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Ideal-Based Wijsman Convergence in Intuitionistic Fuzzy 2-Metric Frameworks

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ABSTRACT: Motivated by [20]'s notion of intuitionistic fuzzy 2-metric space (briefly, IF-2MS), we conduct a thorough investigation and analysis of Wijsman ideal convergence within an IF-2MS.This exploration provides a novel framework for assessing the convergence behavior of set sequences in 2-metric spaces. Additionally, we elucidate the relationship between Wijsman ideal Cauchy sequences and Wijsman ideal convergence within this innovative framework. Furthermore, we analyze $\mathcal{J}-$ limit points and $\mathcal{J}-$ cluster ponits, and establish the inclusion relationship between them in this context.

Keywords: Ideal, filter, $\mathcal{J}$–convergence, Wijsman convergence, intuitionistic fuzzy 2-metric space.

Contents

1. Introduction

An effective technique for examining the convergence of a sequence of numerical problems based on the idea of density is statistical convergence. This convergence is introduced by Fast [5] in 1951. Nuray and Rhoades [22] researched the sets theory and examined Wijsman statistical convergence in sequence of sets in 2012. Furthur, Ulusu and Nuray [28] extended this concept and analyzed Wijsman convergence of sequence of sets via lacunary statistical. Wijsman λ- statistical convergence of interval numbers studied by Esi et al. [4]. Kostyrko et al. [19] proposed the ideal of $\mathcal{I}-$ convergence which is generalized version of statistical convergence. Further, its properties were examined by Salát et al. [26,27]. Kisi and Nuray [17] investigated Wijsman $\mathcal{I}$-convergence in the set sequences. In [6], Gümüş examined Wijsman convergent the set sequences via ideal by employing the Orlicz function. Additionally, Hazarikaa and Esi [10] used ideal and Wijsman convergence to study asymptotically lacunary statistical convergence of set sequences.

Zadeh presented fuzzy set theory in 1965 [30]. Karmosil et al. [15] later defined a fuzzy metric space. George and Veermani [8,9] also studied Hausdorff topology and fuzzy metric space. Atanassov [1] proposed intuitionistic fuzzy sets, a generalized form of fuzzy sets, in 1986. Park [23] explored the concept of intuitionistic metric space, and subsequently, Sadaati and Park [25] delved into the study of intuitionistic fuzzy topological spaces. In a later study, Mursaleen et al. [20] introduced the concept of an intuitionistic fuzzy 2-metric space and further developed the idea of an intuitionistic fuzzy metric space. Güner et al. [7] introduced some fundamental definitions and fixed-point theorems concerning intuitionistic fuzzy 2-metric spaces. Esi et al. [3] put forward the concept of Wijsman ideal convergence within intuitionistic fuzzy metric spaces. Kisi [18] explored Wijsman lacunary statistical convergence in intuitionistic fuzzy metric spaces through the lens of ideals. Khan and Ahmad [11,12] analyzed Zweier ideal convergence and Tauberian Theorems for intuitionistic fuzzy normed spaces. Additionally, other authors have examined the statistical, ideal convergent, and other characteristics of sequences in various fields [2,13,14,21,24,29].

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2. Preliminieries

Some essential definitions, remark, and lemma that are necessary for our primary findings have been covered in this section:

Definition 2.1 *[19] A non-void subset $\mathcal{J}$ of $P(\mathbb{N})$ is termed an ideal on $\mathbb{N}$ if it satisfies the following criteria: $\emptyset$ is an element of $\mathcal{J}$, for any $\mathcal{J}_1$ and $\mathcal{J}_2$ in $\mathcal{J}$, the union $\mathcal{J}_1 \cup \mathcal{J}_2$ is also in $\mathcal{J}$, and for any $\mathcal{J}_1$ in $\mathcal{J}$, if $\mathcal{J}_2$ is a subset of $\mathcal{J}_1$, then $\mathcal{J}_2$ is also in $\mathcal{J}$.*

Remark 2.1 *[19] An ideal $\mathcal{J} \subsetneq P(\mathbb{N})$ is referred to as nontrivial. We refer to a nontrivial ideal $\mathcal{J}$ as admissible provided that all singleton subsets of $\mathbb{N}$ belong to $\mathcal{J}$, that is $\{\{v\} : v \in \mathbb{N}\} \subseteq \mathcal{J}$.*

Definition 2.2 *[19] A non-void collection $\mathcal{F} \subseteq P(\mathbb{N})$ is termed as filter on $\mathbb{N}$ if it must meet the following conditions: $\emptyset \notin \mathcal{F}$, for any $\mathcal{F}_1$ and $\mathcal{F}_1$ in $\mathcal{F}$, the intersection $\mathcal{F}_1 \cap \mathcal{F}_2$ is also in $\mathcal{F}$, for any $\mathcal{F}_1$ in $\mathcal{F}$, $\mathcal{F}_1$ is a subset $\mathcal{F}_2$, then $\mathcal{F}_2$ is also in $\mathcal{F}$.*

Definition 2.3 *[19] The filter $\mathcal{F}$ associated with an ideal $\mathcal{J}$ is represented by*

$$\mathcal{F}(\mathcal{J}) = \{B \subset \mathbb{N} : \mathbb{N} \setminus B \in \mathcal{J}\}$$

Definition 2.4 *[19] Let $\{\mathcal{A}_1, \mathcal{A}_2, \dots\}$ be the mutually exclusive sets sequence of $\mathcal{J}$. Subsequently, there exits $\{\mathcal{B}_1, \mathcal{B}_2, \dots\}$ sets sequence with the result that $\cup_{K=1}^{\infty} \mathcal{B}_j \in \mathcal{J}$ and $\mathcal{A}_j \triangle \mathcal{B}_j (j = 1, 2, 3, 4 \dots)$ is finite. Under these circumstances, an admissible ideal $\mathcal{J}$ is referred to as having property (AP).*

Lemma 2.1 *[16] Assume $\mathcal{J}$ be the admissible ideal with (AP) property. Also assume that a countable family of subsets $\{\mathcal{A}_k\}_{k=1}^{\infty}$ of $\mathbb{N}$ so that $\mathcal{A}_k \in \mathcal{F}(\mathcal{J})$. Therefore, there is the set $\mathcal{A} \subset \mathbb{N}$ in order that $\mathcal{A} \setminus \mathcal{A}_k, \quad \forall \quad \mathcal{A} \in \mathcal{F}(\mathcal{J})$ is finite.*

Definition 2.5 *[17] Suppose that $(\mathcal{W}, \psi)$ is a metric space. Let $\{\mathcal{A}_k\}$ be a sequence of closed, non-void subsets of $\mathcal{W}$. This sequence is said to Wijsman $\mathcal{J}-$ convergent to $\mathcal{A}$ that is, non-void closed set in $\mathcal{W}$, if*

$$\lim_{k \to \infty} \psi(u, \mathcal{A}_k) = \psi(u, \mathcal{A}) \text{ for every } u \in \mathcal{W}.$$

We denote $\mathcal{W} - \lim_{k \to \infty} \mathcal{A}_k = \mathcal{A}$.

Definition 2.6 *[17] Suppose that $(\mathcal{W}, \psi)$ is a metric space. Let $\{\mathcal{A}_k\}$ be a sequence of closed, non-void subsets of $\mathcal{W}$. This sequence is said to Wijsman $\mathcal{J}-$ convergent to $\mathcal{A}$, that is, non-void closed set in $\mathcal{W}$ if, for ecah $u \in \mathcal{W}$, the set*

$$\left\{k \in \mathbb{N} :\mid \psi(u, \mathcal{A}_k) - \psi(u, \mathcal{A}) \mid \geq \epsilon \right\} \in \mathcal{J}.$$

We denote $\mathcal{W}_{\mathcal{J}} - \lim_{k \to \infty} \mathcal{A}_k = \mathcal{A}$.

Definition 2.7 *[17] Suppose that $(\mathcal{W}, \psi)$ is a metric space. Let $\{\mathcal{A}_k\}$ be a sequence of closed, non-void subsets of $\mathcal{W}$. This sequence is said to Wijsman $\mathcal{J}-$ Cauchy if, for every $u \in \mathcal{W}$ and there exists $N = N(\epsilon)$, the set*

$$\left\{k \in \mathbb{N} :\mid \psi(u, \mathcal{A}_k) - \psi(u, \mathcal{A}_m) \mid \geq \epsilon \right\} \in \mathcal{J}, \qquad \forall \quad m \geq N.$$

Definition 2.8 *[17] Let $(\mathcal{W}, \psi)$ is a separable metric space. A set of closed, non-void subsets $\{\mathcal{A}_k\}$ of $\mathcal{W}$ is referred to as Wijsman $\mathcal{J}^*-$ convergent to $\mathcal{A}$, that is, non-void closed set in $\mathcal{W}$ iff $\quad \exists \ T \in \mathcal{F}(\mathcal{J})$ and $T = \{t = (t_j < t_{j+1}, j \in \mathbb{N})\} \subset \mathbb{N}$ in order that*

$$\lim_{k \to \infty} \psi(u, \mathcal{A}_{t_k}) = \psi(u, \mathcal{A}), \qquad \forall \quad u \in \mathcal{W}.$$

Definition 2.9 *[17] Let $(\mathcal{W}, \psi)$ is a separable metric space. A set of closed, non-void subsets $\{\mathcal{A}_k\}$ of $\mathcal{W}$ is referred to as Wijsman $\mathcal{J}^*-$ Cauchy if, $\exists T \quad \in \mathcal{F}(\mathcal{J})$, where $T = \{t = (t_j < t_{j+1}, i \in \mathbb{N})\}$ in order that subsequence $\mathcal{A}_T = \{\mathcal{A}_{t_k}\}$ the Wijsman Cauchy in $\mathcal{W}$; i.e.,*

$$\lim_{k,l\to\infty} \mid \psi(u, \mathcal{A}_{t_k}) - \psi(u, \mathcal{A}_{t_l}) \mid = 0.$$

Remark 2.2 *[17] The convergent sequence $\{\mathcal{A}_k\}$ has subsequences that all converge to the same limit, provided that $(\mathcal{W}, \psi)$ is a separable metric space.*

Definition 2.10 *[20] A function $\psi : \mathcal{W} \times \mathcal{W} \times \mathcal{W} \to \mathbb{R}$ is known as 2-metric if ψ fullfil the following terms :*

1. *To every pair of the elements $w, v \in \mathcal{W}(w \neq v)$, there is poin $u \in \mathcal{W}$ in such a way that $\psi(w, v, u) \neq 0$,*
2. *$\psi(w, v, u) = 0$, when atleat two of w, v, u are equal.*
3. *$\psi(w, v, u) = \psi(w, u, v) = \psi(v, u, v)$ for all $w, v, u \in \mathcal{W}$,*
4. *$\psi(w, v, u) \leq \psi(w, v, z) + \psi(w, z, u) + \psi(z, v, u)$.*

Definition 2.11 *[20] Let $\mathcal{W}$ is a non-void set, $\mathcal{P}$ and $\mathcal{Q}$ are fuzzy sets on $\mathcal{W}^3 \times (0, \infty)$, $*$ and $\diamond$ are continuous t-norm and t-conorm respectively. Then, if the following criteria are met, $(\mathcal{W}, \mathcal{P}, \mathcal{Q}, *, \diamond)$ is referred to as an IF-2MS. For all $t, s, s_1, s_2, s_3 > 0$ and for every $w, v, u \in \mathcal{W}$;*
(i) $\mathcal{P}(w, v, u; s) + \mathcal{Q}(w, v, u; s) \leq 1$,
(ii) $\mathcal{P}(w, v, u; 0) = 0$
(iii) $\mathcal{P}(w, v, u; s) = 1 \quad \forall s > 0$ if any two of the following w, v, u are equal.
(iv) $\mathcal{P}(w, v, u; s) = \mathcal{P}(u, w, v; s) = \mathcal{P}(v, w, u; s) = \mathcal{P}(w, u, v; s)$
(v) $\mathcal{P}(w, v, u; s_1 + s_2 + s_3) \geq \mathcal{P}(w, v, u; s_1) * \mathcal{P}(w, v, u; s_2) * \mathcal{Q}(w, v, u; s_3)$
(vi) $\mathcal{P}(w, v, u; .) : (0, \infty) \to [0, 1]$ is left continuous,
(vii) $\mathcal{Q}(w, v, u; 0) = 1$,
(viii) $\mathcal{Q}(w, v, u; s) = 0 \quad \forall s > 0$ when at least two of w, v, u are equal,
(ix) $\mathcal{Q}(w, v, u; s) = \mathcal{Q}(u, w, v; s) = \mathcal{Q}(v, w, u; s) = \mathcal{Q}(w, u, v; s)$
(x) $\mathcal{Q}(w, v, u; s_1 + s_2 + s_3) \leq \mathcal{Q}(w, v, u; s_1) \diamond \mathcal{Q}(w, v, u; s_2) \diamond \mathcal{Q}(w, v, u; s_3)$
(xi) $\mathcal{Q}(w, v, u; .) : (0, \infty) \to [0, 1]$ is right continuous.

Example [20] Suppose $(\mathcal{W}, \psi)$ be a 2-metric space. Define $a_1 \diamond a_2 = \min(a_1 + 2, 1)$ and $a_1 * a_2 = a_1 a_2, \forall \quad a_1, a_2 \in [0, 1]$ and assume $\mathcal{P}$ and $\mathcal{Q}$ are fuzzy sets on $\mathcal{W}^3 \times (0, \infty)$ define as:

$$\mathcal{P}(w, v, u; s) = \frac{s}{s + \psi(w, v, u)}, \quad \mathcal{Q}(w, v, u; s) = \frac{\psi(w, v, u)}{s + \psi(w, v, u)}.$$

Then $(\mathcal{W}, \mathcal{P}, \mathcal{Q}, *, \diamond)$ is an IF-2MS.

Definition 2.12 *[20] Let $(\mathcal{W}, \mathcal{P}, \mathcal{Q}, *, \diamond)$ be an IF-2MS. A sequence (u_i) in $\mathcal{W}$ is known as*

1. *converge to $u \in \mathcal{W}$ if $\lim_{n\to\infty} \mathcal{P}(u_n, u, w; \eta) = 1$ and $\lim_{n\to\infty} \mathcal{Q}(u_n, u, w; \eta) = 0, \forall\ w \in \mathcal{W}$ and $\eta > 0$.*
2. *Cauchy if $\lim_{n,m\to\infty} \mathcal{P}(u_n, u_m, w; \eta) = 1$ and $\lim_{n,m\to\infty} \mathcal{Q}(u_n, u_m, w; \eta) = 0, \ \forall \ \ w \in \mathcal{W}$ and $\eta > 0$.*

3. Main Results

We adopt $\mathcal{J}$ to be an admissible ideal in $\mathbb{N}$ throughout present section, and we begin by providing the required definitions.

Definition 3.1 *Consider the IF-2MS $(\mathcal{W}, \mathcal{P}, \mathcal{Q}, *, \diamond)$. Let $\mathcal{A} \subset \mathcal{W}$ and $u, w \in \mathcal{W}$, then the membeship and non-membership are respectively defined by*

$$\mathcal{P}(u, \mathcal{A}, w; s) := \sup_{a \in A} \mathcal{P}(u, a, w; s) \text{ and}$$

$$\mathcal{Q}(u, \mathcal{A}, w; s) := \inf_{a \in A} \mathcal{Q}(u, a, w; s).$$

This measures the intuitive closeness of u and w to the set A.

Definition 3.2 *Consider the IF-2MS $(\mathcal{W}, \mathcal{P}, \mathcal{Q}, *, \diamond)$. Let $\{\mathcal{A}_k\}$ be a sequence of closed, non-void subsets of $\mathcal{W}$. This sequence is said to converge to $\mathcal{A}$, that is, non-void closed set in $\mathcal{W}$ in the Wijsman sense if there exists $k_0 \in \mathbb{N}$, for all $u, w \in \mathcal{W}$ and any $s > 0$ in a manner that*

$$\lim_{k \to \infty} \mathcal{P}(u, \mathcal{A}_k, w; s) = \mathcal{P}(u, \mathcal{A}, w; s) \text{ and}$$

$$\lim_{k \to \infty} \mathcal{Q}(u, \mathcal{A}_k, w; s) = \mathcal{Q}(u, \mathcal{A}, w; s) \text{ for all } k \geq k_0.$$

We represent by $\mathcal{L}_{\{\mathcal{A}_k\}}$ the set of all Wijsman limit points.

Definition 3.3 *Consider the IF-2MS $(\mathcal{W}, \mathcal{P}, \mathcal{Q}, *, \diamond)$. Let $\{\mathcal{A}_k\}$ be a sequence of closed, non-void subsets of $\mathcal{W}$. This sequence is said to Wijsman $\mathcal{J}-$ convergent to $\mathcal{A}$ that is, non-void closed set in $\mathcal{W}$, if for each $0 < \epsilon < 1$, $s > 0$ and for all $u, w \in \mathcal{W}$ in a manner that*

$$\Big\{k \in \mathbb{N} :\mid \mathcal{P}(u, \mathcal{A}_k, w; s) - \mathcal{P}(u, \mathcal{A}, w; s) \mid \leq 1 - \epsilon \quad or \quad \mid \mathcal{Q}(u, \mathcal{A}_k, w; s) - \mathcal{Q}(u, \mathcal{A}, w; s) \mid \geq \epsilon\Big\} \in \mathcal{J}.$$

We denote $\mathcal{W}_{\mathcal{J}} - \lim\limits_{k \to \infty} \mathcal{A}_k = \mathcal{A}$.

Example Let $(\mathcal{W}, \mathcal{P}, \mathcal{Q}, *, \diamond)$ be an IF-2MS. We define membership and non-membership functions as : for all $u, v, w \in \mathcal{W}$ and $s > 0$

$$\mathcal{P}(u, v, w; s) = \frac{s}{s + \mid (u - v)(v - w)(w - u) \mid} \qquad \text{and}$$

$$\mathcal{Q}(u, v, w; s) = \frac{\mid (u - v)(v - w)(w - u) \mid}{s + \mid (u - v)(v - w)(w - u) \mid}.$$

Consider the sequence of singleton sets in $\mathbb{R}$ is $A_k = \{\frac{1}{k}\}$ and limit be the set $A = \{0\}$. Therefore for any fix ponts $u, w \in \mathcal{W}$ to the set A_k as follows

$$\mathcal{P}(u, \mathcal{A}_k, w; s) = \sup_{a \in A} \mathcal{P}(u, a, w; s) = \mathcal{P}(u, \frac{1}{k}, w; s)$$

$$\mathcal{Q}(u, \mathcal{A}_k, w; s) = \inf_{a \in A} \mathcal{Q}(u, a, w; s) = \mathcal{Q}(u, \frac{1}{k}, w; s)$$

Let's select $u = 2, w = 3, s = 1$, we obtain

$$\mathcal{P}(2, \mathcal{A}_k, 3; 1) = \mathcal{P}(2, \frac{1}{k}, 3; 1) = \frac{1}{1 + \mid (2 - \frac{1}{k})(\frac{1}{k} - 3)(3 - 2) \mid} \to \frac{1}{7} \quad \text{as} \quad k \to \infty$$

$$\mathcal{Q}(2, \mathcal{A}_k, 3; 1) = \mathcal{Q}(2, \frac{1}{k}, 3; 1) = \frac{\mid (2 - \frac{1}{k})(\frac{1}{k} - 3)(3 - 2) \mid}{1 + \mid (2 - \frac{1}{k})(\frac{1}{k} - 3)(3 - 2) \mid} \to \frac{6}{7} \quad \text{as} \quad k \to \infty.$$

Also

$$\mathcal{P}(2,\mathcal{A},3;1)=\mathcal{P}(2,0,3;1)=\frac{1}{1+\mid(2-0)(0-3)(3-2)\mid}=\frac{1}{7}\quad\text{and}$$

$$\mathcal{Q}(2,\mathcal{A},3;1)=\mathcal{Q}(2,0,3;1)=\frac{\mid(2-0)(0-3)(3-2)\mid}{1+\mid(2-0)(0-3)(3-2)\mid}=\frac{6}{7}$$

Therefore, for all $u,w\in\mathcal{W}$ and $s>0$:

$$\lim_{k\to\infty}\mathcal{P}(u,\mathcal{A}_k,w;s)=\mathcal{P}(u,\mathcal{A},w;s)\text{ and }\lim_{k\to\infty}\mathcal{Q}(u,\mathcal{A}_k,w;s)=\mathcal{Q}(u,\mathcal{A},w;s)$$

Since,

$$\lim_{k\to\infty}\Big|\Big\{n\le k:\mid\mathcal{P}(u,\mathcal{A}_k,w;s)-\mathcal{P}(u,\mathcal{A},w;s)\mid\le 1-\epsilon\ \text{ or }\ \mid\mathcal{Q}(u,\mathcal{A}_k,w;s)-\mathcal{Q}(u,\mathcal{A},w;s)\mid\ge\epsilon\Big\}\Big|=0.$$

Thus, $\{\mathcal{A}_k\}$ is a Wijsman statistical convergence to $\mathcal{A}$.
Consider $V(\epsilon)$ define as:

$$V(\epsilon)=\Big\{k\in\mathbb{N}:\mid\mathcal{P}(u,\mathcal{A}_k,w;s)-\mathcal{P}(u,\mathcal{A},w;s)\mid\le 1-\epsilon\ \text{ or }\ \mid\mathcal{Q}(u,\mathcal{A}_k,w;s)-\mathcal{Q}(u,\mathcal{A},w;s)\mid\ge\epsilon\Big\}$$

If one take $\mathcal{J}=\mathcal{J}_d$ then Wijsman statistical and ideal convergence coincides.
Hence,

$$\Big\{k\in\mathbb{N}:\mid\mathcal{P}(u,\mathcal{A}_k,w;s)-\mathcal{P}(u,\mathcal{A},w;s)\mid\le 1-\epsilon\ \text{ or }\ \mid\mathcal{Q}(u,\mathcal{A}_k,w;s)-\mathcal{Q}(u,\mathcal{A},w;s)\mid\ge\epsilon\Big\}\in\mathcal{J}.$$

Definition 3.4 *An IF-2MS $(\mathcal{W},\mathcal{P},\mathcal{Q},*,\diamond)$ is separable if it possesses a countable dense subset. Concretely, there exists a countable collection there $\{\mathcal{A}_k\}_{k=1}^{\infty}\subset\mathcal{W}$ such that: For every $\mathcal{A}\subset\mathcal{W}$, any positive radius $s>0$ and for all $w\in\mathcal{W}$, one can find set $\mathcal{A}_n$ in this countable set satisfying:*

$$\mathcal{P}(\mathcal{A}_n,\mathcal{A},w;s)\ge 1-\epsilon \textit{ and } \mathcal{Q}(\mathcal{A}_n,\mathcal{A},w;s)\le\epsilon,\ \textit{for each } \epsilon\in(0,1).$$

Definition 3.5 *Let $(\mathcal{W},\mathcal{P},\mathcal{Q},*,\diamond)$ is a separable IF-2MS. Let $\{\mathcal{A}_k\}$ be a sequence of closed, non-void subsets of $\mathcal{W}$. This sequence is said to Wijsman $\mathcal{J}-$ Cauchy, if for each $1>\epsilon>0$, for each $u,w\in\mathcal{W}$ and for all $0<s,\exists\quad l=l(\epsilon)$ so that,*

$$\Big\{k\in\mathbb{N}:\mid\mathcal{P}(u,\mathcal{A}_k,w;s)-\mathcal{P}(u,\mathcal{A}_l,w;s)\mid\le 1-\epsilon\quad\textit{or}\quad\mid\mathcal{Q}(u,\mathcal{A}_k,w;s)-\mathcal{Q}(u,\mathcal{A}_l,w;s)\mid\ge\epsilon\Big\}\in\mathcal{J}.$$

Definition 3.6 *Let $(\mathcal{W},\mathcal{P},\mathcal{Q},*,\diamond)$ be a separable IF-2MS. Let $\{\mathcal{A}_k\}$ be a sequence of closed, non-void subsets of $\mathcal{W}$. This sequence is said to Wijsman $\mathcal{J}^*-$ Cauchy, if there exists $T=\{t=(t_j):t_j<t_{j+1},j\in\mathbb{N}\}\subset\mathbb{N}$ and $T\in\mathcal{F}(\mathcal{J})$, the subsequence $\mathcal{A}_T=\{\mathcal{A}_{t_k}\}$ is a Wijsman Cauchy in $\mathcal{W}$; that is*

$$\lim_{k,l\to\infty}\mid\mathcal{P}(u,\mathcal{A}_{t_k},w;s)-\mathcal{P}(u,\mathcal{A}_{t_l},w;s)\mid=1\qquad\textit{and}$$

$$\lim_{k,l\to\infty}\mid\mathcal{Q}(u,\mathcal{A}_{t_k},w;s)-\mathcal{Q}(u,\mathcal{A}_{t_l},w;s)\mid=0,\ \forall\ u,w\in\mathcal{W}.$$

Definition 3.7 *Let $(\mathcal{W}, \mathcal{P}, \mathcal{Q}, *, \diamond)$ be a separable IF-2MS. Let $\{\mathcal{A}_k\}$ be a sequence of closed, non-void subsets of $\mathcal{W}$. This sequence is said to Wijsman $\mathcal{J}^*-$ convergent to $\mathcal{A}$ that is, non-void closed set in $\mathcal{W}$, if there exists $T \in \mathcal{F}(\mathcal{J})$, where, $T = \{t = (t_j) : t_j < t_{j+1}, j \in \mathbb{N}\} \subset \mathbb{N}$ so that, for each $0 < s$, one obtain*

$$\lim_{k\to\infty} \mathcal{P}(u, \mathcal{A}_{t_k}, w; s) = \mathcal{P}(u, \mathcal{A}, w; s) \qquad \textit{and}$$

$$\lim_{k\to\infty} \mathcal{Q}(u, \mathcal{A}_{t_k}, w; s) = \mathcal{Q}(u, \mathcal{A}, w; s), \ \forall \ u, w \in \mathcal{W}.$$

We denote $\mathcal{W}_{\mathcal{J}^*} = \lim_{k\to\infty} \mathcal{A}_k = \mathcal{A}$.

Theorem 3.1 *Let $(\mathcal{W}, \mathcal{P}, \mathcal{Q}, *, \diamond)$ be a separable IF-2MS, and let $\mathcal{J}$ be any admissible ideal. If a sequence of closed sets $\{\mathcal{A}_k\}$ converges to a limit in the Wijsman $\mathcal{J}-$ sense, then that sequence is necessarily Wijsman $\mathcal{J}-$ Cauchy.*

Proof: Suppose $\mathcal{W}_{\mathcal{J}} - \lim_{k\to\infty} \mathcal{A}_k = \mathcal{A}$. Then for each $1 > \epsilon > 0$, $0 < s$ and for all $u, w \in \mathcal{W}$, the set

$$M(\epsilon, s) = \Big\{k \in \mathbb{N} :| \mathcal{P}(u, \mathcal{A}_k, w; s) - \mathcal{P}(u, \mathcal{A}, w; s) |\leq 1 - \epsilon \text{ or}$$
$$| \mathcal{Q}(u, \mathcal{A}_k, w; s) - \mathcal{Q}(u, \mathcal{A}, w; s) |\geq \epsilon\Big\} \in \mathcal{J}.$$

Being $\mathcal{J}$ is a admissible ideal consequently there is a positive integer k_0 in such a manner that, $k_0 \notin M(\epsilon, s)$. Now, assume that

$$N(\epsilon, s) = \Big\{k \in \mathbb{N} :| \mathcal{P}(u, \mathcal{A}_k, w; s) - \mathcal{P}(u, \mathcal{A}_{k_0}, w; s) |\leq (1 - 2\epsilon) \text{ or}$$
$$| \mathcal{Q}(u, \mathcal{A}_k, w; s) - \mathcal{Q}(u, \mathcal{A}_{k_0}, w; s) |\geq 2\epsilon\Big\}.$$

Considering the inequality

$$| \mathcal{P}(u, \mathcal{A}_k, w; s) - \mathcal{P}(u, \mathcal{A}_{k_0}, w; s) |\leq| \mathcal{P}(u, \mathcal{A}_k, w; s) - \mathcal{P}(u, \mathcal{A}, w; s) |$$
$$+ | \mathcal{P}(u, \mathcal{A}_{k_0}, w; s) - \mathcal{P}(u, \mathcal{A}, w; s) |$$

and

$$| \mathcal{Q}(u, \mathcal{A}_k, w; s) - \mathcal{Q}(u, \mathcal{A}_{k_0}, w; s) |\leq| \mathcal{Q}(u, \mathcal{A}_k, w; s) - \mathcal{Q}(u, \mathcal{A}, w; s) |$$
$$+ | \mathcal{Q}(u, \mathcal{A}_{k_0}, w; s) - \mathcal{Q}(u, \mathcal{A}, w; s) |$$

Observe that, if $k \in N(\epsilon, s)$, therefore

$$| \mathcal{P}(u, \mathcal{A}_k, w; s) - \mathcal{P}(u, \mathcal{A}, w; s) | + | \mathcal{P}(u, \mathcal{A}_{k_0}, w; s) - \mathcal{P}(u, \mathcal{A}, w; s) |\leq (1 - 2\epsilon)$$

and

$$| \mathcal{Q}(u, \mathcal{A}_k, w; s) - \mathcal{Q}(u, \mathcal{A}, w; s) | + | \mathcal{Q}(u, \mathcal{A}_{k_0}, w; s) - \mathcal{Q}(u, \mathcal{A}, w; s) |\geq 2\epsilon.$$

From another point of view, since $k_0 \notin N(\epsilon, s)$, we obtain

$$| \mathcal{P}(u, \mathcal{A}_{k_0}, w; s) - \mathcal{P}(u, \mathcal{A}, w; s) |> 1 - \epsilon \ \text{ and } \ | \mathcal{Q}(u, \mathcal{A}_{k_0}, w; s) - \mathcal{Q}(u, \mathcal{A}, w; s) |< \epsilon.$$

We achieve that

$$| \mathcal{P}(u, \mathcal{A}_k, w; s) - \mathcal{P}(u, \mathcal{A}, w; s) |\leq 1 - \epsilon \text{ or } | \mathcal{Q}(u, \mathcal{A}_k, w; s) - \mathcal{Q}(u, \mathcal{A}, w; s) |\geq \epsilon.$$

Hence $k \in M(\epsilon, s)$. This imply that $M(\epsilon, s) \subset N(\epsilon, s) \in \mathcal{J}$ for each $1 > \epsilon > 0$ and for all $0 < s$ and $u, w \in \mathcal{W}$. Therefore, $N(\epsilon, s) \in \mathcal{J}$, consequently $\{\mathcal{A}_k\}$ is a Wijsman $\mathcal{J}$–Cauchy. □

Theorem 3.2 *Let $(\mathcal{W}, \mathcal{P}, \mathcal{Q}, *, \diamond)$ is a separable IF-2MS and $\mathcal{J}$ be an admissible ideal. The sequence $\{\mathcal{A}_k\}$ is Wijsman $\mathcal{J}-$ Cauchy if $\{\mathcal{A}_k\}$ is Wijsman $\mathcal{J}^*-$ Cauchy.*

Proof: Let $\{\mathcal{A}_k\}$ be a Wijsman $\mathcal{J}^*-$ Cauchy sequence. Subsequently, for every $u, w \in \mathcal{W}$ and for every $\epsilon \in (0,1)$, there exists $T \in \mathcal{F}(\mathcal{J})$, where, $T = \{(t_j) : t_j < t_{j+1}, j \in \mathbb{N}\}$ in order that

$$| \mathcal{P}(u, \mathcal{A}_{t_k}, w; s) - \mathcal{P}(u, \mathcal{A}_{t_l}, w; s) | \leq 1 - \epsilon$$

and

$$| \mathcal{Q}(u, \mathcal{A}_{t_k}, w; s) - \mathcal{Q}(u, \mathcal{A}_{t_l}, w; s) | \geq \epsilon, \quad \forall \quad k, l > k_0 = k_0(\epsilon).$$

Suppose $N = N(\epsilon) = t_{k_0+1}$. Therefore for each $\epsilon > 0$, one obtain

$$| \mathcal{P}(u, \mathcal{A}_{t_k}, w; s) - \mathcal{P}(u, \mathcal{A}_N, w; s) | \leq 1 - \epsilon$$

and

$$| \mathcal{Q}(u, \mathcal{A}_{t_k}, w; s) - \mathcal{Q}(u, \mathcal{A}_N, w; s) | \geq \epsilon \text{ for all } k > k_0.$$

Consider, $S = \mathbb{N} \setminus T$. Evidently $S \in \mathcal{J}$ and

$$H(\epsilon, s) = \Big\{k \in \mathbb{N} : | \mathcal{P}(u, \mathcal{A}_k, w; s) - \mathcal{P}(u, \mathcal{A}_N, w; s) | \leq 1 - \epsilon \text{ or } | \mathcal{Q}(u, \mathcal{A}_k, w; s) - \mathcal{Q}(u, \mathcal{A}_N, w; s) | \geq \epsilon\Big\}$$

$$H(\epsilon, s) \subset S \cup \{t_1, t_2, ..., t_{k_0}\} \in \mathcal{J}.$$

On that account, for each $1 > \epsilon > 0$ and for all $0 < s$, one can find out $N = N(\epsilon)$ in order $H(\epsilon, s) \in \mathcal{J}$. Hence, $\{\mathcal{A}_k\}$ is the Wijsman $\mathcal{J}-$ Cauchy. □

Theorem 3.3 *Let $(\mathcal{W}, \mathcal{P}, \mathcal{Q}, *, \diamond)$ is a separable IF-2MS, and let $\mathcal{J}$ be an admissible ideal satisfying the additive property (AP). Then, Wijsman$\mathcal{J}^*-$ Cauchy and Wijsman $\mathcal{J}-$ Cauchy sequence of sets are conceptually equivalent.*

Proof: Theorem 3.2 already proved the immediate implication.
The sequence of closed sets $\{\mathcal{A}_k\}$ is now assumed to be Wijsman $\mathcal{J}-$ Cauchy. Then, for each $1 > \epsilon > 0$, for every $u, w \in \mathcal{W}$ and for all $0 < s$, there exists a $n = n(\epsilon)$ in such a manner that

$$C(\epsilon, s) = \Big\{k \in \mathbb{N} : | \mathcal{P}(u, \mathcal{A}_k, w; s) - \mathcal{P}(u, \mathcal{A}_n, w; s) | \leq 1 - \epsilon \text{ or } | \mathcal{Q}(u, \mathcal{A}_k, w; s) - \mathcal{Q}(u, \mathcal{A}_n, w; s) | \geq \epsilon\Big\} \in \mathcal{J}$$

Now, suppose that

$$D_j(\epsilon, s) = \Big\{k \in \mathbb{N} : | \mathcal{P}(u, \mathcal{A}_k, w; s) - \mathcal{P}(u, \mathcal{A}_{N_j}, w; s) | > 1 - \frac{1}{j} \text{ or } | \mathcal{Q}(u, \mathcal{A}_k, w; s) - \mathcal{Q}(u, \mathcal{A}_{N_j}, w; s) | < \frac{1}{j}\Big\},$$

where $N_j = N(\frac{1}{j})$, $j = 1, 2 \cdots$. Evidently, for $j = 1, 2 \cdots$, $D_j(\epsilon, s) \in \mathcal{F}(\mathcal{J})$. By utilizing Lemma 2.1, there is a set $T \subset \mathbb{N}$ in order that, $T \setminus D_j$ be a finite set for all j and $T \in \mathcal{F}(\mathcal{J})$.
Now, we need to show that

$$\lim_{k,l \to \infty} | \mathcal{P}(u, \mathcal{A}_k, w; s) - \mathcal{P}(u, \mathcal{A}_l, w; s) | = 1$$

and

$$\lim_{k,l\to\infty} \mid \mathcal{Q}(u,\mathcal{A}_k,w;s)-\mathcal{Q}(u,\mathcal{A}_l,w;s)\mid = 0.$$

Let $\eta\in\mathbb{N}$ and $0<\epsilon$ in such a manner that $\eta>\frac{2}{\epsilon}$. If k,l in T subsequently $T\setminus T_j$ is finite, then there exists $m=m(\eta)$ in such a manner that

$$\mid \mathcal{P}(u,\mathcal{A}_k,w;s)-\mathcal{P}(u,\mathcal{A}_{l_\eta},w;s)\mid > 1-\frac{1}{\eta}$$

$$\mid \mathcal{P}(u,\mathcal{A}_l,w;s)-\mathcal{P}(u,\mathcal{A}_{l_\eta},w;s)\mid > 1-\frac{1}{\eta}$$

and

$$\mid \mathcal{Q}(u,\mathcal{A}_k,w;s)-\mathcal{Q}(u,\mathcal{A}_{l_\eta},w;s)\mid < \frac{1}{\eta}$$

$$\mid \mathcal{Q}(u,\mathcal{A}_l,w;s)-\mathcal{Q}(u,\mathcal{A}_{l_\eta},w;s)\mid < \frac{1}{\eta}$$

for every $k,l>m(\eta)$. Then, for $k,l>m(\eta)$ the above inequalities follows as

$$\mid \mathcal{P}(u,\mathcal{A}_k,w;s)-\mathcal{P}(u,\mathcal{A}_l,w;s)\mid \leq \mid \mathcal{P}(u,\mathcal{A}_k,w;s)-\mathcal{P}(u,\mathcal{A}_{l_\eta},w;s)\mid + \mid \mathcal{P}(u,\mathcal{A}_l,w;s)-\mathcal{P}(u,\mathcal{A}_{l_\eta},w;s)\mid$$

$$\begin{aligned}\mid \mathcal{P}(u,\mathcal{A}_k,w;s)-\mathcal{P}(u,\mathcal{A}_l,w;s)\mid &> (1-\frac{1}{\eta})+(1-\frac{1}{\eta})\\ &> 1-\epsilon\end{aligned}$$

and

$$\mid \mathcal{Q}(u,\mathcal{A}_k,w;s)-\mathcal{Q}(u,\mathcal{A}_l,w;s)\mid \leq \mid \mathcal{Q}(u,\mathcal{A}_k,w;s)-\mathcal{Q}(u,\mathcal{A}_{l_j},w;s)\mid + \mid \mathcal{Q}(u,\mathcal{A}_l,w;s)-\mathcal{Q}(u,\mathcal{A}_{l_j},w;s)\mid$$

$$\begin{aligned}\mid \mathcal{Q}(u,\mathcal{A}_k,w;s)-\mathcal{Q}(u,\mathcal{A}_l,w;s)\mid &< \frac{1}{\eta}+\frac{1}{\eta}\\ &< \epsilon\end{aligned}$$

Hence, for every $0<\epsilon, \exists\ \ m=m(\epsilon)$ and $k,l\in P$, one obtain

$$\Big\{k\in\mathbb{N}: \mid \mathcal{P}(u,\mathcal{A}_k,w;s)-\mathcal{P}(u,\mathcal{A}_l,w;s)\mid \leq 1-\epsilon \text{ or } \mid \mathcal{Q}(u,\mathcal{A}_k,w;s)-\mathcal{Q}(u,\mathcal{A}_l,w;s)\mid \geq \epsilon\Big\}\in\mathcal{J}.$$

Hence, $\{\mathcal{A}_k\}$ is a Wijsman $\mathcal{J}^*-$ Cauchy sequence. □

Theorem 3.4 *Assume $\mathcal{J}$ ia an arbitrary admissible ideal, and let $(\mathcal{W},\mathcal{P},\mathcal{Q},*,\diamond)$ is a separable IF-2MS. Therefore,*

$$\mathcal{W}^*_{\mathcal{J}}-\lim_{k\to\infty}\mathcal{A}_k=\mathcal{A}$$

it follows that sequence $\{\mathcal{A}_k\}$ is Wijsman $\mathcal{J}-$ Cauchy.

Proof: Let $\mathcal{W}^*_{\mathcal{J}} - \lim_{k\to\infty} \mathcal{A}_k = \mathcal{A}$. According to the definition 2.8, a set T exists satisfying the required condition $T = \{t = (t_j) : t_j < t_{j+1}, j \in \mathbb{N}\} \subset \mathbb{N}$ with $T \in \mathcal{F}(\mathcal{J})$ in such a way that $\mathcal{A}_T = \{\mathcal{A}_{t_k}\}$

$$\lim_{k\to\infty} \mathcal{P}(u, \mathcal{A}_{t_k}, w; s) = \mathcal{P}(u, \mathcal{A}, w; s) \quad \text{and}$$

$$\lim_{k\to\infty} \mathcal{Q}(u, \mathcal{A}_{t_k}, w; s) = \mathcal{Q}(u, \mathcal{A}, w; s),\ k, l > k_0.$$

Assume $\delta \in \mathbb{N}$ and $0 < \epsilon$ so that $\delta > \frac{2}{\epsilon}$. If $k, l \in T$ therefore $T \setminus T_j$ is finite set, then there exists $k(\delta) = k$ in order that

$$\mid \mathcal{P}(u, \mathcal{A}_{t_k}, w; s) - \mathcal{P}(u, \mathcal{A}_{t_l}, w; s) \mid \leq \mid \mathcal{P}(u, \mathcal{A}_{t_k}, w; s) - \mathcal{P}(u, \mathcal{A}, w; s) \mid + \mid \mathcal{P}(u, \mathcal{A}_{t_l}, w; s) - \mathcal{P}(u, \mathcal{A}, w; s) \mid$$

$$\mid \mathcal{P}(u, \mathcal{A}_{t_k}, w; s) - \mathcal{P}(u, \mathcal{A}_{t_l}, w; s) \mid > (1 - \frac{1}{\delta}) + (1 - \frac{1}{\delta}) > 1 - \epsilon$$

and

$$\mid \mathcal{Q}(u, \mathcal{A}_{t_k}, w; s) - \mathcal{Q}(u, \mathcal{A}_{t_l}, w; s) \mid < \mid \mathcal{Q}(u, \mathcal{A}_{t_k}, w; s) - \mathcal{Q}(u, \mathcal{A}, w; s) \mid + \mid \mathcal{Q}(u, \mathcal{A}_{t_l}, w; s) - \mathcal{Q}(u, \mathcal{A}, w; s) \mid$$

$$\mid \mathcal{Q}(u, \mathcal{A}_{t_k}, w; s) - \mathcal{Q}(u, \mathcal{A}_{t_l}, w; s) \mid < \frac{1}{\delta} + \frac{1}{\delta} < \epsilon$$

Then,

$$\lim_{k,l\to\infty} \mid \mathcal{P}(u, \mathcal{A}_{t_k}, w; s) - \mathcal{P}(u, \mathcal{A}_{t_l}, w; s) \mid = 1$$

and

$$\lim_{k,l\to\infty} \mid \mathcal{Q}(u, \mathcal{A}_{t_k}, w; s) - \mathcal{Q}(u, \mathcal{A}_{t_l}, w; s) \mid = 0.$$

Hence, sequence $\{\mathcal{A}_k\}$ is Wijsman $\mathcal{J}-$ Cauchy. □

Definition 3.8 *Assume $(\mathcal{W}, \mathcal{P}, \mathcal{Q}, *, \diamond)$ is a separable IF-2MS. Any non-void and closed set $\mathcal{A} \in \mathcal{W}$ is known as*

1. *Wijsman $\mathcal{J}$-cluster point of $\{\mathcal{A}_k\}$ iff for each $\epsilon > 0$, $s > 0$ and for all $u, w \in \mathcal{W}$, the set*

$$\Big\{k \in \mathbb{N} : \mid \mathcal{P}(u, \mathcal{A}_k, w; s) - \mathcal{P}(u, \mathcal{A}, w; s) \mid < 1 - \epsilon \ \text{or} \ \mid \mathcal{Q}(u, \mathcal{A}_k, w; s) - \mathcal{Q}(u, \mathcal{A}, w; s) \mid > \epsilon\Big\} \notin \mathcal{J}.$$

   We represent by $\mathcal{W}_{\mathcal{J}}(\Gamma \mathcal{A}_k)$ the set of all Wijsman $\mathcal{J}$-cluster points.

2. *Wijsman $\mathcal{J}$-limit point of $\{\mathcal{A}_k\}$ if $T = \{t = (t_j) : t_j < t_{j+1}; j \in \mathbb{N}\} \subset \mathbb{N}$ in a manner that $T \notin \mathcal{J}$, for all $u, w \in \mathcal{W}$ and $0 < s$, one have*

$$\lim_{k\to\infty} \mathcal{P}(u, \mathcal{A}_{t_k}, w; s) = \mathcal{P}(u, \mathcal{A}, w; s) \ \text{and} \ \lim_{k\to\infty} \mathcal{Q}(u, \mathcal{A}_{t_k}, w; s) = \mathcal{Q}(u, \mathcal{A}, w; s).$$

   We represent by $\mathcal{W}_{\mathcal{J}}(\Lambda_{\{\mathcal{A}_k\}})$ the set of all Wijsman $\mathcal{J}$-limit points.

Theorem 3.5 *In separable IF-2MS $(\mathcal{W}, \mathcal{P}, \mathcal{Q}, *, \diamond)$, for any sequence of closed sets, $\{\mathcal{A}_k\} \subset \mathcal{W}$; we have $\mathcal{W}_{\mathcal{J}}(\Lambda_{\{\mathcal{A}_k\}}) \subset \mathcal{W}_{\mathcal{J}}(\Gamma_{\{\mathcal{A}_k\}})$.*

Proof: Assume $\mathcal{A} \in \mathcal{W}_{\mathcal{J}}(\Lambda_{\{\mathcal{A}_k\}})$, therefore there is set $T = \{t_1 < t_2 < \dots\} \subset \mathbb{N}$ in such a way that $T = \{t = (t_j) : t_j < t_{j+1}, j \in \mathbb{N}\} \notin \mathcal{J}$. For all $u, w \in \mathcal{W}$ and for each $s > 0$, one obtain

$$\lim_{k\to\infty} \mathcal{P}(u, \mathcal{A}_{t_k}, w; s) = \mathcal{P}(u, \mathcal{A}, w; s) \tag{3.1}$$

and

$$\lim_{k\to\infty} \mathcal{Q}(u, \mathcal{A}_{t_k}, w; s) = \mathcal{Q}(u, \mathcal{A}, w; s). \tag{3.2}$$

Following the above equations (3.1) and (3.2), there is positive integer k_0 in such a way that for all $u, w \in \mathcal{W}$ and for any $\epsilon > 0$, and $k_0 < k$

$$\mid \mathcal{P}(u, \mathcal{A}_{t_k}, w; s) - \mathcal{P}(u, \mathcal{A}, w; s) \mid > 1 - \epsilon$$

and

$$\mid \mathcal{Q}(u, \mathcal{A}_{t_k}, w; s) - \mathcal{Q}(u, \mathcal{A}, w; s) \mid < \epsilon.$$

Then,

$$\{k \in \mathbb{N} : \mid \mathcal{P}(u, \mathcal{A}_{t_k}, w; s) - \mathcal{P}(u, \mathcal{A}, w; s) \mid > 1 - \epsilon, \mid \mathcal{Q}(u, \mathcal{A}_{t_k}, w; s) - \mathcal{Q}(u, \mathcal{A}, w; s) \mid < \epsilon\}$$
$$\supseteq P \setminus \{t_1, t_1, \dot{;} t_{k_0}\} \notin \mathcal{J}$$

Hence,

$$\{k \in \mathbb{N} : \mid \mathcal{P}(u, \mathcal{A}_{p_k}, w; s) - \mathcal{P}(u, \mathcal{A}, w; s) \mid > 1 - \epsilon, \mid \mathcal{Q}(u, \mathcal{A}_{p_k}, w; s) - \mathcal{Q}(u, \mathcal{A}, w; s) \mid < \epsilon\} \notin \mathcal{J}$$

This inplies that $\mathcal{A} \in W_{\mathcal{J}}(\Gamma_{\{\mathcal{A}_k\}})$. □

Theorem 3.6 *In separable IF-2MS $(\mathcal{W}, \mathcal{P}, \mathcal{Q}, *, \diamond)$, for every sequence of closed sets $\{\mathcal{A}_k\} \subset \mathcal{W}$, one obtain $\mathcal{W}_{\mathcal{J}}(\Gamma_{\{\mathcal{A}_k\}}) \subset \mathcal{L}_{\{\mathcal{A}_k\}}$.*

Proof: Assume $\mathcal{A} \in \mathcal{W}_{\mathcal{J}}(\Gamma_{\{\mathcal{A}_k\}})$. Then for all $u, w \in \mathcal{W}$, for any $0 < \epsilon < 1$ and $0 < s$, the set

$$\{k \in \mathbb{N} : \mid \mathcal{P}(u, \mathcal{A}_k, w; s) - \mathcal{P}(u, \mathcal{A}, w; s) \mid < 1 - \epsilon \text{ or } \mid \mathcal{Q}(u, \mathcal{A}_k, w; s) - \mathcal{Q}(u, \mathcal{A}, w; s) \mid > \epsilon\} \notin \mathcal{J}.$$

Let,

$$U_k := \{k \in \mathbb{N} : \mid \mathcal{P}(u, \mathcal{A}_k, w; s) - \mathcal{P}(u, \mathcal{A}, w; s) \mid > 1 - \frac{1}{k}, \mid \mathcal{Q}(u, \mathcal{A}_k, w; s) - \mathcal{Q}(u, \mathcal{A}, w; s) \mid < \frac{1}{k}\}.$$

$\{U_k\}_{k=1}^{\infty}$ of infinite subsets of $\mathbb{N}$ such that $U_{k+1} \subseteq U_k$ for all k. Therefore,
$U = \{k = (k_i) : k_i < k_{i+1}, i \in \mathbb{N}\} \notin \mathcal{J}$ in order that

$$\lim_{k\to\infty} \mathcal{P}(u, \mathcal{A}_{k_i}, w; s) = \mathcal{P}(u, \mathcal{A}, w; s)$$

and

$$\lim_{k\to\infty} \mathcal{Q}(u, \mathcal{A}_{k_i}, w; s) = \mathcal{Q}(u, \mathcal{A}, w; s)$$

which implies $\mathcal{A} \in \mathcal{L}_{\{\mathcal{A}_k\}}$. □

4. Conclusion

In this paper, we have expanded the exploration of convergence within intuitionistic fuzzy 2-metric spaces by examining Wijsman ideal convergence. Additionally, the introduction of $\mathcal{J}-$ limits and $\mathcal{J}-$ cluster points, offers a more profound insight into limit behavior along with their inclusion relationship. These findings not only broaden existing concepts of convergence but also pave the way for further research, such as investigating various classes of approximation methods, ideals, extending to probabilistic or neutrosophic or 2-metric spaces, and exploring potential applications in decision theory.

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