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# An Integrated Pythagorean Fuzzy TOPSIS Framework for Supplier Selection with Uncertain Attribute Weights

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ABSTRACT: The significance of attribute weights is paramount in supplier selection, a classic multi-criteria decision-making (MCDM) problem. Conventional MCDM models typically treat these weights as predetermined, which does not reflect real-world ambiguity. To bridge this gap, our work incorporates scenarios where weights are known, entirely unknown, or partially specified as intervals. We introduce a novel framework operating within a Pythagorean fuzzy environment, which provides a more powerful and less restrictive way to capture vagueness. Within this framework, we apply the TOPSIS technique adapted for Pythagorean fuzzy sets, utilizing spherical distance to calculate each supplier's proximity to the ideal solution. The proposed model's applicability and strength are demonstrated through a numerical example involving five potential suppliers.

Keywords: Pythagorean fuzzy sets, exponential score function, spherical distance measurement, TOPSIS method, unknown weight, partial known weight.

## Contents



## 1. Introduction

In the era of globalized supply chains and heightened competitive pressure, the selection of optimal suppliers has evolved from a simple cost-based procurement activity to a complex strategic decision that directly impacts a firm's profitability, risk exposure, and sustainability footprint. An effective supplier selection decision enhances product quality, ensures timely delivery, fosters innovation, and mitigates operational and reputational risks. This selection process is inherently a Multi-Criteria Decision-Making (MCDM) problem, involving both quantitative and qualitative factors such as cost, quality, service, and environmental compliance. However, decision-makers (DMs) often provide evaluations laden with

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imprecision, vagueness, and personal hesitancy, especially for subjective criteria. Traditional crisp number approaches fail to capture this inherent uncertainty.

To address uncertainty in practical problems, Zadeh [22] introduced the concept of fuzzy sets (FS). Building on this idea, several extensions of fuzzy sets have been developed to better capture vagueness and imprecision. One such extension is the intuitionistic fuzzy set (IFS), proposed by Atanassov [5], which incorporates both membership and nonmembership degrees for each element.

Yager [20] introduced a Pythagorean fuzzy set (PFS). Yager overcomes the situation when sum of membership degree and nonmembership degree greater than 1. PFS is an extension of IFS with the condition that the square sum of the membership degree and the nonmembership degree is less than or equal to 1. The concept of Pythagorean fuzzy sets (PFS) gives the larger preference domain for decision makers (DM). DMs can define their support and against the degree of membership as $\alpha = 0.7$, $\beta(x) = 0.5$. In this case, $0.7 + 0.5 > 1$ is not valid in IFS but squaring $0.7^2 + 0.5^2 < 1$ implies the Pythagorean fuzzy set is more suitable than the intuitionistic fuzzy set.

Adak et al., [1,2] investigated loan prediction under pythagorean fuzzy environment. Some problem on assignment management presented by Adak et. al. [3]. Dai et. al., [9] proposed an agricultural product supplier selection algorithm based on the Pythagorean fuzzy power Bonferroni mean operator under Pythagorean fuzzy environment. Khan et. al., [13] introduced a novel Nonlinear Programming (NLP) approach that utilizes the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method to identify the most suitable green supplier within cubic Pythagorean fuzzy (CPF) environments. Shahab et. al., [18] focused on development of soft-max based fuzzy aggregation operators (AOs) for Pythagorean fuzzy sets (PyFS), capitalizing on the benefits provided by the soft-max function. Çalık et. al., [8] discussed a fuzzy multi-objective linear programming (FMOLP) model in the context of I4.0 enablers and resilience strategies to trade-off between resilience, I4.0, and operations cost for the supplier selection process. Bisht et. al., [6] developed a new Trigonometric Operational Laws, a weight determination method, and a novel score function for group decision-making (GDM) problems in the HPF environment. Liu et. al., [15] established suitable suppliers in the Sustainable circular supplier selection (SCSS) and introduced an extended framework using the evaluation based on distance from average solution (EDAS) with PFSs and implemented it to solve the SCSS in the manufacturing sector. Boltürk et. al., [7] developed the Pythagorean fuzzy extension of CODAS method. Dehshiri et al. [10] studied on sustainable supplier selection based on a comparative decision-making approach. Gocer [11] developed a state-of-the-art MCDM method for sustainable supplier selection by integrating AHP and TOPSIS techniques within the Pythagorean Fuzzy Sets (PFSs) linguistic setting. Kolour et al. [14] proposed a hybrid MCDM approach for supplier selection in public manufacturing. Lo et al. [16] presented a multi-year performance evaluation and trend tracking of sustainable suppliers model. Tufan et al. [19] investigated on supplier selection in the food sector using LODECI and CORASO methods. Zhou et. al., [25] proposed an integrated approach for green supplier selection under Pythagorean fuzzy scenarios.

In response to this gap, this article proposes an integrated MCDM framework for sustainable supplier selection within a Pythagorean fuzzy context. The primary contributions of this work are threefold:
(1) It develops a comprehensive set of criteria encompassing economic, operational, green, and social dimensions for holistic supplier evaluation.
(2) It integrates PF-TOPSIS for ranking within a unified Pythagorean fuzzy framework, effectively handling expert hesitancy.
(3) the weight of the attributes are taken as known, completely unknown and partial known i.e., weights of the attributes are taken in an interval.
(4) It provides a practical, step-by-step case study to demonstrate the viability and stability of the proposed approach.

Following this introduction, the paper proceeds in six sections. First, Section 2 establishes the foundational concepts of Pythagorean Fuzzy Sets (PFS) and surveys the existing literature. Building on this, Section 3 develops novel spherical and normalized spherical distance measures for Pythagorean fuzzy numbers. These measures are then integrated into the methodology presented in Section 4. The practical application of this methodology is illustrated through a supplier selection case study in Section 5, validating its accuracy and effectiveness. Subsequently, Section 6 analyzes the results and presents a comparative discussion of three distinct scenarios. The paper concludes in Section 7 by summarizing the findings, acknowledging limitations, and outlining potential avenues for future research.

## 2. Theoretical Aspects

The fundamental concepts of intuitionistic and Pythagorean fuzzy sets are reviewed here. Along with that, we cover some score function of Pythagorean fuzzy numbers that are required for this article.

**Definition 2.1** *[5] An intuitionistic fuzzy set (IFS) $I$ in $X$ is of the form*

$$I = \{\langle \varsigma, \varphi_I(\varsigma), \varrho_I(\varsigma)\rangle : \varsigma \in X\},$$

*where $\varphi_I : X \to [0,1]$, $\varrho_I : X \to [0,1]$ are membership grade (MG) and non-membership grade (NMG) with $0 \le \varphi_I(\varsigma) + \varrho_I(\varsigma) \le 1$, for all $\varsigma \in X$.*

*Indeterminacy $\pi_I(\varsigma) = 1 - \varphi_I(\varsigma) - \varrho_I(\varsigma)$.*

**Definition 2.2** *[21] A pythagorean fuzzy set $P$ is*

$$P = \{\langle \varsigma, \varphi_P(\varsigma), \varrho_P(\varsigma)\rangle | \varsigma \in X\},$$

*where $\varphi_P(\varsigma) : X \to [0,1]$ and $\varrho_P(\varsigma) : X \to [0,1]$ are MG and NMG respectively with $0 \le (\varphi_P(\varsigma))^2 + (\varrho_P(\varsigma))^2 \le 1$.*

*The indeterminacy is $\varpi_P(\varsigma) = \sqrt{1 - (\varphi_P(\varsigma))^2 - (\varrho_P(\varsigma))^2}$. The order pair $(\varphi, \varrho)$ denoted as pythagorean fuzzy number (PFN).*

### 2.1. Some Operations on PFNs

Consider three PFNs $p = \langle \varphi, \varrho\rangle$, $p_1 = \langle \varphi_1, \varrho_1\rangle$ and $p_2 = \langle \varphi_2, \varrho_2\rangle$. Then

(i) $\bar{p} = \langle \varrho, \varphi\rangle$

(ii) $p_1 \cup p_2 = \langle \max\{\varphi_1, \varphi_2\}, \min\{\varrho_1, \varrho_2\}\rangle$

(iii) $p_1 \cap p_2 = \langle \min\{\varphi_1, \varphi_2\}, \max\{\varrho_1, \varrho_2\}\rangle$

Different types of functions based on MG and NMG are used to rank PFNs (PFNs). This is particularly important in decision-making applications, where ranking alternatives plays a critical role.

**Definition 2.3** *The score function $s(p)$ of $p = \langle \varphi, \varrho\rangle$ is*

$$s(p) = (\varphi)^2 - (\varrho)^2 \tag{2.1}$$

*where $s(p) \in [-1, 1]$.*

**Definition 2.4** *The accuracy function $a(p)$ of $p = \langle \varphi, \varrho\rangle$ is*

$$a(p) = (\varphi)^2 + (\varrho)^2 \tag{2.2}$$

*where $h(p) \in [0, 1]$.*

**Definition 2.5** *[17] Let $p = \langle \alpha, \beta\rangle$ be a pythagorean fuzzy number. The exponential score function $S_{pd}$ of $p$ is defined as*

$$S_{pd}(p) = \frac{e^{\alpha^2 - \beta^2}}{\pi^2 + 1}. \tag{2.3}$$

## 3. Distance Measurement Method for PFNs

In this section, we introduce spherical, normalized spherical and weighted spherical distance measurement method for PFSs and PFNs. These distance measure are used in proposed integrated TOPSIS method.

### 3.1. Spherical Distance Measurement Method for PFNs

Assume that $p = \langle\varphi, \varrho\rangle$ be a PFN that satisfies the condition $0 \le \varphi^2 + \varrho^2 \le 1$ and hesitancy grade $\varpi = \sqrt{1-\varphi^2-\varrho^2}$ i.e., $\varphi^2 + \varrho^2 + \varpi^2 = 1$.

Consider $(\varphi, \varrho, \varpi)$ reside on spherical surface of unit-radius with origin as its center. According to this concept, spherical distance between two PFNs on confined spherical surface can be defined as follows.

**Definition 3.1** *[1] The spherical distance between two locations $A$ and $C$ on a spherical surface, where $A$ is located at $(x_1, y_1, z_1)$ and $C$ is located at $(x_2, y_2, z_2)$ on the same surface is*

$$D_{SP}(A,C) = \arccos\left\{1 - \frac{1}{2}\left[(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2\right]\right\} \tag{3.1}$$

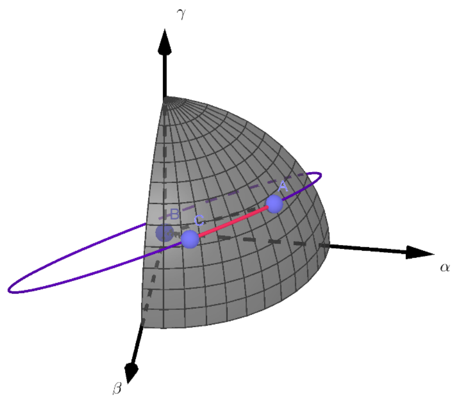


Figure 1: Spherical distance

Incorporating this expression, spherical distance between two PFNs is as follows:

**Definition 3.2** *[1] Consider $p_1 = \langle\varphi_1, \varrho_1\rangle$ & $p_2 = \langle\varphi_2, \varrho_2\rangle$ represent two PFNs, each associated with hesitation functions $\varpi_1$ and $\varpi_2$, respectively. The spherical distance calculated as*

$$D_{SP}(p_1,p_2) = \frac{2}{\pi}\arccos\left\{1 - \frac{1}{2}[(\varphi_1-\varphi_2)^2 + (\varrho_1-\varrho_2)^2 + (\varpi_1-\varpi_2)^2]\right\} \tag{3.2}$$

*The factor $\frac{2}{\pi}$ is introduced to obtain the distance value within the range $[0,1]$.*
*Given that $\varphi_1^2+\varrho_1^2+\varpi_1^2 = 1$ and $\varphi_2^2+\varrho_2^2+\varpi_2^2 = 1$, the simplification of equation (3.2) yields the following results.*

$$D_{SP}(p_1,p_2) = \frac{2}{\pi}\arccos\left[\varphi_1\varphi_2 + \varrho_1\varrho_2 + \varpi_1\varpi_2\right] \tag{3.3}$$

**Definition 3.3** *[1] Let $P = \{\varsigma_i, \langle\varphi_P(\varsigma_i), \varrho_P(\varsigma_i)\rangle : \varsigma_i \in X\}$ and $Q = \{\varsigma_i, \langle\varphi_Q(\varsigma_i), \varrho_Q(\varsigma_i)\rangle : \varsigma_i \in X\}$ within the universe of discourse $X = \{\varsigma_1, \varsigma_2, \ldots, \varsigma_n\}$. The spherical and normalized spherical distances between these sets are as bellows:*

**Spherical Distance:**

$$D_{SP}(P,Q) = \frac{2}{\pi}\sum_{i=1}^{n}\arccos\left[\varphi_P(\varsigma_i)\varphi_Q(\varsigma_i) + \varrho_P(\varsigma_i)\varrho_Q(\varsigma_i) + \varpi_P(\varsigma_i)\varpi_Q(\varsigma_i)\right] \tag{3.4}$$

*where $0 \le D_{SP}(P,Q) \le n$.*

**Normalized Spherical Distance:**

$$D_{NSP}(P,Q) = \frac{2}{n\pi}\sum_{i=1}^{n}\arccos\left[\varphi_P(\varsigma_i)\varphi_Q(\varsigma_i) + \varrho_P(\varsigma_i)\varrho_Q(\varsigma_i) + \varpi_P(\varsigma_i)\varpi_Q(\varsigma_i)\right] \tag{3.5}$$

*where* $0 \leq D_{NSP}(P,Q) \leq 1$.

**Definition 3.4** *Let* $P_1 = (\varphi_{1j}, \varrho_{1j})$, $P_2 = (\varphi_{2j}, \varrho_{2j})$, $j = 1,2,\ldots,n$, *be two sets of PFNs.* $w_j$ *is the weight of* $j-th$ *criteria, i.e.,* $w = (w_1, w_2, \ldots, w_n)^T$, *where* $0 \leq w_i \leq 1$ *and* $\sum\limits_{i=1}^{n} w_i = 1$.
*The weighted normalized spherical distance between* $P_1$ *and* $P_2$ *is defined in the form of*

$$D'_{NSP}(P_1, P_2) = \frac{2}{n\pi}\sum_{j=1}^{n} w_j \arccos\left[\varphi_{1j}\varphi_{2j} + \varrho_{1j}\varrho_{2j} + \varpi_{1j}\varpi_{2j}\right] \tag{3.6}$$

## 4. Methodology

This section presents a multi-criteria decision-making issue using information represented by PFNs. For each criterion and alternative, construct a decision matrix using PFSs. Each entry of matrix represents MG and NMG of each alternative concerning each criterion.

This study employs TOPSIS method to evaluate the applicability of MADM techniques in selecting the optimal supplier from a range of available alternatives, each characterised by multiple attributes.

Let $S = \{S_1, S_2, \ldots, S_m\}$, where $m \geq 2$ and $\Gamma = \{C_1, C_2, \ldots, C_n\}$, with $n \geq 2$, denote set of alternatives and criteria respectively.

Let the PFNs $\langle \varphi_{ij}, \varrho_{ij} \rangle$ represent the assessment value of i-th alternative and j-th criterion, such that $C_j(\varsigma_i) = \langle \varphi_{ij}, \varrho_{ij} \rangle$ and $R = (C_j(\varsigma_i))_{m \times n}$, where

$$R = \begin{bmatrix} \langle \varphi_{11}, \varrho_{11} \rangle & \langle \varphi_{12}, \varrho_{12} \rangle & \cdots & \langle \varphi_{1n}, \varrho_{1n} \rangle \\ \langle \varphi_{21}, \varrho_{21} \rangle & \langle \varphi_{22}, \varrho_{22} \rangle & \cdots & \langle \varphi_{2n}, \varrho_{2n} \rangle \\ \cdots & \cdots & \cdots & \cdots \\ \langle \varphi_{m1}, \varrho_{m1} \rangle & \langle \varphi_{m2}, \varrho_{m2} \rangle & \cdots & \langle \varphi_{mn}, \varrho_{mn} \rangle \end{bmatrix}.$$

### 4.1. TOPSIS Method

The fundamental concept of the TOPSIS method is that the optimal choice should exhibit the minimum distance from the positive ideal solution while simultaneously maximising the distance from the negative ideal solution.

This approach entails calculating the Pythagorean fuzzy positive ideal solution (PFPIS) and the Pythagorean fuzzy negative ideal solution (PFNIS). Classify $J_1$ and $J_2$ into benefit criteria and cost criteria categories. The determination of PFPIS and PFNIS was conducted utilizing a modified accuracy function $S_{pd}$ by using Equation (2.3). $\varsigma^+$ and $\varsigma^-$ represent PFPIS and PFNIS, respectively. They are computed utilizing subsequent formulas

$$\varsigma^+ = \begin{cases} \max(S(C_j(\varsigma_i)))|j \in J_1 \\ \min(S(C_j(\varsigma_i)))|j \in J_2 \end{cases} \tag{4.1}$$

$$\varsigma^- = \begin{cases} \min(S(C_j(\varsigma_i)))|j \in J_1 \\ \max(S(C_j(\varsigma_i)))|j \in J_2 \end{cases} \tag{4.2}$$

Subsequently, we compute $D_{NSP}$ from each alternative to PFPIS, $D_{NSP}(\varsigma_i, \varsigma^+)$, and to PFNIS, $D_{NSP}(\varsigma_i, \varsigma^-)$.

Let $w = (w_1, w_2, \ldots, w_n)$ be the weight of all attributes, where $0 \leq w_i \leq 1$, $i = 1,2,\ldots,n$ is weight of each attribute and $\sum\limits_{i=1}^{n} w_i = 1$. The attribute weights information is usually unknown or partially known due to the insufficient knowledge or limitation of time of decision makers in the decision making process. Therefore, the determination of attribute weights is an important issue in MCDM problems. In this paper, we will discuss two methods to determine the attribute weights for the unknown and partially known weight.

Case-I: Weight of the attribute known.

Case-II: Weight of the attribute unknown.
When the information about the attribute weights is completely unknown, we can use maximizing deviation method to derive the weights of attributes with the following formula

$$w_j = \frac{\sum_{i=1}^{n}\sum_{k=1}^{n}\mathcal{D}_{NSP}(\varsigma_{ij},\varsigma_{kj})}{\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{k=1}^{n}\mathcal{D}_{NSP}(\varsigma_{ij},\varsigma_{kj})} \tag{4.3}$$

where $\mathcal{D}_{NSP}(\varsigma_{ij},\varsigma_{kj})$ is the normalized spherical distance.
Case-III: Weight of the attribute partially known.
Utilize the principle of maximization of spherical distance measurement to get the attribute weight vector by considering the following programming:

$$\begin{aligned} \max S &= \sum_{j=1}^{n} w_j \mathcal{D}_{NSP}(\varsigma_{ij},\varsigma_j^+) \\ & \quad \alpha_j \le w_j \le \beta_j, j=1,2,\ldots,n \\ \text{s.t.} & \quad \sum_{j=1}^{n} w_j = 1. \end{aligned} \tag{4.4}$$

We now calculated weighted $D_{NSP}$ of alternative $\varsigma_i$ from PFPIS $\varsigma^+$ according to (3.6), which can be articulated as follows.

$$\begin{aligned} D_{NSP}(\varsigma_i,\varsigma^+) &= \sum_{j=1}^{n} D_{NSP}(C_j(\varsigma_i),C_j(\varsigma^+)) \\ &= \frac{2}{n\pi}\sum_{j=1}^{n} w_j \arccos(\varphi_{ij}\varphi_j^+ + \varrho_{ij}\varrho_j^+ + \varpi_{ij}\varpi_j^+), \quad i=1,2,\ldots,n. \end{aligned} \tag{4.5}$$

Smaller $D_{NSP}(\varsigma_i,\varsigma^+)$ is the better alternative $\varsigma_i$.
Let

$$D_{\min}(\varsigma_i,\varsigma^+) = \min_i D_{NSP}(\varsigma_i,\varsigma^+), i=1,2,\ldots,n$$

Similarly, for PFNIS

$$\begin{aligned} D_{NSP}(\varsigma_i,\varsigma^-) &= \sum_{j=1}^{n} D_{NSP}(C_j(\varsigma_i),C_j(\varsigma^-)) \\ &= \frac{2}{n\pi}\sum_{j=1}^{n} w_j \arccos(\varphi_{ij}\varphi_j^- + \varrho_{ij}\varrho_j^- + \varpi_{ij}\varpi_j^-), \quad i=1,2,\ldots,n. \end{aligned} \tag{4.6}$$

Greater $D_{NSP}(\varsigma_i,\varsigma^-)$ is better alternative $\varsigma_i$.
Let

$$D_{\max}(\varsigma_i,\varsigma^+) = \max_i D_{NSP}(\varsigma_i,\varsigma^+), i=1,2,\ldots,n.$$

We will now compute the relative closeness coefficient of the alternative $\varsigma_i$ in relation to the PFPIS ($\varsigma^+$) and PFNIS ($\varsigma^-$) using the fundamental principles of the conventional TOPSIS technique as

$$RC(\varsigma_i) = \frac{D_{NSP}(\varsigma_i,\varsigma^-)}{D_{NSP}(\varsigma_i,\varsigma^+) + D_{NSP}(\varsigma_i,\varsigma^-)} \tag{4.7}$$

We use Lingo technique to solve this model and get the optimal attribute weight vector.

The optimal solution is defined as shortest distance from PIS and the farthest distance from NIS. As a result, Zhang and Xu [23] investigated a modified index, represented by $\xi(\varsigma_i)$, to establish the ranking sequence. The formula for the index is articulated as follows

$$\xi(\varsigma_i) = \frac{D_{NSP}(\varsigma_i, \varsigma^-)}{D_{\max}(\varsigma_i, \varsigma^-)} - \frac{D_{NSP}(\varsigma_i, \varsigma^+)}{D_{\min}(\varsigma_i, \varsigma^+)} \tag{4.8}$$

Based on $RC(\varsigma_i)$ or $\xi(\varsigma_i)$, we derive ranking of alternatives $\varsigma_i$, which is utilized to identify optimal solution by maximizing value of $RC(\varsigma_i)$ or $\xi(\varsigma_i)$.

### 4.2. Algorithm for TOPSIS method

The conventional TOPSIS method, as presented by Hwang and Yoon [12], serves as a foundational and effective approach for addressing MCDM problems involving precise numerical values. Zhang and Xu [23] proposed an enhanced TOPSIS approach to address MCDM challenges involving Pythagorean fuzzy data. The procedure consists of the subsequent phases:

**Step 1.** In addressing a MCDM problem involving PFNs, we formulate decision matrix $R = (C_j(\varsigma_i))_{m\times n}$. $C_j(\varsigma_i)$, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$, represent evaluations of $\varsigma_i$ in relation to $C_j$.

**Step 2.** New score function used to ascertain PFPIS ($\varsigma^+$) and PFNIS ($\varsigma^-$).

**Step 3.** Utilize equations (4.5) and (4.6) to compute the weighted spherical distances for each alternative $\varsigma_i$ in relation to PFPIS ($\varsigma^+$) and PFNIS ($\varsigma^-$).

**Step 4.** Equations (4.7) and (4.8) to compute the relative closeness $RC(\varsigma_i)$ and closeness $\xi(\varsigma_i)$ for $\varsigma_i$.

**Step 5.** Evaluate options and identify the optimal choice(s) based on descending order of relative proximity. The values of $RC(\varsigma_i)$ and revised closeness $\xi(\varsigma_i)$ are derived from Step 4.

The larger the $RC(\varsigma_i)$, the more favorable the $\varsigma_i$, where $(i = 1, 2, \ldots, m)$, will become.

## 5. Illustrative Example

A fast-growing online retailer specializing in organic household goods, faces a critical strategic decision: selecting a primary supplier for its proprietary line of eco-friendly packaging. The packaging a blend of recycled cardboard and biodegradable cushioning is a key part of its brand promise. With six potential suppliers (S1 to S6) shortlisted after an RFQ process. The challenge is to employ a transparent, objective Multi-Criteria Decision-Making (MCDM) model that avoids subjective weight assignment and effectively handles both benefit and cost criteria. The integrated TOPSIS method is used to solve this real-world supplier selection problem for goods.

### 5.1. Defining Criteria and Supplier Alternatives

A cross-functional team identified five critical criteria for evaluating packaging material suppliers. These reflect the triple bottom line: economic, operational, and environmental/social performance.

**Criteria ($C_1$ to $C_5$):**

$C_1$ Sustainability Certification Score: Benefit Criterion

$C_2$ Unit Cost: Cost Criterion (Lower is better)

$C_3$ Defect Rate: Cost Criterion (Lower is better)

$C_4$ On-Time Delivery Rate: Benefit Criterion (Higher is better)

$C_5$ Innovation & R&D Support: Benefit Criterion (Willingness to co-develop new materials)

**Supplier Alternatives:** Six suppliers from different regions ($S_1$: Domestic, $S_2$: Northern Europe, $S_3$: Southeast Asia, $S_4$: South America, $S_5$: Eastern Europe, $S_6$: Western Europe) were evaluated.

Data are compiled from RFQ responses and they are uncertain in nature. In this study, the data are pythagorean fuzzy. The initial decision matrix is presented below.

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---|---|---|---|---|---|
| $S_1$ | $\langle 0.82, 0.41\rangle$ | $\langle 0.64, 0.46\rangle$ | $\langle 0.91, 0.31\rangle$ | $\langle 0.72, 0.44\rangle$ | $\langle 0.83, 0.37\rangle$ |
| $S_2$ | $\langle 0.93, 0.21\rangle$ | $\langle 0.83, 0.33\rangle$ | $\langle 0.65, 0.47\rangle$ | $\langle 0.87, 0.41\rangle$ | $\langle 0.64, 0.39\rangle$ |
| $S_3$ | $\langle 0.76, 0.38\rangle$ | $\langle 0.59, 0.42\rangle$ | $\langle 0.87, 0.42\rangle$ | $\langle 0.63, 0.48\rangle$ | $\langle 0.90, 0.15\rangle$ |
| $S_4$ | $\langle 0.65, 0.43\rangle$ | $\langle 0.90, 0.29\rangle$ | $\langle 0.72, 0.46\rangle$ | $\langle 0.56, 0.48\rangle$ | $\langle 0.72, 0.51\rangle$ |
| $S_5$ | $\langle 0.84, 0.27\rangle$ | $\langle 0.76, 0.42\rangle$ | $\langle 0.91, 0.22\rangle$ | $\langle 0.87, 0.32\rangle$ | $\langle 0.63, 0.38\rangle$ |
| $S_6$ | $\langle 0.91, 0.30\rangle$ | $\langle 0.63, 0.41\rangle$ | $\langle 0.72, 0.40\rangle$ | $\langle 0.72, 0.31\rangle$ | $\langle 0.54, 0.42\rangle$ |

Table 1: Decision matrix

where for alternative $\varsigma_1$ and criterion $C_1$ the membership degree is 0.82 and the non-membership degree is 0.41.

Using exponential score function (2.3), modified decision matrix will be as follows:

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---|---|---|---|---|---|
| $S_1$ | 0.1523 | 0.1121 | 0.1912 | 0.1273 | 0.1597 |
| $S_2$ | 0.2090 | 0.1643 | 0.1125 | 0.1657 | 0.1190 |
| $S_3$ | 0.1418 | 0.1092 | 0.1643 | 0.1086 | 0.2022 |
| $S_4$ | 0.1166 | 0.1901 | 0.1250 | 0.1089 | 0.1191 |
| $S_5$ | 0.1732 | 0.1374 | 0.2006 | 0.1770 | 0.1184 |
| $S_6$ | 0.1924 | 0.1156 | 0.1316 | 0.1403 | 0.1032 |

Table 2: Modified Decision matrix

We classify the attributes in two category viz., beneficial attributes and non-beneficial attributes according to the impact on users. Beneficial attributes ($J_1$) are $J_1 = \{C_1, C_4, C_5\}$, where as $J_2 = \{C_2, C_3\}$ are considered as non-beneficial attributes ($J_2$).

Formula (4.1) and (4.2) are used to calculate PFPIS ($\varsigma^+$) and PFNIS ($\varsigma^-$). The values are

$$\varsigma^+ = \{\langle 0.93, 0.21\rangle, \langle 0.59, 0.42\rangle, \langle 0.65, 0.47\rangle, \langle 0.87, 0.32\rangle, \langle 0.90, 0.15\rangle\}$$
$$\varsigma^- = \{\langle 0.65, 0.43\rangle, \langle 0.83, 0.33\rangle, \langle 0.91, 0.22\rangle, \langle 0.56, 0.38\rangle, \langle 0.54, 0.42\rangle\}$$

### 5.2. Known Weight

First, we consider the weight of the criteria are known i.e., weight $w = (0.25, 0.20, 0.20, 0.15, 0.20)^T$, where $0 \le w_i \le 1$ for all $i$, that satisfies $\sum_{i=1}^{5} w_i = 1$.

Next, utilize equation (4.5) and (4.6), normalized weighted spherical distances $D_{NSP}(\varsigma_i, \varsigma^+)$ and $D_{NSP}(\varsigma_i, \varsigma^-)$ are calculated for each alternatives $\varsigma_i$ from PFPIS and PFNIS as

| | $D_{NSP}(\varsigma_i, \varsigma^+)$ | $D_{NSP}(\varsigma_i, \varsigma^-)$ |
|---|---|---|
| $S_1$ | 0.0324 | 0.0.0352 |
| $S_2$ | 0.0226 | 0.0157 |
| $S_3$ | 0.0286 | 0.0371 |
| $S_4$ | 0.0504 | 0.0210 |
| $S_5$ | 0.0378 | 0.0255 |
| $S_6$ | 0.0266 | 0.0346 |

Table 3: Normalized Spherical distance from PFPIS & PFNIS

Equation (4.7) and (4.8) used to calculate $RC(\varsigma_i)$ and $\xi(\varsigma_i)$ listed bellow:

| | $RC(\varsigma_i)$ (Rank) | $\xi(\varsigma_i)$(Rank) |
|---|---|---|
| $S_1$ | 0.5207(3) | −0.0.4846(3) |
| $S_2$ | 0.4099(4) | −0.5768(4) |
| $S_3$ | 0.5646(2) | −0.2654(2) |
| $S_4$ | 0.2941(6) | −1.6640(6) |
| $S_5$ | 0.4028(5) | −0.9852(5) |
| $S_6$ | 0.5653(1) | −0.2443(1) |

Table 4: Rank of the alternatives

Based on $RC(S_i)$ rank of the alternatives are $S_6 \succ S_3 \succ S_1 \succ S_2 \succ S_5 \succ S_4$ and $S_6$ is the best alternative. With respect to $\xi(S_i)$ ranking of the alternatives are $S_6 \succ S_3 \succ S_1 \succ S_2 \succ S_5 \succ S_6$. Here also, the best alternative is $S_6$.

### 5.3. Unknown Weight

First, we have calculate weight of the attribute from the decision matrix.

$$w_j = \frac{\sum\limits_{i=1}^{5}\sum\limits_{k=1}^{5} \mathcal{D}_{NSP}(a_{ij}, a_{kj})}{\sum\limits_{j=1}^{5}\sum\limits_{i=1}^{5}\sum\limits_{k=1}^{5} \mathcal{D}_{NSP}(a_{ij}, a_{kj})}$$

Utilizing the above formula weight of the different criteria are
$w = (0.182, 0.188, 0.216, 0.208, 0.206)^T$, where $0 \leq w_i \leq 1$ for all $i$, that satisfies $\sum\limits_{i=1}^{5} w_i = 1$.

Next, utilize equation (4.5) and (4.6), normalized weighted spherical distances $D_{NSP}(S_i, \varsigma^+)$ and $D_{NSP}(S_i, \varsigma^-)$ are calculated for each alternatives $S_i$ from PFPIS and PFNIS as

| | $D_{NSP}(S_i, \varsigma^+)$ | $D_{NSP}(S_i, \varsigma^-)$ |
|---|---|---|
| $S_1$ | 0.0331 | 0.0.0350 |
| $S_2$ | 0.0235 | 0.0416 |
| $S_3$ | 0.0295 | 0.0375 |
| $S_4$ | 0.0497 | 0.0218 |
| $S_5$ | 0.0368 | 0.0185 |
| $S_6$ | 0.0285 | 0.0326 |

Table 5: Normalized Spherical distance from PFPIS & PFNIS

Equation (4.7) and (4.8) used to calculate $RC(S_i)$ and $\xi(S_i)$ listed bellow:

| | $RC(S_i)$ (Rank) | $\xi(S_i)$(Rank) |
|---|---|---|
| $S_1$ | 0.5139(4) | −0.0.5671(4) |
| $S_2$ | 0.6390(1) | 0.0(1) |
| $S_3$ | 0.5597(2) | −0.3538(3) |
| $S_4$ | 0.3048(6) | −1.65908(6) |
| $S_5$ | 0.3345(5) | −1.1212(5) |
| $S_6$ | 0.5335(3) | −0.2163(2) |

Table 6: Rank of the alternatives

Based on $RC(S_i)$ rank of the alternatives are $S_2 \succ S_3 \succ S_6 \succ S_1 \succ S_5 \succ S_4$ and $S_2$ is the best alternative. With respect to $\xi(S_i)$ ranking of the alternatives are $S_2 \succ S_6 \succ S_3 \succ S_1 \succ S_5 \succ S_4$. Here also, the best alternative is $S_2$.

### 5.4. Weight of Attribute Partially Known

Utilize Equation (4.4), calculate weight of the attribute as follows:

$$\begin{aligned} \max S \quad &= \quad 1.3612w_1 + 1.352w_2 + 1.529w_3 + 1.535w_4 + 2.0895w_5 \\ \text{s.t.} \quad & \quad 0.17 \leq w_1 \leq 0.25 \\ & \quad 0.10 \leq w_2 \leq 0.15 \\ & \quad 0.22 \leq w_3 \leq 0.30 \\ & \quad 0.15 \leq w_4 \leq 0.20 \\ & \quad 0.21 \leq w_5 \leq 0.28 \\ & \quad \sum_{j=1}^{n} w_j = 1 \end{aligned}$$

Using Lingo Technique, we solve the above problems and we get weights of the attribute as $w = (0.17, 0.10, 0.27, 0.20, 0.26)^T$, where $0 \leq w_i \leq 1$ for all $i$, that satisfies $\sum_{i=1}^{5} w_i = 1$.

Next, utilize equation (4.5) and (4.6), normalized weighted spherical distances $D_{NSP}(\varsigma_i, \varsigma^+)$ and $D_{NSP}(\varsigma_i, \varsigma^-)$ are calculated for each alternatives $S_i$ from PFPIS and PFNIS as

| | $D_{NSP}(\varsigma_i, \varsigma^+)$ | $D_{NSP}(\varsigma_i, \varsigma^-)$ |
|---|---|---|
| $S_1$ | 0.0360 | 0.0.0348 |
| $S_2$ | 0.0224 | 0.0441 |
| $S_3$ | 0.0316 | 0.0385 |
| $S_4$ | 0.0464 | 0.0248 |
| $S_5$ | 0.0387 | 0.0244 |
| $S_6$ | 0.0321 | 0.0306 |

Table 7: Normalized Spherical distance from PFPIS & PFNIS

Equation (4.7) and (4.8) used to calculate $RC(\varsigma_i)$ and $\xi(\varsigma_i)$ listed bellow:

| | $RC(\varsigma_i)$ (Rank) | $\xi(\varsigma_i)$(Rank) |
|---|---|---|
| $S_1$ | 0.4915(3) | −0.0.818(3) |
| $S_2$ | 0.6631(1) | 0.0(1) |
| $S_3$ | 0.5492(2) | −0.5376(2) |
| $S_4$ | 0.3483(6) | −1.509(6) |
| $S_5$ | 0.3866(5) | −1.1174(5) |
| $S_6$ | 0.4880(4) | −0.7391(4) |

Table 8: Rank of the alternatives

Based on $RC(\varsigma_i)$ rank of the alternatives are $S_2 \succ S_3 \succ S_1 \succ S_6 \succ S_5 \succ S_4$ and $S_2$ is the best alternative. With respect to $\xi(\varsigma_i)$ ranking of the alternatives are $S_2 \succ S_3 \succ S_1 \succ S_6 \succ S_5 \succ S_4$. Here also, the best alternative is $S_2$.

## 6. Discussion & Analysis

To assess the validity and practicality of the proposed technique, the decision matrix was analyzed under three distinct weight conditions: fully known, completely unknown, and partially known attribute weights. The alternative rankings produced by both the relative closeness method and the revised index method are shown in Figures 2 and 3, respectively.

| Alternatives | Case-I(Known) | Case-II(Unknown) | Case-III (Partial Known) |
|---|---|---|---|
| $S_1$ | 0.5207(3) | 0.5139(4) | 0.4915(3) |
| $S_2$ | 0.4099(4) | 0.6390(1) | 0.6631(1) |
| $S_3$ | 0.5646(2) | 0.5597(2) | 0.5492(2) |
| $S_4$ | 0.2941(6) | 0.3048(6) | 0.3483(6) |
| $S_5$ | 0.4028(5) | 0.3345(5) | 0.3866(5) |
| $S_6$ | 0.5653(1) | 0.5335(3) | 0.4880(4) |

Table 9: Comparative Rank w.r.to Relative Closeness

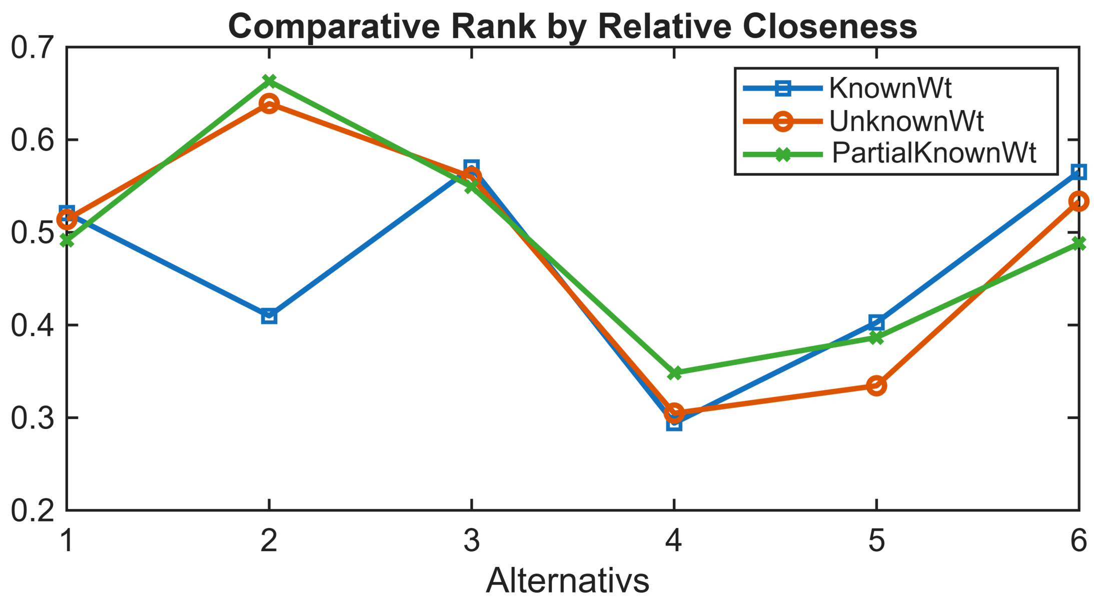


Figure 2: Comparative Rank w.r.to Relative Closeness

| Alternatives | Case-I(Known) | Case-II(Unknown) | Case-III (Partial Known) |
|---|---|---|---|
| $S_1$ | $-0.4846(3)$ | $-0.5671(4)$ | $-0.818(3)$ |
| $S_2$ | $-0.5768(4)$ | $0.0(1)$ | $0.0(1)$ |
| $S_3$ | $-0.2654(2)$ | $-0.3538(3)$ | $-0.5376(2)$ |
| $S_4$ | $-1.6640(6)$ | $-1.5908(6)$ | $-1.509(6)$ |
| $S_5$ | $-0.9852(5)$ | $-1.1212(5)$ | $-1.174(5)$ |
| $S_6$ | $-0.2443(1)$ | $-0.2163(2)$ | $-0.7391(4)$ |

Table 10: Comparative Rank w.r.to Revised Index

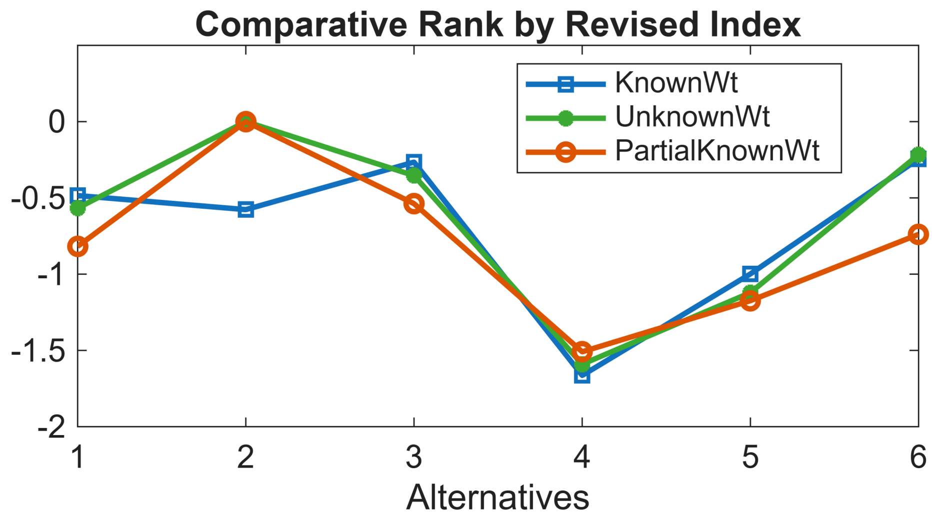


Figure 3: Comparative Rank w.r.to Revised Index

The outcomes demonstrate that the ranking generated with partially known weights is more robust and reliable than those derived from the other two scenarios.

## 7. Conclusion

Selecting the optimal supplier is a critical, multi-faceted strategic decision that directly impacts an organization's resilience, efficiency, and competitive edge. This study has addressed the inherent complexity of this process in real-world environments, where decision-makers must operate with a mixture of known, partially known, and completely unknown attribute information substantial ambiguity and subjective judgment. By operating within the expressive Pythagorean fuzzy (PF) environment, the model adeptly captures expert hesitation and linguistic uncertainty. The methodology combines maximizing deviation for deriving objective weights for unknown criteria with a constraint-based model for partially known weights, leading to a comprehensive PF ranking of suppliers. Future research may focus on other complex decision-making domains, such as sustainable technology selection, healthcare management, or financial portfolio optimization, where mixed information and uncertainty are prevalent.


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### Conflicts of Interest

The authors declare that there is no competing of interests.

### Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

### Author Contribution

Amal Kumar Adak: Visualization, Validation, Conceptualization. Dragan Pamucar : Supervision, Investigation, Writing original draft. Vladimir Simic: Methodology, Formal analysis.

## References


1. Adak, A. K., Kumar, D. (2023). Spherical Distance Measurement Method for Solving MCDM Problems under Pythagorean Fuzzy Environment. *Journal of fuzzy extension and applications* 4 (4), 28-39. https://doi.org/10.22105/jfea.2022.351677.1224

2. Adak, A.K., & Nilkamal. (2025). Pythagorean Fuzzy Sets for Credit Risk Assessment: A Novel Approach to Predicting Loan Default. *Control and Optimization in Applied Mathematics- COAM*, 10(1), 175-191 DOI. 10.30473/coam.2025.73505.1287

3. Adak, A., Pamucar, D., & Ali, W. (2025). Solving Pythagorean Fuzzy Assignment Problems in Management: A Framework Based on Spherical Distance Measures. Management Science Advances, 3(1), 84-95. https://doi.org/10.31181/msa31202636

4. Atanassov, K. (1983). A second type of intuitionistic fuzzy sets. BUSEFAL, 56, 66-70.

5. Atanassov, K. (1986). Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20, 87-96.

6. Bisht, G., & Pal, A.K. (2023). A novel multi-criteria group decision-making approach using aggregation operators and weight determination method for supplier selection problem in hesitant Pythagorean fuzzy environment. Decision Science Letters. DOI:10.5267/j.dsl.2023.4.009

7. Boltürk, E. (2018). Pythagorean fuzzy CODAS and its application to supplier selection in a manufacturing firm. J. Enterp. Inf. Manag., 31, 550-564. DOI:10.1108/JEIM-01-2018-0020

8. Çalık, A. (2023). A novel resilient supplier and order allocation model with Pythagorean fuzzy sets based on Industry 4.0 initiatives. Environment, Development and Sustainability, 1-29. DOI:10.1007/s10668-023-03608-z

9. Dai, L., & Bai, S. (2020). An Approach to Selection of Agricultural Product Supplier Using Pythagorean Fuzzy Sets. Mathematical Problems in Engineering. 20, 1-7, DOI:10.1155/2020/1816028

10. Dehshiri, S. J. H. . (2026). Sustainable Supplier Selection Based on a Comparative Decision-Making Approach Under Uncertainty. Spectrum of Operational Research, 3(1), 238-251. https://doi.org/10.31181/sor31202644

11. Göçer, F. (2021). Improving sustainable supplier evaluation by an integrated MCDM method under pythagorean fuzzy environment. Cumhuriyet Science Journal. DOI:10.17776/CSJ.735674

12. Hwang, C. L. & Yoon, K. (1981). Multiple attribute decision methods and applications: A state of the art survey, *Springer Verlag*, New York.

13. Khan M, Chao W, Rahim M, Amin F (2024) Enhancing green supplier selection: A nonlinear programming method with TOPSIS in cubic Pythagorean fuzzy contexts. PLoS ONE 19(12): e0310956. https://doi.org/10.1371/journal.pone.0310956

14. Kolour, H. R., Momayezi, V., & Momayezi, F. (2026). Enhancing Supplier Selection in Public Manufacturing: A Hybrid Multi-Criteria Decision-Making Approach. Spectrum of Decision Making and Applications, 3(1), 1-20. https://doi.org/10.31181/sdmap31202629

15. Liu, C., Rani, P., & Pachori, K. (2021). Sustainable circular supplier selection and evaluation in the manufacturing sector using Pythagorean fuzzy EDAS approach. J. Enterp. Inf. Manag., 35, 1040-1066. DOI:10.1108/jeim-04-2021-0187

16. Lo, H.-W., Li, K.-Y., & Lin, S.-W. (2025). Multi-Year Performance Evaluation and Trend Tracking of Sustainable Suppliers: An Application of a Hybrid Decision Analysis Model. Journal of Intelligent Decision Making and Granular Computing, 1(1), 89-105. https://doi.org/10.31181/jidmgc1120256

17. Peng X, Dai J. (2017). Approaches to Pythagorean fuzzy stochastic multi-criteria decision making based on prospect theory and regret theory with new distance measure and score function. International Journal Intelligent System. 32, 1187–1214, https://doi.org/10.1002/int.21896

18. Shahab, S., Anjum, M., Dutta, A.K., & Ahmad, S. (2024). Gamified approach towards optimizing supplier selection through Pythagorean Fuzzy soft-max aggregation operators for healthcare applications. AIMS Mathematics. DOI:10.3934/math.2024329

19. Tufan, D., & Ulutaş, A. (2026). Supplier Selection in the Food Sector: An Integrated Approach Using LODECI and CORASO Methods. Spectrum of Decision Making and Applications, 3(1), 40-51. https://doi.org/10.31181/sdmap31202631

20. Yager, R. R. (2013). Pythagorean fuzzy subsets. In: Proceedings of joint IFSA world congress and NAFIPS annual meeting, Edmonton, Canada, pp 57-61.

21. Yager, R. R. (2016). Properties and applications of Pythagorean fuzzy sets. *Springer, Berlin*

22. Zadeh, L. A. (1965). Fuzzy sets, *Information and Control*, 8, 338-353.

23. Zhang, X. L. & Xu, Z. S. (2014). Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets, *Int. J. Intell. Syst.* 29, 1016-1078.

24. Zhang, J., Guo, L., & Lyu, T. (2021). An enhanced personal credit identification coin-day destruction model based on blockchain technology fuzzy sets for region of China pearl river delta. Journal of Intelligent and Fuzzy Systems, 41(3), 4519–4525. https://doi.org/10.3233/JIFS-189712

25. Zhou, F., & Chen, T. (2020). An Integrated Multicriteria Group Decision-Making Approach for Green Supplier Selection Under Pythagorean Fuzzy Scenarios. IEEE Access, 8, 165216-165231. DOI:10.1109/ACCESS.2020.3022377

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