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New Topological Group Construction via $^\star h$-Set

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ABSTRACT: This paper introduces the class of $^\star h$- topological groups, applying $^\star h$-continuous group operations. We establish several basic properties and characterizations that distinguish them from classical topological groups. A collection of examples is provided to illustrate these differences and to demonstrate the scope of the framework. Also we define $^\star h$- irresolute topological groups and investigate their connection with $^\star h$- topological groups, proving that every $^\star h$- irresolute topological group is $^\star h$- topological, whereas the converse does not generally hold. Collectively, these results extend the framework of generalized topologies and offer a broader foundation for analyzing algebraic structures equipped with non-standard notions of openness.

Keywords: $^\star h$-open sets, $^\star h$-continuous, $^\star h$-topological group, $^\star h$-irresolute topological group.

Contents

1. Introduction

The study of generalized topological structures has become an active area of research due to their ability to extend classical topological notions and provide greater flexibility in describing continuity and openness. Since the introduction of generalized closed and generalized open sets by Levine [3] and the corresponding generalized closure and interior operators developed by Dunham [2], many authors have explored various generalized forms of continuity, mappings, and algebraic structures equipped with non-standard topologies. In the classical setting, a topological group [4] is characterized by the continuity of its group operation and inversion. This combination of algebraic and topological structure has been widely studied and forms a foundational concept in modern topology and group theory. However, when continuity is replaced or weakened through generalized notions, the resulting group structures exhibit significantly different and often more intricate behavior. Such generalized topological groups have been investigated in varying forms, leading to several interesting extensions of the classical theory.

Motivated by these developments, this paper introduces a new class of generalized topological groups called $^\star h$- topological groups, defined through $^\star h$- open sets and $^\star h$- continuous mappings. We establish fundamental properties of $^\star h$- topological groups and provide examples that highlight their distinction from classical topological groups. To further extend the framework, we introduce $^\star h$- irresolute topological groups and examine their relationship with $^\star h$- topological groups. Overall, this work contributes to the broader study of generalized topologies by providing a systematic development of $^\star h$- topological and $^\star h$- irresolute topological groups, thereby enriching the interaction between algebraic structures and generalized continuity.

2. Preliminaries

This chapter consists of some basic definitions and fundamental which are needed to build this thesis. Throughout the study (X, τ), (Y, σ) (or simply X, Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, $cl(A)$, $int(A)$,$cl^*(A)$, $int^*(A)$ and

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X-A denote the closure of A, the interior of A, generalized closure of A, generalized interior of A and the complement of A respectively.

Definition 2.1 [3] A subset A of a topological space (X, τ) is called

(i) generalized closed (briefly g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

(ii) generalized open (briefly g-open) if $X \setminus A$ is g-closed in X.

Definition 2.2 [2] Let A be a subset of X. Then

(i) generalized closure of A is defined the intersection of all g-closed sets containing A, and is denoted by $cl^*(A)$.

(ii) generalized interior of A is defined as the union of all g-open subsets of A and is denoted by $int^*(A)$.

Definition 2.3 [5] A subset A of a topological space X is called $^\star h$-open set if for every non-empty set G in X, $G \neq X$ and $G \in \tau$ such that $A \subseteq int^*(A \cup G)$

Definition 2.4 [6] A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is said to be

(i) $^\star h$- continuous if $f^{-1}(U)$ is $^\star h$- open set in X for every open set U in Y.

(ii) $^\star h$- irresolute if $f^{-1}(U)$ is $^\star h$- open set in X for every $^\star h$- open set U in Y.

Definition 2.5 [4] Let S be a subset of a group G is said to be symmetric with respect to the group operation and the inverse operation if $S = S^{-1}$, where $S^{-1} = \{x^{-1} : x \in S\}$. For any subset S of a group G, $S \cup S^{-1}$ and $S \cap S^{-1}$ are symmetric.

3. $^\star h$-Topological Groups

In this section, we are defining the concept of $^\star h$-topological groups and derives its some properties and characterizations.

Definition 3.1 Let $(G, *)$ be group equipped with topology τ on G. A triple $(G, *, \tau)$ is said to be an $^\star h$- topological group if the following conditions are satisfied.

(i) The group operation $* : G \times G \to G$ is defined by $*(x, y) = x * y$, for each x, y in G is $^\star h$-continuous.

(ii) The inverse operation $i : G \to G$ is defined by $i(x) = x^{-1}$ for each x in G is $^\star h$-continuous.

In short, $(G, *, \tau)$ is denoted by G, a $^\star h$-TG.

Example 3.1 Consider the group $G = (Z_3, +_3)$ and $\tau = \{\phi, \{0, 1\}, Z_3\}$
$\tau_{\star h} = \{\phi, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, Z_3\}$. Then $(G, +_3, \tau)$ is $^\star h$-topological group. It is not an topological group because the inverse mapping $i(x) = -x(mod\ 3)$ gives $i(0) = 0$,$i(1) = 2$ and $i(2) = 1$. Now $i^{-1}(\{0, 1\}) = \{0, 2\}$ which is not open whenever $\{0, 1\}$ is open. The inverse mapping i is not continuous.

Theorem 3.1 Let $(G, *)$ be group endowed with topology τ on it. Then $(G, *, \tau)$ is $^\star h$-topological group iff

(i) for each open set W of $x * y$, there exists a $^\star h$-open set U contains x and a $^\star h$-open set V containing y such that $U * V \subseteq W$.

(ii) for each open set W of x^{-1}, there exists an $^\star h$-open set U containing x such that $U^{-1} \subseteq W$.

Proof: (i) Assume that $(G,*,\tau)$ is an $^\star h$-topological group. Let $x,y \in G$ and let W be any open set containing $x*y$. Since $*: G\times G \to G$ by $*(x,y)=x*y$ is $^\star h$- continuous, there exists an $^\star h$- open sets P and Q such that $(x,y)\in P\times Q$ and $*(P\times Q)\subseteq W$. Since P and Q are $^\star h$- open sets containing x and y respectively, there exists $^\star h$- open sets U containing x and V containing y such that $x\in U\subseteq P$ and $y\in V\subseteq Q$. Then $(x,y)\in U\times V\subseteq P\times Q$. Hence $U*V = *(P\times Q)\subseteq W$.

(ii) The inverse map $i: G\to G$ by $i(x)=x^{-1}$ is $^\star h$- continuous. For any $x\in\ G$, gives an open neighbourhood W of x^{-1}, there exists a $^\star h$- open set U containing x such that $i(U)\subseteq W \Longrightarrow U^{-1}\subseteq W$. Conversely assume that the given two conditions hold. Let $(x,y)\in G\times G$ be any point and W be any open set in G containing the point $x*y$. By hypothesis there exists a $^\star h$- open set U containing x and y containing V such that $U*V\subseteq W$. Then $U*V$ is a $^\star h$- open set in $G\times G$ containing the point (x,y) such that $*(U\times V)=U*V\subseteq W$. Hence the group operation is $^\star h$- continuous. Thus $(G,*,\tau)$ is $^\star h$-topological group. ☐

Theorem 3.2 If $(G,*)$ is a group and $\tau_{\star_h}$ is a discrete topology on G, then $(G,*,\tau)$ forms an $^\star h$-topological group.

Proof: Let $(G,*)$ be a group and $\tau_{\star_h}$ be a discrete topology on G.
If cardinality of G is 1, then the proof is trivial. Suppose the cardinality of G is greater than one. Let x,y be any two elements of G and W be an open neighbourhood of $x*y$ then by Theorem 3.1, there exists a $^\star h$- open neighbourhood U and V containing x and y respectively such that $U*V\subseteq W$. Also let W be any open neighbourhood of x^{-1}, then there exists an $^\star h$- open neighbourhood of U containing x such that $U^{-1}\subseteq W$. Hence $(G,*,\tau)$ is an $^\star h$- topological group. ☐

Remark 3.1 *The converse of the above theorem is not true in general as shown in the following example.*

Example 3.2 *Consider the set* $G=\{e,a,a^2\}$ *with the operation* $*$ *is defined as* $a^2*a=e$ *and* e *is the identity element.* $(G,*)$ *is a cyclic group endowed with the topology* $\tau=\{\phi,\{e\},G\}$ *and* $\tau_{\star_h}=\{\phi,\{e\},G\}$. *Clearly* $(G,*,\tau)$ *is a* $^\star h$- *topological group but* $\tau_{\star_h}$ *is not discrete.*

Theorem 3.3 Let $(G,+)$ be a group and τ be a topology on G. Then $(G,+,\tau)$ is an $^\star h$- topological group iff for any elements x,y of G and open set W with $x-y\in W$, there exists a $^\star h$- open set U and V containing x and y respectively such that $U-V\subseteq W$.

Proof: Let $x,y\in G$. Then $x-y\in G$. Let W be any open set containing $x-y$. Then there are $^\star h$- open set U containing and R containing $-y$ such that $U+R\subseteq W$ and also there is an $^\star h$- open set V containing y such that $-V\subseteq R$. Therefore $U-V\subseteq W$.

Conversely assume that $x\in G$ and W be an open set containing $-x$. Then W also contains the element $0-x$ and hence there exists $^\star h$- open sets U containing 0 and V containing a such that $U-V\subseteq W$ and $-V=0-V\subseteq U-V\subseteq W$. Now let $x,y\in G$ and W be an open set containing $x+y$. Then by hypothesis there exists $^\star h$- open sets U and R containing x and $-y$ respectively such that $U-R\subseteq W$ and there is an $^\star h$-open set V containing y such that $-V\subseteq R$. Now $U+V=U+(-(-V))\subseteq U+(-R)=U-R\subseteq W$. Hence $(G,+,\tau)$ is a $^\star h$- topological group. ☐

Theorem 3.4 Let $(G,*,\tau)$ be an $^\star h$- topological group. If A is open in $(G,*,\tau)$. Then the following are true.

(i) $x*A$ is $^\star h$- open for each $x\in G$

(ii) $A*x$ is $^\star h$- open for each $x\in G$

(iii) A^{-1} is $^\star h$- open

Proof: (i) Let $y \in x * A$. Then $y = x * a$ for some $a \in A$. Then there exists $^{\star}h$- open sets U and V containing x^{-1} and y respectively such that $U * V \subseteq A$. This implies that $V \subseteq x * A \Longrightarrow y \in int_{^{\star}h}(x * A)$. Therefore $x * A \subseteq int_{^{\star}h}(x * A)$. Always $int_{^{\star}h}(x * A) \subseteq x * A$. Hence $x * A = int_{^{\star}h}(x * A)$. Thus $x * A$ is $^{\star}h$- open set in $(G, *, \tau)$.
(ii) The proof is analogous to part (i).
(iii) For any element $y^{-1} \in A^{-1}$, we have $y \in A$. Since A is open in $^{\star}h$- topological group, there exists an $^{\star}h$- open set V containing y such that $V^{-1} \subseteq A$. This implies that $V \subseteq A^{-1}$. Hence $y^{-1} \in int_{^{\star}h}(A^{-1})$. Therefore $A^{-1} \subseteq int_{^{\star}h}(A^{-1})$. Always $int_{^{\star}h}(A^{-1}) \subseteq A^{-1}$. So $A^{-1} = int_{^{\star}h}(A^{-1})$. Hence A^{-1} is $^{\star}h$- open. □

Theorem 3.5 $(G, *, \tau)$ be a $^{\star}h$- topological group. If A is open subset of $^{\star}h$- topological group then $A * B$ is a $^{\star}h$- open set in $(G, *, \tau)$, then for any subset B of $^{\star}h$- topological group.

Proof: Let G be a $^{\star}h$- topological group. Given A is any open subset of G. Let B be any open subset of G. Then by Theorem 4.2, $y * A$ is a $^{\star}h$- open set in G for all $y \in B$. Now $A * B = \bigcup_{y \in B}(A * y)$. Since arbitrary union of $^{\star}h$- open set is $^{\star}h$- open, $A + B = \bigcup_{y \in B}(A * y)$ is $^{\star}h$- open in G. That is, $A * B$ is $^{\star}h$- open. □

Theorem 3.6 Every open subgroup of an $^{\star}h$- topological group $(G, *, \tau)$ is also $^{\star}h$- topological group.

Proof: Let G be any $^{\star}h$- topological group and H be the subgroup of G. Let x, y be any two distinct points of H. Let W be any open set containing the point $x * y$ in the subgroup H. By definition of $^{\star}h$- topological group, there exists a $^{\star}h$- set U of x and a $^{\star}h$- open set V of y such that $U * V \subseteq W$. Since H is open in G, the sets $A = K \cap U$ and $B = K \cap V$ are $^{\star}h$- open sets in H containing x and y respectively. Also $A * B = (K \cap U) * (K \cap V) = (U * V) \cap K \subseteq U * V \subseteq W$. Hence $*$ is $^{\star}h$- continuous. Next we can prove continuity of inverse of $*$. Now let $x \in H$. Since H is subgroup of $G, x^{-1} \in H \subseteq G$. Let W be any open set of x^{-1}. Since G is $^{\star}h$- topological group, there exists an $^{\star}h$- open neighbourhood of U containing x such that $U^{-1} \subseteq W$. Hence $(H, *, \tau)$ is $^{\star}h$- topological group. □

4. $^{\star}h$- Irresolute Topological Groups

We prove that $^{\star}h$- irresolute topological group is $^{\star}h$- topological group Also it's basic properties have been derived.

Definition 4.1 Let $(G, *)$ be group equipped with topology τ on G. A triple $(G, *, \tau)$ is said to be an $^{\star}h$- irresolute topological group if the following conditions are satisfied.

(i) for each $^{\star}h$- open neighbourhood W of $x * y$, there exists a $^{\star}h$- open neighbourhoods U and V containing x and y respectively such that $U * V \subseteq W$.

(ii) for each $^{\star}h$- open neighbourhood W of x^{-1}, there exists an $^{\star}h$-open neighbourhood U containing x such that $U^{-1} \subseteq W$.

Theorem 4.1 Every $^{\star}h$- irresolute topological group is $^{\star}h$- topological group

Proof: Let G be an $^{\star}h$- irresolute topological group. Consider $x, y \in G$ and W be any open set containing $x * y$. We know that every open set is $^{\star}h$- open, so W is $^{\star}h$- open containing $x * y$. Since G is $^{\star}h$- irresolute topological group, there exists an $^{\star}h$- open neighbourhoods U and V containing x and y respectively such that $U * V \subseteq W$. Now let W be any open neighbourhood of x^{-1}. Since every open set is $^{\star}h$- open, W is $^{\star}h$- open neighbourhood of x^{-1} also since G is $^{\star}h$- irresolute topological group, there exists an $^{\star}h$- open neighbourhood U containing x such that $U^{-1} \subseteq W$. Hence G is $^{\star}h$- topological group. □

Remark 4.1 The converse of the above theorem is not true in general as shown in the following example.

Example 4.1 Let $G = (Z_3, \oplus)$ and $\tau = \{\phi, \{0\}, G\}; \tau_{\star_h} = P(G)$.
The multiplication map $*(x, y) = x + y$. Here $*^{-1}(\phi) = \phi, *^{-1}(G) = G \times G$ and $*^{-1}(\{0\} = \{(0,0),(1,2),(2,1)\}$ which are $^\star h$- open sets in $G \times G$. Therefore multiplication map is $^\star h$- continuous. The inverse mapping $i(x) = -x$. Since the $^\star h$- opens are the discrete topology, the inverse of any element in Z_3 is also in Z_3. Hence both inverse and multiplication maps are $^\star h$- continuous. Thus $(G, +_3, \tau)$ is $^\star h$- topological group. But it is not $^\star h$- irresolute because the inverse image of $\{1\}$ under multiplication is $*^{-1}(\{1\}) = \{(0,1),(1,0),(2,2)\}$. The nonempty open sets of the product space are $\{0\} \times \{0\}, \{0\} \times G$ and $G \times G$. Take $W = \{0\} \times \{0\}$,

$$\begin{aligned} *^{-1}(\{1\}) &\subseteq int_{\star h}(*^{-1}(\{1\}) \cup W) \\ &= int_{\star h}(*^{-1}(\{1\}) \cup (\{0\} \times \{0\})) \\ &= int_{\star h}(*^{-1}(\{0,1\},(1,0),(2,2)) \cup (\{(0,0)\}) \\ &= int_{\star h}(*^{-1}(\{0,0\},(0,1),(1,0),(2,2)) \\ &= \phi \end{aligned}$$

Since $*^{-1}(\{1\}) \nsubseteq \phi$, the inverse image of $\{1\}$ under multiplication is not $^\star h$- open set whenever $\{1\}$ is $^\star h$- open set.

Theorem 4.2 In a $^\star h$- topological group $(G, *, \tau)$, for any $^\star h$- is open set A such that $x * A, A * x$ and A^{-1} are $^\star h$- open set for all $x \in G$.

Proof: (i) Let $y \in x * A$. Then $y = x * a$ for some $a \in A$. Since A is $^\star h$- open set and G is $^\star h$- irresolute topological group, then there exists $^\star h$- open sets U and V containing x^{-1} and y respectively such that $U * V \subseteq A$. This implies that $V \subseteq x * A \Longrightarrow y \in int_{\star h}(x * A)$. Therefore $x * A \subseteq int_{\star h}(x * A)$. Always $int_{\star h}(x * A) \subseteq x * A$. Hence $x * A = int_{\star h}(x * A)$. Thus $x * A$ is $^\star h$- open set in $(G, *, \tau)$.
Similarly we can prove $A * x$ is also $^\star h$- open.

Next we shall prove A^{-1} is $^\star h$- open. Let $y^{-1} \in A^{-1}$, for some $y \in A$. Since A is $^\star h$- open neighbourhood of G and since G is $^\star h$- irresolute topological group, there exists an $^\star h$- open set V containing y such that $V^{-1} \subseteq A$. This implies that $V \subseteq A^{-1}$. Hence $y^{-1} \in int_{\star h}(A^{-1})$. Therefore $A^{-1} \subseteq int_{\star h}(A^{-1})$. Always $int_{\star h}(A^{-1}) \subseteq A^{-1}$. Hence $A^{-1} = int_{\star h}(A^{-1})$. Thus A^{-1} is $^\star h$- open. □

Theorem 4.3 If G is an $^\star h$- irresolute topological group and any subset A of G then for all $x \in G$, we have

(i) $int_{\star h}(x * A) = x * int_{\star h}(A)$

(ii) $int_{\star h}(A^{-1}) = (int_{\star h}(A))^{-1}$

Proof: (i) Let $y \in int_{\star h}(x * A)$. Then $y = x * a$ for some $a \in A$. Since G is $^\star h$- irresolute topological group, there exists $^\star h$-open sets U and V containing x and a respectively such that $U \times V \subseteq x * A$. Hence $y \in x * int_{\star h}(A)$. Thus

$$int_{\star h}(x * A) \subseteq x * int_{\star h}(A). \tag{4.1}$$

Next let $y \in x * int_{\star h}(A)$, then $y = x * s$ for some $a \in int_{\star h}(A)$. Since A is $^\star h$- open set and since G is $^\star h$- irresolute topological group, there exists $^\star h$- open sets U and V containing x^{-1} and y respectively such that $U * V \subseteq A$. Hence $y \in int_{\star h}(x * A)$. So

$$x * int_{\star h}(A) \subseteq int_{\star h}(x * A). \tag{4.2}$$

From (1) and (2), $int_{\star h}(x * A) = x * int_{\star h}(A)$.
(ii) Let $y \in int_{\star h}(A^{-1})$. Then there exists an $^\star h$- open set U such that $y \in U \subseteq A^{-1}$. This implies $y^{-1} \in U^{-1} \subseteq A^{-1}$. Since G is $^\star h$- irresolute topological group, the inverse operation is $^\star h$- irresolute, so U^{-1} is $^\star h$- open set. Thus $y^{-1} \in int_{\star h}(A) \Longrightarrow y \in (int_{\star h}(A))^{-1}$.
Conversely, let $y \in (int_{\star h}(A))^{-1}$, then $y^{-1} \in int_{\star h}(A)$. Then there exists an $^\star h$- open set V such that $y^{-1} \in V \subseteq A$. Then $y \in V^{-1} \subseteq A^{-1}$ and V^{-1} is $^\star h$- open set. Hence $y \in int_{\star h}(A^{-1})$. Therefore $int_{\star h}(A^{-1}) = (int_{\star h}(A))^{-1}$. □

Theorem 4.4 Consider an ${}^\star h$- irresolute topological group G and H be an open subgroup of G. Then H is also an ${}^\star h$- irresolute topological group.

Proof: Let x and y be any two arbitrary elements of H and W be an ${}^\star h$- open neighbourhood containing $x * y$ in the subgroup of H. By definition of ${}^\star h$-irresolute topological group, there exists ${}^\star h$- open neighbourhood U and V containing x and y respectively such that $U * V \subseteq W$. Since H is open in G, the sets $A = H \cap U$ and $B = H \cap V$ are ${}^\star h$- open sets in H containing x and y respectively and satisfying $A * B \subseteq U * V \subseteq W$. Hence $*$ is ${}^\star h$- irresolute. Now we shall prove ${}^\star h$- irresoluteness of inverse of $*$. Now let $x \in H$. Since H is a subgroup of G, $x^{-1} \in H \subseteq G$. Let W be an ${}^\star h$- open neighbourhood containing x^{-1}. Since G is ${}^\star h$- irresolute topological group, there exists an ${}^\star h$- open neighbourhood U containing x such that $U^{-1} \subseteq W$. Hence $(H, *, \tau)$ is ${}^\star h$- irresolute topological group. □

Theorem 4.5 In a ${}^\star h$-irresolute topological group $(G, \cdot, \tau)$, for any ${}^\star h$- open set U containing the identity element e, there exists a symmetric ${}^\star h$- open set V containing e such that $V \cdot V \subseteq U$.

Proof: Let U be ${}^\star h$- open set containing $e = e \cdot e$. By the definition of ${}^\star h$- irresolute topological group, there exists ${}^\star h$- open sets V_1 and V_2 containing e such that $V_1 \cdot V_2 \subseteq U$. By the definition of symmetric, $(V_1 \cap V_1^{-1})$ and $(V_2 \cap V_2^{-1})$ are symmetric sets containing e. Since intersection of two symmetric sets is always symmetric, $[V_1 \cap V_1^{-1}] \cap [V_2 \cap V_2^{-1}] = V$ is a symmetric set containing e. Clearly $V = [V_1 \cap V_1^{-1}] \cap [V_2 \cap V_2^{-1}] \subseteq V_1$ and $V = [V_1 \cap V_1^{-1}] \cap [V_2 \cap V_2^{-1}] \subseteq V_2$. Then $V \cdot V \subseteq V_1 \cdot V_2 \subseteq U$. □

Remark 4.2 Analogous result holds for additive groups with 0 as identity and $V + V \subseteq U$.

Theorem 4.6 Let $(G, +, \tau)$ be a ${}^\star h$- irresolute topological group. If U is ${}^\star h$- open set in G then there exists a ${}^\star h$- open set V containing the identity element 0 such that $u + V \subseteq U$ for all $u \in U$.

Proof: Let U be any ${}^\star h$- open set in $(G, +, \tau)$. By Theorem 4.2, $x + U$ is ${}^\star h$- open set for all x in G. In particular, $U - u$ is a ${}^\star h$- open set in G containing 0 for all $u \in U$. By taking $V = U - u$, we get a ${}^\star h$- open set V containing 0 such that $u + V \subseteq U$. □

The theorem remains valid if the group operation is denoted as multiplication $(\cdot)$ instead of addition $(+)$.

Theorem 4.7 Let H be the non-empty subgroup of an ${}^\star h$- irresolute topological group $(G, *, \tau)$. Then the following are equivalent.

(i) H is ${}^\star h$- open

(ii) $int_{\star h}(H)$ is non-empty.

Proof: $(i) \Rightarrow (ii)$ If H is ${}^\star h$- open then $int_{\star h}(H) = H$. Since H is non-empty, $int_{\star h}(H)$ is non-empty. $(ii) \Rightarrow (i)$ Suppose $int_{\star h}(H) \neq \phi$. Let $x \in int_{\star h}(H)$. Then there exists an ${}^\star h$- open set U such that $x \in U \subseteq H$. Clearly $x * U \subseteq H$. Now $y * U = y * x^{-1} * x * U \subseteq H$, for all $y \in H$. Hence $H = \bigcup_{y \in H} y * U$ is ${}^\star h$- open because union of ${}^\star h$- open set is ${}^\star h$- open. □

References

1. Bosan.M.S, Khan.M.D, Kocinac.L.D.R, On s- Topological groups, Math. Morav.18 (2014), 35-44. https://doi.org/10.5937/matmor1402035b.
2. Dunham.W, A new closure operator for Non-T_1 toplogies, kyungpook Math.J, 22(1) (1982), 55-60.
3. Levine.N, Generalized closed sets in Topology, Rend. Circ. Mat. Palermo, 19(2)(1970), 89-89.
4. Pontryagin.L. S, Topological Groups, 2nd ed., Gordon and Breach Taylor and Francis, New York London Paris, (1986).

5. SHYAMALA.G.R, MEERA DEVI.B, $^{\star}h$- Open Sets in Topological Spaces, Proceedings of International Conference on Recent Advances in Algebra, ISBN; 978-93-341-2459-0, (2024), 74-78.

6. SHYAMALA.G.R, MEERA DEVI.B, New Class of Mappings Via $^{\star}h$- Set, Proceedings of International Conference on Recent Trends in Applied Mathematics and Computer Science, ISBN; 978-81-974481-3-3, (2025), 264-268.

7. TKACHENKO.M, Paratopological and semitopological groups vs topological groups, Recent Progress in General Toplogy III, Atlantis Press, (2014), 825-882.

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