,R\}$ and $\{S,R\}$ are compatible random operators.

Then there exists a measurable mapping $\zeta:\Omega\to K$ such that

$$R\left(\omega,\zeta\left(\omega\right)\right)=T\left(\omega,\zeta\left(\omega\right)\right)=S\left(\omega,\zeta\left(\omega\right)\right)$$

for each $\omega\in\Omega$.

With choice $\mathcal{I}_{\mathrm{G}}(s,t,r)=\frac{s}{1+t}$ in Theorem 4 we have the following:

Corollary 2.4 *Let $(X, d, \preceq)$ be a separable ordered metric space and K be a nonempty Polish subspace of X. Let $T, S, R : \Omega \times K \to K$ be random operators satisfying the following inequality, for every pair $(x, y) \in K \times K$ with $R(\omega, x)$ and $R(\omega, y)$ comparable,*

$$\begin{aligned}\phi_1\left(d\left(S\left(\omega,x\right),T\left(\omega,y\right)\right)\right) \le\ & \psi_1(d\left(R\left(\omega,x\right),R\left(\omega,y\right)\right),d\left(R\left(\omega,x\right),S\left(\omega,x\right)\right),\\ & d\left(R\left(\omega,y\right),T\left(\omega,y\right)\right),\frac{1}{2}[d\left(R\left(\omega,x\right),T\left(\omega,y\right)\right)+\\ & d\left(R\left(\omega,y\right),S\left(\omega,x\right)\right)])\\ & /[1+\psi_2(d\left(R\left(\omega,x\right),R\left(\omega,y\right)\right),d\left(R\left(\omega,x\right),S\left(\omega,x\right)\right),\\ & d\left(R\left(\omega,y\right),T\left(\omega,y\right)\right),\frac{1}{2}[d\left(R\left(\omega,x\right),T\left(\omega,y\right)\right)+\\ & d\left(R\left(\omega,y\right),S\left(\omega,x\right)\right)])] \end{aligned} \tag{2.46}$$

where F is C-class function , ψ_1 is a generalized altering distance function and $\phi_1(x) = \psi_1(x, x, x, x)$ and. ψ_2 is a generalized ultra altering distance function . Assume that
(i)T, S and R are continuous random operators,
(ii) $T(\omega, X) \subseteq R(\omega, X)$ and $S(\omega, X) \subseteq R(\omega, X)$ for each $\omega \in \Omega$,
(iii) $T(\omega, .)$ and $S(\omega, .)$ are weakly increasing with respect to $R(\omega, .)$ for each $\omega \in \Omega$,
(iv) the pairs $\{T, R\}$ and $\{S, R\}$ are compatible random operators.
Then there exists a measurable mapping $\zeta : \Omega \to K$ such that

$$R\left(\omega,\zeta\left(\omega\right)\right)=T\left(\omega,\zeta\left(\omega\right)\right)=S\left(\omega,\zeta\left(\omega\right)\right)$$

for each $\omega \in \Omega$.

With choice $\mathcal{I}_{\mathrm{G}}(s, t, r) = ks$, $0 < k < 1$, in Theorem 2.1, we have the following:

Corollary 2.5 *Let $(X, d, \preceq)$ be a separable ordered metric space and K be a nonempty Polish subspace of X. Let $T, S, R : \Omega \times K \to K$ be random operators satisfying the following inequality, for every pair $(x, y) \in K \times K$ with $R(\omega, x)$ and $R(\omega, y)$ comparable,*

$$\begin{aligned}\phi_1\left(d\left(S\left(\omega,x\right),T\left(\omega,y\right)\right)\right) \le\ & k\psi_1(d\left(R\left(\omega,x\right),R\left(\omega,y\right)\right),d\left(R\left(\omega,x\right),S\left(\omega,x\right)\right),\\ & d\left(R\left(\omega,y\right),T\left(\omega,y\right)\right),\frac{1}{2}[d\left(R\left(\omega,x\right),T\left(\omega,y\right)\right)+\\ & d\left(R\left(\omega,y\right),S\left(\omega,x\right)\right)])\end{aligned}$$

where , ψ_1 is generalized altering distance function and $\phi_1(x) = \psi_1(x, x, x, x)$. Assume that
(i)T, S and R are continuous random operators,
(ii) $T(\omega, X) \subseteq R(\omega, X)$ and $S(\omega, X) \subseteq R(\omega, X)$ for each $\omega \in \Omega$,
(iii) $T(\omega, .)$ and $S(\omega, .)$ are weakly increasing with respect to $R(\omega, .)$ for each $\omega \in \Omega$,
(iv) the pairs $\{T, R\}$ and $\{S, R\}$ are compatible random operators.
Then there exists a measurable mapping $\zeta : \Omega \to K$ such that

$$R\left(\omega,\zeta\left(\omega\right)\right)=T\left(\omega,\zeta\left(\omega\right)\right)=S\left(\omega,\zeta\left(\omega\right)\right)$$

for each $\omega \in \Omega$.

With choice $\mathcal{I}_{\mathrm{G}}(s, t, r) = s\beta(s)$, $\beta : [0, \infty) \to [0, 1)$ in Theorem 2.1, we have the following:

Corollary 2.6 *Let $(X, d, \preceq)$ be a separable ordered metric space and K be a nonempty Polish subspace of X. Let $T, S, R : \Omega \times K \to K$ be random operators satisfying the following inequality, for every pair*

$(x,y) \in K \times K$ with $R(\omega,x)$ and $R(\omega,y)$ comparable,

$$\begin{aligned}\phi_1\left(d\left(S\left(\omega,x\right),T\left(\omega,y\right)\right)\right) \leq\ & \psi_1(d\left(R\left(\omega,x\right),R\left(\omega,y\right)\right),d\left(R\left(\omega,x\right),S\left(\omega,x\right)\right),\\ & d\left(R\left(\omega,y\right),T\left(\omega,y\right)\right),\frac{1}{2}[d\left(R\left(\omega,x\right),T\left(\omega,y\right)\right)+\\ & d\left(R\left(\omega,y\right),S\left(\omega,x\right)\right)])\\ & \beta(\psi_1(d\left(R\left(\omega,x\right),R\left(\omega,y\right)\right),d\left(R\left(\omega,x\right),S\left(\omega,x\right)\right),\\ & d\left(R\left(\omega,y\right),T\left(\omega,y\right)\right),\frac{1}{2}[d\left(R\left(\omega,x\right),T\left(\omega,y\right)\right)+\\ & d\left(R\left(\omega,y\right),S\left(\omega,x\right)\right)]))\end{aligned}$$

where , ψ_1 is generalized altering distance function and $\phi_1(x) = \psi_1(x,x,x,x)$. Assume that
(i)T,S and R are continuous random operators,
(ii) $T(\omega,X) \subseteq R(\omega,X)$ and $S(\omega,X) \subseteq R(\omega,X)$ for each $\omega \in \Omega$,
(iii) $T(\omega,.)$ and $S(\omega,.)$ are weakly increasing with respect to $R(\omega,.)$ for each $\omega \in \Omega$,
(iv) the pairs $\{T,R\}$ and $\{S,R\}$ are compatible random operators.
Then there exists a measurable mapping $\zeta : \Omega \to K$ such that

$$R\left(\omega,\zeta\left(\omega\right)\right) = T\left(\omega,\zeta\left(\omega\right)\right) = S\left(\omega,\zeta\left(\omega\right)\right)$$

for each $\omega \in \Omega$.

With choice $\mathcal{I}_G(s,t,r) = \omega(s)$, here $\omega : [0,\infty) \to [0,\infty)$ is an upper semicontinuous function such that $\omega(0) = 0$, and $\phi(t) < t$ for $t > 0$, in Theorem 2.1, we have the following:

Corollary 2.7 *Let $(X,d,\preceq)$ be a separable ordered metric space and K be a nonempty Polish subspace of X. Let $T,S,R : \Omega \times K \to K$ be random operators satisfying the following inequality, for every pair $(x,y) \in K \times K$ with $R(\omega,x)$ and $R(\omega,y)$ comparable,*

$$\begin{aligned}\phi_1\left(d\left(S\left(\omega,x\right),T\left(\omega,y\right)\right)\right) \leq\ & \omega(\psi_1(d\left(R\left(\omega,x\right),R\left(\omega,y\right)\right),d\left(R\left(\omega,x\right),S\left(\omega,x\right)\right),\\ & d\left(R\left(\omega,y\right),T\left(\omega,y\right)\right),\frac{1}{2}[d\left(R\left(\omega,x\right),T\left(\omega,y\right)\right)+\\ & d\left(R\left(\omega,y\right),S\left(\omega,x\right)\right)]))\end{aligned}$$

where ψ_1 is a generalized altering distance function and $\phi_1(x) = \psi_1(x,x,x,x)$ and $\omega : [0,\infty) \to [0,\infty)$ is an upper semicontinuous function such that $\omega(0) = 0$, and $\phi(t) < t$ for $t > 0$. Assume that
(i)T,S and R are continuous random operators,
(ii) $T(\omega,X) \subseteq R(\omega,X)$ and $S(\omega,X) \subseteq R(\omega,X)$ for each $\omega \in \Omega$,
(iii) $T(\omega,.)$ and $S(\omega,.)$ are weakly increasing with respect to $R(\omega,.)$ for each $\omega \in \Omega$,
(iv) the pairs $\{T,R\}$ and $\{S,R\}$ are compatible random operators.
Then there exists a measurable mapping $\zeta : \Omega \to K$ such that

$$R\left(\omega,\zeta\left(\omega\right)\right) = T\left(\omega,\zeta\left(\omega\right)\right) = S\left(\omega,\zeta\left(\omega\right)\right)$$

for each $\omega \in \Omega$.

Corollary 2.8 *Let $(X,d,\preceq)$ be a separable ordered metric space and K be a nonempty Polish subspace of X. Let $T,S,R : \Omega \times K \to K$ be random operators satisfying (2.1). Assume that*
(i)T,S and R are continuous random operators,
(ii) $T(\omega,X) \subseteq R(\omega,X)$ and $S(\omega,X) \subseteq R(\omega,X)$ for each $\omega \in \Omega$,
(iii) $T(\omega,.)$ and $S(\omega,.)$ are weakly increasing with respect to $R(\omega,.)$ for each $\omega \in \Omega$,
(iv) the pairs $\{T,R\}$ and $\{S,R\}$ are commuting random operators.
Then there exists a measurable mapping $\zeta : \Omega \to K$ such that

$$R\left(\omega,\zeta\left(\omega\right)\right) = T\left(\omega,\zeta\left(\omega\right)\right) = S\left(\omega,\zeta\left(\omega\right)\right)$$

for each $\omega \in \Omega$.

Corollary 2.9 *Let $(X,d,\preceq)$ be a separable ordered metric space and K be a nonempty Polish subspace of X. Let $T,S:\Omega\times K\to K$ be random operators satisfying the following inequality, for every pair $(x,y)\in K\times K$ with x and y comparable,*

$$\begin{aligned}\phi_1\left(d\left(S\left(\omega,x\right),T\left(\omega,y\right)\right)\right) \leq\ & \mathcal{I}_{\rm G}(\psi_1(d\left(x,y\right),d\left(x,S\left(\omega,x\right)\right),d\left(y,T\left(\omega,y\right)\right), \\ & \frac{1}{2}[d\left(x,T\left(\omega,y\right)\right)+d\left(y,S\left(\omega,x\right)\right)]) \\ & ,\psi_2(d\left(x,y\right),d\left(x,S\left(\omega,x\right)\right),d\left(y,T\left(\omega,y\right)\right), \\ & \frac{1}{2}[d\left(x,T\left(\omega,y\right)\right)+d\left(y,S\left(\omega,x\right)\right)]), \\ & \psi_3(d\left(x,y\right),d\left(x,S\left(\omega,x\right)\right),d\left(y,T\left(\omega,y\right)\right), \\ & \frac{1}{2}[d\left(x,T\left(\omega,y\right)\right)+d\left(y,S\left(\omega,x\right)\right)])),\end{aligned}$$

where $\mathcal{I}_{\rm G}$ is a generalized $C_{\rm G}$-class function, ψ_1 is a generalized altering distance function and $\phi_1(x)=\psi_1(x,x,x,x)$ and ψ_2, ψ_3 are generalized ultra altering distance functions. Assume that T and S are continuous random operators and $T(\omega,.)$ and $S(\omega,.)$ are weakly increasing for each $\omega\in\Omega$. Then, there exists a measurable mapping $\zeta:\Omega\to K$ such that

$$T\left(\omega,\zeta\left(\omega\right)\right)=S\left(\omega,\zeta\left(\omega\right)\right)\ \textit{for each}\ \omega\in\Omega.$$

Definition 2.3 *Let $(X,d,\preceq)$ be a separable partially ordered metric space and K be a nonempty Polish subspace of X. We say that K is regular if and only if the following hypothesis holds: if $\{\zeta_n(\omega)\}$ is a nondecreasing sequence in K with respect to $\leq$ and $\zeta:\Omega\to K$ such that $\zeta_n(\omega)\to\zeta(\omega)\in K$ as $n\to+\infty$, then $\zeta_n(\omega)\leq\zeta(\omega)$ for all n*
$in\mathbb{N}$ and for each $\omega\in\Omega$.

Theorem 2.2 *Let $(X,d,\preceq)$ be a separable ordered metric space and K be a nonempty Polish subspace of X. Let $T,S,R:\Omega\times K\to K$ be random operators satisfying inequality (2.1). Assume that*

(i) $T(\omega,X)\subseteq R(\omega,X)$ and $S(\omega,X)\subseteq R(\omega,X)$ for each $\omega\in\Omega$,

(ii) $T(\omega,.)$ and $S(\omega,.)$ are weakly increasing with respect to $R(\omega,.)$ for each $\omega\in\Omega$,

(iii) K is regular,

(iv) $R(\omega,K)$ is a complete subspace of K.

Then there is a measurable mapping $\zeta:\Omega\to K$ such that

$$R\left(\omega,\zeta\left(\omega\right)\right)=T\left(\omega,\zeta\left(\omega\right)\right)=S\left(\omega,\zeta\left(\omega\right)\right)\ \textit{for each}\ \omega\in\Omega.$$

Proof. As in the proof of Theorem 4, we have $\{R\left(\omega,\zeta_n\left(\omega\right)\right)\}$ is a Cauchy sequence in the complete metric space $(R(\omega,K),d)$, therefore there is $\theta\left(\omega\right)=R\left(\omega,\zeta\left(\omega\right)\right)$, $\zeta\left(\omega\right)\in K$ such that

$$\lim_{n\to+\infty}\{R\left(\omega,\zeta_n\left(\omega\right)\right)\}=\theta\left(\omega\right)=R\left(\omega,\zeta\left(\omega\right)\right)\quad\text{for each }\omega\in\Omega. \tag{2.47}$$

Since $\{R\left(\omega,\zeta_n\left(\omega\right)\right)\}$ is a nondecreasing sequence and K is regular, it follows from (2.49) that $R\left(\omega,\zeta_n\left(\omega\right)\right)\preceq R\left(\omega,\zeta\left(\omega\right)\right)$ for all n
$in\mathbb{N}$ and for each $\omega\in\Omega$. So, by contractive condition (2.1), we have

$$\phi_1\left(d\left(S\left(\omega,\zeta\left(\omega\right)\right),R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right)\right)=\phi_1\left(d\left(S\left(\omega,\zeta\left(\omega\right)\right),T\left(\omega,\zeta_{2n}\left(\omega\right)\right)\right)\right)$$

$$\begin{aligned}\leq\ & \mathcal{I}_{\rm G}(\psi_1(d\left(R\left(\omega,\zeta\left(\omega\right)\right),R\left(\omega,\zeta_{2n}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta\left(\omega\right)\right),S\left(\omega,\zeta\left(\omega\right)\right)\right), \\ & d\left(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right),\frac{1}{2}[d\left(R\left(\omega,\zeta\left(\omega\right)\right),T\left(\omega,\zeta_{2n}\left(\omega\right)\right)\right)\end{aligned}$$

$$+d\left(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),S\left(\omega,\zeta\left(\omega\right)\right)\right)]),\psi_2(d\left(R\left(\omega,\zeta\left(\omega\right)\right),R\left(\omega,\zeta_{2n}\left(\omega\right)\right)\right),$$

$$d\left(R\left(\omega,\zeta\left(\omega\right)\right),S\left(\omega,\zeta\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right),$$
$$\frac{1}{2}[d\left(R\left(\omega,\zeta\left(\omega\right)\right),T\left(\omega,\zeta_{2n}\left(\omega\right)\right)\right)+d\left(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),S\left(\omega,\zeta\left(\omega\right)\right)\right)])$$

$$,\psi_3(d\left(R\left(\omega,\zeta\left(\omega\right)\right),R\left(\omega,\zeta_{2n}\left(\omega\right)\right)\right),$$

$$d\left(R\left(\omega,\zeta\left(\omega\right)\right),S\left(\omega,\zeta\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right),$$
$$\frac{1}{2}[d\left(R\left(\omega,\zeta\left(\omega\right)\right),T\left(\omega,\zeta_{2n}\left(\omega\right)\right)\right)+d\left(R\left(\omega,\zeta_{2n}\left(\omega\right)\right),S\left(\omega,\zeta\left(\omega\right)\right)\right)])).$$

Letting $n\to+\infty$ in the above inequality and using (2.7), (2.47) and the properties of ψ_1 and ψ_2, we obtain

$$\begin{aligned}
&\phi_1\left(d\left(S\left(\omega,\zeta\left(\omega\right)\right),R\left(\omega,\zeta\left(\omega\right)\right)\right)\right)\leq\\
&\mathcal{I}_{\mathrm{G}}(\psi_1(0,d\left(R\left(\omega,\zeta\left(\omega\right)\right),S\left(\omega,\zeta\left(\omega\right)\right)\right),0,\\
&\frac{1}{2}d\left(R\left(\omega,\zeta\left(\omega\right)\right),S\left(\omega,\zeta\left(\omega\right)\right)\right))\\
&,\psi_2(0,d\left(R\left(\omega,\zeta\left(\omega\right)\right),S\left(\omega,\zeta\left(\omega\right)\right)\right),0,\\
&\frac{1}{2}d\left(R\left(\omega,\zeta\left(\omega\right)\right),S\left(\omega,\zeta\left(\omega\right)\right)\right),\psi_3(0,d\left(R\left(\omega,\zeta\left(\omega\right)\right),S\left(\omega,\zeta\left(\omega\right)\right)\right),0,.))\\
\leq\ &\mathcal{I}_{\mathrm{G}}(\phi_1\left(d\left(S\left(\omega,\zeta\left(\omega\right)\right),R\left(\omega,\zeta\left(\omega\right)\right)\right)\right)\\
&,\psi_2(0,d\left(R\left(\omega,\zeta\left(\omega\right)\right),S\left(\omega,\zeta\left(\omega\right)\right)\right),0,\\
&\frac{1}{2}d\left(R\left(\omega,\zeta\left(\omega\right)\right),S\left(\omega,\zeta\left(\omega\right)\right)\right),\psi_3(0,d\left(R\left(\omega,\zeta\left(\omega\right)\right),S\left(\omega,\zeta\left(\omega\right)\right)\right),0,.).
\end{aligned}\tag{2.48}$$

This implies that $\psi_2(0,d\left(R\left(\omega,\zeta\left(\omega\right)\right),S\left(\omega,\zeta\left(\omega\right)\right)\right),0,\frac{1}{2}d\left(R\left(\omega,\zeta\left(\omega\right)\right),S\left(\omega,\zeta\left(\omega\right)\right)\right)=0$, which gives that $d\left(R\left(\omega,\zeta\left(\omega\right)\right),S\left(\omega,\zeta\left(\omega\right)\right)\right)=0$, that is,

$$R\left(\omega,\zeta\left(\omega\right)\right)=S\left(\omega,\zeta\left(\omega\right)\right).\tag{2.49}$$

Again, by (2.1), we have

$$\phi_1\left(d\left(R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right)\right)=\phi_1\left(d\left(S\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right)\right)$$

$$\begin{aligned}
\leq\ &\mathcal{I}_{\mathrm{G}}(\psi_1(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),\\
&d\left(R\left(\omega,\zeta\left(\omega\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right),\frac{1}{2}[d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right)\\
&+d\left(R\left(\omega,\zeta\left(\omega\right)\right),S\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right)]),\psi_2(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta\left(\omega\right)\right)\right),
\end{aligned}$$

$$d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta\left(\omega\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right),$$
$$\frac{1}{2}[d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right)+d\left(R\left(\omega,\zeta\left(\omega\right)\right),S\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right)]).$$

$$,\psi_3(d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta\left(\omega\right)\right)\right),$$

$$d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),R\left(\omega,\zeta_{2n+2}\left(\omega\right)\right)\right),d\left(R\left(\omega,\zeta\left(\omega\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right),$$
$$\frac{1}{2}[d\left(R\left(\omega,\zeta_{2n+1}\left(\omega\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right)+d\left(R\left(\omega,\zeta\left(\omega\right)\right),S\left(\omega,\zeta_{2n+1}\left(\omega\right)\right)\right)])).$$

Letting $n\to+\infty$, we get

$$\begin{aligned}
&\phi_1\left(d\left(R\left(\omega,\zeta\left(\omega\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right)\right)\leq\\
&\mathcal{I}_{\mathrm{G}}(\psi_1(0,0,d\left(R\left(\omega,\zeta\left(\omega\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right),\\
&\frac{1}{2}d\left(R\left(\omega,\zeta\left(\omega\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right))\\
&,\psi_2(0,0,d\left(R\left(\omega,\zeta\left(\omega\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right),\\
&\frac{1}{2}d\left(R\left(\omega,\zeta\left(\omega\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right)),\psi_3(0,0,d\left(R\left(\omega,\zeta\left(\omega\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right),.)\\
\leq\ &\mathcal{I}_{\mathrm{G}}(\phi_1\left(d\left(R\left(\omega,\zeta\left(\omega\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right)\right)\\
&,\psi_2(0,0,d\left(R\left(\omega,\zeta\left(\omega\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right))\\
&\frac{1}{2}d\left(R\left(\omega,\zeta\left(\omega\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right)),,\psi_3(0,0,d\left(R\left(\omega,\zeta\left(\omega\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right),.). \qquad (2.50)
\end{aligned}$$

This implies that $\psi_2(0,0,d\left(R\left(\omega,\zeta\left(\omega\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right),\frac{1}{2}d\left(R\left(\omega,\zeta\left(\omega\right)\right),T\left(\omega,\zeta\left(\omega\right)\right)\right))=0$ and so,

$$R\left(\omega,\zeta\left(\omega\right)\right)=T\left(\omega,\zeta\left(\omega\right)\right). \qquad (2.51)$$

Now, combining (2.51) and (2.53), we obtain

$$R\left(\omega,\zeta\left(\omega\right)\right)=T\left(\omega,\zeta\left(\omega\right)\right)=S\left(\omega,\zeta\left(\omega\right)\right)$$

for each $\omega\in\Omega$. Hence, $\zeta\left(\omega\right)$ is a random coincidence point of T, S, and R.

Corollary 2.10 *Let $(X,d,\preceq)$ be a separable ordered metric space and K be a nonempty Polish subspace of X. Let $T,S:\Omega\times K\to K$ be random operators satisfying the following inequality for every pair $(x,y)\in K\times K$ with x and y comparable,*

$$\begin{aligned}
\phi_1\left(d\left(S\left(\omega,x\right),T\left(\omega,y\right)\right)\right)\ \leq\ &\mathcal{I}_{\mathrm{G}}(\psi_1(d\left(x,y\right),d\left(x,S\left(\omega,x\right)\right),d\left(y,T\left(\omega,y\right)\right),\\
&\frac{1}{2}[d\left(x,T\left(\omega,y\right)\right)+d\left(y,S\left(\omega,x\right)\right)])\\
&,\psi_2(d\left(x,y\right),d\left(x,S\left(\omega,x\right)\right),d\left(y,T\left(\omega,y\right)\right),\\
&\frac{1}{2}[d\left(x,T\left(\omega,y\right)\right)+d\left(y,S\left(\omega,x\right)\right)])\\
&,\psi_3(d\left(x,y\right),d\left(x,S\left(\omega,x\right)\right),d\left(y,T\left(\omega,y\right)\right),\\
&\frac{1}{2}[d\left(x,T\left(\omega,y\right)\right)+d\left(y,S\left(\omega,x\right)\right)]))
\end{aligned}$$

where $\mathcal{I}_{\mathrm{G}}$ is a generalized C_{G}-class function, ψ_1 is a generalized altering distance function and $\phi_1\left(x\right)=\psi_1\left(x,x,x,x\right)$ and ψ_2, ψ_3 are generalized ultra altering distance functions. Assume that $T(\omega,.)$ and $S(\omega,.)$ are weakly increasing. If K is regular, then there is a measurable mapping $\zeta:\Omega\to K$ such that

$$T\left(\omega,\zeta\left(\omega\right)\right)=S\left(\omega,\zeta\left(\omega\right)\right)\ \text{for each}\ \omega\in\Omega.$$

Corollary 2.11 *Let $(X,d,\preceq)$ be a separable ordered metric space and K be a nonempty Polish subspace of X. Let $T:\Omega\times K\to K$ be a random operator satisfying the following inequality, for every comparable pair $(x,y)\in K\times K$,*

$$\begin{aligned}
\phi_1\left(d\left(T\left(\omega,x\right),T\left(\omega,y\right)\right)\right)\ \leq\ &\mathcal{I}_{\mathrm{G}}(\psi_1(d\left(x,y\right),d\left(x,T\left(\omega,x\right)\right),d\left(y,T\left(\omega,y\right)\right),\\
&\frac{1}{2}[d\left(x,T\left(\omega,y\right)\right)+d\left(y,T\left(\omega,x\right)\right)])\\
&,\psi_2(d\left(x,y\right),d\left(x,T\left(\omega,x\right)\right),d\left(y,T\left(\omega,y\right)\right),\\
&\frac{1}{2}[d\left(x,T\left(\omega,y\right)\right)+d\left(y,T\left(\omega,x\right)\right)])\\
&,\psi_3(d\left(x,y\right),d\left(x,T\left(\omega,x\right)\right),d\left(y,T\left(\omega,y\right)\right),\\
&\frac{1}{2}[d\left(x,T\left(\omega,y\right)\right)+d\left(y,T\left(\omega,x\right)\right)]))
\end{aligned}$$

where $\mathcal{I}_\mathrm{G}$ is a generalized C_G-class function, ψ_1 is a generalized altering distance function and $\phi_1(x) = \psi_1(x,x,x,x)$ and ψ_2, ψ_3 are generalized ultra altering distance functions. Suppose $T(\omega,.)$ is weakly increasing for each $\omega \in \Omega$. Assume that either T is a continuous random operator, or K is regular, then T has a random fixed point, that is, there exists a measurable mapping $\zeta : \Omega \to K$ such that $\zeta(\omega) = T(\omega, \zeta(\omega))$ for each $\omega \in \Omega$.

Remark 2.1 (1) Theorem 2.1 is a generalization of Theorem 3.3 of [26] for three maps considering a generalized altering distance function.

(2) Corollary 2.1 is Theorem 3.3 of [26].

(3) Our results present a random version improvement, extension and generalization of results from [12], [22] and [28].

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