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Reliability Analysis and Cost-Effectiveness of Large Commercial HVAC Systems with Regenerating Points and Semi-Markov Processes

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ABSTRACT: Global change in climatic conditions has led to increase in global temperatures, and hence the need for heating, ventilation, and air-conditioning (HVAC) systems has increased significantly. These systems play a vital role in regulating indoor environments and ensuring comfortable temperatures in both residential and commercial spaces. This paper delves into the intricate analysis of large commercial HVAC systems comprising two similar cooling or heating units and a dissimilar unit dedicated to ventilation. These systems play a pivotal role in maintaining comfortable climate conditions, ensuring occupant comfort and operational efficiency. The study begins with three functioning units, where any two units are sufficient to sustain system operation. One crucial aspect addressed is the warranty coverage of these units, guaranteeing that any malfunctioning unit undergoes restoration at no additional cost within the stipulated warranty period. The restoration process is assumed to be flawless, conducted by a proficient service provider, with restoration times following an exponential distribution. Employing advanced methodologies such as the regenerating point technique and semi-Markov processes, this paper formulates equations to evaluate key reliability metrics and the profit function. These metrics encompass constant failure rates and consider the duration of the warranty periods, providing valuable insights into system reliability and cost-effectiveness.

Keywords: Semi-Markov process, repairable units, regenerative point technique, dissimilar system, HVAC, warranty.

Contents

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1. Introduction

Heating, Ventilation, and Air Conditioning (HVAC) systems play crucial role in ensuring good indoor air quality, maintaining thermal comfort, and supporting energy-efficient operation in a wide range of building settings. The growing global demand for sustainable infrastructure and operational optimization has led to extensive research in HVAC system modelling, energy analysis, maintenance strategies, and cost efficiency. Data-driven approaches are increasingly being used for fault detection and diagnostics in large-scale HVAC systems. A comprehensive review by Mirnaghi and Haghighat [2] highlights how these techniques improve operational efficiency and reduce unplanned downtimes. Integrating Building Information Modelling (BIM) into HVAC design is another innovative step, allowing accurate energy simulations and improved system design, as demonstrated by Mohd et al. [3]. In climate-responsive applications, Nadeem et al. [5] proposed an HVAC system design based on the Cooling Load Temperature Difference method by hourly analysis, specifically tailored for hot and humid climatic regions. However, Raad [6] cautions that existing HVAC modelling approaches have several practical shortcomings, underscoring the need for adaptable frameworks.

To ensure continuous functionality, reliability modelling of repairable HVAC systems has gained traction. Rathee et al. [4] and Kumar et al. [1] analysed warm standby and parallel systems under operational priority constraints. These efforts are extended by Kour et al. [7] and Kadyan et al. [8], who incorporated dissimilar units, switching devices, and simultaneous working conditions into their models. EL-Sherbery [12] analysed the stochastic behaviour of systems exposed to Poisson shocks, while Yusuf and Hussaini [9] studied 2-out-of-3 standby configurations under perfect restoration assumptions. These stochastic approaches support long-term planning and failure mitigation.

Preventive maintenance (PM) and warranty policies are also integral to HVAC reliability. Kumar et al. [10] introduced Weibull-based failure and restoration distributions for redundant systems, while Yusuf et al. [11] developed a dual maintenance strategy incorporating both online and offline PM. Alqahtani and Gupta [13] emphasized warranty as a marketing and assurance tool for remanufactured equipment, and Simin et al. [25] proposed a novel warranty policy tied to customer maintenance investment behaviour. Niwas and Kadyan [14], and Niwas and Garg [15], analysed reliability under degradation and cost-free warranty conditions. These studies bridge the gap between technical performance and consumer confidence.

The economic aspects of HVAC reliability have also been extensively explored. Ambekar and Jagtap [16] conducted cost modelling for warranty plans, while Huang et al. [26] examined the financial implications of two-dimensional warranties involving periodic maintenance. EL-Sherbery and Hussain [19] included administrative delays in system models, affecting both cost and uptime. Gitanjali [24] offered a cost-benefit analysis for parallel systems with constrained restoration times. These contributions underscore the financial impact of system downtime, warranty strategies, and restoration logistics. Other modelling studies focus on restoration policies, expert maintenance, and abnormal degradation. Malik and Gitanjali [17], and Rathee [18], explored systems with expert restoration teams and constraints on maximum operation or restoration time. Kumar and Saini [20] compared semi-Markov systems under degraded environments, while Kumar et al. [21] assessed profit in systems using software rejuvenation. Singh and Poonia [22] investigated two-unit systems with corresponding lifetimes under inspection regimes, utilizing regenerative point techniques. Cha et al. [23] proposed an optimal warranty strategy for stochastically degrading products, showing significant cost savings in diversified system environments.

Together with these works which reflects a growing convergence of reliability engineering, maintenance economics, degradation modelling, and environmentally adaptive HVAC design. However, an integrated framework that considers stochastic reliability, climate-based demand, maintenance scheduling, and cost modelling simultaneously is still lacking. This paper addresses this research gap by proposing a comprehensive stochastic model for HVAC systems that integrates operational reliability, preventive maintenance, cost analysis, and warranty policy evaluation, supporting sustainable and cost-effective design decisions.

2. Model Assumptions and Description

Here, a repairable HVAC system comprising of two similar units either heating or cooling units and one dissimilar unit as the ventilation unit is being analysed.

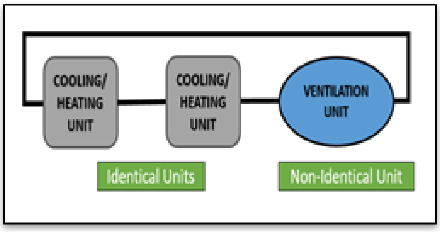


Fig.1 State transition diagram

- In the initial condition, the system runs on the two similar units, whereas the dissimilar unit is kept in cold standby mode, ready to be activated when required.
- The system maintains full functional as long as at least two of the three units are operational.
- The efficiency of the system reduces when it operates with one similar unit and other dissimilar unit, because of the difference in their capacities and functional behaviour create an imbalance in system performance.
- A server is available all the time for the system.
- After failure, the failed unit undergoes for restoration.
- Within the warranty period, all restoration costs are paid by the manufacturer, whereas beyond the warranty period, all the restoration expenses are paid by customer.
- The time required for restoring (repairing) the units is assumed to be exponential.
- For analysis purposes, the failure rates of the units and the duration of the warranty period are assumed to be arbitrarily constant.

These assumptions can help in establishing the foundation for analyzing the HVAC system's reliability and performance in relation to warranty conditions, restoration costs, and server availability. Examining these factors will help to understand the system's working behaviour and effectiveness, and will support in making informed decisions on maintenance planning and cost management.

3. State Specification

To analyse the system's behaviour at any given time, we can represent it using the following states:

S_0/S_1	The system is operational with both similar units active while the dissimilar unit remains on backup mode, both during and after the warranty period.
S_2/S_8	The system is operational with one similar unit and the dissimilar unit active, while the other similar unit is under restoration, whether within or beyond the warranty period.
S_3/S_7	The system is non-operational, as one similar unit is already under restoration and the dissimilar unit is holding on for restoration, whether within or beyond the warranty period.

S_4/S_9	The system is non-operational, as one similar unit is under restoration, and the other similar unit is holding on for restoration, whether within or beyond the warranty period.
S_5/S_{10}	The system is operational on two similar units, with the dissimilar unit under restoration under or beyond warranty.
S_6/S_{11}	The system is non-operational with the dissimilar unit under ongoing restoration, and one similar unit is holding on for restoration, whether within or beyond the warranty period.

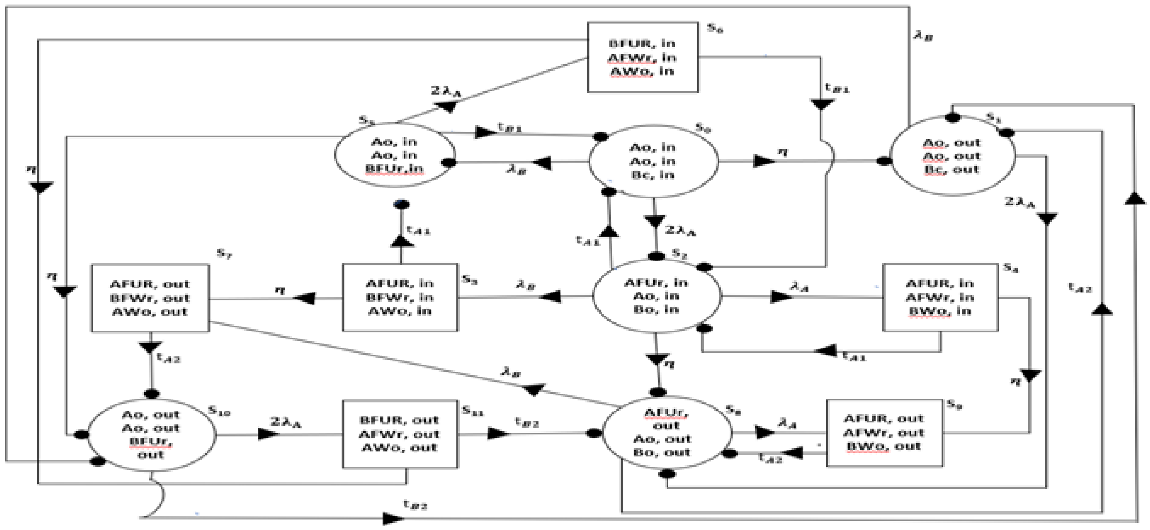
By categorizing the system into these specific states, we can effectively track its operational status, ongoing restoration activities, and warranty conditions. This classification enhances our ability to understand and evaluate the overall reliability and performance of the system.

4. Notations

A_O/B_O	Unit A/B in fully functional and operational mode
A_C/B_C	Unit A/B in backup mode
AW_o/BW_o	Unit A/B is awaiting operation
λ_A/λ_B	failure rate of the unit A/B
η	warranty period
$t_{A1}/A_2(t)$ / $T_{A1}/A_2(t)$	pdf/cdf of the restoration rate of the similar units A within/beyond the warranty period
$t_{B1}/B_2(t)$ / $T_{B1}/B_2(t)$	pdf/cdf of the restoration rate of the dissimilar unit B within/beyond the warranty period
$AFUr_{in}/BFUr_{in}$	Unit A/B is going through restoration within the warranty period following failure
$AFUr_{out}/BFUr_{out}$	Unit A/B is going through restoration beyond the warranty period following failure
$AFUR_{in}/BFUR_{in}$	unit A/B is going through restoration since previous state under warranty period following failure
$AFUR_{out}/BFUR_{out}$	Unit A/B is going through restoration since previous state beyond warranty period following failure
AWO_{out}/BWO_{out}	Unit A/B is holding on for restoration beyond warranty period following failure
AWO_{in}/BWO_{in}	Unit A/B is holding on for restoration under warranty period following failure
$r_{ij}(t)/\theta_{ij}(t)$	pdf/cdf of the transit time from a regenerative state i to another regenerative state j or to a failed state j, without passing through any other regenerative state during the interval $(0, t]$.
m_{ij}	Contribution to mean sojourn time (μ_i) in state S_i when system transits directly to state S_j so that, $\mu_i = \sum_j m_{ij}$ and $m_{ij} = \int t\, d\theta_{ij}(t) = -r_{ij}^{*\prime}(0)$
μ_i	mean sojourn time in state S_i which is given by $\mu_i = E(T_i) = \int_0^\infty P(T_i > t)\, dt = \sum_j m_{ij}$ where T_i denotes the time to system failure
$^{**}/^{*}$	Sign for Laplace–Stieltjes transform / Laplace transform
$\otimes/©$	Sign for Stieltjes convolution / Laplace convolution

The model diagram for the transition of states under study is:

Fig.1 State transition diagram



5. Reliability Measures

5.1. Steady State Transition Probabilities and Mean Sojourn Times

The following expressions for steady state transition probabilities from state i to j:

$$p_{ij} = \theta_{ij}(\infty) = \int_0^\infty r_{ij}(t)\,dt, \qquad d\theta_{ij}(t) = r_{ij}(t)\,dt.$$

$$d\theta_{01}(t) = r_{01}(t)\,dt = \eta e^{-(2\lambda_A+\lambda_B+\eta)t}\,dt$$

Taking LST of above equation we get,

$$\theta_{01}^{**}(s) = \int_0^\infty e^{-st}\,d[\theta_{01}(t)] = \eta\int_0^\infty e^{-(2\lambda_A+\lambda_B+\eta+s)t}\,dt = \frac{\eta}{2\lambda_A+\lambda_B+\eta+s}$$

Taking limit $s \to 0$ the steady state probability,

$$p_{01} = \frac{\eta}{2\lambda_A+\lambda_B+\eta}$$

Similarly, we obtain the following steady-state transition probabilities:

$$p_{02} = \frac{2\lambda_A}{2\lambda_A+\lambda_B+\eta}, \quad p_{05} = \frac{\lambda_B}{2\lambda_A+\lambda_B+\eta}, \quad p_{18} = \frac{2\lambda_A}{2\lambda_A+s}, \quad p_{20} = t_{A1}^*(\lambda_A+\lambda_B+\eta),$$

$$p_{23} = \frac{\lambda_B}{\lambda_A+\lambda_B+\eta}\left(1-t_{A1}^*(\lambda_A+\lambda_B+\eta)\right), \quad p_{24} = \frac{\lambda_A}{\lambda_A+\lambda_B+\eta}\left(1-t_{A1}^*(\lambda_A+\lambda_B+\eta)\right),$$

$$p_{28} = \frac{\eta}{\lambda_A+\lambda_B+\eta}\left(1-t_{A1}^*(\lambda_A+\lambda_B+\eta)\right), \quad p_{35} = t_{A1}^*(\eta), \quad p_{37} = 1-t_{A1}^*(\eta),$$

$$p_{42} = t_{A1}^*(\eta), \quad p_{49} = 1-t_{A1}^*(\eta), \quad p_{50} = t_{B1}^*(2\lambda_A+\eta), \quad p_{56} = \frac{2\lambda_A}{2\lambda_A+\eta}$$

$$p_{5.10} = \frac{\eta}{\lambda_B+\eta}\left(1-t_{B1}^*(\lambda_B+\eta)\right), \quad p_{62} = t_{B1}^*(\eta), \qquad p_{6.11} = 1-t_{B1}^*(\eta), \quad p_{7,10} = t_{A2}^*(0) = 1 = p_{98},$$

$$p_{81} = t_{A2}^*(\lambda_A+\lambda_B), \quad p_{87} = \frac{\lambda_B}{\lambda_A+\lambda_B}\left(1-t_{A2}^*(\lambda_A+\lambda_B)\right), \quad p_{89} = \frac{\lambda_A}{\lambda_A+\lambda_B}\left(1-t_{A2}^*(\lambda_A+\lambda_B)\right),$$

$$p_{10,1} = t_{B2}^*(2\lambda_A), \quad p_{10,11} = 1-t_{B2}^*(2\lambda_A), \quad p_{11,8} = t_{B2}^*(0) = 1.$$

It can be verified that

$$p_{01}+p_{02}+p_{05}=1,\quad p_{18}=1p_{20}+p_{23}+p_{24}+p_{28}=1,\quad p_{35}+p_{37}=1,\quad p_{42}+p_{49}=1,$$
$$p_{50}+p_{56}+p_{5,10}=1,\quad p_{62}+p_{6,11}=1,\quad p_{7,10}=1,\quad p_{87}+p_{81}+p_{89}=1,$$
$$p_{98}=1,\quad p_{10,1}+p_{10,11}=1.$$

The mean sojourn times (μ_i) in state S_i are given by:

$$\mu_0=l_{01}+l_{02}+l_{05},\quad \mu_1=l_{18},\quad \mu_2=l_{20}+l_{23}+l_{24}+l_{28},\quad \mu_3=l_{35}+l_{37},$$
$$\mu_4=l_{42}+l_{49},\quad \mu_5=l_{50}+l_{56}+l_{5,10},\quad \mu_6=l_{62}+l_{6,11},\quad \mu_7=l_{7,10},\quad \mu_8=l_{81}+l_{89}+l_{87}.$$
$$\mu_2'=l_{20}+l_{22;4}+l_{28}+l_{28;49}+l_{25;3}+l_{2,10;37}$$
$$\mu_5'=l_{50}+l_{5,10}+l_{52;6}+l_{58;6,11},$$
$$\mu_8'=l_{81}+l_{88;9}+l_{8,10;7},\quad \mu_{10}'=l_{10,1}+l_{10,8;11}$$

5.2. Mean Time to System Failure (MTSF)

Mean time to system failure is defined as the reliability metric depicting the average time a system operates before its first failure. A regenerative state in a system describes a state where the system restores its own condition. The cumulative distribution function (cdf) representing the first time a regenerative state S_i transitions into any failed state is incorporated into the recurrence relations and is denoted as $\phi_i(t)$.

$$\phi_0=r_{01}\otimes\phi_1+r_{02}\otimes\phi_2+r_{05}\otimes\phi_5 \tag{1}$$
$$\phi_1=r_{18}\otimes\phi_8+r_{1,10}\otimes\phi_{10} \tag{2}$$
$$\phi_2=r_{28}\otimes\phi_8+r_{20}\otimes\phi_0+r_{23}+r_{24} \tag{3}$$
$$\phi_5=r_{50}\otimes\phi_0+r_{5,10}\otimes\phi_{10}+r_{56} \tag{4}$$
$$\phi_8=r_{81}\otimes\phi_1+r_{87}+r_{89} \tag{5}$$
$$\phi_{10}=r_{10,1}\otimes\phi_1+r_{10,11} \tag{6}$$

Solving for $\phi_0^{**}(s)$, applying the Laplace–Stieltjes transformation to these equations yields:

$$\phi_0^{**}(s)=\frac{-r_{01}^*r_{18}^*(r_{87}^*+r_{89}^*)-r_{02}^*(r_{23}^*+r_{24}^*)(r_{18}^*r_{81}^*-1-r_{28}^*(r_{87}^*+r_{89}^*))}{\left(r_{18}^*r_{81}^*+r_{1,10}^*r_{10,1}^*-1\right)\left(r_{02}^*r_{20}^*+r_{50}^*r_{05}^*-1\right)}$$
$$+\frac{r_{05}^*\Big(r_{5,10}^*\big(-r_{18}^*r_{81}^*r_{10,11}^*+r_{18}^*r_{10,1}^*(r_{87}^*+r_{89}^*)+r_{10,11}^*\big)+r_{56}^*(r_{18}^*r_{81}^*-1)\Big)}{\left(r_{18}^*r_{81}^*+r_{1,10}^*r_{10,1}^*-1\right)\left(r_{02}^*r_{20}^*+r_{50}^*r_{05}^*-1\right)} \tag{7}$$

MTSF is given by:

$$\text{MTSF}=\lim_{s\to0}\frac{1-\phi_0^{**}(s)}{s}=\frac{N_0}{D_0}$$

where

$$N_0=(r_{87}+r_{89})\mu_0-r_{02}(r_{87}+r_{89})\mu_2+r_{05}(r_{87}+r_{89})\mu_5-(r_{01}+r_{02}r_{28}r_{81}+r_{05}r_{5,10}r_{10,1})\mu_1$$
$$+(r_{01}-r_{02}r_{28}+r_{05}r_{5,10}r_{10,1})\mu_8+r_{05}r_{5,10}(r_{87}+r_{89})\mu_{10}$$
$$D_0=\left(r_{18}r_{81}+r_{1,10}r_{10,1}-1\right)\left(r_{02}r_{20}+r_{50}r_{05}-1\right)$$

5.3. Availability

Availability is a reliability metric which determine how long a system remains operational with respect to its intended operating time. Suppose the system enters the regenerative state S_i at time $t=0$. Let

$A_i(t)$ denote the probability that the system is functioning at time t. The following recurrence relations describe $A_i(t)$:

$$A_0 = M_0 + r_{01} \otimes A_1 + r_{02} \otimes A_2 + r_{05} \otimes A_5 \tag{8}$$
$$A_1 = M_1 + r_{18} \otimes A_8 + r_{1,10} \otimes A_{10} \tag{9}$$
$$A_2 = M_2 + r_{20} \otimes A_0 + r_{22;4} \otimes A_2 + r_{28} \otimes A_8 + r_{25;3} \otimes A_5 \tag{10}$$
$$A_5 = M_5 + r_{50} \otimes A_0 + r_{52;6} \otimes A_2 + r_{5,10} \otimes A_{10} \tag{11}$$
$$A_8 = M_8 + r_{81} \otimes A_1 + r_{8,10;9} \otimes A_8 + r_{8,8;7} \otimes A_{10} \tag{12}$$
$$A_{10} = M_{10} + r \otimes A_1 + r_{10,8;11} \otimes A_8 \tag{13}$$

where

$$M_0(t) = e^{-(2\lambda_A+\lambda_B+\eta)t}, \qquad M_1(t) = e^{-2\lambda_A t},$$
$$M_2(t) = e^{-(\lambda_A+\lambda_B+\eta)t}\,\overline{T_{A1}(t)}, \qquad M_5(t) = e^{-(2\lambda_A+\eta)t}\,\overline{T_{B1}(t)},$$
$$M_8(t) = e^{-(\lambda_A+\lambda_B)t}\,\overline{T_{A2}(t)}, \qquad M_{10}(t) = e^{-(2\lambda_A)t}\,\overline{T_{B2}(t)}.$$

By working on the equations (8)–(13), relating the Laplace–Stieltjes transformation, and subsequently using Cramer's rule, we obtain:

for $A_0^{**}(s)$. For a steady-state system, the availability can be calculated as:

$$A = \lim_{s\to 0} sA_0^{**}(s) = \frac{N_A}{D_A} + \frac{N_B}{D_B}, \tag{14}$$

where

$$N_A = M_1^*\big(r_{81}^* - r_{8,10;7}^*\, r_{10,1}\big) + M_8^*\big(r_{18}^* - r_{1,10}^*\, r_{10,8;11}^*\big) + M_{10}^*\big(r_{88;7}^* - r_{1,10}^*\, r_{10,1}\, r_{81}^*\big)\,,$$
$$D_A = \mu_1\big(r_{81}^* - r_{88;7}^*\, r_{10,1}\big) + \mu_8'\big(r_{18}^* - r_{1,10}^*\, r_{10,8;11}^*\big) + \mu_{10}'\big(r_{88;7}^* - r_{1,10}^*\, r_{81}^*\big)\,,$$
$$N_B = M_0^*\left(r_{25;3}^* r_{52;6}^* - r_{22;4}^* - 1\right) + M_2^*\big(r_{02}^* - r_{05}^* r_{52;6}^*\big) + M_5^*\left(r_{25;3}^* r_{02}^* - r_{05}^* r_{22;4}^* - r_{05}^*\right),$$
$$D_B = r_{02}^*\big(r_{20}^* - r_{25;3}^* r_{50}^*\big) + (r_{22;4}^* + 1)(r_{05}^* r_{50}^* - 1) - r_{05}^* r_{20}^* r_{52;6}^* + r_{25;3}^* r_{52;6}^*.$$

5.4. Busy Period of Server Due to Restoration in Warranty Period

Busy period of server in a system determine the continuous stretch of time where the server is actively working in restorationing the failed unit, till the moment the server finishes all its tasks. Let $B_i(t)$ denote the possibility that the server is engaged with restoration at time t, given that the HVAC system entered regenerative state i at $t = 0$. Let W_i represent the waiting time as the server is busy due to ongoing restoration activities. The recursive relations for $B_i(t)$ are then given by:

$$B_0 = r_{01} \otimes B_1 + r_{02} \otimes B_2 + r_{05} \otimes B_5 \tag{15}$$
$$B_1 = r_{18} \otimes B_8 + r_{1,10} \otimes B_{10} \tag{16}$$
$$B_2 = W_2 + r_{20} \otimes B_0 + r_{22;4} \otimes B_2 + r_{25;3} \otimes B_5 + r_{28} \otimes B_8 \tag{17}$$
$$B_5 = W_5 + r_{50} \otimes B_0 + r_{52;6} \otimes B_2 + r_{5,10} \otimes B_{10} \tag{18}$$
$$B_8 = r_{81} \otimes B_1 + r_{8,10;9} \otimes B_8 + r_{88;7} \otimes B_{10} \tag{19}$$
$$B_{10} = r_{10,1} \otimes B_1 + r_{10,8;11} \otimes B_8 \tag{20}$$

where

$$W_2 = (\lambda_A + \lambda_B + \eta)\, e^{-(\lambda_A+\lambda_B+\eta)t} \copyright\, \overline{T}_{A1}(t),$$
$$W_5 = (\lambda_A + \eta)\, e^{-(\lambda_A+\eta)t} \copyright\, \overline{T}_{B1}(t).$$

By relating the Laplace–Stieltjes transformation to the above equations (15)–(20) and working for $B_0^*(s)$, the steady-state time during which the server remains busy with restoration is obtained as:

$$B_0^1(\infty) = \lim_{s\to 0} sB_0^*(s) = \frac{N_B^1}{D_B}, \tag{21}$$

where

$$N_B^1 = M_2^*\left(r_{02}^* - r_{05}^* r_{52;6}^*\right) + M_5^*\left(r_{25;3}^* r_{02}^* - r_{05}^* r_{22;4}^* - r_{05}^*\right),$$
$$D_B = r_{02}^*\left(r_{20}^* - r_{25;3}^* r_{50}^*\right) + (r_{22;4}^* + 1)(r_{05}^* r_{50}^* - 1) - r_{05}^* r_{20}^* r_{52;6}^* + r_{25;3}^* r_{52;6}^*.$$

5.5. Busy Period of Server Due to Restoration Beyond Warranty Period

Let $B_i(t)$ denote the possibility that the server is engaged with restoration at some particular time t, given that the HVAC system entered the regenerative state i at $t = 0$. Let W_i represent the waiting time as the server is busy due to restoration activities. The recursive relations for $B_i(t)$ are given by:

$$B_0 = r_{01} \otimes B_1 + r_{02} \otimes B_2 + r_{05} \otimes B_5 \tag{22}$$
$$B_1 = r_{18} \otimes B_8 + r_{1,10} \otimes B_{10} \tag{23}$$
$$B_2 = r_{20} \otimes B_0 + r_{22;4} \otimes B_2 + r_{25;3} \otimes B_5 + r_{28} \otimes B_8 \tag{24}$$
$$B_5 = r_{50} \otimes B_0 + r_{52;6} \otimes B_2 + r_{5,10} \otimes B_{10} \tag{25}$$
$$B_8 = W_8 + r_{81} \otimes B_1 + r_{8,10;9} \otimes B_8 + r_{88;7} \otimes B_{10} \tag{26}$$
$$B_{10} = W_{10} + r_{10,1} \otimes B_1 + r_{10,8;11} \otimes B_8 \tag{27}$$

where

$$W_8 = (\lambda_A + \lambda_B)\, e^{-(\lambda_A+\lambda_B)t} \copyright \overline{T_{A2}(t)}, \qquad W_{10} = \lambda_B e^{-\lambda_B t} \copyright \overline{T_{B2}(t)}.$$

By relating the Laplace–Stieltjes transformation to the above equations (22)–(27) and working on the linear equations for $B_0^*(s)$, the steady-state duration for which the server remains busy due to restoration is obtained as:

$$B_0^2(\infty) = \lim_{s\to 0} sB_0^*(s) = \frac{N_B^2}{D_A} \tag{28}$$

where

$$N_B^2 = W_8^*\left(r_{18}^* - r_{1,10}^* r_{10,8;11}^*\right) + W_{10}^*\left(r_{88;7}^* - r_{1,10}^* r_{81}^*\right),$$
$$D_A = \mu_1\left(r_{81}^* - r_{88;7}^* r_{10,1}\right) + \mu_8'\left(r_{18}^* - r_{1,10}^* r_{10,8;11}^*\right) + \mu_{10}'\left(r_{88;7}^* - r_{1,10}^* r_{81}^*\right).$$

5.6. Expected Number of Restorations of Identical Unit

Calculating the expected number of restorations is essential in cost analysis for a restorationable system. Let $R_i(t)$ denote the expected number of restorations conducted on the similar units, cooling/heating units, of the HVAC system by the server at any time t, it is assumed that the system entered the regenerative state S_i at $t = 0$. The recursive relations are given by:

$$R_0 = r_{01} \otimes R_1 + r_{02} \otimes (1 + R_2) + r_{05} \otimes R_5 \tag{29}$$
$$R_1 = r_{18} \otimes (1 + R_8) + r_{1,10} \otimes R_{10} \tag{30}$$
$$R_2 = r_{20} \otimes R_0 + r_{22;4} \otimes R_2 + r_{28} \otimes R_8 + r_{25;3} \otimes R_5 \tag{31}$$
$$R_5 = r_{50} \otimes R_0 + r_{52;6} \otimes R_2 + r_{5,10} \otimes R_{10} \tag{32}$$
$$R_8 = r_{81} \otimes R_1 + r_{8,10;9} \otimes R_8 + r_{88;7} \otimes R_{10} \tag{33}$$
$$R_{10} = r_{10,1} \otimes R_1 + r_{10,8;11} \otimes R_8 \tag{34}$$

By relating the Laplace–Stieltjes transformation to the set of above equations (29)–(34) and working on the linear equations for $R_0^{**}(s)$, the expected number of restorations performed by the server per unit time in the steady state is obtained as:

$$R_0^1 = \lim_{s\to 0} sR_0^{**}(s) = \frac{N_R^1}{D_A} \tag{35}$$

where

$$N_R^1 = r_{18}^*\left(r_{81}^* - r_{8,10;7}^* r_{10,1}\right) + r_{02}^*\left(r_{25;3}^* r_{52;6}^* - r_{22;4}^* - 1\right).$$

5.7. Expected Number of Restorations of Non-Identical Unit

Let $R_i(t)$ denote the expected number of restorations conducted on the dissimilar unit, ventilation unit, of the HVAC system by the server at any time t, it is assumed that the system entered the regenerative state S_i at $t = 0$. The recursive relations are given by:

$$R_0 = r_{01} \otimes R_1 + r_{02} \otimes R_2 + r_{05} \otimes (1 + R_5) \quad (36)$$
$$R_1 = r_{18} \otimes R_8 + r_{1,10} \otimes (1 + R_{10}) \quad (37)$$
$$R_2 = r_{20} \otimes R_0 + r_{22;4} \otimes R_2 + r_{28} \otimes R_8 + r_{25;3} \otimes R_5 \quad (38)$$
$$R_5 = r_{50} \otimes R_0 + r_{52;6} \otimes R_2 + r_{5,10} \otimes R_{10} \quad (39)$$
$$R_8 = r_{81} \otimes R_1 + r_{8,10;9} \otimes R_8 + r_{88;7} \otimes R_{10} \quad (40)$$
$$R_{10} = r_{10,1} \otimes R_1 + r_{10,8;11} \otimes R_8 \quad (41)$$

By relating the Laplace–Stieltjes transformation to the set of above equations (36)–(41) and working on the linear equations for $R_0^{**}(s)$ using Cramer's rule, the expected number of restorations performed by the server per unit time in the steady state is obtained as:

$$R_0^2 = \lim_{s \to 0} sR_0^{**}(s) = \frac{N_R^2}{D_A} \quad (42)$$

where

$$N_R^2 = r_{1,10}^*\left(r_{81}^* - r_{8,10;7}^* r_{10,1}\right) + r_{05}^*\left(r_{25;3}^* r_{52;6}^* - r_{22;4}^* - 1\right).$$

5.8. Profit Analysis

In steady state, the HVAC system model profit is evaluated using the equations (14), (21), (28), (35) and (42) in the following equation:

$$P = K_0 A_0 - K_1 B_0^1 - K_2 B_0^2 - K_3 R_0^1 - K_4 R_0^2,$$

where

P = Profit estimation by HVAC system model,

K_0 = Cost gained by availability of the system,

K_1 = Cost experienced when the server is occupied with restoration activities during the warranty period,

K_2 = Cost experienced when the server is occupied with restoration activities beyond the warranty period,

K_3 = Cost experienced for restorationing similar (cooling/heating) unit of the HVAC system,

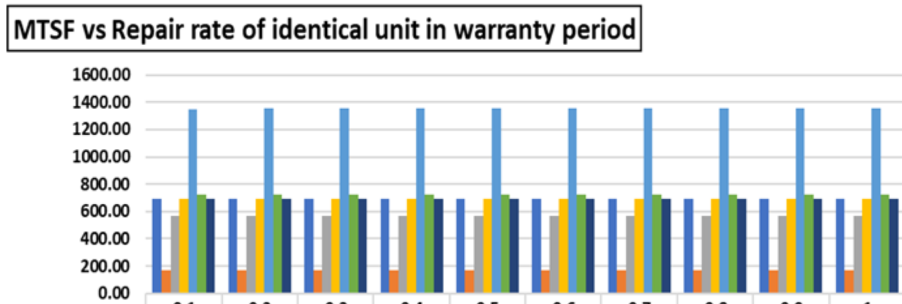
K_4 = Cost experienced for restorationing dissimilar (ventilation) unit of the HVAC system.

6. Numerical Representations of Various Reliability Measures

To analyse the behaviour of the HVAC system model, numerical computations are performed to assess the Mean Time to System Failure (MTSF), Steady-State Availability, and the Profit function under varying restoration rates, warranty periods and failure rates. The restoration times of dissimilar units A and B are assumed to follow a negative exponential distribution with rates α_1 and α_2, respectively, expressed as

$$t_{\beta_1}(t) = \alpha_1 e^{-\alpha_1 t} \quad \text{and} \quad t_{\beta_2}(t) = \alpha_2 e^{-\alpha_2 t}.$$

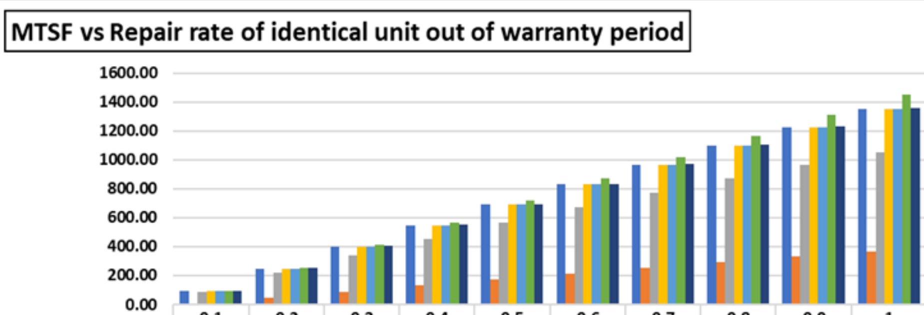
For the numerical evaluation, specific values are assigned to the failure rates (λ_a, λ_b) and the parameters used in the profit function $(K_0, K_1, K_2, K_3, K_4)$. This analysis provides insights into the dependability of the HVAC system—particularly its availability and profitability—across different operational scenarios.



	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Λa=0.005,Λb=0.001, β1=0.1, α2=0.05, β2=0.1, η=2	690.79	691.13	691.44	691.73	691.99	692.24	692.46	692.67	692.86	693.05
Λa=0.01, Λb=0.001, β1=0.1, α2=0.05, β2=0.1, η=2	171.16	171.35	171.53	171.69	171.83	171.97	172.10	172.21	172.32	172.42
Λa=0.005, Λb=0.002, β1=0.1, α2=0.05, β2=0.1, η=2	565.22	565.50	565.76	565.99	566.20	566.40	566.58	566.75	566.91	567.06
Λa=0.005, Λb=0.001, β1=0.2, α2=0.05, β2=0.1, η=2	690.79	691.13	691.45	691.73	691.99	692.24	692.46	692.67	692.87	693.05
Λa=0.005, Λb=0.001, β1=0.1, α2=0.1, β2=0.1, η=2	1350.33	1350.96	1351.53	1352.05	1352.53	1352.98	1353.39	1353.78	1354.13	1354.47
Λa=0.005, Λb=0.001, β1=0.1, α2=0.05, β2=0.2, η=2	721.35	721.71	722.03	722.33	722.61	722.86	723.10	723.32	723.52	723.71
Λa=0.005, Λb=0.001, β1=0.1, α2=0.05, β2=0.1, η=4	694.73	694.82	694.91	695.00	695.08	695.15	695.23	695.30	695.36	695.43

Figure 1

The Figure 1 visualizes how the Mean Time to System Failure (MTSF) varies with the restoration rate of similar cooling/heating units of HVAC system for different parameter combinations related to system reliability and maintenance. It can be visualized that MTSF increases with higher in-warranty (α_1, β_1) and out-of-warranty (α_2, β_2) restoration rates, but the in-warranty rates have a more significant impact. Longer warranty consistently gives the best reliability improvement.



	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Λa=0.005, Λb=0.001, α1=0.05, β1=0.1, β2=0.1, η=2	92.91	248.92	400.44	547.67	690.79	829.97	965.36	1097.13	1225.40	1350.33
Λa=0.01, Λb=0.001, α1=0.05, β1=0.1, β2=0.1, η=2	2.19	45.37	87.92	129.84	171.16	211.89	252.03	291.61	330.63	369.10
Λa=0.005, Λb=0.002, α1=0.05, β1=0.1, β2=0.1, η=2	89.42	217.04	338.60	454.53	565.22	671.02	772.23	869.15	962.05	1051.18
Λa=0.005, Λb=0.001, α1=0.1, β1=0.1, β2=0.1, η=2	93.00	249.07	400.66	547.95	691.13	830.37	965.82	1097.64	1225.97	1350.96
Λa=0.005, Λb=0.001, α1=0.05, β1=0.2, β2=0.1, η=2	92.91	248.92	400.44	547.68	690.79	829.97	965.36	1097.13	1225.41	1350.33
Λa=0.005, Λb=0.001, α1=0.05, β1=0.1, β2=0.2, η=2	95.05	255.28	413.03	568.37	721.35	872.02	1020.44	1166.65	1310.70	1452.64
Λa=0.005, Λb=0.001, α1=0.05, β1=0.1, β2=0.1, η=4	94.06	250.80	403.03	550.95	694.73	834.56	970.58	1102.96	1231.84	1357.35

Figure 2

The Figure 2 shows how the Mean Time to System Failure (MTSF) varies with change in the restoration rate beyond warranty (α_2) for similar cooling/heating units of HVAC system. For all parameter combinations, MTSF rises sharply as α_2 increases from 0.1 to 1.0. This confirms that better restoration support after the warranty period greatly extends system life. Clearly, extending the warranty period has a large, beneficial impact even if other restoration rates remain unchanged.

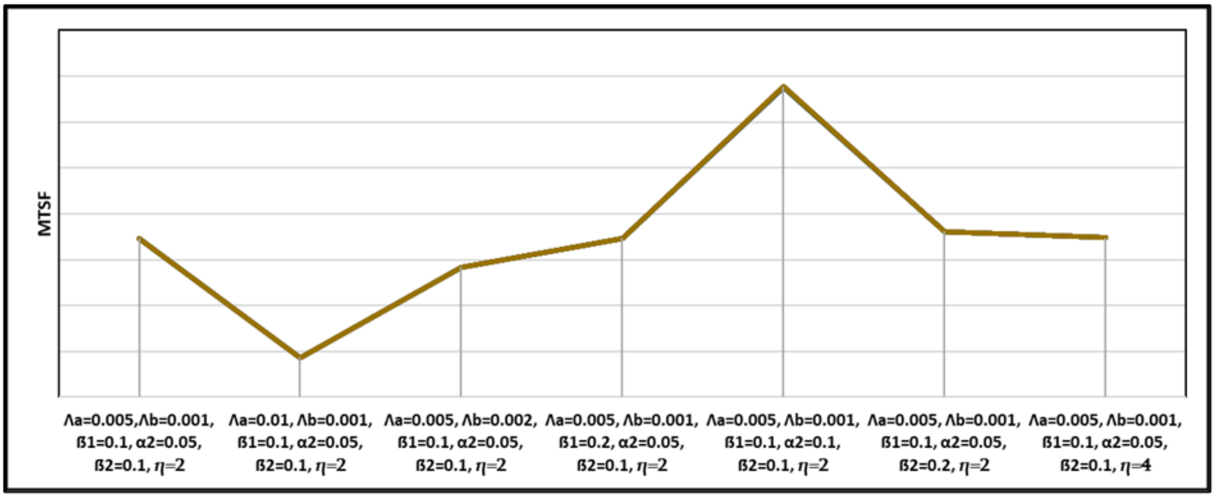
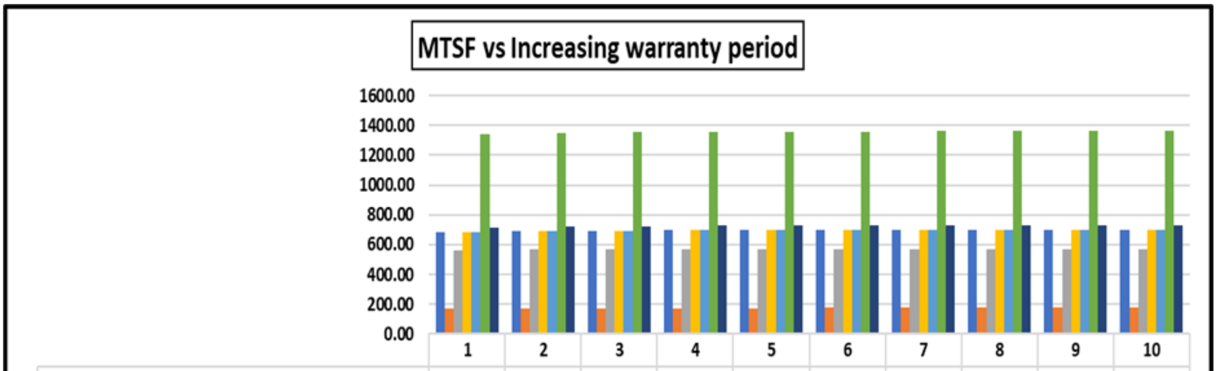


Figure 3

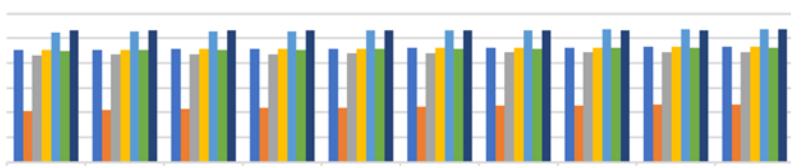
The line chart in Figure 3 presents a focused analysis of MTSF (Mean Time to System Failure) across different parameter configurations, particularly varying combinations of failure rates (λ_a, λ_b), restoration rates (β_1 in warranty, β_2 beyond warranty) of non-identical ventilation unit of HVAC system, identical unit restoration rate beyond warranty (α_2), and warranty duration (η). It can be inferred that high failure rates (λ_a or λ_b) strongly reduce MTSF. Improved restoration rates during the warranty period boost MTSF substantially. Extending warranty duration (η) can help, but only if supported by good restoration infrastructure.



	1	2	3	4	5	6	7	8	9	10
Λa=0.005, Λb=0.001, α1=0.1, ß1=0.1, α2=0.05, ß2=0.1	683.49	690.79	693.40	694.73	695.54	696.09	696.48	696.78	697.01	697.20
Λa=0.01, Λb=0.001, α1=0.1, ß1=0.1, α2=0.05, ß2=0.1	166.90	171.16	172.69	173.48	173.95	174.28	174.51	174.68	174.82	174.93
Λa=0.005, Λb=0.002, α1=0.1, ß1=0.1, α2=0.05, ß2=0.1	559.15	565.22	567.39	568.50	569.17	569.63	569.95	570.20	570.39	570.54
Λa=0.005, Λb=0.001, α1=0.2, ß1=0.1, α2=0.05, ß2=0.1	684.66	691.13	693.55	694.82	695.60	696.13	696.51	696.80	697.03	697.21
Λa=0.005, Λb=0.001, α1=0.1, ß1=0.2, α2=0.05, ß2=0.1	683.49	690.79	693.40	694.73	695.54	696.09	696.48	696.78	697.01	697.20
Λa=0.005, Λb=0.001, α1=0.1, ß1=0.1, α2=0.1, ß2=0.1	1337.34	1350.33	1354.97	1357.35	1358.80	1359.77	1360.47	1361.00	1361.41	1361.74
Λa=0.005, Λb=0.001, α1=0.1, ß1=0.1, α2=0.05, ß2=0.2	713.73	721.35	724.06	725.46	726.30	726.87	727.28	727.59	727.83	728.03

Figure 4

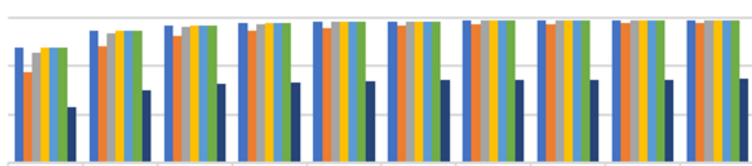
The Figure 4 analysed how the Mean Time to System Failure (MTSF) changes as the warranty period increases from 1 to 10 under various system configurations. Longer warranty periods improve MTSF, especially with good in-warranty restoration support. High failure rates (λ_a) limit the benefit of longer warranties. The best results come from combining long warranty with strong restoration policies both during and after warranty.



	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Λa=0.005,Λb=0.001, ß1=0.1, α2=0.05, ß2=0.1, η=2	0.9852	0.9854	0.9856	0.9857	0.9859	0.9861	0.9862	0.9863	0.9865	0.9866
Λa=0.01, Λb=0.001, ß1=0.1, α2=0.05, ß2=0.1, η=2	0.9606	0.9610	0.9614	0.9618	0.9621	0.9624	0.9627	0.9629	0.9632	0.9634
Λa=0.005, Λb=0.002, ß1=0.1, α2=0.05, ß2=0.1, η=2	0.9832	0.9834	0.9836	0.9838	0.9839	0.9841	0.9842	0.9844	0.9845	0.9846
Λa=0.005, Λb=0.001, ß1=0.2, α2=0.05, ß2=0.1, η=2	0.9852	0.9854	0.9856	0.9858	0.9859	0.9861	0.9862	0.9864	0.9865	0.9866
Λa=0.005, Λb=0.001, ß1=0.1, α2=0.1, ß2=0.1, η=2	0.9924	0.9927	0.9928	0.9930	0.9932	0.9933	0.9935	0.9936	0.9937	0.9938
Λa=0.005, Λb=0.001, ß1=0.1, α2=0.05, ß2=0.2, η=2	0.9849	0.9851	0.9853	0.9855	0.9857	0.9858	0.9860	0.9861	0.9862	0.9863
Λa=0.005, Λb=0.001, ß1=0.1, α2=0.05, ß2=0.1, η=4	0.9932	0.9933	0.9933	0.9934	0.9934	0.9934	0.9934	0.9935	0.9935	0.9935

Figure 5

The Figure 5 shows how availability of HVAC system changes with increasing restoration rate (α_1) of a similar cooling/heating unit of HVAC system during the warranty period, under different system configurations. Higher restoration rate (α_1) during warranty significantly improves system availability. System with low failure rate (λ_a) and good restoration support maintains high availability. High failure rate systems ($\lambda_a = 0.01$) show poor availability even when restoration rate of similar unit (α_1) is increased. Extending warranty period (η) adds a noticeable availability boost when restoration rates are already strong.



	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Λa=0.005, Λb=0.001, α1=0.05, ß1=0.1, ß2=0.1, η=2	0.8757	0.9483	0.9707	0.9803	0.9852	0.9880	0.9898	0.9910	0.9918	0.9924
Λa=0.01, Λb=0.001, α1=0.05, ß1=0.1, ß2=0.1, η=2	0.7758	0.8809	0.9256	0.9480	0.9606	0.9684	0.9736	0.9772	0.9797	0.9816
Λa=0.005, Λb=0.002, α1=0.05, ß1=0.1, ß2=0.1, η=2	0.8563	0.9396	0.9661	0.9774	0.9832	0.9866	0.9887	0.9901	0.9911	0.9919
Λa=0.005, Λb=0.001, α1=0.1, ß1=0.1, ß2=0.1, η=2	0.8759	0.9485	0.9709	0.9805	0.9854	0.9882	0.9900	0.9912	0.9920	0.9927
Λa=0.005, Λb=0.001, α1=0.05, ß1=0.2, ß2=0.1, η=2	0.8757	0.9483	0.9707	0.9803	0.9852	0.9880	0.9898	0.9910	0.9918	0.9925
Λa=0.005, Λb=0.001, α1=0.05, ß1=0.1, ß2=0.2, η=2	0.8753	0.9481	0.9705	0.9800	0.9849	0.9878	0.9895	0.9907	0.9916	0.9922
Λa=0.005, Λb=0.001, α1=0.05, ß1=0.1, ß2=0.1, η=4	0.6295	0.7022	0.7246	0.7341	0.7390	0.7419	0.7436	0.7448	0.7457	0.7463

Figure 6

The Figure 6 presents how availability of HVAC system changes with increasing restoration rate α_2 of a similar unit after the warranty period under different configurations. System availability strongly improves with increasing post-warranty restoration rate α_2. High failure rate ($\lambda_a = 0.01$) restricts availability growth despite good restoration. Extending warranty (η) alone does not guarantee better availability unless paired with strong post-warranty restoration support.

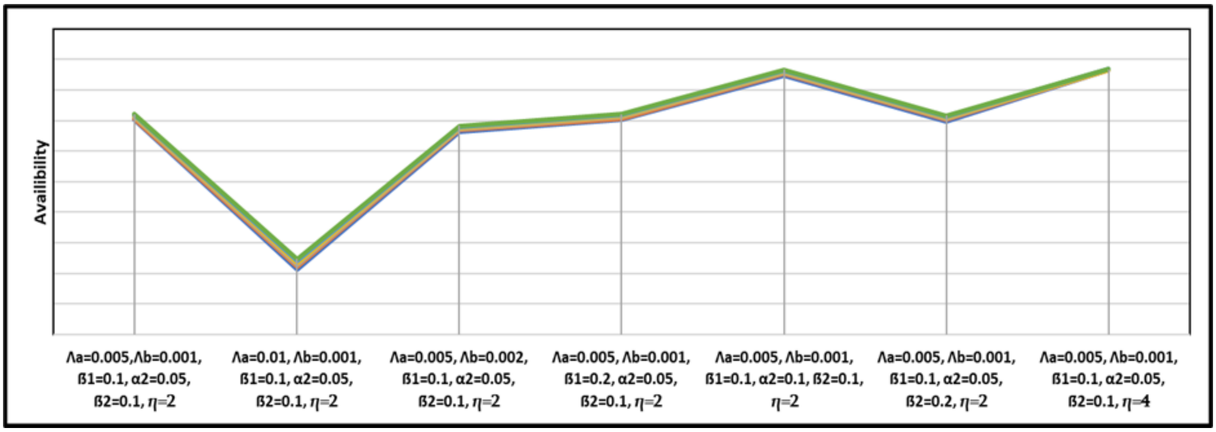
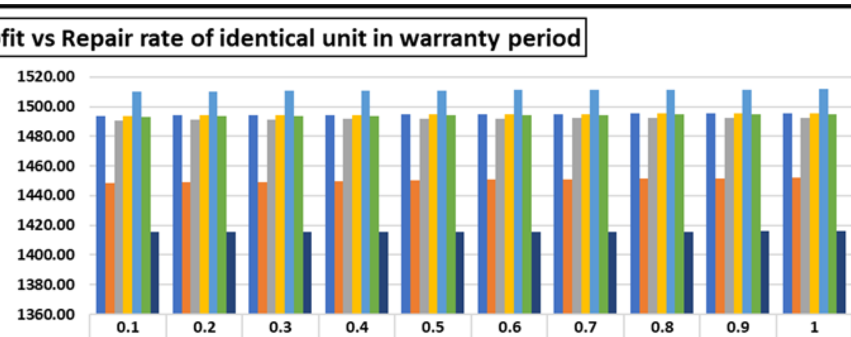


Figure 7

This line chart presents a comparison of system availability under different parameter configurations. There is a clear dip at the second parameter set indicating lower availability due to increased failure rate. Availability is most sensitive to the failure rate of similar units.



	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Λa=0.005,Λb=0.001, ß1=0.1, α2=0.05, ß2=0.1, η=2	1493.73	1494.01	1494.27	1494.50	1494.72	1494.92	1495.11	1495.28	1495.44	1495.59
Λa=0.01, Λb=0.001, ß1=0.1, α2=0.05, ß2=0.1, η=2	1448.35	1448.90	1449.41	1449.88	1450.30	1450.70	1451.07	1451.41	1451.73	1452.02
Λa=0.005, Λb=0.002, ß1=0.1, α2=0.05, ß2=0.1, η=2	1490.78	1491.06	1491.32	1491.55	1491.77	1491.97	1492.16	1492.33	1492.49	1492.64
Λa=0.005, Λb=0.001, ß1=0.2, α2=0.05, ß2=0.1, η=2	1493.76	1494.04	1494.30	1494.53	1494.75	1494.95	1495.14	1495.31	1495.47	1495.62
Λa=0.005, Λb=0.001, ß1=0.1, α2=0.1, ß2=0.1, η=2	1509.91	1510.19	1510.45	1510.69	1510.91	1511.11	1511.29	1511.46	1511.63	1511.78
Λa=0.005, Λb=0.001, ß1=0.1, α2=0.05, ß2=0.2, η=2	1493.19	1493.47	1493.73	1493.96	1494.18	1494.38	1494.57	1494.74	1494.90	1495.05
Λa=0.005, Λb=0.001, ß1=0.1, α2=0.05, ß2=0.1, η=4	1415.57	1415.61	1415.66	1415.70	1415.74	1415.78	1415.82	1415.86	1415.89	1415.93

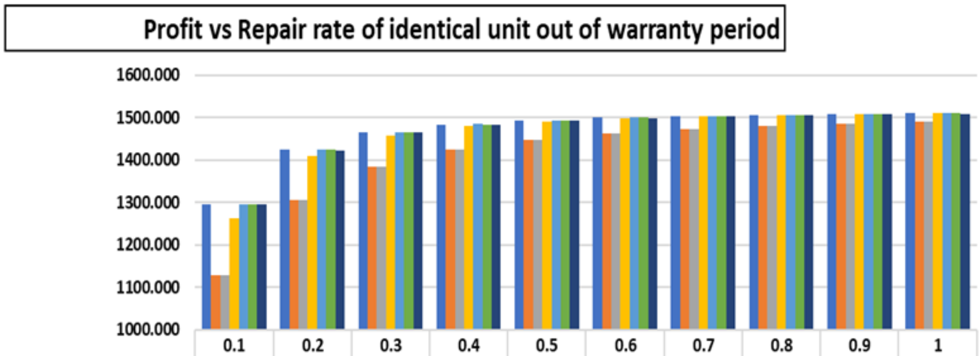
Figure 8

This chart compares profit as a function of restoration rate (α_1) for similar cooling/heating units during the warranty period, under various configurations of system parameters. Longer warranty periods increase costs, hence reduce profit. Improving restoration efficiency during warranty (higher α_1) improves profit. The cost impact of warranty support, redundancy, and failure rates significantly shapes profitability.

The Figure 9 examines how profit changes with restoration rate α_2 of similar cooling/heating units after the warranty period across different system configurations. This indicates that restoration strategies after the warranty period are more financially critical for the company (perhaps due to service contracts or cost-sharing with users). This suggests that post-warranty restoration efficiency is crucial for profitability, likely because customers bear restoration costs or system uptime improves significantly.

7. Discussion

The conjecture outlined in the study offer valuable understanding into the mechanism of the HVAC system, particularly concerning the Mean Time to System Failure (MTSF), steady-state availability,



	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Λa=0.005, Λb=0.001, α1=0.05, ß1=0.1, ß2=0.1, η=2	1295.706	1423.668	1465.245	1483.784	1493.728	1499.733	1503.669	1506.409	1508.405	1509.913
Λa=0.01, Λb=0.001, α1=0.05, ß1=0.1, ß2=0.1, η=2	1127.944	1306.299	1384.566	1424.917	1448.349	1463.180	1473.192	1480.292	1485.527	1489.510
Λa=0.005, Λb=0.002, α1=0.05, ß1=0.1, ß2=0.1, η=2	1127.944	1306.299	1384.566	1424.917	1448.349	1463.180	1473.192	1480.292	1485.527	1489.510
Λa=0.005, Λb=0.001, α1=0.1, ß1=0.1, ß2=0.1, η=2	1263.103	1409.038	1457.543	1479.199	1490.780	1497.745	1502.294	1505.448	1507.738	1509.462
Λa=0.005, Λb=0.001, α1=0.05, ß1=0.2, ß2=0.1, η=2	1295.988	1423.950	1465.526	1484.066	1494.010	1500.014	1503.950	1506.690	1508.687	1510.195
Λa=0.005, Λb=0.001, α1=0.05, ß1=0.1, ß2=0.2, η=2	1295.734	1423.696	1465.273	1483.812	1493.756	1499.761	1503.697	1506.437	1508.433	1509.941
Λa=0.005, Λb=0.001, α1=0.05, ß1=0.1, ß2=0.1, η=4	1294.954	1423.204	1464.780	1483.282	1493.188	1499.162	1503.073	1505.792	1507.771	1509.265

Figure 9

and overall system profitability. These factors are analysed within a framework comprising two similar cooling/heating units and one dissimilar ventilation unit, taking into account the restoration rates of the similar units both during and after the warranty period.

- Tables in Figure 1 and Figure 2 illustrate that Mean Time to System Failure (MTSF) increases with higher restoration rates, suggesting that frequent restorations extend the intervals between failures.
- Table in Figure 4 shows longer warranty periods result in an increase in MTSF, indicating a positive relationship between warranty duration and the average time between system failures.
- Tables in Figure 5 and Figure 6 plot availability versus restoration rates, indicating system availability decreases beyond the warranty period but improves with higher restoration rates. However, extending the warranty period leads to a decrease in system availability.
- Tables in Figure 8 and Figure 9 show the relationship between profit and other parameters. Profit rises with an increase in restoration rates, showing the financial benefit of maintaining higher restoration efficiency. Conversely, profit decreases as failure rates increase, reflecting the cost implications of more frequent failures.

These findings underscore the significant impact of the restoration rates of similar cooling/heating units of the system on the performance of the HVAC system. Within the warranty period, recurring restorations not only enhance the MTSF of the system but also improve availability and profitability. Beyond the warranty period, frequent restoration rates continue to enhance availability and profit, though the effect is comparatively less pronounced.

8. Conclusion

According to the results from the analysis of the HVAC system with two similar cooling/heating units and one dissimilar ventilation unit, the following suggestions can be made to improve system reliability:

1. Focus on continuing restorations within the warranty period to maximize the system's operational life. This approach will help extend the MTSF, improve system availability, and enhance overall profitability. Timely restorations can reduce the likelihood of costly breakdowns.

2. Introduction to systematic inspections and preventive maintenance routines to identify and address major issues before they lead to failures. Focus on proactive strategies can reduce restoration rates, enhance the longevity of components, and optimize system reliability. This preventive approach can decrease downtime and improve the efficiency of the HVAC system.

3. Use data from the observed trends to adjust warranty periods strategically. Extending warranty periods can positively affect system performance by improving MTSF, availability, and profitability, as indicated in the tables. Longer warranties can also enhance customer satisfaction and trust, leading to potentially higher system sales and retention.

These strategies will ensure the HVAC system runs efficiently, minimizes downtime, and maximizes profitability while considering long-term performance.

9. Future Scope

The future research of this study on reliability analysis for HVAC systems under warranty condition offers several promising directions for deeper exploration and refinement:

1. Employing techniques like Monte Carlo simulations can provide more robust insights into the uncertainties involved, helping to predict long-term system reliability under varying conditions.

2. By incorporating a cost analysis that evaluates not only restoration rates but also other cost factors such as installation, operation, and potential revenue generation. This holistic approach would allow for a better understanding of the total economic impact of various maintenance strategies, providing a clearer picture of cost-efficiency.

3. The model can be broadened to incorporate a variety of warranty periods across different units. By exploring systems with both shorter and longer warranties, one can gain a more comprehensive understanding of how warranty duration impacts system performance and reliability. This will enable the development of more personalized maintenance and warranty management strategies that align with the unique needs of each system, offering better ways to optimize uptime, restoration costs, and overall performance.

These potential future directions will enhance the reliability and profitability analysis of HVAC systems and also provide valuable insights for decision-makers in maintenance, warranty management, and cost optimization.

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