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Intelligent Decision Making Using a Chi-Squared Based Similarity Measure for Neutrosophic Soft Set

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ABSTRACT: Data, knowledge, and even queries in the real world are not necessarily accurate. The majority of them are imprecise, ambiguous, or both. In many situations, the data may be inconsistent also. Mathematical frameworks that cope with such imperfect or inconsistent data include neutrosophic set and neutrosophic soft set. Fabrication of database models that can handle imprecise information more precisely than existing models is ongoing in literature. The notion of similarity measure plays a significant role in comparing alternatives for imprecise data or queries and is often used in decision-making for uncertain queries. This paper introduces a novel similarity measure on neutrosophic soft set based on the Chi-Squared metric. The applicability of the predicted measure in decision making is exhibited with suitable real-life examples in diversified domains. It is also demonstrated that the proposed measure can identify differences in crucial situations when existing measures fail and thus can help in intelligent decision making.

Keywords: Neutrosophic set, neutrosophic soft set, similarity measure, Chi-Squared similarity measure, decision making.

Contents

1. Introduction

The information or data generated from real life are not always accurate. Such data are often ambiguous, or imprecise in nature, and in certain situations may be inconsistent too. So there remains always a need to develop database models that can handle inaccurate data. Furthermore, there is no certainty that the queries that a database model deals with will always be precise. Consequently, database models capable of handling imprecise, vague, or inconsistent data as well as unclear queries are always in great demand. In 1965, Zadeh [1] initiated the concept of fuzzy logic. Sets built on fuzzy logic are quite well-equipped to handle uncertain data. Here, a set member is paired with its membership value which signifies the extent to which the data lies in the set. Usually, this membership function lies between 0 and 1. In 1975, Zadeh [2] further extended the fuzzy set to interval-valued fuzzy set, where the membership values are represented by intervals. Since then, many authors have put forth different generalizations of this set to deal with uncertainty more accurately. In 1986, intuitionistic fuzzy set was introduced by

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Atanassov [3]. Here, every element of the set has not only a membership value but also a non-membership value. In 1993, Gau and Buehrer [4] developed the concept of vague set. In a vague set, an element is associated with a truth as well false membership values and is characterized by an interval membership. All these sets have their own limitations in processing imprecise or inconsistent information generated from real-world problems. In 1998, Smarandache [5] pitched a new concept of neutrosophic logic that can also deal with indeterminacy. Here, an element of the set is linked with three types of membership values: truthness t_{N_s}, indeterminacy i_{N_s}, and falsehood f_{N_s} . Each of these components can fetch values from the real standard or non-standard subsets of $]0^-, 1^+[$. Also, their sum adheres to the condition

$$0^- \leq \sup\, t_{N_s} + \sup\, i_{N_s} + \sup\, f_{N_s} \leq 3^+$$

This concept was modified by Wang et al. [6] from a technical viewpoint to define single-valued neutrosophic set. Each membership function is now a subset of [0,1] and

$$0 \leq \sup\, t_{N_s} + \sup\, i_{N_s} + \sup\, f_{N_s} \leq 3$$

Many researchers [7,8,9] have pointed out in their studies that models based on neutrosophic logic can handle imprecise data and queries in a better way.

Following the inception of neutrosophic set, works on the extension of the existing generalized and hybrid set models based on fuzzy or intuitionistic fuzzy logic to neutrosophic logic grabbed the interest of many researchers. In 2015, Deli et al. [10] proposed the concept of bipolar neutrosophic set which was actually an extension of the existing bipolar fuzzy set, and neutrosophic set. In 2017, Ali and Smarandache [11] introduced complex neutrosophic set to overcome the limitations of complex fuzzy set and complex intuitionistic fuzzy set in handling imprecise information with periodic nature. In 2019, Jansi et al. [12] presented the Pythagorean neutrosophic set. In 2021, Sweety and Jansi [13] extended the Fermatean fuzzy set to the Fermatean neutrosophic set. Recently, in 2024, Voskoglou et al. [14] proposed the notion of a q-rung orthopair neutrosophic set as an extension of q-rung orthopair fuzzy set to a neutrosophic environment.

In 1999, Molodtsov [15] proposed a novel concept of soft set that can provide better decisions under different parameters. Soft set theory is capable of making decisions with uncertain parameter values as well. Since the formation of the soft set, scholars have paired the existing set theoretical logics with soft set theory. In 2013, Maji [16] amalgamated the concepts of neutrosophy and soft set to introduce the new theory of neutrosophic soft set. In literature, this kind of set has been shown to be very effective in decision-making. Subsequently, many researchers started to come up with different hybrid sets involving soft set theory and other existing set theories. In 2017, Ali et al. [17] presented the idea of bipolar neutrosophic soft set. Complex neutrosophic soft set was also introduced in 2017 by Broumi et al. [18]. In 2021, Radha et al. [19] proposed the notion of neutrosophic Pythagorean soft set.

Though the existing fuzzy, vague, and neutrosophic logics have proved their distinct capabilities, each of them has its own limitations in tackling decision-making problems under uncertain, imprecise, or inconsistent environments. Hence, continuous development of different techniques for solving decision-making problems is in progress and many such hybrid sets with varied approaches have been proposed in literature over recent years. A neutrosophic soft set method was applied by Jha et al. [20] for stock trending analysis to make better investment plans. Saqlain et al. [21] used it in the selection of the right smartphone by a customer. Relations on this set were studied and applied to decision-making by Dalkılıç et al. [22]. Priyadarsini et al. [23] introduced a generalization of neutrosophic soft set to solve multiple expert decision-making problems in 2022. The sequence of neutrosophic soft set was introduced in the same year by Bui et al. [24] and it was applied to a medical diagnosis problem. Shanmugam et al. [25] employed a generalized q-rung neutrosophic soft set in a decision-making problem for agricultural production. Mohanty & Tripathy [26] applied it to an MCDM problem to select faculty members through an interview process. The effectiveness of the neutrosophic soft set framework was shown in their work by Karatas et al. [27]. In 2024, a method was proposed to select bionic thin-wall structures with interval-valued neutrosophic soft set by Zhang et al. [28]. Thus it is widely accepted that neutrosophic soft set is one of the most effective and powerful extension of soft sets. This has instigated the present authors to explore neutrosophic soft sets and their applicability in decision-making.

In making decisions, the conception of similarity measure is often used to choose the correct or the most appropriate alternative by computing similarity between the available alternatives. It is represented numerically by a value between 0 and 1. A value nearer to 1 represents higher similarity, while that closer to 0 indicates higher dissimilarity. To measure similarity, the dual nature of similarity and distance is often used. More the distance, less is the similarity. Several authors have put forward different similarity measures based on fuzzy [29,30,31], vague [32,33,34], and neutrosophic sets [35,36]. Such measures are usually built up using the standard distance metrics such as Euclidean [36,37], Hamming [36,38,39], Hausdorff [35,40,41], distance by cosine [42] method, Manhattan [43] distance, etc. Continuous studies are going on for the development of new similarity measures to bring more accuracy in decision-making in a wide range of real-life applications in various fields including clustering [44], pattern recognition, medical diagnosis [45], risk evaluation problems [46,47] and many more.

As similarity measures are formed based on popular distance measures, there have been ongoing efforts by several researchers to improve these distance metrics for better accuracy in different frameworks. Szmidt et al. [48] applied Hausdorff distance measure to intuitionistic fuzzy sets. Ye [49] used the cosine method. Broumi et al. [50] proposed new measures of distance and similarity for interval neutrosophic sets. In [51], Ye showed that the Hamming and Euclidean distances based similarity measures can be very useful in multi-criteria decision-making problems with interval neutrosophic sets. In [52,53,54], authors used Hamming and Euclidean distances to measure the similarity of neutrosophic soft sets. Khan et al. [55] used vector similarity measure for a simplified neutrosophic hesitant fuzzy set. A new similarity measure based on the Euclidean distance metric for single-valued and interval-valued neutrosophic sets was proposed by Liu et al. [37]. A novel weighted distance metric for single-valued neutrosophic set was proposed by Wang [38]. Hausdorff distance metric was again deployed in [41] with its application to multi-criteria decision-making for single-valued neutrosophic sets. The present authors [56] explored the Hausdorff similarity measure in the context of neutrsphic soft sets. Dice and hybrid vector similarity measures were proposed in [57] to explain the applicability of bipolar neutrosophic sets in multi-criteria decision-making. In [58,59,60], authors explored several similarity measures as exponential and non-exponential based generalized measures, hybrid vector, and cosine similarity measures on complex hesitant fuzzy sets and examined their utility in the fields of pattern recognition and medical diagnosis. Similarity measures based on Euclidean and Hamming distances were also studied for interval-complex neutrosophic soft set [61]. Such distance measures were investigated for neutrosophic fuzzy soft set [62] and possibility neutrosophic soft expert set as well [63]. Very recently, in 2024, researchers [64,65,66,67] have again focused attention on intuitionistic fuzzy sets and their variants and explored improved similarity measures with application to various domains such as pattern recognition, medical diagnosis, software quality evaluation, face recognition, and many more real-world problems. In particular, circular and interval intuitionistic fuzzy sets have been examined in [65] and [66]. The notion of strict similarity or distance measure has been introduced for intuitionistic fuzzy set in [67] which enables one to rank this kind of set in a better manner.

In the present work, a similarity measure based on the Chi-Squared formula, proposed by Ren et al. [68] for the neutrosophic set, has been extended to the neutrosophic soft set environment. It has been established by the authors in [68] that the similarity measure based on Chi-Squared distance overcomes the limitations of existing popular similarity measures. Also, in [45], the authors presented a comparative analysis where it has been shown that the Chi-squared-based similarity measure generates values that are more reasonable and logical for different scenarios. However, the Chi-Squared-based similarity measure has not yet been explored in literature for neutrosophic soft sets. This led the present authors to propose a new measure of similarity for the neutrosophic soft set that uses the Chi-Squared metric.

Section 2 explains the basic terminologies that are required to develop a proper understanding of the context. The proposed measure is put forth in section 3 while its mathematical validation is shown in section 4. The significance and applicability of the recommended measure in decision making are established in section 5 through real life examples in diverse situations. The conclusion and scope of further studies are discussed in section 6.

2. Basic Terminologies

Here, we present few definitions interrelated to this work.

Let U be the universe under consideration and u denote an element in U.

2.1 Fuzzy Set [1]

A fuzzy set F_s in the universe U is described in terms of a membership function $\mu_{F_s} : U \to [0,1]$ and we define $F_s = \{ (u, \ \mu_{F_s}(u)) : u \in U\}$ where $\mu_{F_s}(u)$ denotes the degree of belongingness of each $u \in U$ in the fuzzy set F_s.

2.2 Vague Set [4]

A vague set V_s in the universe U is described in terms of two membership functions, a truth membership function $t_{V_s} : U \to [0,1]$ and a false membership function $f_{V_s} : U \to [0,1]$, such that $0 \le t_{V_s}(u)+f_{V_s}(u) \le 1$. The vague set V_s is written as

$$V_s = \left\{ \left(u, [t_{V_s}(u), 1 - f_{V_s}(u)]\right) : u \in U \right\},$$

where the interval $[t_{V_s}(u), 1 - f_{V_s}(u)]$ characterises the membership grade $\mu_{V_s}(u)$ of u in the vague set V_s and and is termed as the vague value of u.

2.3 Neutrosophic Set [5]

A neutrosophic set N_s in the universe U is expressed with three membership functions; truth membership $t_{N_s} : U \to [0,1]$, indeterminacy $i_{N_s} : U \to [0,1]$, and a false membership $f_{N_s} : U \to [0,1]$. and is represented as

$$N_s = \{(u, \ t_{N_s}(u), \ i_{N_s}(u), \ f_{N_s}(u)) : u \in U\}.$$

The neutrosophic value of the element u will be now written as the triplet $(t_{N_s}(u), \ i_{N_s}(u), \ f_{N_s}(u))$.

2.4 Neutrosophic Soft Set [16]

Consider the universe of discourse to be U and a set containing all the parameters be called E. Let a subset of E be B and $P(U)$ represent the set of all single-valued neutrosophic sets of U. Then the pair (F, B), where F denotes a mapping from B to $P(U)$, is called a neutrosophic soft set on U.
Let us understand the concept through an example.
Suppose an investor is choosing different companies to invest in. He considers different domains for investment and evaluates the performance of the companies based on some parameters.
Let U be the universal set of three elements x_1, x_2, x_3 that denote three investment areas like investment in Technology, Real Estate, Green Energy, on which the investor may invest.
Suppose $E = \{e_1, e_2, e_3, e_4\}$ be a parameter set based on which the investor will opt for the companies. Let us take the parameters as follows:
e_1: Modestly High Growth Potential, e_2: Reasonably Competent Management Quality, e_3: Moderate Market Stability, e_4 : Reasonably Sound Financial Health
Note that the parameters involve neutrosophic words. A neutrosophic soft set is essentially a function with neutrosophic values as an outcome. It takes elements from a subset of the parameter set and maps them to the elements of a neutrosophic set on the universal set U.
Then, the following is a neutrosophic soft set (F, E), presented in a matrix form that describes the performance of a company P according to each parameter for each concerned domain.

$$P = \begin{bmatrix} & F(e_1) & F(e_2) & F(e_3) & F(e_4) \\ x_1 & (1, 0.2, 0.1) & (0.6, 0.1, 0.1) & (0.9, 0.3, 0.6) & (0.7, 0.3, 0.4) \\ x_2 & (0.8, 0.1, 0.2) & (0.7, 0.2, 0.3) & (0.6, 0.6, 0.3) & (0.9, 0.3, 0.1) \\ x_3 & (0.7, 0.2, 0.4) & (0.6, 0.4, 0.3) & (1, 0, 0.1) & (0.7, 0.4, 0.2) \end{bmatrix}$$

Here, $F(e_1)$ will generate the output of domains with Modestly High Growth Potential and represent the neutrosophic set $\{(x_1, 1, 0.2, 0.1), (x_2, 0.8, 0.1, 0.2), (x_3, 0.7, 0.2, 0.4)\}$.

2.5 Similarity Measure

It is employed to ascertain the extent of similarity between alternatives. The current work is based on a similarity measure presented by Ren et al. [68] for the neutrosophic set which is shown to generate more reasonable values.

The similarity measure for two neutrosophic data A and B as proposed by Ren et al. [68] is the following:
Let $X = \{x_1, x_2, \ldots, x_n\}$ be a universe set. Then for two single-valued neutrosophic sets
$A = \{< x_i,\ T_A(x_i), I_A(x_i), F_A(x_i) > |\, x_i \in X\}$
and $B = \{< x_i,\ T_B(x_i), I_B(x_i), F_B(x_i) > |\, x_i \in X\}$,
the Chi-Squared distance based similarity is

$$S = 1 - \frac{1}{2n}\sum_{i=1}^{n}\left[\frac{(T_A(x_i)-T_B(x_i))^2}{2+T_A(x_i)+T_B(x_i)} + \frac{(I_A(x_i)-I_B(x_i))^2}{2+I_A(x_i)+I_B(x_i)} + \frac{(F_A(x_i)-F_B(x_i))^2}{2+F_A(x_i)+F_B(x_i)} + |m_A(x_i)-m_B(x_i)|\right] \tag{2.1}$$

where $m_k(x_i) = \frac{1+T_k(x_i)-F_k(x_i)}{2}$ for $k = A, B$.

3. Proposed Similarity Measure for Neutrosophic Soft Set

We now extend the measure given in (2.1) for the neutrosophic soft set.
Let X be the universe of discourse with elements $x_1,\ x_2\ ,\ \ldots\ ,\ x_n$ and E be the parameter set with elements $e_1,\ e_2\ ,\ \ldots\ ,\ e_m$.
Let (A, E) and (B, E) be neutrosophic soft sets on X, where

$$T_{A(e_j)}(x_i),\ T_{B(e_j)}(x_i),\ I_{A(e_j)}(x_i),\ I_{B(e_j)}(x_i),\ F_{A(e_j)}(x_i),\ F_{B(e_j)}(x_i)$$

respectively denote the truthness, indeterminacy and falsehood membership functions for the neutrosophic soft sets (A, E) and (B, E) corresponding to the element x_i and parameter e_j.
We then define the **Chi-Squared distance** based similarity between the two **neutrosophic soft sets** as

$$SM(A, B) = 1 - D(A, B) \tag{3.1}$$

where

$$D(A,B) = \frac{1}{2mn}\sum_{i=1}^{n}\sum_{j=1}^{m}\left[\frac{(T_{A(e_j)}(x_i)-T_{B(e_j)}(x_i))^2}{2+T_{A(e_j)}(x_i)+T_{B(e_j)}(x_i)} + \frac{(I_{A(e_j)}(x_i)-I_{B(e_j)}(x_i))^2}{2+I_{A(e_j)}(x_i)+I_{B(e_j)}(x_i)} + \frac{(F_{A(e_j)}(x_i)-F_{B(e_j)}(x_i))^2}{2+F_{A(e_j)}(x_i)+F_{B(e_j)}(x_i)} + \frac{1}{2}\left|m_{A(e_j)}(x_i)-m_{B(e_j)}(x_i)\right|\right] \tag{3.2}$$

and $m_{k(e_j)}(x_i) = \frac{1+T_{k(e_j)}(x_i)-F_{k(e_j)}(x_i)}{2}$ for $k = A, B$.

3.1. Computational Complexity of the Proposed Similarity Measure

Consider all the steps required to measure the distance $D(A,B)$ for the calculation of computational complexity.
n: number of elements in the universe of discourse, used in the outer sum
m: number of parameters, used in the inner sum
The values of these following components need to be known for each pair (i,j)

$$T_{A(e_j)}(x_i), T_{B(e_j)}(x_i), \; I_{A(e_j)}(x_i), \; I_{B(e_j)}(x_i), \; F_{A(e_j)}(x_i), \; F_{B(e_j)}(x_i)$$

The basic operations then used are as follows:
i) The following terms are computed.

$$\frac{\left(T_{A(e_j)}(x_i) - T_{B(e_j)}(x_i)\right)^2}{2 + T_{A(e_j)}(x_i) + T_{B(e_j)}(x_i)}, \frac{\left(I_{A(e_j)}(x_i) - I_{B(e_j)}(x_i)\right)^2}{2 + I_{A(e_j)}(x_i) + I_{B(e_j)}(x_i)}, \frac{\left(F_{A(e_j)}(x_i) - F_{B(e_j)}(x_i)\right)^2}{2 + F_{A(e_j)}(x_i) + F_{B(e_j)}(x_i)},$$

$$\frac{1}{2}\left|m_{A(e_j)}(x_i) - m_{B(e_j)}(x_i)\right|$$

ii) These terms mentioned above are then added and the summation is run for all values of i and j.
iii) Next, the sum value is divided by the factor $2mn$ and is subtracted from 1 to get the similarity value.

Complexity Analysis:

- The values of each $T_{A(e_j)}(x_i)$, $T_{B(e_j)}(x_i)$, $I_{A(e_j)}(x_i)$, $I_{B(e_j)}(x_i)$, $F_{A(e_j)}(x_i)$ and $F_{B(e_j)}(x_i)$ takes constant time $6 \times \mathcal{O}(1) = \mathcal{O}(1)$ for each (i,j) pair.
- Each term mentioned in the above point (i) also takes constant time $\mathcal{O}(1)$. There are 4 such terms so the total time taken is $4 \times \mathcal{O}(1) = \mathcal{O}(1)$
- Summation over each pair (i,j) takes $\mathcal{O}(mn)$ addition time.
- To find the similarity value at the last stage the division by the factor $2mn$ and the subtraction from 1 also takes constant time, $\mathcal{O}(1)$.

Then, the net computational complexity is $\mathcal{O}(mn)$ which is comparable to that of the standard measures such as Hamming, Euclidean [52], or Hausdorff [56].

3.2. Advantage of the Proposed Similarity Measure

In literature, similarity measures for neutrosophic soft sets are mostly built on well-known distance measures as Euclidean, Hamming [52], and Hausdorff [56]. All these measures have their own limitations. To illustrate the advantage of the proposed method in decision making, let us consider the following scenario where a person compares the performance of two investment companies with an ideal performance of a company.
Let U be the universal set of two elements x_1, x_2, where x_1, x_2 denote the investment areas like Technology and Green Energy, on which the companies may invest.
Suppose $E = \{e_1, e_2\}$ be a parameter set based on which the investor will opt for the companies. Let us take the parameters as follows:
e_1: Modestly High Growth Potential, e_2: Moderate Market Stability
Let (A,E) , (B,E) and (C,E) be three neutrosophic soft sets representing the performance of investment companies as shown in Tables 1-3. Let (A,E) represent the ideal performance of a company and the person wants to choose his preferred company between (B,E) and (C,E).

We now determine the similarity of the neutrosophic soft sets (B,E) and (C,E) with (A,E) using different existing similarity measures and compare the results with that of the Chi-squared metric based similarity measure proposed in this work. It may be observed that the sets (B,E) and (C,E) are only slightly different.

Table 1: Neutrosophic soft set (A, E)

(A, E)	e_1	e_2
x_1	$(0.6, 0.6, 0.7)$	$(0.7, 0.7, 0.8)$
x_2	$(0.3, 0.3, 0.4)$	$(0.2, 0.2, 0.3)$

Table 2: Neutrosophic soft set (B, E)

(B, E)	e_1	e_2
x_1	(0.4,0.3,0.4)	(0.4,0.3,0.4)
x_2	(0.4,0.3,0.4)	(0.4,0.3,0.4)

Table 3: Neutrosophic soft set (C, E)

(C, E)	e_1	e_2
x_1	(0.3,0.4,0.4)	(0.3,0.4,0.4)
x_2	(0.3,0.4,0.4)	(0.3,0.4,0.4)

The results shown below in Table 4 clearly demonstrate that only the proposed measure that uses Chi-squared metric is able to spot the difference in the neutrosophic soft sets (B, E) and (C, E) while measuring their similarity with the set A. All other measures return exactly the same similarity value and thus fails to recognize the differences between the sets. Thus, using the proposed measure, the person can choose company C as his preferred investment company because it has higher similarity to the ideal case A and is expected to serve better.

Table 4: Similarity values by different measures

Measure	**Similarity Value $(\boldsymbol{A}, \boldsymbol{B})$**	**Similarity Value $(\boldsymbol{A}, \boldsymbol{C})$**
Eulcidean	0.6743	0.6743
Hamming	0.5556	0.5556
Hausdorff	0.75	0.75
Proposed Method	**0.95843**	**0.97093**

Let us again inspect the neutrosophic soft sets (P, E) and (Q, E) over the universal set U under the same parameter set E as displayed in Tables 5 and 6 and measure their similarity with (A, E). It may be noted that the two sets differ only for x_1 corresponding to the parameter e_1. The similarity values portrayed in Table 7 once again confirm that the existing measures are unable to recognize the difference between P and Q. However, the similarity values obtained by the proposed measure are different. Thus, the measure based on Chi-squared metric can identify differences in neutrosophic soft sets when existing measures fail and hence may provide fruitful decision in critical situations.

Table 5: Neutrosophic soft set$(\boldsymbol{P}, \boldsymbol{E})$

(P, E)	e_1	e_2
x_1	(0.4,0.3,0.4)	(0.1,0.1,0.2)
x_2	(0.5,0.5,0.6)	(0.2,0.2,0.3)

4. Mathematical Validation

It is widely accepted that the following properties must be satisfied by a valid similarity measure. In this section, the properties are verified for the proposed similarity measure between the neutrosophic soft sets (A, E) and (B, E).

Table 6: Neutrosophic soft set $(\boldsymbol{Q},\boldsymbol{E})$

(Q,E)	e_1	e_2
x_1	(0.3,0.4,0.4)	(0.1,0.1,0.2)
x_2	(0.5,0.5,0.6)	(0.2,0.2,0.3)

Table 7: Similarity values by different measures

Measure	**Similarity Value $(\boldsymbol{A},\boldsymbol{P})$**	**Similarity Value $(\boldsymbol{A},\boldsymbol{Q})$**
Eulcidean	0.5924198	0.5924198
Hamming	0.4838710	0.4838710
Hausdorff	0.725	0.725
Proposed Method	**0.9353190**	**0.9384440**

Properties of Similarity Measures:

1. $0 \le SM(A,B) \le 1$
2. $SM(A,B) = 1 \ iff \ A = B$
3. $SM(A,B) = SM(B,A)$
4. If $A \subseteq B \subseteq C$, then $SM(A,C) \le SM(A,B)$ and $SM(A,C) \le SM(B,C)$ where (C,E) is another neutrosophic soft set on U.

Proof : i) To prove $0 \le SM(A,B) \le 1$, i.e, $0 \le D(A,B) \le 1$
Now, for the Neutrosophic soft sets (A,E) and (B,E) corresponding to the element x_i and the parameter e_j,

$$0 \le T_{A(e_j)}(x_i),\ T_{B(e_j)}(x_i) \le 1$$

$$0 \le I_{A(e_j)}(x_i),\ I_{B(e_j)}(x_i) \le 1$$

$$0 \le F_{A(e_j)}(x_i),\ F_{B(e_j)}(x_i) \le 1$$

Therefore , $0 \le T_{A(e_j)}(x_i) + T_{B(e_j)}(x_i) \le 2$
So,

$$\frac{1}{4} \le \frac{1}{2 + T_{A(e_j)}(x_i) + T_{B(e_j)}(x_i)} \le \frac{1}{2} \tag{4.1}$$

Also,

$$0 \le \left(T_{A(e_j)}(x_i) - T_{B(e_j)}(x_i)\right)^2 \le 1 \tag{4.2}$$

From, (4.1) and (4.2),

$$0 \le \frac{\left(T_{A(e_j)}(x_i) - T_{B(e_j)}(x_i)\right)^2}{2 + T_{A(e_j)}(x_i) + T_{B(e_j)}(x_i)} \le \frac{1}{2} \tag{4.3}$$

Similarly,

$$0 \le \frac{\left(I_{A(e_j)}(x_i) - I_{B(e_j)}(x_i)\right)^2}{2 + I_{A(e_j)}(x_i) + I_{B(e_j)}(x_i)} \le \frac{1}{2} \tag{4.4}$$

And

$$0 \leq \frac{\left(F_{A(e_j)}\left(x_i\right)-F_{B(e_j)}\left(x_i\right)\right)^2}{2+F_{A(e_j)}\left(x_i\right)+F_{B(e_j)}\left(x_i\right)} \leq \frac{1}{2} \tag{4.5}$$

Now, for the Neutrosophic soft set (A, E),

$$0 \leq T_{A(e_j)}\left(x_i\right) \leq 1$$
$$0 \leq F_{B(e_j)}\left(x_i\right) \leq 1$$

So, $0 \leq 1+T_{A(e_j)}\left(x_i\right)- \; F_{A(e_j)}\left(x_i\right) \leq 2$

Therefore ,

$$0 \leq \frac{1+T_{A(e_j)}\left(x_i\right)-F_{A(e_j)}\left(x_i\right)}{2} \leq 1 \tag{4.6}$$

Similarly, for the Neutrosophic soft set (B, E)

$$0 \leq \frac{1+T_{B(e_j)}\left(x_i\right)-F_{B(e_j)}\left(x_i\right)}{2} \leq 1 \tag{4.7}$$

From (4.6) and (4.7) , we have ,

$$0 \leq \frac{1}{2}\left|m_{A(e_j)}\left(x_i\right)-m_{B(e_j)}\left(x_i\right)\right| \leq \frac{1}{2} \tag{4.8}$$

Now, adding, (4.3), (4.4), (4.5) and (4.8)

$$\begin{aligned} 0 \leq \; & \frac{\left(T_{A(e_j)}\left(x_i\right)-T_{B(e_j)}\left(x_i\right)\right)^2}{2+T_{A(e_j)}\left(x_i\right)+T_{B(e_j)}\left(x_i\right)}+\frac{\left(I_{A(e_j)}\left(x_i\right)-I_{B(e_j)}\left(x_i\right)\right)^2}{2+I_{A(e_j)}\left(x_i\right)+I_{B(e_j)}\left(x_i\right)}+ \\ & \frac{\left(F_{A(e_j)}\left(x_i\right)-F_{B(e_j)}\left(x_i\right)\right)^2}{2+F_{A(e_j)}\left(x_i\right)+F_{B(e_j)}\left(x_i\right)}+\frac{1}{2}\left|m_{A(e_j)}\left(x_i\right)-m_{B(e_j)}\left(x_i\right)\right| \leq 2 \end{aligned} \tag{4.9}$$

Now for every pair of (x_i, e_j), we add (4.9) to have

$$\begin{aligned} 0 \leq \sum_{i=1}^{n} \sum_{j=1}^{m} & \left[\frac{(T_{A(e_j)}(x_i)-T_{B(e_j)}(x_i))^2}{2+T_{A(e_j)}(x_i)+T_{B(e_j)}(x_i)}+\frac{(I_{A(e_j)}(x_i)-I_{B(e_j)}(x_i))^2}{2+I_{A(e_j)}(x_i)+I_{B(e_j)}(x_i)}\right. \\ & \left.+\frac{(F_{A(e_j)}(x_i)-F_{B(e_j)}(x_i))^2}{2+F_{A(e_j)}(x_i)+F_{B(e_j)}(x_i)}+\frac{1}{2}\left|m_{A(e_j)}(x_i)-m_{B(e_j)}(x_i)\right|\right] \leq 2mn \end{aligned} \tag{4.10}$$

$$\begin{aligned} \text{Hence, } 0 \leq D(A, B)=\frac{1}{2mn} \sum_{i=1}^{n} \sum_{j=1}^{m} & \left[\frac{\left(T_{A(e_j)}(x_i)-T_{B(e_j)}(x_i)\right)^2}{2+T_{A(e_j)}(x_i)+T_{B(e_j)}(x_i)}\right. \\ & +\frac{\left(I_{A(e_j)}(x_i)-I_{B(e_j)}(x_i)\right)^2}{2+I_{A(e_j)}(x_i)+I_{B(e_j)}(x_i)} \\ & +\frac{\left(F_{A(e_j)}(x_i)-F_{B(e_j)}(x_i)\right)^2}{2+F_{A(e_j)}(x_i)+F_{B(e_j)}(x_i)} \\ & \left.+\frac{1}{2}\left|m_{A(e_j)}(x_i)-m_{B(e_j)}(x_i)\right|\right] \leq 1. \end{aligned} \tag{4.11}$$

ii) and iii) these two properties follow directly from the definition of $D(A,B)$.
iv) To prove this property, we first prove the following lemmas.

Lemma1 : If $0 \leq a \leq b \leq c \leq 1$, then $\frac{(a-c)^2}{2+a+c} \geq \frac{(a-b)^2}{2+a+b}$
Proof : To prove $(2+a+b)\,(a-c)^2 \geq (2+a+c)\,(a-b)^2$
i.e, $(2+a)\,(a-c)^2 + b(a-c)^2 \geq (2+a)\,(a-b)^2 + c(a-b)^2$
Now, $0 \leq a \leq b \leq c \leq 1$, So, $(a-c)^2 \geq (a-b)^2$
Therefore,

$$(2+a)(a-c)^2 \geq (2+a)\,(a-b)^2 \tag{4.12}$$

Let us now calculate $b(a-c)^2 - \ c(a-b)^2$

$$b(a-c)^2 - \ c(a-b)^2 = ba^2 - 2abc + bc^2 - ca^2 + 2abc - cb^2$$

$$= ba^2 + bc^2 - ca^2 - cb^2 = a^2(b-c) + bc\,(c-b) = (bc - a^2)(c-b) \tag{4.13}$$

Now, $(c-b) \geq 0$. Also since $a \leq b \leq c$, $(bc - a^2) \geq 0$
Therefore from (4.13) we have,

$$b(a-c)^2 \geq \ c(a-b)^2 \tag{4.14}$$

Adding (4.12) and (4.14), we obtain $(2+a+b)\,(a-c)^2 \geq (2+a+c)\,(a-b)^2$

Lemma2 : If $0 \leq a \leq b \leq c \leq 1$, then $\frac{(a-c)^2}{2+a+c} \geq \frac{(b-c)^2}{2+b+c}$
Proof: Since , $0 \leq a \leq b \leq c \leq 1$, So,

$$(a-c)^2 \geq (a-b)^2 \tag{4.15}$$

Also, $2+a+c \leq 2+b+c$
Therefore,

$$\frac{1}{2+a+c} \geq \frac{1}{2+b+c} \tag{4.16}$$

Multiplying (4.15) and (4.16) , we obtain $\frac{(a-c)^2}{2+a+c} \geq \frac{(b-c)^2}{2+b+c}$

Now, To prove property (iv) we have to show the following,
If $A \subseteq B \ \subseteq C$, then $a)$ $D\,(A,C) \geq D\,(A,B)$ and b) $D\,(A,C) \geq D\,(B,C)$ where (C,E) is another neutrosophic soft set on U.
Now, if $A \subseteq B \ \subseteq C$, we have then

$$T_{A(e_j)}\,(x_i) \leq T_{B(e_j)}\,(x_i) \leq T_{C(e_j)}\,(x_i)\,, \tag{4.17}$$

$$I_{A(e_j)}\,(x_i) \geq I_{B(e_j)}\,(x_i) \geq I_{C(e_j)}\,(x_i)\,, \tag{4.18}$$

$$F_{A(e_j)}\,(x_i) \geq F_{B(e_j)}\,(x_i) \geq F_{C(e_j)}\,(x_i) \tag{4.19}$$

$a)$ To prove, $D\,(A,C) \geq D\,(A,B)$, we proceed as follows
From (4.17) and lemma (1), we have

$$\frac{\left(T_{A(e_j)}\,(x_i) - T_{B(e_j)}\,(x_i)\right)^2}{2 + T_{A(e_j)}\,(x_i) + T_{B(e_j)}\,(x_i)} \leq \frac{\left(T_{A(e_j)}\,(x_i) - T_{C(e_j)}\,(x_i)\right)^2}{2 + T_{A(e_j)}\,(x_i) + T_{C(e_j)}\,(x_i)} \tag{4.20}$$

From (4.18) and lemma (2), we have

$$\frac{\left(I_{A(e_j)}\,(x_i) - I_{B(e_j)}\,(x_i)\right)^2}{2 + I_{A(e_j)}\,(x_i) + I_{B(e_j)}\,(x_i)} \leq \frac{\left(I_{A(e_j)}\,(x_i) - I_{C(e_j)}\,(x_i)\right)^2}{2 + I_{A(e_j)}\,(x_i) + I_{C(e_j)}\,(x_i)} \tag{4.21}$$

From (4.19) and lemma (2), we have

$$\frac{(F_{A(e_j)}(x_i) - F_{B(e_j)}(x_i))^2}{2 + F_{A(e_j)}(x_i) + F_{B(e_j)}(x_i)} \leq \frac{(F_{A(e_j)}(x_i) - F_{C(e_j)}(x_i))^2}{2 + F_{A(e_j)}(x_i) + F_{C(e_j)}(x_i)} \tag{4.22}$$

Now, from (4.17) we have,

$$T_{A(e_j)}(x_i) - T_{C(e_j)}(x_i) \leq T_{A(e_j)}(x_i) - T_{B(e_j)}(x_i) \leq 0 \tag{4.23}$$

From (4.19) we have,

$$F_{A(e_j)}(x_i) - F_{C(e_j)}(x_i) \geq F_{A(e_j)}(x_i) - F_{B(e_j)}(x_i) \geq 0 \tag{4.24}$$

From (4.23) and (4.24),

$$\frac{\left|\left(T_{A(e_j)}(x_i) - T_{C(e_j)}(x_i)\right) - \left(F_{A(e_j)}(x_i) - F_{C(e_j)}(x_i)\right)\right|}{2} \geq \frac{\left|\left(T_{A(e_j)}(x_i) - T_{B(e_j)}(x_i)\right) - \left(F_{A(e_j)}(x_i) - F_{B(e_j)}(x_i)\right)\right|}{2}$$

Therefore,

$$\frac{1}{2}\left|m_{A(e_j)}(x_i) - m_{B(e_j)}(x_i)\right| \leq \frac{1}{2}\left|m_{A(e_j)}(x_i) - m_{C(e_j)}(x_i)\right| \tag{4.25}$$

Adding (4.20), (4.21), (4.22) and (4.25) for a particular x_i and e_j, we have,

$$\begin{aligned}
&\frac{(T_{A(e_j)}(x_i) - T_{C(e_j)}(x_i))^2}{2 + T_{A(e_j)}(x_i) + T_{C(e_j)}(x_i)} + \frac{(I_{A(e_j)}(x_i) - I_{C(e_j)}(x_i))^2}{2 + I_{A(e_j)}(x_i) + I_{C(e_j)}(x_i)} \\
&\quad + \frac{(F_{A(e_j)}(x_i) - F_{C(e_j)}(x_i))^2}{2 + F_{A(e_j)}(x_i) + F_{C(e_j)}(x_i)} + \frac{1}{2}\left|m_{A(e_j)}(x_i) - m_{C(e_j)}(x_i)\right| \\
&\geq \frac{(T_{A(e_j)}(x_i) - T_{B(e_j)}(x_i))^2}{2 + T_{A(e_j)}(x_i) + T_{B(e_j)}(x_i)} + \frac{(I_{A(e_j)}(x_i) - I_{B(e_j)}(x_i))^2}{2 + I_{A(e_j)}(x_i) + I_{B(e_j)}(x_i)} \\
&\quad + \frac{(F_{A(e_j)}(x_i) - F_{B(e_j)}(x_i))^2}{2 + F_{A(e_j)}(x_i) + F_{B(e_j)}(x_i)} + \frac{1}{2}\left|m_{A(e_j)}(x_i) - m_{B(e_j)}(x_i)\right|
\end{aligned}$$

Then, adding this result for each pair of x_i and e_j,

$$\begin{aligned}
&\frac{1}{2mn}\sum_{i=1}^{n}\sum_{j=1}^{m}\left[\frac{(T_{A(e_j)}(x_i) - T_{C(e_j)}(x_i))^2}{2 + T_{A(e_j)}(x_i) + T_{C(e_j)}(x_i)} + \frac{(I_{A(e_j)}(x_i) - I_{C(e_j)}(x_i))^2}{2 + I_{A(e_j)}(x_i) + I_{C(e_j)}(x_i)}\right. \\
&\quad \left. + \frac{(F_{A(e_j)}(x_i) - F_{C(e_j)}(x_i))^2}{2 + F_{A(e_j)}(x_i) + F_{C(e_j)}(x_i)} + \frac{1}{2}\left|m_{A(e_j)}(x_i) - m_{C(e_j)}(x_i)\right|\right] \\
&\geq \frac{1}{2mn}\sum_{i=1}^{n}\sum_{j=1}^{m}\left[\frac{(T_{A(e_j)}(x_i) - T_{B(e_j)}(x_i))^2}{2 + T_{A(e_j)}(x_i) + T_{B(e_j)}(x_i)} + \frac{(I_{A(e_j)}(x_i) - I_{B(e_j)}(x_i))^2}{2 + I_{A(e_j)}(x_i) + I_{B(e_j)}(x_i)}\right. \\
&\quad \left. + \frac{(F_{A(e_j)}(x_i) - F_{B(e_j)}(x_i))^2}{2 + F_{A(e_j)}(x_i) + F_{B(e_j)}(x_i)} + \frac{1}{2}\left|m_{A(e_j)}(x_i) - m_{B(e_j)}(x_i)\right|\right]
\end{aligned}$$

Therefore , $D(A, C) \geq D(A, B)$.
b)To prove $D(A, C) \geq D(B, C)$, we can proceed as above.

5. Real Life Application

To demonstrate the applicability and usefulness of the similarity measure designed in this study, a number of real life scenarios and case studies are presented and analysed.

5.1. Decision Making in Medical Diagnosis-I

In this section, we consider a real-life example of the risk of a person being affected by poliovirus (taken from Sinha and Majumdar [54]). India had been declared to be polio-free by WHO in 2014. However, as this is a highly contagious disease, there remains the chance of getting affected by the virus from other countries through international travel. Hence, government initiative continues to maintain a polio-free environment which includes vaccination as well as regular surveillance. In the present work, we introduce a similarity measure technique to determine if a patient under consideration is infected by poliovirus where the available data and the related problem are set as neutrosophic soft sets.

The approach includes the following steps.

Firstly, the data of an infected patient is obtained. Then the data of the suspected patients are considered. Next, the similarity in the data of each suspected patient with the already affected patient is evaluated using the proposed measure. Finally, it is concluded that a patient who exhibits greater resemblance or similarity with an infected patient has a higher likelihood of being affected.

Suppose U be the universal set of three elements $u_{a_1},\ u_{a_2},\ u_{a_3}$ where $u_{a_1},\ u_{a_2}, u_{a_3}$ denote the symptoms of high growth, moderate growth, and low growth of polio disease respectively.

Suppose $E = \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ be the parameter set where the parameters are as follows:

ε_1: literacy factor, ε_2: high population, ε_3: socio-economic background, ε_4 : government initiative.

Now consider the following matrix P, represented in Table 8 by the neutrosophic soft set (F_1, E) , for a patient infected by polio on the basis of data available from a government report :

Table 8: Neutrosophic Soft Matrix for the Patient $\boldsymbol{P}$

(P, E)	$F_1(\varepsilon_1)$	$F_1(\varepsilon_2)$	$F_1(\varepsilon_3)$	$F_1(\varepsilon_4)$
u_{a_1}	(0.7,0.2,0.3)	(0.6,0.1,0.3)	(0.8,0.3,0.5)	(0.7,0.2,0.4)
u_{a_2}	(0.6,0.3,0.2)	(0.1,0.5,0.5)	(0.4,0.3,0.3)	(0.4,0.7,0.3)
u_{a_3}	(0.2,0.6,0.7)	(0.2,0.4,0.4)	(0,1,0)	(0.3,0.2,0.7)

Let the matrices Q and R displayed in Tables 9 and 10 represent the data of the two suspected patients through neutrosophic soft sets (F_2, E) and (F_3, E).

Table 9: Neutrosophic Soft Matrix for the Patient $\boldsymbol{Q}$

(Q, E)	$F_2(\varepsilon_1)$	$F_2(\varepsilon_2)$	$F_2(\varepsilon_3)$	$F_2(\varepsilon_4)$
u_{a_1}	(0.8,0.3,0.5)	(0.7,0.4,0.3)	(0.8,0.6,0.7)	(1,0,0)
u_{a_2}	(0.2,0.5,0.6)	(0.1,0.1,0.8)	(0.4,0.1,0.5)	(0.3,0.3,0.4)
u_{a_3}	(0,0,0)	(0.1,0.3,0.3)	(1,1,0)	(0,0,0)

Table 10: Neutrosophic Soft Matrix for the Patient $\boldsymbol{R}$

(R, E)	$F_3(\varepsilon_1)$	$F_3(\varepsilon_2)$	$F_3(\varepsilon_3)$	$F_3(\varepsilon_4)$
u_{a_1}	(0.8,0.4,0.8)	(0.6,0.3,0.1)	(0.5,0.6,0.5)	(0.7,0.2,0.4)
u_{a_2}	(0,0,0)	(0,1,1)	(0.3,0.1,0.1)	(0.2,0.5,0.4)
u_{a_3}	(0.2,0.6,0.2)	(0.2,0.6,0.6)	(0,0,0)	(0.4,0.2,0.8)

The similarity calculated by the proposed measure between the patients P and Q as well as between P and R are

$$\boldsymbol{SM}\,(\boldsymbol{P}, \boldsymbol{Q}) = \mathbf{0.893}$$

$$\boldsymbol{SM}\,(\boldsymbol{P}, \boldsymbol{R}) = \mathbf{0.925}$$

Observation - The above result indicates that patient R has more chances of being infected by the poliovirus than patient Q as $\boldsymbol{SM}\,(\boldsymbol{P}, \boldsymbol{R})$ is higher than $\boldsymbol{SM}\,(\boldsymbol{P}, \boldsymbol{Q})$. The same result was obtained by

Sinha and Majumdar [54] as well as Sarkar and Ghosh [56] using different similarity measures which confirms the reliability of the suggested measure.

5.2. Decision Making in Medical Diagnosis-II

To establish the applicability of the proposed measure in decision making, we apply it in another scenario described in the work of Das, Mukherjee and Tripathy [39].

Here, the measure is applied to identify a patient suffering from COVID-19 by calculating the similarity to an ideal case.

Suppose U be the universal set of four experts u_{a_1}, u_{a_2}, u_{a_3}, u_{a_4} and $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6$, be the parameter set representing the parameters fever, dry cough, exhaustion, headache, loss of taste or smell, and difficulty in breathing respectively.

Let (I, E) be the neutrosophic soft set that represents the data of an ideal case i.e. an infected patient in Table 11.

Table 11: Neutrosophic Soft Matrix for the ideal case $\boldsymbol{I}$

(I,E)	$I(\varepsilon_1)$	$I(\varepsilon_2)$	$I(\varepsilon_3)$	$I(\varepsilon_4)$	$I(\varepsilon_5)$	$I(\varepsilon_6)$
u_{a_1}	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)
u_{a_2}	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)
u_{a_3}	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)
u_{a_4}	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)	(1, 0, 0)

Let (P_1, E), (P_2, E), (P_3, E) be the neutrosophic soft representation of three suspected patients as displayed in Tables 12-14.

Table 12: Neutrosophic Soft Matrix for the Patient P_1

(P_1,E)	$P_1(\varepsilon_1)$	$P_1(\varepsilon_2)$	$P_1(\varepsilon_3)$	$P_1(\varepsilon_4)$	$P_1(\varepsilon_5)$	$P_1(\varepsilon_6)$
u_{a_1}	(0.1,0.2,0.7)	(0.2,0.3,0.5)	(0.1,0.1,0.9)	(0.2,0.3,0.5)	(0.21,0.2,0.9)	(0.1,0.3,0.9)
u_{a_2}	(0.1,0.6,0.6)	(0.2,0.5,0.7)	(0.1,0.2,0.8)	(0.1,0.5,0.4)	(0.3, 0.2, 0.5)	(0.1,0.2,0.6)
u_{a_3}	(0.3,0.2,0.6)	(0.3,0.4,0.6)	(0.1,0.2,0.7)	(0.2,0.3,0.5)	(0.2, 0.6, 0.9)	(0,0.6,0.9)
u_{a_4}	(0.1,0.1,0.9)	(0.2,0.3,0.5)	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.2, 0.5, 0.3)	(0.4,0.1,0.5)

Table 13: Neutrosophic Soft Matrix for the Patient P_2

(P_2,E)	$P_2(\varepsilon_1)$	$P_2(\varepsilon_2)$	$P_2(\varepsilon_3)$	$P_2(\varepsilon_4)$	$P_2(\varepsilon_5)$	$P_2(\varepsilon_6)$
u_{a_1}	(0.5,0.4,0.2)	(0.4,0.4,0.3)	(0.5,0.5,0)	(0.1,0.4,0.5)	(0.7,0.3,0)	(0.8,0.2,0)
u_{a_2}	(0.4,0.5,1)	(0.3,0.4,0.3)	(0.5,0.2,0.3)	(0.2,0.3,0.4)	(0.8,0,0.2)	(0.7,0,0.3)
u_{a_3}	(0.6,0.4,0)	(0.5,0.5,0)	(0.4,0.2,0.4)	(0.1,0.2,0.7)	(0.5,0.2,0.3)	(0.6,0.1,0.3)
u_{a_4}	(0.5,0.2,0.3)	(0.2,0.3,0.5)	(0.4,0.3,0.3)	(0.3,0.2,0.5)	(0.6,0.2,0.2)	(0.8,0.1,0.1)

Table 14: Neutrosophic Soft Matrix for the Patient P_3

(P_3,E)	$P_3(\varepsilon_1)$	$P_3(\varepsilon_2)$	$P_3(\varepsilon_3)$	$P_3(\varepsilon_4)$	$P_3(\varepsilon_5)$	$P_3(\varepsilon_6)$
u_{a_1}	(0.8,0,0.2)	(0.7,0.1,0.2)	(0.8,0.1,0.1)	(0.7,0.2,0.1)	(0.6,0.2,0.2)	(0.7,0.2,0.1)
u_{a_2}	(0.7,0.3,0)	(0.8,0,0.2)	(0.9,0.1,0)	(0.8,0,0.2)	(0.7,0.2,0.1)	(0.6,0.1,0.3)
u_{a_3}	(0.8,0.2,0)	(0.7,0.2,0.1)	(0.6,0.1,0.3)	(0.7,0.1,0.2)	(0.6,0.4,0)	(0.7,0.1,0.2)
u_{a_4}	(0.7,0.1,0.2)	(0.6,0.2,0.2)	(0.7,0.1,0.2)	(0.6,0.2,0.2)	(0.6,0.2,0.2)	(0.6,0.3,0.1)

The similarity values calculated by the proposed measure between the infected person I and the suspected patients P_1, P_2, P_3 are as follows

$$\boldsymbol{SM}\left(\boldsymbol{I}, P_1\right) = \mathbf{0.59037}$$

$$\boldsymbol{SM}(\boldsymbol{I}, P_2) = \mathbf{0.80259}$$

$$\boldsymbol{SM}(\boldsymbol{I}, P_3) = \mathbf{0.91551}$$

Observation - The above result indicates that the symptoms of the patients $\boldsymbol{P_2}$ and $\boldsymbol{P_3}$ have more similarity with that of the infected patient $\boldsymbol{I}$ than $\boldsymbol{P_1}$. Thus it can be concluded that the patients $\boldsymbol{P_2}$ and $\boldsymbol{P_3}$ have much higher chances of being COVID – 19 positive and hence needs proper treatment and care. The same result was obtained by the authors in [39].
This kind of study can actually lead to early detection of many diseases, thus enhancing diagnostic accuracy and enabling faster treatment options.

5.3. Application in Pattern Recognition

Here we consider a plausible application of the proposed similarity measure in pattern recognition. Let us suppose, three sample patterns represented through neutrosophic soft sets are to be identified. For this purpose, an ideal neutrosophic soft set pattern is considered. The following Tables 15-18 represent the ideal pattern I and the three given patterns $P1$, $P2$, $P3$. Let the neutrosophic soft sets representing the patterns be built upon the parameter set $E = \{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$ and the elements $\{\alpha_1, \alpha_2, \alpha_3\}$ of the universal set U.

Table 15: Ideal pattern $(\boldsymbol{I}, \boldsymbol{E})$

(I, E)	ε_1	ε_2	ε_3
α_1	(0.65,0.15,0.45)	(0.85,0.25,0.55)	(0.55,0.20,0.70)
α_2	(0.55,0.05,0.35)	(0.30,0.15,0.65)	(0.45,0.25,0.55)
α_3	(0.75,0.35,0.25)	(0.75,0.45,0.40)	(0.75,0.10,0.60)

Table 16: Pattern 1 $(\boldsymbol{P1}, \boldsymbol{E})$

$(P1, E)$	ε_1	ε_2	ε_3
α_1	(0.25,0.45,0.65)	(0.25,0.65,0.90)	(0.90,0.55,0.15)
α_2	(0.15,0.65,0.80)	(0.85,0.45,0.25)	(0.05,0.65,0.85)
α_3	(0.35,0.05,0.75)	(0.15,0.25,0.75)	(0.15,0.35,0.25)

Table 17: Pattern 2 $(\boldsymbol{P2}, \boldsymbol{E})$

$(P2, E)$	ε_1	ε_2	ε_3
α_1	(0.70,0.20,0.40)	(0.80,0.15,0.45)	(0.475,0.25,0.80)
α_2	(0.50,0.10,0.45)	(0.35,0.10,0.60)	(0.45,0.20,0.50)
α_3	(0.70,0.25,0.25)	(0.975,0.35,0.45)	(0.75,0.15,0.50)

Table 18: Pattern 3 $(\boldsymbol{P3}, \boldsymbol{E})$

$(P3, E)$	ε_1	ε_2	ε_3
α_1	(0.60,0.20,0.525)	(0.85,0.175,0.60)	(0.625,0.25,0.70)
α_2	(0.55,0.10,0.30)	(0.40,0.20,0.70)	(0.50,0.30,0.60)
α_3	(0.80,0.225,0.225)	(0.825,0.30,0.475)	(0.65,0.20,0.50)

Applying the proposed similarity measure between the soft set I and the three patterns $P1$, $P2$, $P3$, the following similarity values are obtained:

$$\boldsymbol{SM}(\boldsymbol{I}, \boldsymbol{P1}) = \mathbf{0.81170}$$

$$\boldsymbol{SM}\left(\boldsymbol{I},\boldsymbol{P2}\right)=\mathbf{0.98382}$$

$$\boldsymbol{SM}\left(\boldsymbol{I},\boldsymbol{P3}\right)=\mathbf{0.99137}$$

Here, as $P2$ and $P3$ are having very high similarity (> 0.98) with the ideal pattern I, we may contemplate that $P2$ and $P3$ belong to the family of ideal pattern.

5.4. Decision Making in Pixel Clustering

Here we investigate the potential application of the similarity measure in Medical MRI Image clustering.

Suppose, the neutrosophic soft sets are built upon the parameter set E = {Intensity, Texture, Spatial Proximity} and the universal set U = {Smoothness, Sharpness, Colorfulness} . Three clusters, namely, Bone (B) , Muscle (M), Fluid (F) are represented through neutrosophic soft sets as shown in Tables 19-21.

Table 19: Bone Cluster (B,E)

(B,E)	Intensity	Texture	Spatial Proximity
Smoothness	(0.8,0.1,0.1)	(0.7,0.2,0.1)	(0.9,0.05,0.05)
Sharpness	(0.5,0.3,0.2)	(0.4,0.4,0.2)	(0.6,0.2,0.2)
Colorfulness	(0.7,0.2,0.1)	(0.6,0.3,0.1)	(0.8,0.1,0.1)

Table 20: Muscle Cluster (M,E)

(M,E)	Intensity	Texture	Spatial Proximity
Smoothness	(0.3,0.2,0.5)	(0.5,0.3,0.2)	(0.4,0.3,0.3)
Sharpness	(0.7,0.1,0.2)	(0.6,0.2,0.2)	(0.7,0.2,0.1)
Colorfulness	(0.4,0.4,0.2)	(0.5,0.3,0.2)	(0.5,0.3,0.2)

Table 21: Fluid Cluster (F,E)

(F,E)	Intensity	Texture	Spatial Proximity
Smoothness	(0.2,0.3,0.5)	(0.3,0.4,0.3)	(0.3,0.3,0.4)
Sharpness	(0.5,0.3,0.2)	(0.6,0.2,0.2)	(0.5,0.3,0.2)
Colorfulness	(0.4,0.3,0.3)	(0.5,0.3,0.2)	(0.6,0.2,0.2)

A pixel P which is to be identified as one of the clusters mentioned above, is represented through a neutrosophic soft set in Table 22.

Table 22: Pixel P (P,E)

(P,E)	Intensity	Texture	Spatial Proximity
Smoothness	(0.8,0.4,0.8)	(0.6,0.3,0.1)	(0.5,0.6,0.5)
Sharpness	(0,0,0)	(0,1,1)	(0.3,0.1,0.1)
Colorfulness	(0.2,0.6,0.2)	(0.2,0.6,0.6)	(0,0,0)

Applying the proposed similarity measure between the soft set P and the three clusters B, M, F, the following similarity values are obtained:

$$S_M\left(\mathrm{B},\mathrm{P}\right)=0.8347$$

$$S_M\left(\mathrm{M},\mathrm{P}\right)=0.8729$$

$$S_M\left(\mathrm{F},\mathrm{P}\right)=0.8720$$

6. Conclusion

Here, a similarity measure utilizing the Chi-Squared distance metric is presented in the framework of neutrosophic soft sets. This study provides a new technique to measure similarity between uncertain or ambiguous data sets. The advantage of this measure lies in its theoretical supremacy over the existing measures as shown by Chai et al. [45] and Ren et al. [68]. It is also established in this work that the proposed measure can detect differences in neutrosophic soft sets in situations when the existing measures fail to do so and hence may be considered to be more dependable than the existing measures. Further, as demonstrated by real life examples, the Chi-Squared-based similarity measure plays a reliable role in decision-making process in a neutrosophic soft set environment. The proposed measure may be used successfully in future in diverse domains including medical diagnostics, image processing, pattern recognition, clustering and many more, with other extended or hybrid set frameworks.

Declarations

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