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$H^{p}_{\gamma,\omega}(I)$ Space a New Contribution to Weighted Space Theory and Smoothness Analysis

Safa Turki Atiyah and Nada Zuhair Abd Al-Sada

ABSTRACT: In this survey, we will consideration a new weighted norm space, a style of Sobolev weighted space, which we will symbolize as $H^{p}_{\gamma,\omega}(I)$, wherever I=[a,b]. The functions of this space are functions that reign ordinary conventional derivatives of order m, and they are integrable, since every function $\mathfrak{f}$ in the norm space $H^{p}_{\gamma,\omega}(I)$ is a product of a weight function ω. We also debate in this research the substantial properties of this space that recognize it from other space, comprehensives smoothness, integrability, and ordinary conventional differentility. Likewise, this research infer that the weighted norm space $H^{p}_{\gamma,\omega}(I)$ is a complete (Banach) space under the weighted norm $L_{\omega,p}(I)$. Additionally, a relationship was assured between the weighted norm space $H^{p}_{\gamma,\omega}(I)$ and the weighted Sobolev space $W^{k,p}(I)$. This demonstrates to the reader that the weighted makeup using the weight function ω affects the convergence and approximation feature. All these results demonstrate the paramount and material turn of space in mathematical anatomy , numerical anatomy, and differential equations

Keywords: Sobolev space, weighted function, weighted normed space, Banach space.

Contents

1 Introduction **1**

2 Definitions and Example **2**
2.1 Definition "The space $H^{p}_{\gamma,\omega}(I)$" . . . 2
2.2 Definition "Convergence in $H^{p}_{\gamma,\omega}(I)$" . . . 3
2.3 Definition "The Cauchy sequence in the space $H^{p}_{\gamma,\omega}(I)$" . . . 3
2.4 Example . . . 3

3 Properties of the Space $H^{p}_{\gamma,\omega}(I)$ **4**
3.1 Smooth Functions . . . 4
3.2 Property of Integration . . . 4
3.3 Continuity and Smoothness Property . . . 5
3.4 The Norm in the Space $H^{p}_{\gamma,\omega}(I)$ . . . 5
3.5 Methods of Integration . . . 5

4 Relationship Between the Space $H^{p}_{\gamma,\omega}(I)$ and the Sobolev Space $W^{k,p}(\Omega)$ **5**

5 Main Results **5**

1. Introduction

The Sobolev space is behold one of the substantial spaces that plays a significant role in the study of many subjects, inclusive differential equations, numerical analysis, and mathematics. Functions belonging to this space stock ordinary, classical derivatives that can be manipulated and are readily available. These derivatives can be continuous (smooth) to a confirmed order, ensuring the widespread use of this space in analysis.

In this context, the Sobolev space $W^{k,p}(I)$, is widely used to demonstrate that its function with weak derivatives to a certain order belong to the Lebesgue space $L_{\omega,p}(I)$, [1].

This helps in finding solutions for dealing with discontinuous function, and this has been proven in many implementation. This advantage of Sobolev function has given them significance as a tool for comprehension the monarchy of smoothness and approximation of functions.

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$$W^{k,p}(I) = \{u \in L^p(I) : D^\alpha u \in L^p(I), \forall \alpha \text{ with } |\alpha| \le k\}$$

The norm in a Sobolev space is defined as follows:

$$\|\mathfrak{f}\|_{W^{k,p}(I)} = \left(\int_I |\mathfrak{f}(x)|^p \, dx + \sum_{|\alpha| \le k} \int_I |D^\alpha \mathfrak{f}(x)|^p \, dx \right)^{\frac{1}{p}}$$

Where $D^\alpha \mathfrak{f}(x)$, is the weak derivative of the function of order α α is the multi-index of differentiation [2,6].

Let W be the set of all weighted functions defined by $W : I \to R^+, I = [-d, d]$. The set W is positive and decreasing function such that all $\mathfrak{w} \neq 0$, where $\mathfrak{w} \in W$. The weighted function $\mathfrak{w} \in W$, in our work accelerates the approximate process.

Let $X = L_{\mathfrak{w},p}(I)$ be a vector space, the norm on X is a nonnegative function $\mathfrak{f}$ such that $\mathfrak{f} \ge 0$, and $\| \cdot \|_{L_{\mathfrak{w},p}(I)} : X \to R$, which is defined by the form [7,9,11].

$$\|\mathfrak{f}\|_{L_{\mathfrak{w},p}(I)} = \left(\int_I |\mathfrak{f}(x)\, \mathfrak{w}(x)|^p \, dx \right)^{\frac{1}{p}}, \forall \mathfrak{f} \in X, \mathfrak{w} \in W, x \in I$$

The weighted Sobolev space $W^{k,p}(\Omega, \mathfrak{w})$ is defined for an open set $\Omega \in \mathbb{R}^n$, a weight $\mathfrak{w}(x) > 0$, and $1 \le p < \infty$. It consists of all functions $\mathfrak{f} \in L_{p,\mathfrak{w}}(\Omega)$ whose weak derivatives of all orders $|\alpha| \le k$ belong to $L_{p,\mathfrak{w}}(\Omega)$ [4,3]. Its norm is

$$\|\mathfrak{f}\|_{W^{k,p}(\Omega,\mathfrak{w})} = \left(\int_\Omega |\mathfrak{f}(x)\, \mathfrak{w}(x)|^p \, dx + \sum_{|\gamma| \le k} \int_\Omega |D^\gamma (\mathfrak{f}(x)\, \mathfrak{w}(x))|^p \, dx \right)^{\frac{1}{p}}$$

The symmetric difference operator of order m is given by:

$$\Delta^{(m)}_{\frac{\varphi h}{m}}(\mathfrak{f}, x)_w = \sum_{i=0}^{m} (-1)^{m-i} \binom{m}{i} \mathfrak{f}\left(x + \frac{\varphi(x)\, hi}{m}\right) \ \mathfrak{w}\left(x + \frac{\varphi(x)\, h}{m}\right)$$

This estimating approximation errors in functional spaces [8]. The work can also be referred to in research [5,10] to connect it with the statement.

2. Definitions and Example

In this section, we present several definitions related to the space $H^p_{\gamma,\mathfrak{w}}(I)$, along with an illustrative example.

2.1. Definition "The space $H^p_{\gamma,\mathfrak{w}}(I)$"

The space $H^p_{\gamma,\mathfrak{w}}(I)$ is defined as a space of functions that are classically smooth up to order m, meaning that the functions in this space possess classical derivatives that exist and are continuous up to order m. This space includes a collection of functions that satisfy the following conditions:

i. The norm in the space $L_{p,\mathfrak{w}}(I)$:

The norm $L_{p,\mathfrak{w}}(I)$ in this space is computed using the $L_{p,\mathfrak{w}}(I)$ as follows:

$$\|\mathfrak{f}\|_{L_{\mathfrak{w},p}(I)} = \left(\int_I |\mathfrak{f}(x)\, \mathfrak{w}(x)|^p \, dx \right)^{\frac{1}{p}}$$

ii. Derivatives of order γ up to order m:The norm is also computed using the derivatives up to order m of the function $\mathfrak{f}$ and these are included in the sum:

$$\sum_{|\gamma| \le m} \int_I |D^\gamma (\mathfrak{f}(x)\, \mathfrak{w}(x))|^p \, dx$$

iii. Holder's condition: The functions in this space satisfy a Holder condition of degree α, which means that the absolute differences of the function (including its derivatives) are smooth of order m and satisfy the Holder condition.

The Subspace of Sobolev Space:

The space $H^p_{\gamma,\mathfrak{w}}(I)$ is a subspace of the Sobolev space. In this context, the full norm of the function $\mathfrak{f}$ in the space $H^p_{\gamma,\mathfrak{w}}(I)$ is defined as:

$$\|\mathfrak{f}\|_{H^p_{\gamma,\mathfrak{w}}(I)} = \left(\int_I |\mathfrak{f}(x)\,\mathfrak{w}(x)|^p\,dx + \sum_{|\gamma|\le m}\int_I |D^\gamma(\mathfrak{f}(x)\,\mathfrak{w}(x))|^p\,dx\right)^{\frac{1}{p}} \tag{2.1}$$

And so it is

$$H^p_{\gamma,\mathfrak{w}}(I) = \left\{\mathfrak{f}\colon \|\mathfrak{f}\|_{H^p_{\gamma,\mathfrak{w}}(I)} < \infty, 1 \le p < \infty, I = [a,b]\right\}$$

2.2. Definition "Convergence in $H^p_{\gamma,\mathfrak{w}}(I)$"

Let $\{\mathfrak{f}\}_{n\in N}$ be a sequence of a functions in $H^p_{\gamma,\mathfrak{w}}(I)$,it is said that $\{\mathfrak{f}_n\}_{n\in N}$ converges to $\mathfrak{f} \in H^p_{\gamma,\mathfrak{w}}(I)$ at the nodes $a_i{}^s_{i=1}$ if

$$\|\mathfrak{f}_n - \mathfrak{f}\|_{H^p_{\gamma,\mathfrak{w}}(I)} \to 0 \text{ as } n \to \infty$$

In an equivalent form, convergence in the space $H^p_{\gamma,\mathfrak{w}}(I)$ means that the sequence $\{\mathfrak{f}_n\}_{n\in N}$ converges to the function $\mathfrak{f}$ if the following condition holds:

i. $\int_I |(\mathfrak{f}_n(x) - \mathfrak{f}(x))\,\mathfrak{w}(x)|^p dx \to 0$

ii. $\int_I |D^\gamma(\mathfrak{f}_n(x)\,\mathfrak{w}(x)) - D^\gamma(\mathfrak{f}(x)\,\mathfrak{w}(x))|^p\,dx \to 0, \forall\,|\gamma| \le m$

2.3. Definition "The Cauchy sequence in the space $H^p_{\gamma,\mathfrak{w}}(I)$"

Let $\{\mathfrak{f}_n\}_{n\in N}$ be a sequence in the space $H^p_{\gamma,\mathfrak{w}}(I)$. The sequence $\{\mathfrak{f}_n\}_{n\in N} \in H^p_{\gamma,\mathfrak{w}}(I)$ is said to be a Cauchy sequence if the following holds:

$$\forall\epsilon > 0, \exists n, m \in N \ni \|\mathfrak{f}_n - \mathfrak{f}_m\|_{H^p_{\gamma,\mathfrak{w}}(I)} <\in$$

In an equivalent form, the sequence $\{\mathfrak{f}_n\}_{n\in N}$ is Cauchy if

i. $\int_I |(\mathfrak{f}_n(x) - \mathfrak{f}_m(x))\,\mathfrak{w}(x)|^p dx \to 0.$

ii. $\int_I |D^\gamma(\mathfrak{f}_n(x)\,\mathfrak{w}(x)) - D^\gamma(\mathfrak{f}_m(x)\,\mathfrak{w}(x))|^p\,dx \to 0, \forall\,|\gamma| \le m$

2.4. Example

Let

1. $\mathfrak{f}(x) = \sin x, \mathfrak{w}(x) = e^{-x^2}$.

2. $I = [-2, 2], m \ge 2, h = 0.1, p = 2, \gamma = 2$

Table 1: The numerical variation in the $L_{p,w}$ norm and the combined norm across different values of xreflects the sensitivity of the norm at points with greater variation.

x	$L_{p,w}$ Norm	Sum of $L_{p,w}$ Norms	Combined Norm
-2.0000	0.1391	0.2458	0.6204
-1.8000	0.3681	0.1254	0.7025
-1.6000	0.4042	0.2170	0.7882
⋮	⋮	⋮	⋮
1.6000	0.1165	0.4225	0.7342
1.8000	0.3980	0.4321	0.9111
2.0000	0.2438	0.2313	0.6893

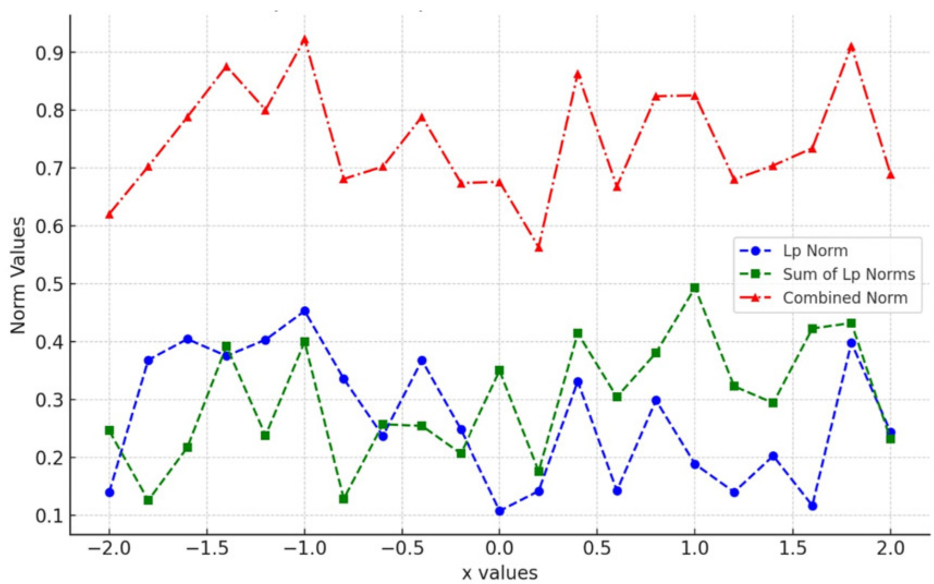

Figure 1: The values of the $L_{p,w}$ norm,the sum of norms,and the comined norm across the points of x reveals the response of these norms to the local variations of the function, with higher values appearing in regions exhibiting greater changes.

3. Properties of the Space $H^p_{\gamma,w}(I)$

The space $H^p_{\gamma,w}(I)$ is characterized by several important properties that make it useful in many mathematical applications.

3.1. Smooth Functions

Functions in an $H^p_{\gamma,w}(I)$ -space are classically smooth up to order m. This means that classical derivatives of functions in an $H^p_{\gamma,w}(I)$ -space exist and are continuous up to order m. Classical derivatives are derivatives that can be conventionally calculated using differential calculus.

3.2. Property of Integration

The functions of this space have ordinary traditional derivatives of order m, for that object the integral of these functions can be establish confer to the known and immediate process for functions and their derivatives .

3.3. Continuity and Smoothness Property

Functions pertinence to space $H^p_{\gamma,\mathfrak{w}}(I)$ exhibit smoothness , which facilitates the use of traditional differentiability . This smoothness also aids in finding derivatives and integrals within the space.

As along as, functions in space $H^p_{\gamma,\mathfrak{w}}(I)$ have steady attitude and do not undergo changes that could hinder analytical calculations of the functions.

3.4. The Norm in the Space $H^p_{\gamma,\mathfrak{w}}(I)$

The definition of space $H^p_{\gamma,\mathfrak{w}}(I)$ expose a balanced grouping of functions and their derivatives up to order.

This definition shows the function, its magnitude ,and its derivatives, and it contributes to demonstrating the degree of regularity, which is shown in the following form:

$$\|\mathfrak{f}\|_{H^p_{\gamma,\mathfrak{w}}(I)} = \left(\int_I |\mathfrak{f}\,(x)\,\mathfrak{w}(x)|^p\,dx + \sum \int_I |D^\gamma \mathfrak{f}\,(x)\,\mathfrak{w}(x)|^p\,dx\right)^{\frac{1}{p}}$$

3.5. Methods of Integration

From the integral construction of space $H^p_{\gamma,\mathfrak{w}}(I)$, standard calculus methods can be used. In individual ,from the standard definition of space $H^p_{\gamma,\mathfrak{w}}(I)$, the calculation of the derivative of a function to order m with its integration is shown.

$$\|\mathfrak{f}\|_{H^p_{\gamma,\mathfrak{w}}(I)} = \left(\int_I |\mathfrak{f}\,(x)\,\mathfrak{w}(x)|^p\,dx + \sum_{|\gamma|\le m} \int_I |D^\gamma \mathfrak{f}\,(x)\,\mathfrak{w}(x)|^p dx\right)^{\frac{1}{p}}$$

Anywhere:

- The formula: $\int_I |\mathfrak{f}(x)\mathfrak{w}(x)|^p\,dx$, display the integration of the function $\mathfrak{f}$ manifold by the weight function $\mathfrak{w}$.
- The formula : $\int_I |D^\gamma\mathfrak{f}(x)\mathfrak{w}(x)|^p\,dx$, display the integration of derivative of the function $\mathfrak{f}$ manifold by the weight function $\mathfrak{w}$ to order m.

4. Relationship Between the Space $H^p_{\gamma,\mathfrak{w}}(I)$ and the Sobolev Space $W^{k,p}(\Omega)$

1. The space $H^p_{\gamma,\mathfrak{w}}(I)$ is a subset of the Sobolev space $W^{k,p}(\Omega)$ when the functions in $H^p_{\gamma,\mathfrak{w}}(I)$ are classically smooth up to order m.
2. In the Sobolev space $W^{k,p}(\Omega)$, weak derivatives are used; however, in the space $H^p_{\gamma,\mathfrak{w}}(I)$, where classical derivatives exist, integrals and derivatives are computed using ordinary derivatives.

Since functions f in the space $H^p_{\gamma,\mathfrak{w}}(I)$ are continuous and possess classical derivatives, they naturally belong to the Sobolev space $W^{k,p}(\Omega)$. In Sobolev space, weak derivatives are computed, but the functions in $H^p_{\gamma,\mathfrak{w}}(I)$ can use the classical derivatives.

5. Main Results

In this section, we demonstrate how the norm of the function $\mathfrak{f} \in H^p_{\gamma,\mathfrak{w}}(I)$, Is expressed, as well as the closure property of the space $H^p_{\gamma,\mathfrak{w}}(I)$, which confirms that it is a Banach space.

Lemma 5.1 *If the function $\mathfrak{f} \in H^p_{\gamma,\mathfrak{w}}(I)$ then the norm of the function in this space can be written as follows:*

$$\|\mathfrak{f}\|_{H^p_{\gamma,\mathfrak{w}}(I)} = \left(\|\mathfrak{f}\|^p_{L_{\mathfrak{w},p}(I)} + o\left(|h|^{\alpha p}\right)\right)^{\frac{1}{p}}$$

Proof: Using the relation

$$\Delta^m_{\frac{\varphi h}{m}}(\mathfrak{f},x)_{p,\mathfrak{w}} \;=\; D^\gamma(\mathfrak{f}(x)\,\mathfrak{w}(x))+o(h^m)$$

$$\|\mathfrak{f}\|^p_{H^p_{\gamma,\mathfrak{w}}(I)} \;=\int_I |\mathfrak{f}(x)\,\mathfrak{w}(x)|^p\,dx+\sum_{\gamma\le m}\int_I \left|\Delta^m_{\frac{\varphi h}{m}}(\mathfrak{f},x)_w-o(h^m)\right|^p dx$$

$$\begin{aligned}
\sum_{\gamma\le m}\int_I \left|A^m_{\frac{\varphi h}{m}}(\mathfrak{f},x)_w-o(h^m)\right|^p dx \le \sum_{\gamma} &\le m\left(\int_I\left|\Delta^m_{\frac{\varphi h}{m}}(f,x)_w\right|^p dx+\int_I |o(h^m)|^p\,dx)\right)\\
&=\sum_{\gamma\le m}\left(\int_I\left|\Delta^m_{\frac{\varphi h}{m}}(\mathfrak{f},x)_w\right|^p dx+\int_I c\,|h|^{mp}\,dx\right)\\
&\le \sum_{\gamma\le m}\int_I c\,|h|^{\alpha p}\,dx+\sum_{\gamma\le m}\int_I c\,|h|^{mp}\,dx\\
&=\sum_{\gamma\le m} c\,|I|\,|h|^{\alpha p}\,dx+\sum_{\gamma\le m} c\,|I|\,|h|^{mp}\,dx\\
&\le \sum_{\gamma\le m} c\,|I|\,|h|^{\alpha p}\,dx+\sum_{\gamma\le m} c\,|I|\,|h|^{\alpha p}\,dx\\
&=\sum_{\gamma\le m} 2c\,|I|\,|h|^{\alpha p}\,dx=o\left(|h|^{\alpha p}\right)
\end{aligned}$$

therefore

$$\|\mathfrak{f}\|^p_{H^p_{\gamma,\mathfrak{w}}(I)} \;=\int_I |\mathfrak{f}(x)\,\mathfrak{w}(x)|^p\,dx+\; o\left(|h|^{\alpha p}\right)$$

Hence

$$\|\mathfrak{f}\|_{H^p_{\gamma,\mathfrak{w}}(I)}=\left(\|\mathfrak{f}\|^p_{L_{\mathfrak{w},p}}+o\left(|h|^{\alpha p}\right)\right)^{\frac{1}{p}} \tag{5.1}$$

□

Note:

1. **Formula (1)**

   The full norm in the space $H^p_{\gamma,\mathfrak{w}}(I)$ is shown, which includes the norm in the space $L_{p,\mathfrak{w}}$ in addition to the sum of the derivatives from degree γ up to order m.

   This formula represents the complete form of the norm that takes into account:

   (a) The norm in the space $L_{p,\mathfrak{w}}$.

   (b) The various derivatives of the function $\mathfrak{f}$ up to order m, which are integrated into the sum in the formula.

2. **Formula (2)**

   This is an approximation for the second term, which includes the derivatives in the first formula. Here, the term $o\left(|h|^{\alpha p}\right)$ is introduced, which shows the convergence of the differences when h is very small. This means the effect of the derivatives becomes very small when h decreases, and thus, it can be ignored for small computations.

Theorem 5.1 *Let $\mathfrak{f}\in H^p_{\gamma,\mathfrak{w}}(I)$, $I=[a,b]$. Then $H^p_{\gamma,\mathfrak{w}}(I)$ is a closed subspace of the weighted sobolev space $W^{k,p}(I)$, consequently $H^p_{\gamma,\mathfrak{w}}(I)$ is a Banach space (i.e complete).*

Proof: Let $\{\mathfrak{f}_n\}_{n\in N}$ be a sequence in the space $H^p_{\gamma,\mathfrak{w}}(I)$ and assume that

$$\mathfrak{f}_n\to\mathfrak{f}$$

in the weighted Sobolev space $W^{m,p}(I)$, this meaning of convergence in the weighted Sobolev space

$$\|\mathfrak{f}_n - \mathfrak{f}\|_{W^{m,p}(I)} \to 0$$

$$\|\mathfrak{f}_n - \mathfrak{f}\|^p{}_{W^{m,p}(I)} = \int_I |(\mathfrak{f}_n(x) - \mathfrak{f}(x))\mathfrak{w}(x)|^p\,dx + \sum_{|\gamma|\le m}\int_I |D^\gamma(\mathfrak{f}_n(x) - \mathfrak{f}(x))\mathfrak{w}(x))|^p\,dx, m \le k$$

From this we obtain that, for every, $|\gamma| \le m$

$$\int_I |(\mathfrak{f}_n(x) - \mathfrak{f}_m(x))\,\mathfrak{w}(x)|^p\,dx \to 0$$

and

$$\sum_{|\gamma|\le m}\int_I |D^\gamma(\mathfrak{f}_n(x) - \mathfrak{f}(x))\mathfrak{w}(x))|^p\,dx \to 0 \ \text{ in } L_{p,\mathfrak{w}}(I).$$

Since the space $L_{p,\mathfrak{w}}(I)$ is a complete space (Banach space), the limits of convergent sequences remain inside it; hence we get

$$\mathfrak{f} \in L_{p,\mathfrak{w}}(I), D^\gamma(\mathfrak{f}(x)\,\mathfrak{w}(x)) \in \ L_{p,\mathfrak{w}}(I)\,\forall\ |\gamma| \le m.$$

Therefore $\mathfrak{f} \in H^p_{\gamma,\mathfrak{w}}(I)$. This means that the limits of convergent sequences in the space $H^p_{\gamma,\mathfrak{w}}(I)$ (under the Sobolev norm $W^{k,p}(I)$) remain inside the space itself. This proves that is closed in $W^{k,p}(I)$.

Since the weighted Sobolev space $W^{k,p}(I)$ is a Banach space, every closed subspace of it is also a Banach space, and hence $H^p_{\gamma,\mathfrak{w}}(I)$ is a Banach space. □

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Safa Turki Atiyah,
Department of Mathematics,
College of Education, University of Al-Qadisiyah,
Iraq.
E-mail address: edu.math.post24.15@qu.edu.iq

and

Nada Zuhair Abd AL-Sada,
Department of Mathematics,
College of Education, University of Al-Qadisiyah,
Iraq.
E-mail address: Nada.abdalsada@qu.edu.iq