



## Jordan Homomorphism of $\Gamma$ -Semirings

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ABSTRACT: The objective of this paper is to define the concept of Jordan mappings for  $\Gamma$ -semirings. We prove that every Jordan homomorphism of  $\Gamma$ -semiring is a homomorphism or anti-homomorphism.

Keywords: Jordan homomorphism,  $\Gamma$ -semiring, semiprime  $\Gamma$ -semiring, prime  $\Gamma$ -semiring.

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### 1. Introduction

In 1950s, Herstein studied Jordan homomorphisms [3] in prime rings. His results have had a notable impact on the study of Jordan structure and Jordan mappings. A while back, the idea of Jordan mappings was considered for semirings [7]. In this paper, we introduce Jordan homomorphism for the algebraic structure of  $\Gamma$ -semiring of a certain type.

By [6], let  $S$  and  $\Gamma$  be additive abelian semigroups then  $S$  is called  $\Gamma$ -semiring if there exist a map  $S \times \Gamma \times S \rightarrow S$  which satisfying the following conditions; (i)  $(s + m)\alpha z = s\alpha z + m\alpha z$  and  $s\alpha(m + z) = s\alpha m + s\alpha z$  (ii)  $s(\alpha + \beta)z = s\alpha z + s\beta z$  (iii)  $s\alpha 0 = 0\alpha s = 0$ . A  $\Gamma$ -semiring  $S$  is called additively inverse  $\Gamma$ -semiring if for each  $s \in S$ , there exists a unique element  $s' \in S$  that such that  $s + s' + s = s$  and  $s' + s + s' = s'$ . The center of  $\Gamma$ -semiring  $Z(S)$  consists of elements in the semiring  $S$  that commute with all other elements, that is,  $Z(S) = \{s : s\alpha m = m\alpha s \quad \forall m \in S, \alpha \in \Gamma\}$ . An inverse  $\Gamma$ -semiring  $S$  is called  $\Gamma$ -MA semiring if for all  $s \in S$ ,  $s + s'$  lies in the center of  $S$ . Throughout this paper, by  $S$ , we mean  $\Gamma$ -MA semiring which satisfying the condition that  $s\alpha m\beta z = s\beta m\alpha z$ . A  $\Gamma$ -semiring  $S$  is prime if  $s\Gamma STm = 0$  implies that either  $s = 0$  or  $m = 0$  and  $S$  is semiprime if the condition  $s\Gamma STs = 0$  implies that  $s = 0$ . The commutator of two elements  $u$  and  $v$  of  $S$  with respect to  $\alpha \in \Gamma$  is defined as  $[u, v]_\alpha = u\alpha v + v\alpha u'$ .

To prove our main result, we first establish the following useful lemmas.

**Lemma 1.1** *Let  $S$  be  $\Gamma$ -semiring. Then for all  $u, v, w \in S$ ,  $\alpha, \delta \in \Gamma$  the following properties are valid.*

- (i)  $[u\delta v, w]_\alpha = u\delta[v, w]_\alpha + [u, w]_\alpha\delta v + u[\delta, \alpha]_w v$
- (ii)  $[u, v\delta w]_\alpha = v\delta[u, w]_\alpha + [u, v]_\alpha\delta w + v[\delta, \alpha]_u w$
- (iii)  $[u\delta v, w]_\alpha = u\delta[v, w]_\alpha + [u, w]_\alpha\delta v$
- (iv)  $[u, v\delta w]_\alpha = v\delta[u, w]_\alpha + [u, v]_\alpha\delta w$ .

**Proof:** We give prove of (i) and (iii), only.

(i):  $[u\delta v, w]_\alpha = u\delta v\alpha w + w\alpha u\delta v' = u\delta v\alpha(w + w' + w) + w\alpha u\delta v' = u\delta v\alpha w + u\delta(w' + w)\alpha v + w\alpha u\delta v' = u\delta(v\alpha w + w'\alpha v) + u\delta w\alpha v + w\alpha u\delta v' = u\delta[v, w]_\alpha + u\delta w\alpha v + w\alpha u\delta v' = u\delta[v, w]_\alpha + u\delta w\alpha(v + v' + v) + w\alpha u\delta v' = u\delta[v, w]_\alpha + u\alpha w\delta v + w\alpha u\delta v' + u\delta w\alpha v + u\alpha w\delta v'$ .

Thus we have,  $[u\delta v, w]_\alpha = u\delta[v, w]_\alpha + [u, w]_\alpha\delta v + u[\delta, \alpha]_w v$ .

(iii): By making use of the assumption that  $s\alpha m\beta z = s\beta m\alpha z$  in (i), we get

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$$[u\delta v, w]_\alpha = u\delta[v, w]_\alpha + [u, w]_\alpha \delta v + u[\delta, \alpha]_w v = u\delta[v, w]_\alpha + (u\alpha w + w\alpha u')\delta v + u(\delta w\alpha + \alpha w\delta')v = u\delta[v, w]_\alpha + u\alpha w\delta v + u\alpha w\delta v + u\alpha w\delta v' + w\alpha u'\delta v = u\delta[v, w]_\alpha + u\alpha w\delta(v+v+v') + w\alpha u'\delta v = u\delta[v, w]_\alpha + [u, w]_\alpha \delta v.$$

Hence proved.  $\square$

**Lemma 1.2** *Let  $S$  be 2-torsion free semiprime  $\Gamma$ -semiring and let  $s, m \in S$ . If  $s\alpha x\beta m + m\alpha x\beta s = 0, \forall \alpha, \beta \in \Gamma$  then  $s\alpha x\beta m = 0 = m\alpha x\beta s$ .*

**Proof:** By assumption,  $s\alpha x\beta m + m\alpha x\beta s = 0$  therefore, we have  $s\alpha x\beta m = m\alpha x\beta s'$ . Consider,  $(m\alpha x\beta s)\alpha m\beta(m\alpha x\beta s) = m\alpha x\beta(s\alpha m\beta m)\alpha x\beta s = m\alpha x\beta(m\alpha m\beta s')\alpha x\beta s = m\alpha(x\beta m\alpha m)\beta s'\alpha x\beta s = s\alpha(x\beta m\alpha m)\beta m\alpha x\beta s$ . That is,  $(m\alpha x\beta s)\alpha m\beta(m\alpha x\beta s) = s\alpha x\beta m\alpha m\beta m\alpha x\beta s$ . Adding  $(m\alpha x\beta s)\alpha m\beta(m\alpha x\beta s)$  on both sides of the last equation, we obtain,  $2(m\alpha x\beta s)\alpha m\beta(m\alpha x\beta s) = (s\alpha x\beta m + m\alpha x\beta s)\alpha m\beta m\alpha x\beta s$ . By hypothesis and using semiprimeness of  $S$ , we get  $m\alpha x\beta s = 0$ . In a similar way, we can show that  $s\alpha x\beta m = 0$ .  $\square$

The following result follows from Lemma 2 and primeness of  $S$ .

**Lemma 1.3** *Let  $S$  be prime  $\Gamma$ -semiring and let  $s, m \in S$  such that for all  $\alpha, \beta \in \Gamma$  we have,  $s\alpha x\beta m + m\alpha x\beta s = 0$ , then either  $s = 0$  or  $m = 0$ .*

## 2. Jordan Homomorphism of $\Gamma$ -Semirings

In this section, we define the concept of Jordan Homomorphism for the structure of  $\Gamma$  – semirings.

**Definition 2.1** *Let  $S$  and  $T$  be  $\Gamma$ -semirings, an additive mapping  $\omega : S \rightarrow T$  is called Jordan homomorphism if*

$$\omega(s\alpha m + m\alpha s) + \omega(s)\alpha\omega(m)' + \omega(m)\alpha\omega(s)' = 0$$

for all  $s, m \in S$  and  $\alpha, \beta \in \Gamma$ .

**Lemma 2.1** *Let  $\omega$  be a Jordan homomorphism of  $\Gamma$ -semiring  $S$  into 2-torsion free  $\Gamma$ -semiring  $T$ . Then for all  $s, m, c \in S$  and  $\alpha, \beta \in \Gamma$ , the following statements are true:*

- (i)  $\omega(s\alpha s) + \omega(s)\alpha\omega(s)' = 0$
- (ii)  $\omega(s\alpha m\beta s) + \omega(s)\alpha\omega(m)\beta\omega(s)' = 0$
- (iii)  $\omega(s\alpha m\beta c + c\alpha m\beta s) + \omega(s)\alpha\omega(m)\beta\omega(c)' + \omega(c)\alpha\omega(m)\beta\omega(s)' = 0$ .

**Proof:** (i) : By definition of Jordan homomorphism, we have

$$\omega(s\alpha m + m\alpha s) + \omega(s)\alpha\omega(m)' + \omega(m)\alpha\omega(s)' = 0 \quad (2.1)$$

Replacing  $m$  by  $s$  in the above equation, we have

$$\omega(2s\alpha s) + 2\omega(s)\alpha\omega(s)' = 0.$$

But  $T$  is 2-torsion free, so we have  $\omega(s\alpha s) + \omega(s)\alpha\omega(s)' = 0$ .

(ii) : Put  $m = s\beta m + m\beta s$  in (2.1), we obtain

$$\omega(s\alpha(s\beta m + m\beta s) + (s\beta m + m\beta s)\alpha s) + \omega(s)\alpha\omega(s\beta m + m\beta s)' + \omega(s\beta m + m\beta s)\alpha\omega(s)' = 0$$

$$\omega(s\alpha s\beta m + m\beta s\alpha s) + 2\omega(s\alpha m\beta s) + \omega(s)\alpha\omega(s)\beta\omega(m)' + \omega(s)\alpha\omega(m)\beta\omega(s)' + \omega(s)\beta\omega(m)\alpha\omega(s)' + \omega(m)\beta\omega(s)\alpha\omega(s)' = 0 \quad (2.2)$$

Replacing  $s$  by  $s\beta s$  in (2.1) and then using (i), we have

$$\omega(s\beta s\alpha m + m\alpha s\beta s) + \omega(s)\beta\omega(s)\alpha\omega(m)' + \omega(m)\alpha\omega(s)\beta\omega(s)' = 0$$

Therefore,

$$\omega(s\alpha s\beta m + m\beta s\alpha s) + \omega(s)\alpha\omega(s)\beta\omega(m)' + \omega(m)\beta\omega(s)\alpha\omega(s)' = 0 \quad (2.3)$$

By (2.2) and (2.3), we get

$$2\omega(s\alpha m\beta s) + 2\omega(s)\alpha\omega(m)\beta\omega(s)' = 0$$

Thus

$$\omega(s\alpha m\beta s) + \omega(s)\alpha\omega(m)\beta\omega(s)' = 0 \quad (2.4)$$

(iii) : Linearize (2.4) and re-arranging the terms, we arrive at

$$\begin{aligned} & \{\omega(s\alpha m\beta s) + \omega(s)\alpha\omega(m)\beta\omega(s)'\} + \{\omega(c\alpha m\beta c) + \omega(c)\alpha\omega(m)\beta\omega(c)'\} + \\ & \omega(s\alpha m\beta c) + \omega(c\alpha m\beta s) + \omega(s)\alpha\omega(m)\beta\omega(c)' + \omega(c)\alpha\omega(m)\beta\omega(s)' = 0 \end{aligned}$$

By using (2.4), we get

$$\omega(s\alpha m\beta c + c\alpha m\beta s) + \omega(s)\alpha\omega(m)\beta\omega(c)' + \omega(c)\alpha\omega(m)\beta\omega(s)' = 0.$$

Hence Proved.  $\square$

#### Notations:

Let us fix some notations :  $(s^m)_\alpha = \omega(s\alpha m) + \omega(s)\alpha\omega(m)'$  and  $(s_m)_\alpha = \omega(s\alpha m) + \omega(m)\alpha\omega(s)'$ .

Thus (1) can be written as

$$(s^m)_\alpha + (m^s)_\alpha = 0 \quad (2.5)$$

**Lemma 2.2** *If  $\omega$  is a Jordan homomorphism from an  $\Gamma$ -semiring  $S$  to 2-torsion free  $\Gamma$ -semiring  $T$  then*

$$(s^m)_\alpha\beta(s_m)_\alpha = 0 \text{ and } (s_m)_\alpha\beta(s^m)_\alpha = 0.$$

**Proof:** By using (i) and (ii) of lemma 4, we have

$$\begin{aligned} (s^m)_\alpha\beta(s_m)_\alpha &= \{\omega(s\alpha m) + \omega(s)\alpha\omega(m)'\}\beta\{\omega(s\alpha m) + \omega(m)\alpha\omega(s)'\} = \omega(s\alpha m)\beta\omega(s\alpha m) + \\ & \omega(s\alpha m)\beta\omega(m)\alpha\omega(s)' + \omega(s)\alpha\omega(m)'\beta\omega(s\alpha m) + \omega(s)\alpha\{\omega(m)'\beta\omega(m)\}\alpha\omega(s)' = \omega\{(s\alpha m)\beta(s\alpha m)\} + \\ & \{\omega(s)'\alpha\omega(m\beta m)\alpha\omega(s)'\} + \omega(s\alpha m)\beta\omega(m)\alpha\omega(s)' + \omega(s)\alpha\omega(m)'\beta\omega(s\alpha m) = \omega(s\alpha m\beta s\alpha m) + \\ & \omega(s\alpha m\beta m\alpha s) + \omega(s\alpha m)\beta\omega(m)\alpha\omega(s)' + \omega(s)\alpha\omega(m)'\beta\omega(s\alpha m) = \omega(s\alpha m\beta s\alpha m + s\alpha m\beta m\alpha s) + \\ & \omega(s\alpha m)\beta\omega(m)\alpha\omega(s)' + \omega(s)\alpha\omega(m)'\beta\omega(s\alpha m). \end{aligned}$$

If we put  $c$  in place of  $s\alpha m$  then we get

$$(s^m)_\alpha\beta(s_m)_\alpha = \omega(s\alpha m\beta c + c\beta m\alpha s) + \omega(c)\beta\omega(m)\alpha\omega(s)' + \omega(s)\alpha\omega(m)\beta\omega(c)'$$

and therefore, by Lemma 4, we get our desired relation.

In a similar fashion, we can show that  $(s_m)_\alpha\beta(s^m)_\alpha = 0$ .  $\square$

**Lemma 2.3** *Let  $\omega$  be a Jordan homomorphism from  $\Gamma$ -semiring  $S$  to 2-torsion free  $\Gamma$ -semiring  $T$ . Then for any  $s, m, r \in S$  and  $\alpha, \beta \in \Gamma$*

$$(s^m)_\alpha\alpha\omega(r)\beta(s^m)_\alpha = (s^m)_\alpha\alpha\omega([s, m]_\alpha\beta r)$$

and

$$(s_m)_\alpha\beta\omega(r)\alpha(s_m)_\alpha = \omega([s, m]_\alpha\beta r)\alpha(s_m)_\alpha.$$

**Proof:** Consider,

$$\omega(r)\beta(s^m)_\alpha = \omega(r)\beta\omega(s\alpha m) + \omega(r)\beta\omega(s)\alpha\omega(m)'$$

But from (iii) of Lemma 4, we have

$$\omega(r)\alpha\omega(s)\beta\omega(m)' = \omega(r\alpha s\beta m + m\alpha s\beta r)' + \omega(m)\alpha\omega(s)\beta\omega(r)$$

So

$$\omega(r)\beta(s^m)_\alpha = \omega(r)\beta\omega(s\alpha m) + \omega(r\alpha s\beta m + m\alpha s\beta r)' + \omega(m)\alpha\omega(s)\beta\omega(r) \quad (2.6)$$

Now, consider,

$$\begin{aligned} \omega(r\alpha s\beta m + m\alpha s\beta r)' &= \omega\{(r + r' + r)\alpha s\beta m + m\alpha s\beta r\}' = \omega\{(r\alpha s\beta m) + (r + r')\alpha s\beta m + m\alpha s\beta r\}' = \\ &= \omega\{(r\alpha s\beta m) + s\alpha m\beta(r + r') + m\alpha s\beta r\}' = \omega\{(r\alpha s\beta m) + s\alpha m\beta r + s\alpha m\beta r' + m\alpha s\beta r\}' = \\ &= \omega\{r\alpha s\beta m + s\alpha m\beta r\}' + \omega\{s\alpha m\beta r' + m\alpha s\beta r\}' = \omega(r)\beta\omega(s\alpha m)' + \omega(s\alpha m)\beta\omega(r)' + \omega(m\alpha s\beta r' + s\alpha m\beta r) \end{aligned}$$

That is,

$$\omega(r\alpha s\beta m + m\alpha s\beta r)' = \omega(r)\beta\omega(s\alpha m)' + \omega(s\alpha m)\beta\omega(r)' + \omega(m\alpha s\beta r' + s\alpha m\beta r)$$

Combining this with (2.6), we obtain

$$\begin{aligned} \omega(r)\beta(s^m)_\alpha &= \omega(r)\beta\{\omega(s\alpha m) + \omega(s\alpha m)'\} + \omega(m)\alpha\omega(s)\beta\omega(r) + \omega(s\alpha m)\beta\omega(r)' \\ &\quad + \omega(m\alpha s'\beta r + s\alpha m\beta r) \end{aligned}$$

But  $\omega(s\alpha m) + \omega(s\alpha m)' \in Z(S)$  so we have

$$\begin{aligned} \omega(r)\beta(s^m)_\alpha &= (\omega(s\alpha m) + \omega(s\alpha m)' + \omega(s\alpha m)')\beta\omega(r) + \omega(m)\alpha\omega(s)\beta\omega(r) + \omega([s, m]_\alpha\beta r) \\ \omega(r)\beta(s^m)_\alpha &= (\omega(s\alpha m)')\beta\omega(r) + \omega(m)\alpha\omega(s)\beta\omega(r) + \omega([s, m]_\alpha\beta r) \\ \omega(r)\beta(s^m)_\alpha &= ((s_m)_\alpha)\beta\omega(r)' + \omega([s, m]_\alpha\beta r) \end{aligned} \quad (2.7)$$

Pre multiply (2.7) by  $(s^m)_\alpha\alpha$

$$(s^m)_\alpha\alpha\omega(r)\beta(s^m)_\alpha = (s^m)_\alpha\alpha(s_m)_\alpha\beta\omega(r)' + (s^m)_\alpha\alpha\omega([s, m]_\alpha\beta r)$$

Thus

$$(s^m)_\alpha\alpha\omega(r)\beta(s^m)_\alpha = (s^m)_\alpha\alpha\omega([s, m]_\alpha\beta r)$$

Post multiply (2.7) by  $\alpha(s_m)_\alpha$

$$\omega(r)\beta(s^m)_\alpha\alpha(s_m)_\alpha = (s_m)_\alpha\beta\omega(r)'\alpha(s_m)_\alpha + \omega([s, m]_\alpha\beta r)\alpha(s_m)_\alpha$$

By Lemma 5, we obtain

$$(s_m)_\alpha\beta\omega(r)\alpha(s_m)_\alpha = \omega([s, m]_\alpha\beta r)\alpha(s_m)_\alpha.$$

Hence Proved. □

**Lemma 2.4** Let  $\omega$  be a Jordan homomorphism from  $\Gamma$ -semiring  $S$  to 2-torsion free  $\Gamma$ -semiring  $T$ . If  $s, m, r \in S$  then

$$\omega(r\beta[s, m]_\alpha) = (s^m)_\alpha\beta\omega(r) + \omega(r)\beta(s_m)_\alpha$$

and

$$\omega([s, m]_\alpha\beta r) = \omega(r)\beta(s^m)_\alpha + (s_m)_\alpha\beta\omega(r).$$

**Proof:** To proof

$$\omega(r\beta[s, m]_\alpha) = (s^m)_\alpha\beta\omega(r) + \omega(r)\beta(s_m)_\alpha$$

Consider,

$$\begin{aligned} (s^m)_\alpha\beta\omega(r) + \omega(r)\beta(s_m)_\alpha &= \omega(s\alpha m)\beta\omega(r) + \omega(s)\alpha\omega(m)'\beta\omega(r) + \omega(r)\beta\omega(s\alpha m) + \omega(r)\beta\omega(m)\alpha\omega(s)' \\ &= \omega(s\alpha m\beta r + r\beta s\alpha m) + \omega(s)'\alpha\omega(m)\beta\omega(r) + \omega(r)\beta\omega(m)\alpha\omega(s)' = \omega(s\alpha m\beta r + r\beta s\alpha m + s\alpha m\beta r' + r\beta m\alpha s') \\ &= \omega(s\alpha m\beta(r+r') + r\beta s\alpha m + r\beta m\alpha s') = \omega\{(r+r')\beta s\alpha m + r\beta s\alpha m + r\beta m\alpha s'\} = \omega\{(r+r'+r)\beta s\alpha m + \\ & r\beta m\alpha s'\} = \omega(r\beta s\alpha m + r\beta m\alpha s') = \omega\{r\beta(s\alpha m + m\alpha s')\} = \omega(r\beta[s, m]_\alpha). \end{aligned}$$

Similarly,  $\omega([s, m]_\alpha\beta r) = \omega(r)\beta(s^m)_\alpha + (s_m)_\alpha\beta\omega(r)$ . This completes the proof.  $\square$

**Lemma 2.5** *Let  $\omega$  be a Jordan homomorphism from  $\Gamma$ -semiring  $S$  to 2-torsion free  $\Gamma$ -semiring  $T$ . If  $s, m, r \in S$  and  $\alpha, \beta \in \Gamma$  then*

$$(s_m)_\alpha\alpha\omega(r)\beta(s^m)_\alpha + (s^m)_\alpha\alpha\omega(r)\beta(s_m)_\alpha = 0.$$

**Proof:** By Lemma 7, we have

$$\omega(r\beta[s, m]_\alpha) = (s^m)_\alpha\beta\omega(r) + \omega(r)\beta(s_m)_\alpha$$

Replace  $r$  by  $[s, m]_\alpha\alpha r$  in above equation, we get

$$\omega([s, m]_\alpha\alpha r\beta[s, m]_\alpha) = (s^m)_\alpha\beta\omega([s, m]_\alpha\alpha r) + \omega([s, m]_\alpha\alpha r)\beta(s_m)_\alpha.$$

By (ii) of Lemma 4 and Lemma 6, we get

$$\omega([s, m]_\alpha)\alpha\omega(r)\beta\omega([s, m]_\alpha) = (s^m)_\alpha\beta\omega(r)\alpha(s^m)_\alpha + (s_m)_\alpha\alpha\omega(r)\beta(s_m)_\alpha. \quad (2.8)$$

Now, by using definition of Jordan homomorphism, we have

$$\begin{aligned} \omega([s, m]_\alpha) &= \omega(s\alpha m + m'\alpha s) = \omega(s\alpha(m+m') + m'\alpha s) = \omega(s\alpha m) + \omega(s\alpha m) + \omega(s)\alpha\omega(m') + \\ & \omega(m')\alpha\omega(s) = \{\omega(s\alpha m) + \omega(s)\alpha\omega(m')\} + \{\omega(s\alpha m) + \omega(m')\alpha\omega(s)\} = (s^m)_\alpha + (s_m)_\alpha. \end{aligned}$$

That is,  $\omega([s, m]_\alpha) = (s^m)_\alpha + (s_m)_\alpha$ .

Thus by (2.8), we obtain

$$\{(s^m)_\alpha + (s_m)_\alpha\}\alpha\omega(r)\beta\{(s^m)_\alpha + (s_m)_\alpha\} = (s^m)_\alpha\beta\omega(r)\alpha(s^m)_\alpha + (s_m)_\alpha\alpha\omega(r)\beta(s_m)_\alpha$$

So we have

$$\begin{aligned} (s^m)_\alpha\alpha\omega(r)\beta(s^m)_\alpha + (s^m)_\alpha\alpha\omega(r)\beta(s_m)_\alpha + (s_m)_\alpha\alpha\omega(r)\beta(s^m)_\alpha + (s_m)_\alpha\alpha\omega(r)\beta(s_m)_\alpha \\ = (s^m)_\alpha\beta\omega(r)\alpha(s^m)_\alpha + (s_m)_\alpha\alpha\omega(r)\beta(s_m)_\alpha \end{aligned}$$

Adding  $(m_s)_\alpha\alpha\omega(r)\beta(s_m)_\alpha + (m^s)_\alpha\alpha\omega(r)\beta(s^m)_\alpha$  on both sides

$$\begin{aligned} (m_s)_\alpha\alpha\omega(r)\beta(s_m)_\alpha + (m^s)_\alpha\alpha\omega(r)\beta(s^m)_\alpha + (s^m)_\alpha\alpha\omega(r)\beta(s^m)_\alpha + (s^m)_\alpha\alpha\omega(r)\beta(s_m)_\alpha \\ + (s_m)_\alpha\alpha\omega(r)\beta(s^m)_\alpha + (s_m)_\alpha\alpha\omega(r)\beta(s_m)_\alpha = (m_s)_\alpha\alpha\omega(r)\beta(s_m)_\alpha + (m^s)_\alpha\alpha\omega(r)\beta(s^m)_\alpha \\ + (s^m)_\alpha\beta\omega(r)\alpha(s^m)_\alpha + (s_m)_\alpha\alpha\omega(r)\beta(s_m)_\alpha. \end{aligned}$$

Therefore,

$$\begin{aligned} \{(m_s)_\alpha + (s_m)_\alpha\}\alpha\omega(r)\beta(s_m)_\alpha + \{(m^s)_\alpha + (s^m)_\alpha\}\alpha\omega(r)\beta(s^m)_\alpha + (s^m)_\alpha\alpha\omega(r)\beta(s_m)_\alpha \\ + (s_m)_\alpha\alpha\omega(r)\beta(s^m)_\alpha = \{(s^m)_\alpha + (m^s)_\alpha\}\alpha\omega(r)\beta(s^m)_\alpha + \{(s_m)_\alpha + (m_s)_\alpha\}\alpha\omega(r)\beta(s_m)_\alpha. \end{aligned}$$

But  $(s^m)_\alpha + (m^s)_\alpha = 0 = (s_m)_\alpha + (m_s)_\alpha$  so we get

$$(s_m)_\alpha\alpha\omega(r)\beta(s^m)_\alpha + (s^m)_\alpha\alpha\omega(r)\beta(s_m)_\alpha = 0.$$

Hence Proved.  $\square$

**Theorem 2.1** *Every Jordan homomorphism  $\omega$  of  $S$  onto 2-torsion free prime  $\Gamma$ -semiring  $T$  is either a homomorphism or an anti-homomorphism.*

**Proof:** By Lemma 8, we have

$$(s_m)_\alpha \alpha \omega(r) \beta (s^m)_\alpha + (s^m)_\alpha \alpha \omega(r) \beta (s_m)_\alpha = 0$$

By Lemma 3, either  $(s_m)_\alpha = 0$  or  $(s^m)_\alpha = 0$ . This implies that  $\omega(s\alpha m) = \omega(s)\alpha\omega(m)$  or  $\omega(s\alpha m) = \omega(m)\alpha\omega(s)$ .

Therefore  $\omega$  is either homomorphism or anti-homomorphism.  $\square$

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