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Multi-Objective Green Vehicle Routing Problem with Uncertain Customer Demand and Carbon Emission Reduction

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ABSTRACT: Vehicle Routing Problem with Time Windows is a combinatorial optimization problem that deals with the fleet of vehicles to find the optimal set of routes while serving the customers at different nodes in a given geographical region within specific time intervals. In this research article, the objective is to minimize the total operational costs, CO_2 emissions and fulfilling the fuzzy customer demand. To calculate the optimal results, a mathematical model consisting of all the restricted constraints is presented and genetic algorithm and particle swarm optimization algorithm is preferred to find the solutions. Moreover, alpha- cut method is applied to calculate the crisp interval for the fuzzy demands. To conclude the experimental results, Solomon dataset (R101) is used to compare the outcomes of genetic algorithm and particle swarm optimization algorithm and it is observed that both genetic algorithm and particle swarm optimization algorithm provides satisfactory results but genetic algorithm provides more optimal outcomes as compared to particle swarm optimization algorithm.

Keywords: Vehicle routing, carbon emissions, uncertainty, evolutionary algorithm, logistics.

Contents

1. Introduction

Vehicle Routing Problem (VRP) is a generalization of Travelling Salesman Problem (TSP) where, there is only one salesman who visit all the customers and fulfil their requirements. But in case of VRP, there are multiple salesman or vehicles that visit multiple customers to fulfil their demands starting and ending the route at a central depot. There are some constraints as well that each vehicle must visit a customer exactly once or other condition can be the vehicle must not overload itself or it can be said that the loading capacity of a vehicle must not be exceeded. VRP first came into existence in 1959 where two researchers (Dantzig and Ramser 1959) worked on a VRP to minimize the overall cost. The authors had also considered some constraints where the loading capacity was limited and time windows were there. VRP is a fundamental combinatorial optimization problem in supply chain and logistics. VRP is the

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base of transportation as there is always a need to transfer goods from one place to another and the objective of industries and companies is always to minimize the overall operational costs and maximize the profits. In VRP, the main motive is to optimize the routes by minimizing transportational costs, this is what the requirement of every industry. Therefore, VRP was introduced to find the optimal solutions. VRP has different variants based on different constraints and conditions. There are some constraints which are usually used by researchers to find the results. First is Capacitated Vehicle Routing Problem (CVRP) where the capacity of the vehicle is limited and it cannot be exceeded. Second is Vehicle Routing Problem with Time Windows (VRPTW) where there are restricted time windows and the salesman must visit the customer in that duration, if it violates, then there are penalty charges for breaking the time windows. Next is Multi Depot Vehicle Routing Problem (MDVRP) where the depots are more than one and multiple vehicles start their routes from multiple depots to visit multiple customers. Another one is Split Delivery Vehicle Routing Problem (SDVRP) where the customers' deliveries are split into multiple deliveries to minimize the total distance travelled. Next is Vehicle Routing Problem with Backhauls (VRPB) where the process is that the vehicle must visit the customer, fulfil his demand, and pick the products in the way while returning to the depot. Another one is Vehicle Routing Problem with Pickup and Delivery (VRPPD) where the vehicle first picks the products and then deliveries it to the customers without exceeding its loading capacity. Next is Open Vehicle Routing Problem (OVRP) in which the vehicle must start its route from the depot but there is no need to return to the depot after completing its deliveries. In this way, there are many more variants of VRP depending on the constraints and real-life scenarios. Figure 1 is describing the different variants of VRP depending on different conditions. If real world conditions are considered there are many more complex variants depending on different conditions concerned with the environment. Green Vehicle Routing Problem (GVRP), and Electric Vehicle Routing Problem (EVRP). These two variants are same but having minor differences of constraints. In GVRP, the objective is to minimize the total operational costs along with minimizing the CO_2 emissions. Traditional and Hybrid vehicles can be used in this case; alternate fuel vehicles can also be preferred that run on Compressed Natural Gas (CNG). In EVRP, electric vehicles are used in which electricity can be generated by many different methods by applying different chemical reactions. This research article is all concerned with minimizing the CO_2 emissions by focusing on reducing pollution. Therefore, it is related to GVRP. GVRP helps in reducing air pollution and maintains environmental sustainability, therefore it also fulfils some sustainable development goals (SDGs) generated by United Nations [30]. There are 17 goals and 169 targets developed by the United Nations to maintain the growth and productivity of the world. Out of 17 SDGs, this research fulfils 4 SDGs. These 4 SDGs involve Goal number 7, Goal number 9, Goal number 11, and Goal number 13. Goal 7 is concerned as Affordable and Clean Energy; it is fulfilled by this research as the research is related to minimize the fuel consumption. Goal 9 is related to Industry, Innovation, and Infrastructure; it is fulfilled in this article as whole research is linked to industries and logistics. Goal 11 is Sustainable Cities and Communities which is fulfilled in this paper as preferring minimization of carbon emissions will make cities more sustainable to survive. Goal 13 is Climate Action which is fulfilled by GVRP by optimizing transportation logistics to reduce greenhouse gas emissions. Apart from this, many other conditions are there including uncertainty in demand of customers which can be improved by using different methods and approaches. As VRP is a complex problem and by adding constraint of green and uncertainty, it becomes more complex. This complexity of the problem makes it harder to solve and cannot be solved manually as there are some large instances as well. These problems are known to be NP- Hard Problems (Non deterministic Polynomial) which means the problems that cannot be solved in polynomial time. Now, NP- Hard is not only type of complexity classes [8]. There are many more, some are deterministic and some are non- deterministic. Basic Time complexity classes are P and NP. P is polynomial time where the class of problems can be solved in polynomial time by using a Turing machine. These types of problems are effectively solvable and give determined results. Second is NP which means nondeterministic polynomial time. In this case, the class of problems cannot be solved in polynomial time but can be verified in polynomial time. Next is, NP Complete which means the problem is the hardest problem in NP and if any NP Complete problem is in P, then P= NP. Traveling Salesman Problem is an example of NP Complete. Last is the NP- Hard problem which is as hard as the hardest problems in NP but not necessarily in NP. Optimization problems are NP- Hard problems as these problems are having nondeterministic constraints where the problems are undecidable. As in this research article, optimization combinatorial problem of VRP is considered, therefore, it is also NP- Hard problem. To solve such kind of complex problems, many exact, heuristics and metaheuristics approaches are developed by many researchers since 1959. If this problem is related to real- life it becomes more complex and constraint of uncertainty increases its complexity. To deal with uncertainty, many methods are developed such as alpha- cut method, Chance-Constrained Programming and Mamdani Fuzzy Inference System (MFIS). This paper consists of different sections. Section 2 deals with the literature review. Section 3 is the mathematical model. Section 4 consists of solution approach. Section 5 gives the experimental results. Section 6 is the comparison of two different algorithms. Section 7 is the conclusion of the whole article.

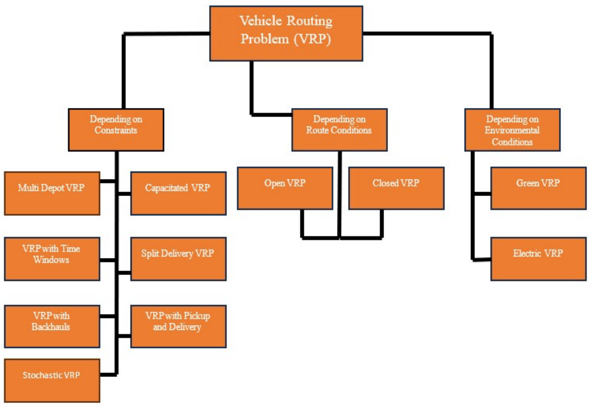

Figure 1: Variants of VRP depending on Different Conditions

2. Literature Review

VRP was first introduced by [18]. Before that, traveling salesman Problem [17] was introduced by researchers in 1954 where the authors studied TSP in a way having large scale instances related to real life data. The researchers have collected data of 49 cities one in each state and the district of Columbia and the objective was to minimize the total distance which means that the researchers found the shortest route. After TSP, the value was given to VRP as in VRP multi vehicles and delivery men were involved. After 1959, there are thousands of papers based on VRP with different constraints and conditions. In 1966, in a dissertation presented by [7], an inventory control problem was considered and heuristic approach was applied to solve the routing problem. A master's thesis was also written by [15] on large scale optimization problem where results were presented and some sample problems were also tested to show the performance of the proposed algorithm which was Clarke and Wright Method. In 1970, [62] introduced Chinese postman problem which was a master thesis as well and the author explained everything in different chapters. First chapter included the general overview of the problem, chapter 2 was related to analysation of problems, Chapter 3 introduced different algorithms which include the Murty's Assignment algorithm, and Decomposition algorithm. In chapter 4, some algorithms were proposed by the author. Chapter 5 involved some computational experiments for large scale cities. In 1974, [5] introduced some technique to solve the problem related to municipal waste collection. The authors elaborated the Clarke and Wright method using an example. In the same year, [53] introduced a fundamental VRP where classic traveling salesman problem and Chinese postman problem were shown as limited cases of generalized routing problem. In 1975, waste collection vehicles were also considered by [68] as the authors have also solved the problem for optimal solutions. Two examples were shown to demonstrate the efficiency of the proposed algorithm which was modified multiple traveling salesmen cost matrix to model. In the next year, [13] preferred some exact as well as approximate methods to solve VRPs and some results were provided based on feasible solution which were helpful in reducing the computational efforts. A multi-depot algorithm was developed by [27] and the authors had compared different heuristics algorithms to check their performance and presented some modifications and extensions to solve large scale problems in a few seconds. Moreover, the developed algorithm was tested on urban newspaper with an evening circulation. VRP with probabilistic demand which was introduced by [28] and Poisson distribution was preferred to show the demand customers at different nodes. The objective was to minimize the total distance travelled with the restriction that probability of the primary error is sufficiently small. [32] worked on VRP and compared two different methods Clarks and Wright method and Districting Analysis to check the performance of both for VRP. The difference in results was as Clarke- Wright method gave exact sequence of stops followed by the vehicles; however, districting analysis grouped the demand points of the total service area into a specified number of compact service districts. In 1981, the researchers [14] used exact algorithms for VRP based on the spanning tree method and shortest path relaxations. The authors found the computational results based on the problems studied in the literature review by them. A branch and bound algorithm was introduced by [40] to find the optimal results. In this paper, the limited loading capacity constraint was also included to check the performance of the proposed method. Exact solutions were found for problems ranging from 15 to 50 cities. After that, in 1984, two exact algorithms were preferred by [41] for a distance- constrained VRP which means there is a restriction of the vehicle cannot exceed the prespecified upper bound of the distance covered by it. Two algorithms used were- Gomory cutting planes and branch and bound method. Numerical results were also computed by the researchers. Six different methods for implementing the Clarke- wright method were recommended in the research article written by [52] where the researchers also compared all the six methods. The methods were checked on both low-density and high-density problems. 55 large test problems were considered to compare the methods. [42] preferred asymmetrical capacitated VRP (CVRP) to do their research and used an exact algorithm for this. Asymmetrical CVRP means multiple traveling salesman problem with capacity constraints. Computational results for 260 nodes were reported by the authors. Different exact algorithms for VRP were introduced by [43] where the authors explained direct tree search algorithms and dynamic programming in detail. The authors found that the exact methods can only handle issues of relatively modest dimensions. Dynamic VRPs were considered in 1988 by [57]. Dynamic means real-time, therefore, Dynamic VRPs (DVRP) are mostly concerned with the real-time problems. The authors focused on DVRP provided differences between dynamic and static VRP and solution methods. [12] considered a VRP and inventory allocation to maximize the profit while satisfying the customers with the product delivery. The scholars developed Lagrangian-based approach to solve the problem and found that this method can generate solutions with small gaps in upper and lower bounds for a range of cost structures. In the next year, [23] worked on VRP where the authors have shown multiple use of vehicles. The problem was based on single depot and time windows constraint. Moreover, vehicle can do multiple trips within a particular time. The computational results were found for large real- life instances. As there were a lot of research articles on VRP since 1959, [44] gathered all the important data and wrote a review article considering an overview of exact and approximate algorithms applied to solve VRPs. The author described many exact algorithms and heuristic algorithms and, in the conclusion, it was found that exact algorithms can only solve small problems but approximate algorithms can give satisfactory results for large problems as well. Two partition methods were used by [63] to solve vehicle routing problem. The first partition method was based on partition into polar regions which was suitable for Euclidean problems where cities are distributed along a central depot. Second partition method was based on the arborescence built from the shortest paths from any city to the depot. A tabu search method for vehicle routing problem was introduced by [26] where the main motive of the paper was to describe the TABUROUTE, where the problem was having capacity and route length restrictions. The results computed on benchmarks proved that tabu search gave better results and TABUROUTE most of the times produces best known solutions. In 1995, a vehicle routing problem with time windows was considered by [38] which was solved by using greedy randomized adaptive search procedure (GRASP) and results were calculated for 417 customers at 100 nodes dataset. Genetic algorithm on vehicle routing problem was applied by [55] and time windows constraint was also considered. This paper also provided the comparison with other heuristics approaches as well. An exact approach was applied by [66] for vehicle routing problem with backhauls. A new integer linear programming model and a Lagrangian lower bound were combined to generate the results. Computational results were done on 100 customers to check the performance of the proposed method. Ant Colony optimization method was applied by [10] on VRP having capacity and distance restrictions. There was one central depot and same type of vehicles. 14 benchmarks were tested and the results were also compared with the different metaheuristics approaches such as tabu search, simulated annealing, and neural networks. [6] preferred to work on Rollon- Rolloff VRP (RRVRP) which means the trucks are loaded to the ferries or ships and then unloaded at the destination. In this RRVRP, tractors moved large trailers between locations and a disposal facility. 4 different algorithms were developed and were tested on 20 different problems. [67] wrote a whole book on vehicle routing problem where the discussion was on different variants of VRP and algorithms and methods to solve those VRPs. These algorithms involve exact, heuristics and metaheuristics used by different researchers to find optimal results for VRPs. Genetic Algorithm (GA) was applied on VRP by [4]. Results for pure genetic algorithm were given and further a hybrid genetic algorithm was generated by combining with neighbourhood search methods and found that this method was competitive with Tabu search and simulated annealing. Again, GA was used by [56] where the author applied GA and showed that no other algorithm could gave better results than hybrid GA and the results were compared with best known 14 classical Christofides instances. A dynamic vehicle routing problem with time dependent travel times was considered by [29] where the performance of genetic algorithm was compared with results of exact methods and vehicles routes were also planned in real times. An open vehicle routing problem was also tested by [45] for large scale instances and the different algorithms were reviewed. The authors had developed a set of 8 large-scale problems that ranged from 200 to 480 nodes and generated solutions were reported. Next, some extensions of periodic vehicle routing problems were discussed by [24] where three different variants of periodic VRP were discussed. These variants were multi-depot periodic vehicle routing problem, periodic vehicle routing problem with time windows and periodic vehicle routing problem with service choice. Case studies were also presented in this research article. Finally, future directions were also discussed by the authors in this research paper. [71] used improved ant colony optimization method for VRP in which a new strategy was generated to update the increased pheromone known as ant- weight strategy and a mutation- operation to solve VRP. 14 benchmarks were reported and results were compared to other metaheuristic methods. VRP with minimization of emissions was also considered by [22] where the authors focused on minimizing the number of vehicles as the prime objective and minimizing the distance travelled without violating the time windows and capacity constraints as the secondary objective. In the same year, a hybrid genetic- particle swarm optimization technique was used by [48]. The algorithm was applied to combinatorial optimization problem and it gave very good results when applied to two benchmark sets of instances. Recharging vehicle routing problem is one of the variants of vehicle routing problem which was introduced by [16], where vehicles with limited range were allowed to recharge at customer locations mid- tour. Results were calculated when the vehicle range was limited and the recharging time was lengthy. In the next year, Green VRP was introduced by [20] and the authors used Modified Clarke and Wright Savings heuristic and the Density-Based Clustering Algorithm, and a customized improvement technique to find the solutions. The motive was to minimize the fuel consumption and total operational costs. In 2013, a review on Dynamic VRP was done by [54] and authors discussed about all the research articles written on Dynamic VRP and the taxonomy was based on quality and evolution of information. Some real- world applications were also discussed related to the DVRP. Furthermore, a survey on green VRP was done by [46] in which the researchers discussed about the past and future trends of GVRP and reviewed the studies on energy consumption, emissions, and reverse logistics. After that, a survey on VRP having multi- depots was done by [50] and the scholars reviewed the papers between 1988 and 2014. The conclusion also included some lines on future trends of vehicle routing problems. In 2016, a research article [47] on Electric vehicle routing was written in which the authors focused on minimizing the total carbon emissions, total traveling costs as well as the number of electric vehicles. The proposed model and method were demonstrated by doing a case study in Austin, Texas. Vehicle routing problem for cities logistics was introduced by [11] where the authors discussed about the challenges for city logistics, different challenges, the main difficulties, and how these difficulties were treated in the literature. [73] introduced electric vehicle routing problem with recharging stations for minimizing energy consumption. Ant colony optimization algorithm was used and the objective was minimization of energy consumption instead of minimizing the total distance travelled. In 2019, [9] introduced two echelon electric vehicle routing problem where the problem was divided into two parts or two echelons, where first echelon involved large trucks used to deliver goods from central depot to intermediate facilities and in second echelon, small vehicles were used to provide the products to the customers. Large neighbourhood search heuristic method was used by the researchers to compute the optimal values and a trade- off was evaluated between battery capacity and detour miles. A taxonomic review on metaheuristic approaches was given by [19], and the authors also discussed the vehicle routing problem and its variants. 299 research articles published from 2009 to 2017 were counted to do this review. The results were based on trends of the algorithms and the solved VRP variants that were most popular at that time and those are promising topics for future research. In 2021, a review article was also written by [64], where the authors discussed about the research done on vehicle routing problems from 2019 to 2021. Classification was done based on exact, heuristic and metaheuristic approaches. One more literature review was done but it was about electric vehicle routing problems by [39], the authors discussed about the EVRP and its variations. Moreover, the existing solution approaches were also summarized in that research article. [51] explained the VRP over time, it was also a literature survey where the authors studied about periodic routing problems, inventory routing problems, vehicle routing problems with release dates, and multi- depot vehicle routing problems. For logistics distribution, a survey on vehicle routing problems was done by [37] and the authors sorted 263 papers on freight transportation to identify the trends of VRP variants and applied approaches during the last 10 years. The qualitative and quantitative results of the literature reviewed were also discussed. A literature review on two- echelon vehicle routing problem was also done by [60] where the researchers studied about the canonical form and real- world variants of the VRP. An overview of exact and heuristic approaches was also proposed in this paper. Arc based and route- based formulations were also noticed by the scholars for the canonical form. [58] worked on a waste management system for vehicle routing problem using heuristic algorithms. Capacitated VRP was considered and the motive was to minimize the cost of transferring the waste material to recycling centres. Reinforcement learning came into existence in 2024 and it was applied by [69] on VRP with backhauls. Deep reinforcement learning was used to solve traditional as well as improved vehicle routing problem with backhauls. The performance was tested on randomly generated instances and some benchmark instances. A review on GVRP and its variants was also done by [25]. 458 papers since 2016 till 2023 were considered to do this survey and different algorithms were also reviewed by the authors. Multi- depot vehicle routing problem was considered by [70] and the authors worked with intelligent recycling prices and transportation resource sharing. A hybrid algorithm with 3D k- means clustering and self-adapting genetic algorithm and particle swarm optimization algorithm was combined to check the optimality and found better results. It was found that the research provided valuable decision-making support for environmental sustainability and resource- efficient city. Electric vehicles totally run on batteries that are needed to be recharged timely. Therefore, a survey was done on battery management in electric vehicles by [49] and many different methodologies, state of the art approaches and strategies for battery management were proposed in the review article. An approach from sustainability strategies was also imposed by [3] where the researchers worked on vehicle routing problem and gave scientific mapping and research perspectives of VRP. In this research article, it was concluded that Yong Wang has the most research articles and Gilbert Laporte leaded in citations and h- index. A 21% annual increment is there in the number of research articles in this field. Now, here comes the recent research articles on VRP and its variants. In 2025, many research articles are published related to this research area. [2] introduced vehicle routing problem with transfers, it is some kind of Split delivery vehicle routing problem where the customers can be visited by multiple vehicles. However, it is kind of different as customer locations can be used as transfer locations to exchange the load among vehicles. A heuristic based on multi start local search was applied and results suggested that transfers were at least as reducing costs as split deliveries. Capacitated vehicle routing was considered by [31] and truck platooning for road networks was the part of it. A 3-stage algorithm with dynamic programming and modified insertion was developed. Platooning benefits were quantified using numerical experiments. Light decoder-based solvers are popular nowadays in solving vehicle routing problem due to their ease of integration with reinforcement learning algorithms. [34] also preferred that to solve the VRP and found that proposed method significantly enhances the out-of-distribution (OOD) settings. Applications of VRP in logistics were also discussed by [74] and modular vehicle routing problem (MVRP) was considered for research. Exact mixed integer linear programming was combined with a tailored tabu search algorithm to solve the MVRP. Genetic Algorithms were combined with Neural Cost Predictor to solve Hierarchical

Vehicle Routing Problems by [61]. Numerical results showed that the proposed algorithm is effective and efficient in generating high-quality solutions for both multi- depot VRP (MDVRP) and Capacitated Location Routing problem (CLRP).

3. Mathematical Model

VRP is based on graph having nodes and arcs defined as $G = (U, V)$, where $U = \{u_0, u_1, u_2, \ldots, u_{n-1}, u_n\}$, u_0= depot and u_i= customer with demand d_i. $V = \{(u_i, u_j) \mid u_i, u_j \in U,\ i \neq j\}$. There is a traveling time for each arc (u_i,u_j) which is represented as t_{ij}. There are some parameters that we have used in our mathematical model of green vehicle routing problem with time windows and fuzzy demand (GVRPTWFD) that are depicted in the Table 1 and Table 2 given below. But, before the mathematical model, there are some constraints that are needed to be considered.

1. Loading capacity of a vehicle cannot be exceeded.
2. Limited number of vehicles can be used to visit the customers and the fleet is homogenous.
3. Customers must be visited in provided time windows; time windows must not be violated.
4. Vehicles must visit all customers exactly once.
5. It is a closed VRP, which means the vehicle must start its tour from the depot and after visiting the customers, it must return to the depot.

Table 1: Symbols and their explanation

Symbols	Explanation
U	Set of nodes u_0 is the depot
N	$N \setminus \{u_0\}$,Set of Customers
V	Collection of Edges
T	Collection of Vehicles
$i, j \in U$	nodes
$a \in P$	Vehicle Index
S_{ij}	Charges from node i to j
t_{ij}	Traveling Time from node i to j
c_{ij}	distance between node i and j
e_t, l_t	Time Window for customer node i
k_i	Service Time at node i
d_i	Triangular $\{m_i, n_i, o_i\}$ demand for customer i
D_a	Loading Capacity of vehicle a
δ_{ij}	Emissions per unit cost in an arc (i, j)
L	Constant used to nullify time windows constraints when arcs are not in use $g_{ija} = 0$
γ	weight to ensure the distance is minimized along with the other components
β	Weighting Parameter between cost and emissions

Table 2: Decision Variables

$g_{ija} \in \{0, 1\}$	$\begin{cases} 1, & \text{if vehicle } a \text{ travels from node } i \text{ to node } j \\ 0, & \text{otherwise} \end{cases}$
$b_{ia} \geq 0$	Arriving time of vehicle a at node i
$q_{ia} \geq 0$	Cumulative load of Vehicle a after serving node i

The methodology applied to solve GVRPTWFD is as follows:
The objective function is to minimize the total travel charges and CO_2 emissions:

Objective Function:

$$min \sum_{a \in T} \sum_{(i,j) \in V} g_{ija}(\gamma c_{ij} + \beta S_{ij} + (1-\beta) \ \cdot \delta_{ij}) \tag{3.1}$$

subject to the constraints:

$$\sum_{a \in P} \sum_{j \in U} g_{ija} = 1, \forall i \in N \tag{3.2}$$

$$\sum_{j \in U \setminus \{0\}} h_{0ja} - \sum_{i \in U \setminus \{0\}} g_{i0a} = 0, \forall a \in T \tag{3.3}$$

$$\sum_{j \in U, j \neq i} g_{ija} = \sum_{j \in U, j \neq i} g_{jia}, \quad \forall i \in U, a \in T \tag{3.4}$$

$$b_{ja} \geq b_{ia} + k_i + t_{ij} - L(1 - g_{ija}), \quad \forall i \neq j, \forall a \in T \tag{3.5}$$

$$e_i \leq b_{ia} \leq l_i, \quad \forall i \in U, a \in T \tag{3.6}$$

$$q_{ja} \geq q_{ia} + d_j - D_a(1 - g_i ja) \quad \forall i \neq j, \forall a \in T \tag{3.7}$$

$$q_{ia} \leq D_a, \forall j \in U, a \in T \tag{3.8}$$

$$q_{0a} = 0, \quad \forall a \in T \tag{3.9}$$

$$g_{ija} \in \{0,1\}, b_{ia} \geq 0, q_{ia} \geq 0 \tag{3.10}$$

The primary objective of the problem is shown in Equation 3.1 which is to reduce the total operational costs and the carbon emissions. Equation 3.2 makes sure that every customer is visited exactly once by a single vehicle. Equation 3.3 is for the closed condition of vehicle routing problem which means they must return to the depot after completing its deliveries. Equation 3.4 ensures that vehicle must stop in the middle of the route, continuity must be maintained. Equation 3.5 is for not violating the time windows. L is the largest constant which acts as a relaxing constraint when g_{ija}= 0. Equation 3.6 restricts the time windows that customer must be visited in the given time.Equation 3.7 is the load propagation of the vehicle after departing from customer i. Equation 3.8 helps in maintaining the loading capacity of the vehicle. Equation 3.9 shows that the load of the vehicle at initial point is zero. Equation 3.10 represents the binary decision variables.

4. Solution Approach

The solution approach to the problem described in this research article is based on genetic algorithm (GA), particle swarm optimization algorithm (PSO) and alpha- cut method to find the optimal values for minimization of operational costs, reduce the number of vehicles, and minimize the carbon emissions and satisfying the uncertain customer demand respectively. In this section, all the methods are described in brief.

4.1. Genetic Algorithm :

Genetic algorithm is an evolutionary algorithm generated by John Holland in the 1970s. John Holland mentioned about the evolutionary algorithms in a book which was written in 1975 and first published by University of Michigan Press and later it was published as [33] reprint in 1992 by MIT Press. This book involved the importance of genetic algorithm in studies of complex adaptive systems. Holland demonstrated the use of genetic algorithm by applying it to economics, psychology, game theory and artificial intelligence and outlined the way where this approach could be useful for mathematical models as well. Since 1975, genetic algorithm was adopted by many researchers of different fields according to their needs to find the optimized results. Mathematicians also adopted genetic algorithm and found that it works well for optimization techniques. As in this research article, the research is all about combinatorial optimization of VRP, therefore, genetic algorithm is a part of this article in solving the problem and finding optimal solutions. Genetic algorithm involves some major steps that are important for solving vehicle routing problems. The steps are discussed as below:

Step1: Initialization of Population: First step is to generate the population of the candidate solutions. Each chromosome reflects complete set of routes for all vehicles, fulfilling the needs of all customers by visiting them exactly once. During initialization, there is need not to violate the constraints of the mathematical model such as capacity constraints, time windows constraint, or the route length. The population size is mostly dependent on the complexity of the problem.

Step 2: Fitness Evaluation: For every individual, there is need to calculate the fitness score to evaluate its equality. In VRP, the fitness function is mostly based on the total operational costs, total distance travelled, total carbon emissions or number of vehicles used. If the constraints are violated, a penalty must be added to decrease the fitness of such solutions.

Step 3: Selection of Best Parent: This step involves the selection of parent having best fitness value for producing the next generation. Some methods are there to choose the best parent. One is Tournament selection where the parent is selected from a randomly chosen group. Second is Roulette Wheel Selection method in which possibility of selecting a parent is directly proportional to the fitness value. Third is Rank selection method where the individuals are ranked and chosen based on their ranks.

Step 4: Crossover: Crossover means recombination where the genetic properties of best selected parents are combined to generate the offsprings. Crossover is further divided into types such as single point crossover, double point crossover, ordered crossover and uniform crossover.

Step 5: Mutation: Mutation involves small random changes to maintain the diversity and avoid the premature convergence. There are some types of this as well such as swap mutation, relocation mutation and inversion mutation.

Step 6: Termination: Repeat the steps until the maximum number of generations is reached or the best solution has not improved over a set of number of generations. Once the termination point is reached, best solution is found for the vehicle routing problem without violating the constraints considered in the mathematical model.

4.2. Algorithm 1 Genetic Algorithm for GVRPTW with Fuzzy Demands :

Algorithm 1 Steps to Calculate Optimal GVRPTW Solution with Fuzzy Demand and ACO

Problem and Inputs
Solve VRPTW for 100 customers (Solomon dataset R101)
Objective: Minimize number of vehicles, distance, and CO_2 emissions
Inputs: R101 dataset, number of Customers, maximum vehicles, population size, generations, mutation rate, vehicle penalty, fuzzy scale
Outputs: Best routes, total distance, vehicle count, feasibility, CO_2 emissions
Constraints: Single visit per customer, vehicle capacity, time windows, Vehicle start and end at the same depot.
Preprocess Data
Load Customer data from `Solomon dataset`
for each customer i do **do**
  Compute fuzzy demand using α - cut method
  Cap fulfilled demand $q_i = \min(fuzzy_i, \bar{q}_i)$
end for
Compute distance matrix d_{ij}
Initialize population with feasible routes
for each individual in individual **do**
  Evaluate fitness: $\sum d_{ij} x_{ijk} + \alpha \sum z_k$
end for
for gen=1 to N **do**
  Select best individual (elitism)
  while new population size ¡ population_size **do**
    Select parents via tournament selection
    Perform order crossover to create child
    Apply mutation with probability Mutation_rate using 2-opt
    Apply 2-opt local search to each route
    Evaluate child fitness
    Add child to new population
  end while
  Update population
end for

4.3. Particle Swarm Optimization Algorithm:

Particle Swarm Optimization Algorithm is also a population-based optimization algorithm which is inspired by flocks of birds or fish schooling. PSO was introduced by [36]. This method is used to find optimal or near optimal solutions for complex problems in optimization by simulating a group of birds that fly from one place to another in search of food and try to find the shortest path towards the food. Each particle depicts a potential solution, and its position is adjusted based on its own experience and experience of its neighbouring particles. PSO is a metaheuristic approach which is mostly preferred to solve complex, non- linear, multi-dimensional or optimization problem where exact or heuristic approaches may fail or become insufficient to calculate the optimal results. The steps involved in PSO are as follows:
Step 1: Initialization of Swarm: Initial step involves defining the number of particles. Each particle is initialized with random position and velocity within the defined bounds of the search space. Every particle has a position vector and a velocity vector denoted by x and v respectively. Position of each particle at initial point is being set as best-known position (*pbest*) and the task is to determine the global

best ($gbest$) position among all the particles.
Step 2: Fitness Evaluation: Calculate the fitness or the objective function value for each particle at its current position. This will show how good results a particle can give. Each particle's pbest is updated if it has better fitness than the previous pbest value. Similarly, update the gbest value if it has better fitness than the previous $gbest$ value.
Step 3: Update Velocity: Adjustment of velocity involves some parameters on which the updating of velocity depends which are: its own best position ($pbest$), its global best position ($gbest$) and its current velocity. The velocity update is shown in Equation 4.1 is as follows:

$$v_i^{(t+1)} = w \cdot v_i^t + c_1 r_1 (pbest_i - x_i^t) + c_2 r_2 (gbest_i - x_i^t) \tag{4.1}$$

where
w= inertia weight
c_1, c_2= acceleration coefficients
r_1, r_2= Random Numbers between 0 and 1
x_i^t, v_i^t= position and velocity of particle i at iteration t

Step 4: Update Position: In this step, use the updated velocity to update the position of the particle using the following Equation 4.2:

$$x_i^{(t+1)} = x_i^t + v_i^(t+1) \tag{4.2}$$

The new positions must be within search space boundaries.
Step 5: Termination Point: The steps from 2 to 4 are needed to be repeated until the stopping criteria is met. Stopping criteria involves the number of iterations or the best fitness value is achieved, or minimal changes in gbest value over number of iterations.

4.4. Alpha- cut Method for Uncertain Customer Demand:

Fuzzy set theory was first introduced by [72] and the authors discussed about different fuzzy methods involving the alpha- Cut method. A separation theorem for convex fuzzy was also proved by them without requiring that the fuzzy sets be disjoint. Alpha- cut method was used by many researchers specially when working with fuzzy logics or fuzzy set theory. This method is applicable in this article as we are working with fuzzy customer demand which means the requirement of the customer is uncertain or it can be said that the demand is indeterministic. Methods like alpha- cut, Chance constrained and Mamdani fuzzy Inference are always used while working in such situations. This method was also used by [35] for vehicle routing problem along with genetic algorithm and the results were also very effective. Alpha-cut method involves some steps as we are working triangular fuzzy number, therefore the demand is divided into 3 different parts.

At a confidence $\alpha \in [0, 1]$. The α-cut of a triangular fuzzy number is represented as in Equation 4.3:

$$d_i^\alpha = [a + (b - a), c - \alpha(c - b)] \tag{4.3}$$

This formula will give crisp interval for customer demand at confidence level α. Steps involved in α - cut method are described below:

Step 1: Depict the Customer Requirement as Triangular Fuzzy Number: For each customer i, the fuzzy demand is shown as $\tilde{d}_i = (a_i, b_i, c_i)$ where (b_i) is the most possible demand and (a_i, c_i) is the full range of uncertainty.
Step 2: Use α-cut to Find Crisp Interval Demands: Use the formula $d_i^\alpha = [a + (b - a), c - \alpha(c - b)]$ for each customer i, to find the interval demand to be used for planning at level α.
Step 3: Update the GVRP Model: While maintaining the GVRP constraints, update the model by

using the upper bounds of α- cut intervals to make sure vehicle capacities are not violated.
Step 4: Analysing the Results: Check the results at different levels of α, so that the results may have optimistic planning at high α level and conservative planning at low α level.

4.5. Fuzzy Membership Function:

In Fuzzy logic, membership function is the degree of truth as an extension of evaluation. Membership functions were first introduced by [72] in their fuzzy logic research paper in 1965. Now, the formal definition of a membership function is written as:
Let $\exists$ a fuzzy set A, such that $A = \{(x, \mu_A(x)) \mid \forall x \in X\}$, is the membership function for the fuzzy set A. X is the universe of discourse. The membership function contains each element $x \in X$ with a value lies in the interval [0,1].
As triangular fuzzy number is used to depict the fuzzy demand of the customers, a brief introduction to TFN's is given as follows:
Triangular fuzzy number shown in Figure 2 is the most popular one among all the fuzzy numbers. It is represented with three points shown above in step 1 as $\tilde{d}_i = (a_i, b_i, c_i)$ and this representation is interpreted as membership functions in Equation 4.4.

$$\mu_{\tilde{d}_i}(x) = \begin{cases} 0, & x < a_i, \\ \dfrac{x - a_i}{b_i - a_i}, & a_i \le x \le b_i, \\ \dfrac{c_i - x}{c_i - b_i}, & b_i \le x \le c_i, \\ 0, & x > c_i. \end{cases} \tag{4.4}$$

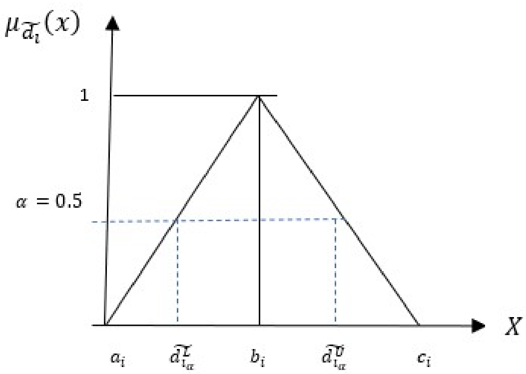

Figure 2: Triangular Fuzzy Number $\tilde{d}_i = (a_i, b_i, c_i)$

where $\mu_{\tilde{d}_i}(x)$ is the membership function and $d_i^\alpha = \left[d_i^L(\alpha),\ d_i^U(\alpha)\right]$ are the corresponding α-cuts.

Now, the crisp value is obtained by applying α - cut operation, then the interval will be obtained as follows $\forall \alpha \in [0, 1]$ shown in Equation 4.5, Equation 4.6, Equation 4.7.

$$\frac{(x - a_i)}{(b_i - a_i)} = \alpha \implies d_i^L(\alpha) = x = \alpha(b_i - a_i) + a_i \tag{4.5}$$

$$\frac{(c_i - x)}{(c_i - b_i)} = \alpha \implies d_i^U(\alpha) = x = c_i - \alpha(c_i - b_i) \tag{4.6}$$

$$\text{Defuzzified Value} = [(1-\alpha)A^L_\alpha + A^U_\alpha)] \tag{4.7}$$

As we are taking the value of α=0.5, therefore, the equations become, $\frac{x-a_i}{b_i-a_i} = 0.5$ and $\frac{c_i-x}{c_i-b_i} = 0.5$ and we get, $x = (b_i - a_i)0.5 + a_i$ and $x = -(c_i - b_i)0.5 + c_i$ and finally, the crisp interval becomes, $d^\alpha_i = [(b_i - a_i)0.5 + a_i, -(c_i - b_i)0.5 + c_i]$.
To illustrate the whole concept of α- cut method, an example can be taken with proper triangular fuzzy number having values as A=(20,30,50), then the membership function $\mu_A(x)$ will be equal to Equation 4.8:

$$\mu_{\tilde{d}_i}(x) = \begin{cases} 0, & x < 20, \\ \dfrac{x-20}{10}, & 20 \leq x \leq 30, \\ \dfrac{50-x}{20}, & 30 \leq x \leq 50, \\ 0, & x > 50. \end{cases} \tag{4.8}$$

And the Figure 3 that reveals the whole example is as follows:

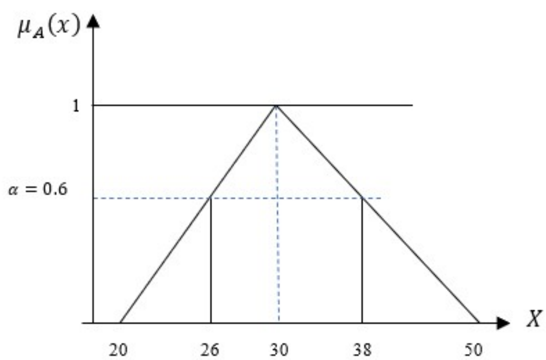

Figure 3: Triangular Fuzzy Number A=(20,30,50) at α =0.6

Here, the value of α=0.6 , therefore the interval will be obtained by taking $\dfrac{x-20}{10} = \alpha$ and $\dfrac{50-x}{20} = \alpha$. Put the value of α and calculate the values which are likely to be x=26 and x=38. Hence, the crisp interval becomes [26,38]. Finally, the defuzzified value or the crisp value can be found by using the equation (17). Therefore, (1-0.6)26+(0.6)38=10.4+22.8=33.2 units is the final defuzzified value. The given formula indicates that with the increase in the value of the α, the defuzzified value moves towards the right-hand bound of the given triangular fuzzy interval.

4.6. Numerical Example:

Let us consider an example having single customer C_1 having the fuzzy demand (Low, Medium, High) = (10,15,25). The crisp demand (CD) is calculated using the Left- Hand Bound and Right-Hand Bound using equation (14) and equation (15) and equation (17) gives different values of fulfilled demand based on different α values shown in the Table 3 below:

Table 3: Interpretation at different values of α :

α	**Left Bound** ($[b_i - a_i)\alpha + a_i]$	**Right Bound** $[c_i - (c_i - b_i)\alpha]$	**Fulfilled Demand**	**Interpretation**
0	(15-10)0+10=10	25-0(25-15) = 25	(1-0)10+0(25) =10	Most Optimistic
0.2	(5) (0.2) +10= 11	25-(0.2) (10) = 23	(1-0.2)11+(0.2) (23) =13.4	Slightly Optimistic
0.5	(5) (0.5) +10= 12.5	25- (0.5) (10) = 24	(1-0.5) (12.5) +(0.5) (24) =18.25	Balanced
0.8	(5) (0.8) +10= 14	25-(0.8) (10) = 17	(1-0.8) (14) +(0.8) (17) =16.4	Conservative
1.0	(5) (1) + 10= 15	25- (1)(10) = 15	(1-1) (15) +(1)(15) =15	Most Pessimistic

Like this, if we consider an example having vehicle capacity limitation of 50 units and fuzzy demand of customers C_1=(10,15,25),C_2=(8,12,20)and C_3=(5,10,15). In the Table 4 below, total defuzzified demand, feasibility and fitness is shown for all the three customers at different α - cut values.

Table 4: Defuzzified Demand at different α-cut values

α	**Fulfilled demand** (C_1, C_2, C_3)	**Total demand**	**Fitness (Cost + CO_2+ Penalties)**
0	(10, 8, 5)	23	Low (Under Utilized Vehicle)
0.2	(13.4, 11.04,6.8)	30.88	Low (Safe but insufficient)
0.5	(18.25, 14, 6.75)	39	Balanced (Optimal)
0.8	(16.4, 13.12,10.6)	40.12	High (Too Conservative)
1.0	(15, 12, 10)	37	Very High (Under Utilization)

5. Experimental Results

The proposed algorithms are written in the Python 3.6 programming language. To test its implementation, a machine with a 2.40 GHz i5 processor and 8 GB RAM was used. The results of the proposed methodology are calculated on some specific constraints involving number of depots, vehicles, customers, vehicle capacity, total distance, crisp demand, and carbon emissions. Solomon dataset (R101)[75] is considered to check the efficiency of the proposed algorithm. Moreover, the parameters used in applied approach are also having some constant values. The results for GA and PSO are shown in Table 5 and Table 7. The fulfilled demand, visited customers, input triangular fuzzy demand, and time for each customer is shown in the Table 6 and Table 8.

Table 5: Results related to routes for R101 using GA for α=0.6

Vehicle Number	**Route Followed**	**Total Distance Covered (units)**	**Total Defuzzified Demand (units)**	**Total CO_2 emissions (kgs)**
1	Depot → C5 → C83 → C88 → C31 → C99 → C96 → C85 → C91 → C98 → C57 → C74 → C55 → C29 → C66 → C46 → Depot	289.55	199.19	634.66
2	Depot → C59 → C72 → C75 → C73 → C87 → C92 → C6 → C37 → C100 → C61 → C89 → C52 → C27 → C25 → C17 → Depot	274.19	199.37	601.48
3	Depot → C95 → C44 → C38 → C16 → C97 → C41 → C4 → C3 → C63 → C49 → C47 → C60 → C70 → Depot	283.80	199.17	622.02
4	Depot → C33 → C40 → C21 → C23 → C15 → C94 → C93 → C8 → C26 → C54 → C24 → C80 → C81 → Depot	269.56	197.83	587.17
5	Depot → C45 → C82 → C19 → C51 → C20 → C32 → C34 → C68 → C12 → C56 → C18 → C58 → Depot	263.24	199.09	576.72
6	Depot → C14 → C39 → C67→ C22→ C43 → C13→ C28 →C76 →C77→ C9→ C35 → C69→ Depot	255.87	198.44	558.93
7	Depot→ C65→ C71→ C79→ C30→ C90 → C10→ C62→ C48→ C84→ C53→ C50→ C1→ Depot	225.88	199.62	496.09
8	Depot→ C36→ C64→ C11→ C7→ C86→ C42→ C2→ C78→ Depot	234.21	81.88	238.60

Table 6 gives the brief explanation of the outputs obtained from the applied algorithm (Genetic Algorithm) which includes the customers, fuzzy input values and fulfilled demand.

Table 6: Results for Vehicle number 5 for GA for α =0.6

Customers (Randomly Picked)	**Fuzzy Input Values**	**Fulfilled Demand**	**Time Windows**
45	(14.28, 16.00, 17.86)	16.17	10
82	(13.75, 16.00, 19.09)	16.38	10
19	(14.11, 17.00, 19.81)	17.21	10
51	(8.92, 10.00, 11.33)	10.15	10
20	(7.41, 9.00, 10.23)	9.04	10
32	(19.35, 23.00, 27.44)	23.48	10
34	(12.03, 14.00, 15.73)	14.10	10
68	(30.58, 36.00, 42.55)	36.71	10
12	(16.14, 19.00, 22.20)	19.31	10
56	(4.97, 6.00, 7.07)	6.09	10
18	(10.15, 12.00, 13.27)	12.01	10
58	(15.73, 18.00, 21.24)	18.42	10

The results for vehicle number 5 show that total distance covered is 263.24 and total defuzzified demand is 199.09. Also, the total carbon emissions are 576.72 kgs.

Table 7: Results related to routes for R101 using PSO for α=0.6

Vehicle Number	**Route Followed**	**Total Distance Covered (units)**	**Total Defuzzified Demand (units)**	**Total CO_2 emissions (kgs)**
1	Depot → C29 → C30 → C9 → C3 → C91 → C77 → C81 → C83 → C33 → C24 → C76 → C5 → C70 → C34 → C51 → C78 → C46 → Depot	320.54	199.32	785.52
2	Depot → C82 → C32 → C18 → C96 → C67 → C14 → C71 → C68 → C20 → C66 → C17 → C55 → Depot	309.40	198.52	769.45
3	Depot → C63 → C2 → C8 → C84 → C27 → C50 → C80 → C28 → C26 → C35 → C47 → C11 → C98 → C59 → C42 → C90 → C6 → Depot	335.87	198.69	762.57
4	Depot → C95 → C44 → C38 → C45 → C13 → C31 → C10 → C62 → C100 → C57 → C92 → C37 → C7 → Depot	268.47	197.10	382.84
5	Depot → C22 → C94 → C86 → C74 → C4 → C89 → C99 → C58 → C65 → C87 → Depot	243.08	198.20	526.80
6	Depot → C39 → C12 → C53 → C69 → C49 → C61 → C41 → C85 → C23 → C56 → C60 → Depot	262.04	198.49	553.88
7	Depot → C21 → C72 → C73 → C54 → C75 → C93 → C43 → C19 → C48 → C64 → C36 → C97 → C15 → Depot	282.99	199.76	509.81
8	Depot → C88 → C52 → C1 → C16 → C79 → C25 → C40 → Depot	206.48	86.05	118.97

Table 7 provides the results generated by applying Particle swarm optimization algorithm. However, in Table 8 the results for randomly chosen Vehicle number 3 are shown where the customers along with fuzzy input values and defuzzified demands are presented.

Table 8: Results for Vehicle number 3 for PSO for α =0.6

Customers (Randomly Picked)	**Fuzzy Input Values**	**Fulfilled Demand**	**Time Windows**
63	(8.50, 10.00, 11.55)	10.13	10
2	(5.85, 7.00, 7.76)	7.00	10
8	(7.79, 9.00, 10.57)	9.19	10
84	(6.18, 7.00, 8.27)	7.18	10
27	(13.43, 16.00, 17.99)	16.07	10
50	(11.69, 13.00, 15.48)	13.39	10
80	(5.07, 6.00, 6.77)	6.04	10
28	(13.71, 16.00, 17.66)	16.03	10
26	(14.69, 17.00, 20.13)	17.38	10
35	(7.10, 8.00, 9.42)	8.20	10
47	(21.61, 27.00, 29.83)	26.82	10
11	(10.42, 12.00, 13.71)	12.16	10
98	(8.28, 10.00, 11.12)	10.00	10
59	(23.35, 28.00, 31.03)	27.98	10
42	(4.06, 5.00, 5.76)	5.03	10
90	(2.55, 3.00, 3.48)	3.04	10
6	(2.44, 3.00, 3.58)	3.05	10

The total capacity of vehicle number 3 is also 200 units while the total distance covered by it is 335.87. Moreover, the total fulfilled demand is 198.69, and total CO_2 emissions are 762.57 kgs.

5.1. Sensitivity Analysis:

A sensitivity analysis is carried out to analyse the changes in total distance and total CO_2 emissions for genetic algorithm and particle swarm optimization algorithm. The outcomes are listed in the Table 9 below. Total distance covered for genetic algorithm is 2096.29 units but when PSO is applied, the total distance covered increases to 2178.87 units which results in a difference of 3.93% in the outcomes of both the approaches. Similarly, for CO_2 emissions, there is a gap of 2.18% in the outcomes of applied algorithms.

Table 9: Sensitivity Analysis for different α- cut values:

α	**Total Distance (GA)**	**Total Distance (PSO)**	**%age error**	**Total CO_2 Emissions (GA)**	**Total CO_2 Emissions (PSO)**	**%age error**	**Total Violations (GA)**	**Total Violations (PSO)**	**%age Error**
0.2	2137.55	2221.98	3.94	4574.32	4661.91	1.91	13705.71	14310.83	4.41
0.4	1966.54	2039.82	3.72	4116.83	4236.42	2.90	11061.24	11419.54	3.23
0.6	2096.29	2178.87	3.93	4315.66	4409.83	2.18	10157.87	10614.15	4.49

The above Table 9 shows the sensitivity analysis of two different algorithms and the percentage error of both the methods. The computational results have shown that genetic algorithm gives better results as compared to the particle swarm optimization algorithm. Moreover, it can be clearly seen that at the highest value of α=0.6 , the results are better and best fitness is also observed at that and there is a huge difference between the values observed at different alphas. After all the tabular results, this paper also includes the figures depicting the routes of all vehicles and the best fitness function for Genetic algorithm and particle swarm optimization algorithm shown in Figure 4, and Figure 5 below.

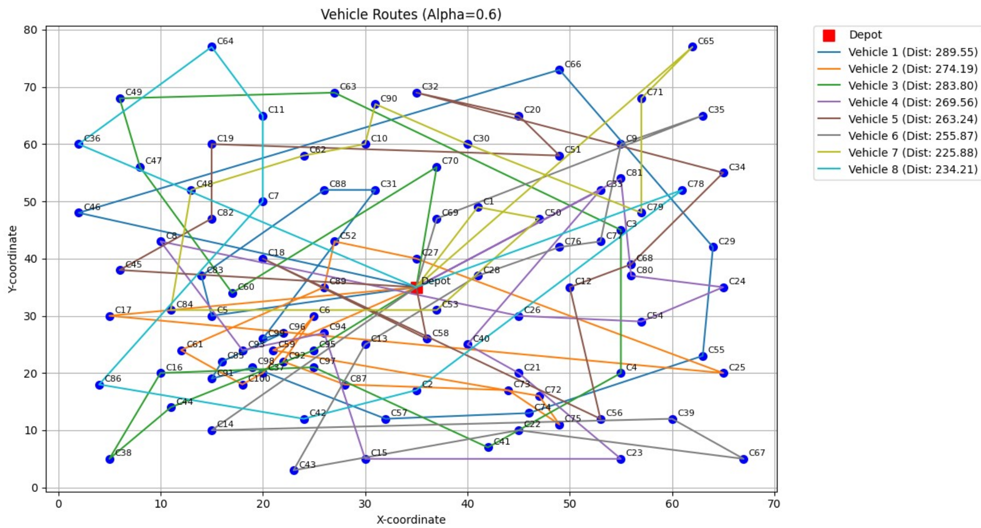

Figure 4: Routes of vehicles for GA at α =0.6

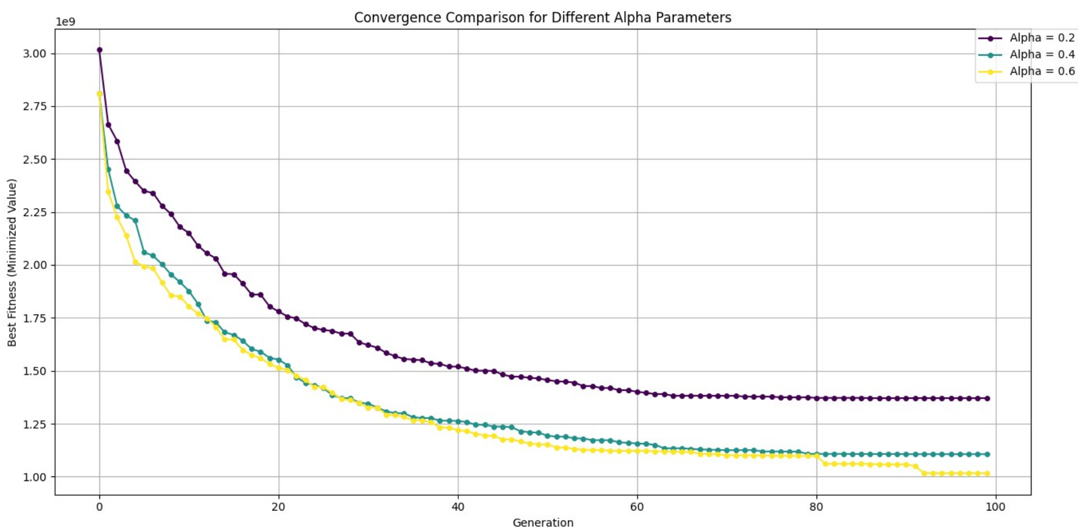

Figure 5: Comparison of Best fitness at different α- cuts (GA)

Figure 4 gives routes for 100 customers picked from Solomon dataset (R101) and figure 5 shows convergence at α=0.6 as it is given the minimized value for the objective function for 100 generations. The plots are shown for Genetic algorithm as it has given better results as compared to particle swarm optimization algorithm.

6. Comparison of Models

In this section, a comparison between different research work done by Fazlollahtabar et al. (2025), Tanash et al. (2025), Adibi et al. (2025), Sayyari et al. (2025), and Kang et al. (2025) is shown in Table 12 and the optimality of the results is compared with the methods and algorithms used in this research article. The proposed work is satisfying all the comparison factors and has also provided satisfactory outcomes.

Table 10: Results for Vehicle number 3 for PSO for α =0.6

Compared Factors	**Fazlollahtabar et al. (2025) [21]**	**Tanash et al. (2025) [65]**	**Adibi et al. (2025) [1]**	**Sayyari et al. (2025) [59]**	**Kang et al. (2025) [35]**	**Proposed Work**
Multi Objective	Yes	Yes	No	No	Yes	Yes
Algorithm	Genetic Algorithm	Priority-Based Ant Colony Optimization	Large Neighborhood Search	General Algebraic Modeling System	Genetic Algorithm	Genetic Algorithm and Particle Swarm Optimization
Fuzzy customer demand	Yes	Yes	Yes	No	No	Yes
α-cut constraints	No	No	No	Yes	Yes	Yes
Customer satisfaction	Yes	Yes	Yes	Yes	Yes	Yes
Traveling cost	Yes	Yes	Yes	Yes	Yes	Yes
Penalty cost	No	Yes	Yes	Yes	Yes	Yes
Minimization of Carbon Emissions	No	No	No	Yes	No	Yes
Low CPU time	Yes	Yes	Yes	Yes	Yes	Yes

From the Table 10 above, metaheuristics are mostly preferred by different researchers as these algorithms give better optimal results as compared to heuristics and exact methods. Moreover, different algorithms have provided different results according to the parameters and constraints considered by the authors. Minimization of CO_2 emissions are considered by some authors but some of them have not considered CO_2 emissions.

7. Conclusion

This research study has addressed the variant of VRP with time windows concerning the environmental sustainability focusing on minimizing the CO_2 emissions along with minimizing total distance without violating the time windows. Genetic algorithm and particle swarm optimization algorithm has provided satisfactory results but the sensitivity analysis has shown that genetic algorithm performed well. Moreover, the uncertain customer demand is also satisfied using the α-cut method as it has given satisfying results at different values of α. Although, all the results at different alphas are satisfying, the results at $\alpha = 0.6$ are better as compared to other values because it is giving more precise and optimal values. The optimal values for $\alpha = 0.6$ are 2096.29 units, and 4315.66 kgs for total distance, and total CO_2 emissions respectively. However, the optimal values at $\alpha = 0.2$ are 2137.55 units, and 4574.32 kgs for total distance, and total CO_2 emissions respectively. These results are for genetic algorithm as it has given better results as compared to particle swarm optimization algorithm. Moreover, the methodology used in the research can also be useful for the future research work as it is totally concerned with the environmental sustainability. To conclude, this research contributes in bridging the gap between environmental sustainability and operational efficiency in modern logistics and transportation systems.

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