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A Hybrid Algorithm for the Simulation of Burgers' Equation near the Inviscid Limit

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ABSTRACT: In this article, a hybrid algorithm has been proposed to solve numerically one dimensional homogeneous Burgers' equation when the viscosity approaches zero i.e, near inviscid limit. Due to its nonlinear nature, Burgers' equation describes a variety of phenomena, including turbulence, shock wave and it has numerous applications in the field of heat conduction, gas dynamics, elasticity, etc. The present hybrid scheme is the combination of method of lines, quasilinearisation technique and implicit trapezoidal rule along with the principle of superposition. Firstly, we apply method of lines to reduce the computational procedure and then linearised the non-linear equation using quasilinearisation method and the linearised equation are integrated by using implicit trapezoidal rule with the principle of superposition to obtain the solution. The majority of numerical methods reported in the literature for solving Burger's equation are unable to capture the physical behaviour when viscosity tends to zero. The main objective of the study to develop a hybrid numerical scheme for the simulation of Burger's equation near the inviscid limit. Problems are chosen to validate our results with the existing numerical data and analytical solutions which are found in the literature. Further, the proposed hybrid technique shows better accuracy with the analytical results, more flexible and computationally cost-effective.

Keywords: Burger equation, method of lines, quasilinearisation method, implicit trapezoidal rule, principle of superposition.

Contents

1. Introduction

The simulation of partial differential equations (PDE) including convective– diffusive term is a major research topic in the domain of computational fluid dynamics (CFD) over several decades in heat transfer and fluid mechanics. Therefore, a deep understanding is necessary to comprehend the mechanism of fluid flow and heat transfer as it plays a major role in advanced engineering system such as application of refrigeration, air conditioning, artificial heart transfer and dialysis systems. These applications are based on the idea of fluid flow and heat transfer. Thus, in a broader perspective, for all the physical phenomena, PDE plays an important role to analyze the system. Bateman [1] first introduced such type of nonlinear PDE along with initial and boundary conditions as follows:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < L, \quad 0 < t < T \tag{1.1}$$

$u(x,0) = \psi(x)$ and $u(0,t) = \xi_1(t)$, $u(L,t) = \xi_2(t)$, $t \in (0,T)$. where ν is kinematic viscosity and $\psi(x)$, $\xi_1(t)$ and $\xi_2(t)$ are known functions.
After that Burger [2] explained the feature of turbulent flow caused by the effects of convection and diffusion that are opposite terms presents in the equation while considering flow in a channel. For this

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contribution of Burger, the equation (1.1) is well known as the Burger equation. The structure of Burger's equation is remarkably similar to that of Navier-Stokes' equation because of the existence of the non-linear convection and diffusion terms along with viscosity term. Therefore, Burger's equation is also seen as a simplified version of Navier-Stokes' equation. The physical behavior of the nonlinear Burgers' equation has captured the interest of many researchers for decades due to its applicability in several domains like shock wave theory, gas dynamics, heat conduction, traffic flow, elasticity, cosmology, and so on [3].
A lot of effort has been invested to capture the phenomena for the solution of the Burgers' equation. Due to the slow convergence rate of the analytical solution containing infinite series representation, various attempts have been made to solve numerically the Burgers' equation. Hassanien et al. [4] formulated a fourth-order finite difference method (FED) for solving the nonlinear one-dimensional Burgers equation. They compared the results with existing methods with great accuracy. Tabatabaei et al. [5] developed multi-symplectic box methods for solving Burgers' equation to get better results with no artificial wiggles. A numerical approach based on Crank-Nicolson finite difference was introduced by Kadalbajoo and Awasthi [6] to solve the one-dimensional Burgers' equation. The generated numerical results were compared with the precise solution for various Reynolds number values. This method has demonstrated unconditionally stable and second-order precision in space and time. Rahman et al.[7] proposed liberalized semi-implicit finite-difference method and predicted results gave excellent agreement with the exact solution. Using the finite element method (FEM), Ozis et al. [8] produced numerical results for various ν values. The results were found to be in good agreement when compared to Cole's analytical solution. Kutluay and Bahadir [9] constructed finite-difference approximations based on the standard explicit method to solve the one-dimensional Burgers equation. Also, an FEM-based Galerkin method was proposed by Dogan [10] for the simulation of Burger's equation. This method is capable of considering a wide range of viscosity values. Mittal and Singhal [11] designed a solution technique to solve Burger's equation by converting it into a system of nonlinear ordinary differential equations. The proposed technique produced very accurate results in comparison to FDM and FEM. Further, a polynomial differential quadrature method (PDQM) was developed by Mittal and Jiwari [19] to determine the numerical solutions of certain nonlinear PDEs of the Burgers' type. The nonlinear PDEs were transformed into a system of nonlinear ODEs using the PDQM, and the Runge-Kutta fourth-order (RK4) method was used to solve the resulting system.. Further, Jiwari et al. [20] proposed a numerical scheme based on the weighted average differential quadrature method (WADQM) where time derivative was discretized by forward difference method, and then, the quasilinearization process was used to tackle the nonlinearity part. Then the fully discretized system of linear equations was solved by the Gauss-elimination method. Furthermore, a numerical scheme to solve the time-dependent Burgers' equation was introduced by Mittal et al. [21] with the combination of quasilinearization to tackle the nonlinearity part and semi-discretization for spatial direction using the differential quadrature method (DQM) to convert a system of first-order ODEs, which are solved by the RK4 method. All the methods [11,19-21] were compared with existing methods and found to have better accuracy. Ali et al.[12] designed a FEM-based collocation with cubic B-splines, Saka and Dag [13] described a collocation method using quartic B-splines, Aksan [14] applied FEM-based quadratic B-splines, and Kutluay et al. [16] created a FEM-based least-squares quadratic B-spline method to solve one-dimensional Burgers' equation. Obtained results for the methods [12-16] were compared with the analytical solutions and were demonstrated a very good level of agreement. Asaithambi [16] computed the solution to the 1D Burgers' equation by using recursive formulas to find the values of the derivatives in the Taylor series. Korkmaz and Dag [17] and Dag introduced the polynomial-based differential quadrature method (PDQ) to simulate nonlinear Burgers' equation. Abdou and Soliman [22] developed a variational iteration (VIM) method to solve the Burger's and coupled Burger's equations and compared it with the Adomian decomposition method (ADM). It showed that VIM is more effective than the ADM method. Further, they used the VIM method to overcome the difficulty arising in calculating Adomian polynomials. Rashidi et al. [23] implemented the homotopy analysis method (HAM) to find predicted results of the Burger equations. The method illustrates great potential in solving and validating the solution. Jiwari [24] invented a numerical scheme based on uniform Haar wavelets and the quasilinearization process for the simulation of the time-dependent nonlinear Burgers' equation. Further, Jiwari [25] formulated a modified numerical hybrid scheme based on the Euler implicit method, quasilinearization, and uniform Haar wavelets for the numerical solutions of Burgers' equation

to overcome the drawback of [24]. He claimed that the use of a uniform Haar wavelet makes the hybrid scheme more accurate, simple, fast, flexible, convenient, and less costly in computations.

Numerical techniques that are reviewed for solving Burgers' equation are unable to capture the physical behaviour when kinematic viscosity ν tends to zero. Recently, Jiwari [25] introduced a hybrid scheme based on the Euler implicit method, which is of first-order accuracy. However, the scheme is unable to predict the numerical results near the inviscid limit. To overcome the drawback, we have developed a new hybrid algorithm based on the trapezoidal rule, a second-order accurate and A-stable [31] implicit method. As the solution of Burgers' equation develops a discontinuity (shock wave) near the inviscid limit (i.e., when viscosity $\nu \to 0$), even from smooth initial conditions. In this situation, the implicit trapezoidal method reduces numerical oscillations and yields smoother and more stable solutions.
Therefore, the main objective of the study is to develop a hybrid, more stable algorithm to predict the numerical solution for the Burger equation for smaller values of ν and to capture the physical phenomena when ν tends to zero. Thus, in this study, we have solved numerically the one-dimensional homogeneous Burgers' equation near the inviscid limit by introducing a new second-order hybrid algorithm, which is the combination of the method of lines, the quasilinearization technique, and the implicit trapezoidal rule along with the principle of superposition.

2. Solution Method

The Burgers' equation (1.1) is a nonlinear and parabolic type partial differential equation. To obtain approximate solutions a numerical algorithm combining with method of lines, quasilinearisation method, implicit trapezoidal rule has been developed. To reduce the computational procedure, firstly method of lines has been introduced and let, L be the left hand side of the equation (1.1), then

$$L = \left(-u\frac{\partial u}{\partial x} + \nu\frac{\partial^2 u}{\partial x^2}\right)$$

and after discretisation at $(i+\frac{1}{2}, j)$, the equation (1.1) can be written as

$$\text{i.e,}\quad \frac{(u_{j+1}-u_j)}{\Delta t} = \frac{1}{2}\left\{\left(-u\frac{\partial u}{\partial x} + \nu\frac{\partial^2 u}{\partial x^2}\right)_{j+1} + L_j\right\}$$

After using simplification, we obtain

$$u^{''}_{j+1} = \frac{1}{\nu}\left(\frac{2u_{j+1}}{\Delta t} + u_{j+1}u^{'}_{j+1}\right) - \frac{1}{\nu}\left(L_j + 2\frac{u_j}{\Delta t}\right) \tag{2.1}$$

where, $u^{'} = \frac{\partial u}{\partial x}$ and $u^{''} = \frac{\partial^2 u}{\partial x^2}$. The non linear term $u_{j+1}u^{'}_{j+1}$ in equation (2.1) is expanded about a nominal solution using quasilinearization method. After simplification and rearranging the equation (2.1) can be written as follows

$$u^{''(k+1)}_{j+1} = A^{(k)}u^{'(k+1)}_{j+1} + B^{(k)}u^{(k+1)}_{j+1} + C^{(k)} \tag{2.2}$$

$$\begin{aligned}
A^{(k)} &= \frac{1}{\nu}\,u^{(k)}_{j+1} \\
B^{(k)} &= \frac{1}{\nu}\left(\frac{2}{\Delta t} + u^{'(k)}_{j+1}\right) \\
C^{(k)} &= -\frac{1}{\nu}u^{(k)}_{j+1}u^{'(k)}_{j+1} - \frac{1}{\nu}\left(L + \frac{2u}{\Delta t}\right)^{(k)}_{j}
\end{aligned}$$

Equation (2.2) can be written in the form of two first order differential equations as,

$$\left[u^{'}\right]^{k+1}_{j+1} = [p]^{k+1}_{j+1} \tag{2.3}$$

$$\left[p^{'}\right]_{j+1}^{k+1} = A^{(k)}\left[p\right]_{j+1}^{k+1} + B^{(k)}\left[u\right]_{j+1}^{k+1} + C^{(k)} \tag{2.4}$$

To obtain required solution, the equations (2.3) and (2.4) are integrated with the help of trapezoidal rule along with the principle of superposition which convert the boundary value problems into initial value problems [26]. For the first-order ordinary differential equation, the implicit trapezoidal rule can be written as

$$z_{m+1} = z_m + \Delta z_m\left(z_{m+1}^{'} + z_m^{'})\right). \tag{2.5}$$

where m denotes the grid points and Δz_m corresponds to the step size. Using the trapezoidal rule (2.5) on the equations (2.3) and (2.4), two equations have been obtained in terms of unknown variables u and $u^{'}$. After solving these equations iteratively and applying the principle of superposition, the desired numerical solution has been obtained for the proposed algorithm.

3. Results and Discussion

In the present study of simulation of the 1D Burgers' equation by using a second-order novel hybrid algorithm, three problems have been considered whose analytical solutions and numerical results are existed in the literature to validate the predicted results and to analyze the characteristics of the predicted solutions for various values of ν at different time steps. Further, the author's interest is to measure the efficiency and performance of the proposed algorithm. All the results have been computed by developing code with the help of Python (version-3.8) software.

3.1. Problem 1

In the section, the Burger's equation (1.1) has been considered with the following conditions: $u(0,t) = 0 = u(1,t)$, $t > 0$ and $u(x,0) = \sin(\pi x)$, $\quad x \in (0,1)$
The analytical solution is given by [27]

$$u(x,t) = \frac{2\pi\nu\sum_{n=1}^{\infty} a_n exp(-n^2\pi^2\nu t)n\sin n\pi x}{a_0 + \sum_{n=1}^{\infty} a_n exp(-n^2\pi^2\nu t)\cos n\pi x} \tag{3.1}$$

where, $a_0 = \int_0^1 e^{-\frac{(1-\cos(\pi x))}{2\pi\nu}} dx$ and $a_n = 2\int_0^1 e^{-\frac{(1-\cos(\pi x))}{2\pi\nu}} \cos(n\pi x)\, dx, \quad n \in \boldsymbol{N}$

The predicted numerical results are illustrated in the Tables 1-3 and in Figure 1 for the problem 1. Table 1 is displayed the numerical results for various grid sizes with $\nu = 1$, $\Delta t = 10^{-5}$ and at time $t = 0.1$. Further, the results has been compared with the exact solution. It is noted that for smaller values of h, the numerical results offer almost the same values for different grid sizes and also predict closer results with the analytical findings. Therefore, we have chosen the grid size $h = 0.0125$ as grid independent, and the remaining calculations in this work have been done with the grid size $h = 0.0125$ for all the calculations. Table 2 represents the comparison between the predicted numerical values and exact results for $\nu = 1$ and $\nu = 0.01$ at different time. From Table 2, it has been cleared that the predicted numerical values are almost nearly the exact results. Table 3 has been displayed for the results comparison with the existing results [28,29,15,30] $\nu = 1.0$ at different time steps. All the result comparisons have demonstrated that the current approach offers better results as compared to others' findings. Figure 1 has been plotted for the predicted numerical results in the time range $0 \leq t \leq 2$ with $\Delta t = 0.0001$, $\nu = 0.1$. The graph indicates that with the increase in time, the approximated solutions become flatter, and near time step $t = 3$, it's almost parallel to the x-axis. This behavior shows that diffusion always wins in the long-time regime when $\nu > 0$. Thus, Figure 1 shows (i) a steep profile at smaller times and a gradually flatter profile as time tends to larger values. This is expected behavior with the physical characteristics.

3.2. Problem 2

The Burger's equation (1.1) has been considered with the following conditions $u(0,t) = 0$ and $u(1,t) = 0$ for $t > 0$ and $u(x,0) = 4x(1-x)$ for $x \in (0,1)$. The analytical solution [27] is same as equation (3.1) with the following coefficients

$a_0 = \int_0^1 e^{-\frac{x^2(3-2x)}{3\nu}} dx$ and $a_n = 2\int_0^1 e^{-\frac{x^2(3-2x}{3\nu}} \cos{(n\pi x)} dx, \quad n \in \mathbf{N}$

All the predicted results are presented in tabular and graphical form in the Table 4-6 and in the Figures 2-3. Table 4 describes the predicted results for the problem 2 and are compared with the analytical solution for $\nu = 1.0$, 0.01 for different time steps. It has been observed that the predicted results have shown very good approximation with the analytical solutions. Table 5 have been presented the results comparison between the numerical outcomes of the proposed hybrid method and the existing results [24] for $\nu = 0.1, 0.01$and 0.005. In Table 6, the computed results for $\nu = 0.004$, 0.003 are compared with the results given in [24,25]. It has been observed from Table 5 and Table 6 that the predicted outcomes of the present work are more accurate as compared to others methods. Figure 2 and 3 have been plotted to see the behavior of the predicted numerical solutions for $\nu = 0.002$ and 0.005 respectively for the problem 2. It shows from the Fig. 2 that the predicted solutions get steeper after a certain time level $t = 0.2$. The physical behavior of the predicted solutions plotted in Fig.3 is quite similar as shown in Fig.2 and both the figures are following the same pattern as Fig.1.

3.3. Problem 3

Here the Burger's equation (1.1) has been considered along with the initial and boundary conditions $u(x, 0) = \frac{2\nu\pi \sin \pi x}{a+\cos \pi x}$, $u(0, t) = 0 = u(1, t)$, $t > 0$. The analytical results [20] is $u(x, t) = \frac{2\nu\pi e^{-\pi^2\nu t} \sin \pi x}{a+2\nu\pi e^{-\pi^2\nu t} \cos \pi x}$, $a > 0$

Table 7 has been illustrated the predicted numerical results near the inviscid limit (i.e., when $\nu \to 0$) for various smaller values of ν =0.001, 0.0001, 0.00001 and 0.000001 with time $t = 0.004$, $a = 2$ and $\Delta t = 0.0001$. It has been observed that with the increase in the time step t, solutions decrease very slowly, whereas with the decrease in ν the approximated solutions, they decrease very fast, i.e., the solutions become flatter. Further, the approximated solutions become closer to zero as ν tending to zero. The numerical predictions for the problem 3 are displayed in Fig. 4 for various values of $\nu = 0.1$, 0.2, 0.5, 0.8, 1.0 with a fixed time step $t = 0.004$. It is obvious from the figure that with the increase in ν the solution, it gradually becomes steeper, and with the decrease of kinematic viscosity, the stiffness becomes flatter, which is expected from the physical characteristics. Figure 5 plotted the physical characteristics of the predicted numerical solution of Problem 3 in a three dimension plot for $\nu = 0.1$, $a = 2$ with $\Delta t = 0.0001$ at time step $t = 0.5$ that is of parabolic type as expected with the physical characteristics. Therefore, the presents study shows that the proposed hybrid method has been illustrated the physical characteristics in the broader range, ν including near inviscid limits, more accurately.

4. Conclusions

In the present article, non-linear Burger equation has been solved numerically by developing a numerical algorithm based on method of lines, quasilinearization technique and implicit trapezoidal rule along with principle of superposition. The proposed algorithm has been tested on three different problems. The predicted results are compared with the exact solution and with other numerical methods exist in the literature. The obtained results of the proposed scheme shows quite satisfactory with the analytical solution as compared to other existing solutions. The following conclusions are made about the proposed algorithm:

1. The use of the present hybrid algorithm to compute the numerical solution of the nonlinear Burger's equation is a novel method as per the author's knowledge.

2. The proposed algorithm is able to describe the natural phenomena of Burger's equation when kinematic viscosity ν tends to zero. There is no development of such a type of numerical algorithm in the literature for $\nu \to 0$.

3. The present algorithm offers more accurate results as compared to other numerical methods as discussed in the present work.

4. The present hybrid scheme is easy to evaluate and can be used widely in different sets of differential equations obtained from practical problems. Further, the scheme can be used for the system of equations when it behaves like stiff differential equations.

5. Further, the proposed hybrid technique is simple, requires less calculation, is more flexible, and is computationally cost-effective.

Table 1: Results comparison between numerical outcomes and analytical solution of Problem 1 with $\nu = 1$ and $\Delta t = 10^{-5}$ for different grid sizes at time $t = 0.1$.

			h		
x	0.05	0.025	0.0125	0.01	Exact
0.1	0.109066	0.109335	0.109402	0.10941	0.109538
0.2	0.208868	0.209403	0.209537	0.209553	0.209792
0.3	0.290563	0.291355	0.291552	0.291576	0.291896
0.4	0.346261	0.347277	0.34753	0.347561	0.347924
0.5	0.369706	0.370883	0.371177	0.371212	0.371577
0.6	0.357137	0.35837	0.358678	0.358714	0.359046
0.7	0.308174	0.309317	0.309602	0.309636	0.309905
0.8	0.226492	0.227381	0.227603	0.22763	0.227817
0.9	0.119966	0.120454	0.120576	0.120591	0.120687

Table 2: Results comparisons between numerical outcomes and analytical solution of Problem 1 for $h = 0.0125$, $\nu = 1.0$, $\nu = 0.01$ and $\Delta t = 10^{-5}$ at different time steps.

x	t	Exact	Result	x	t	Exact	Result
		$\nu = 1.0$				$\nu = 0.01$	
0.25	0.10	0.253638	0.253333	0.25	0.10	0.566328	0.566109
	0.15	0.156601	0.156354		0.15	0.512148	0.511908
	0.20	0.096442	0.096242		0.20	0.466583	0.466346
	0.25	0.059218	0.059063		0.25	0.427995	0.427771
0.5	0.10	0.371577	0.371177	0.5	0.10	0.947414	0.947123
	0.15	0.226824	0.226459		0.15	0.900098	0.899672
	0.20	0.138473	0.138178		0.20	0.848365	0.847878
	0.25	0.084538	0.084312		0.25	0.796762	0.796272
0.75	0.10	0.272582	0.272321	0.75	0.10	0.860134	0.860031
	0.15	0.164369	0.164099		0.15	0.922756	0.922085
	0.20	0.099435	0.099216		0.20	0.961891	0.960572
	0.25	0.060347	0.060183		0.25	0.974689	0.972968

Table 3: Results Comparison between numerical outcomes and analytical solution of Problem 1 for $h = 0.0125$, $\nu = 1.0$, and $\Delta t = 10^{-5}$ at different time step.

x	t	Exact	[28]	[29]	[15]	[30]	Results
0.25	0.4	0.308894	0.317062	0.308776	0.31215	0.30415	0.306386
	0.6	0.240739	0.248472	0.240654	0.2436	0.23629	0.23922
	0.8	0.195676	0.202953	0.195579	0.19815	0.1915	0.194707
	1.0	0.162565	0.169527	0.162513	0.16473	0.15861	0.161911
0.5	0.4	0.569632	0.583408	0.569527	0.57293	0.56711	0.568424
	0.6	0.447206	0.461714	0.447117	0.40588	0.4436	0.446911
	0.8	0.359236	0.3738	0.359161	0.36286	0.35486	0.35878
	1.0	0.291916	0.306184	0.291843	0.29532	0.2871	0.291035
0.75	0.4	0.625438	0.638847	0.625341	0.63038	0.61874	0.629726
	0.6	0.487215	0.506429	0.487089	0.49268	0.47855	0.488624
	0.8	0.373922	0.393565	0.373827	0.37912	0.36467	0.373845
	1.0	0.287474	0.305862	0.029726	0.03038	0.2786	0.286844

Table 4: Results Comparison between numerical outcomes and analytical solution of Problem 2 at different time step for $h = 0.0125$, $\nu = 1.0, 0.01$, and $\Delta t = 10^{-5}$.

x	t	Exact	Results	x	t	Exact	Results
		$\nu = 1$				$\nu = 0.01$	
0.25	0.1	0.26148	0.261257	0.25	0.1	0.607363	0.607353
	0.15	0.161478	0.161255		0.15	0.549421	0.549407
	0.2	0.09947	0.099278		0.2	0.499828	0.499809
	0.25	0.061088	0.06093		0.25	0.457413	0.457384
0.5	0.1	0.383422	0.383026	0.5	0.1	0.956007	0.955986
	0.15	0.234055	0.233694		0.15	0.914426	0.914382
	0.2	0.142888	0.142595		0.2	0.867136	0.867078
	0.25	0.087233	0.087001		0.25	0.818337	0.818274
0.75	0.1	0.281573	0.281221	0.75	0.1	0.886767	0.886654
	0.15	0.169738	0.169445		0.15	0.938437	0.938277
	0.2	0.102655	0.102431		0.2	0.969741	0.969507
	0.25	0.06229	0.062119		0.25	0.979469	0.979193

Table 5: Results Comparison between numerical outcomes and analytical solution of Problem 2 at different time step for various ν values at various time levels with $\Delta t = 0.001$.

x	t	[25]	Results	Exact	[25]	Results	Exact	[25]	Results	Exact
			$\nu = 0.1$			$\nu = 0.01$			$\nu = 0.005$	
0.8	0.4	0.59125	0.590962	0.59135	0.94859	0.948684	0.94863	0.95917	0.959275	0.95849
	0.6	0.45732	0.456945	0.45739	0.82369	0.823740	0.82372	0.83035	0.830411	0.83015
	0.8	0.34555	0.344586	0.34559	0.70194	0.701963	0.70195	0.76656	0.706586	0.76612
	1.0	0.26151	0.261722	0.26153	0.60478	0.604785	0.60478	0.60826	0.608267	0.60826
	3.0	0.02548	0.025498	0.02548	0.24193	0.241937	0.24190	0.24397	0.243951	0.24395
0.9	0.4	0.36528	0.365853	0.36546	0.96342	0.963049	0.96299	0.98311	0.983035	0.98234
	0.6	0.27811	0.277813	0.27823	0.89488	0.894872	0.89477	0.90421	0.904160	0.90398
	0.8	0.20449	0.203948	0.20455	0.77663	0.776537	0.77638	0.78246	0.782493	0.78348
	1.0	0.15094	0.150908	0.15097	0.67360	0.673353	0.67312	0.67810	0.676110	0.67615
	3.0	0.01352	0.013543	0.01352	0.24621	0.245280	0.24563	0.24334	0.273051	0.27291

Table 6: Results Comparison among numerical outcomes with existing numerical solutions to Problem 2 with $\nu = 0.003$ and 0.004 at various time scales with $\Delta t = 0.001$.

x	t	[24]	[25]	Results	Exact	[24]	[25]	Results	Exact
			$\nu = 0.004$				$\nu = 0.003$		
0.25	1	0.19636	0.19640	0.19638	0.19641	0.19668	0.19673	0.196717	0.19673
	5	0.04744	0.04744	0.047438	0.04747	0.04746	0.04747	0.047464	0.04748
	10	0.02434	0.02434	0.024342	0.02434	0.02434	0.02434	0.024350	0.02434
	15	0.01637	0.01637	0.016371	0.01637	0.01637	0.01637	0.016375	0.01637
0.50	1	0.38842	0.38850	0.388491	0.38846	0.38890	0.38898	0.388968	0.38894
	5	0.09491	0.09487	0.094860	0.09493	0.09491	0.09491	0.094911	0.09494
	10	0.04868	0.04868	0.048683	0.04869	0.04870	0.04869	0.048698	0.04871
	15	0.03270	0.03271	0.032708	0.03270	0.03274	0.03275	0.032748	0.03274
0.75	1	0.57312	0.57320	0.573201	0.57315	0.57375	0.57383	0.573832	0.57378
	5	0.14224	0.14225	0.142248	0.14225	0.14232	0.14233	0.142323	0.14234
	10	0.07258	0.07258	0.072590	0.07257	0.07298	0.07299	0.072990	0.07297
	15	0.04696	0.04697	0.046964	0.04695	0.04857	0.04857	0.048574	0.04857

Table 7: Predicted numerical results of Problem 3 at different time step for smaller values of ν with time step $\Delta t = 0.0001$.

	$\nu = 0.001$			$\nu = 0.0001$		
x	$t = 0.1$	$t = 0.5$	$t = 1.0$	$t = 0.1$	$t = 0.5$	$t = 1.0$
0.25	0.00161114	0.00160677	0.0016013	0.00016407	0.00016403	0.00016397
0.5	0.00311991	0.00310744	0.00309193	0.00031789	0.00031776	0.00031759
0.8	0.00307264	0.0030525	0.00302758	0.00029148	0.00029132	0.00029111
	$\nu = 0.00001$			$\nu = 0.000001$		
x	t=0.1	t=0.5	t=1.0	t=0.1	t=0.5	t=1.0
0.25	0.00001927	0.00001927	0.00001927	0.00000479	0.00000479	0.00000479
0.5	0.00003742	0.00003741	0.00003741	0.00000937	0.00000937	0.00000937
0.8	0.0000129	0.0000129	0.00001291	-0.00001496	-0.00001496	0.00001496

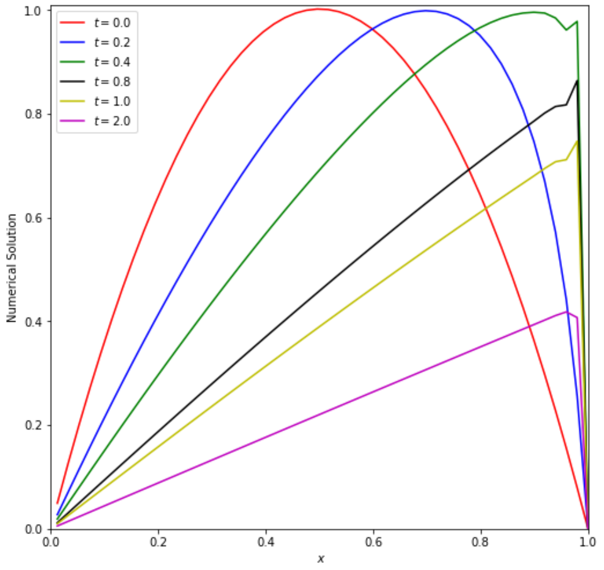

Figure 1: Predicted numerical solution for the velocity profile of Problem 1 with $\Delta t = 0.0001$ for $\nu = 0.1$ at various time level.

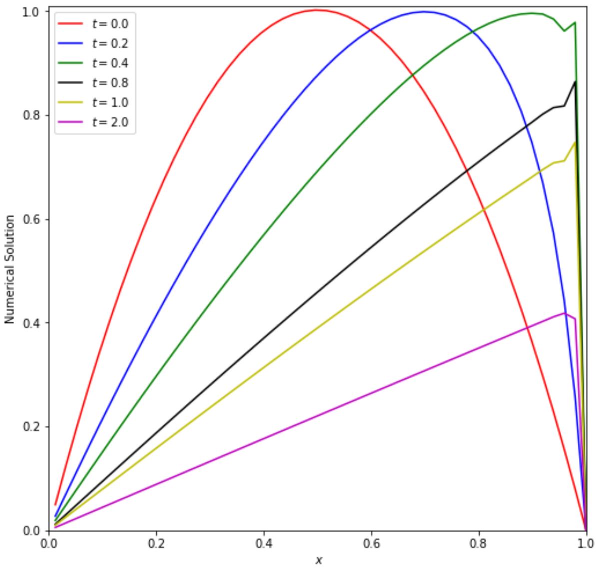

Figure 2: Physical characteristics of the predicted numerical solution of Problem 2 with $\Delta t = 0.0001$ for $\nu = 0.002$.

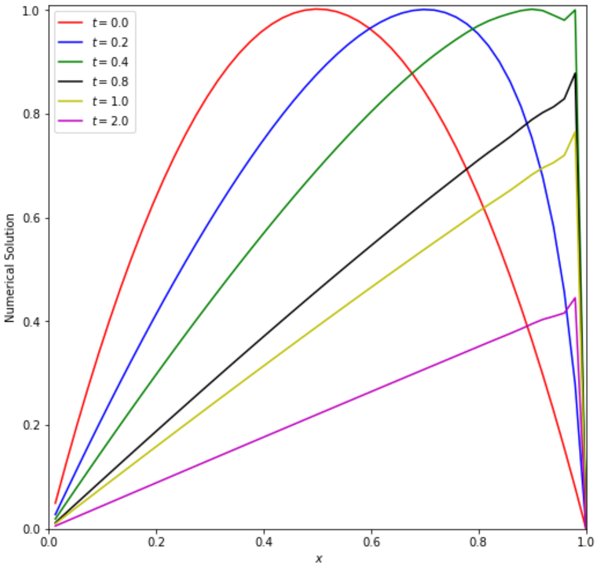

Figure 3: Physical characteristics of the predicted numerical solution of Problem 2 with $\Delta t = 0.0001$ for $\nu = 0.005$.

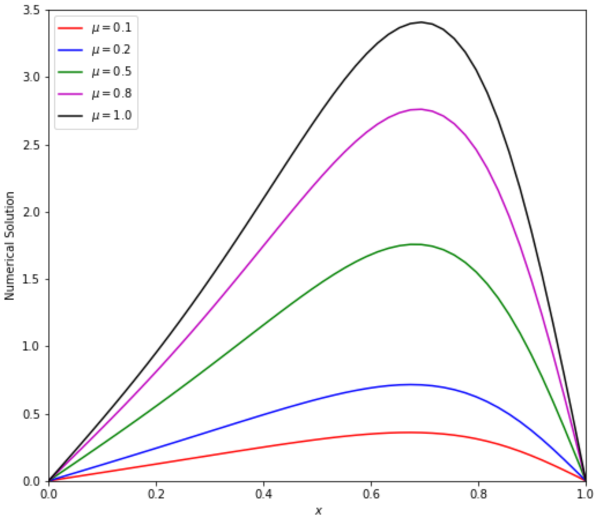

Figure 4: Physical characteristics of the predicted numerical solution of Problem 3 for different ν at time $t = 0.004$ with $a = 2$ with $\Delta t = 0.0001$.

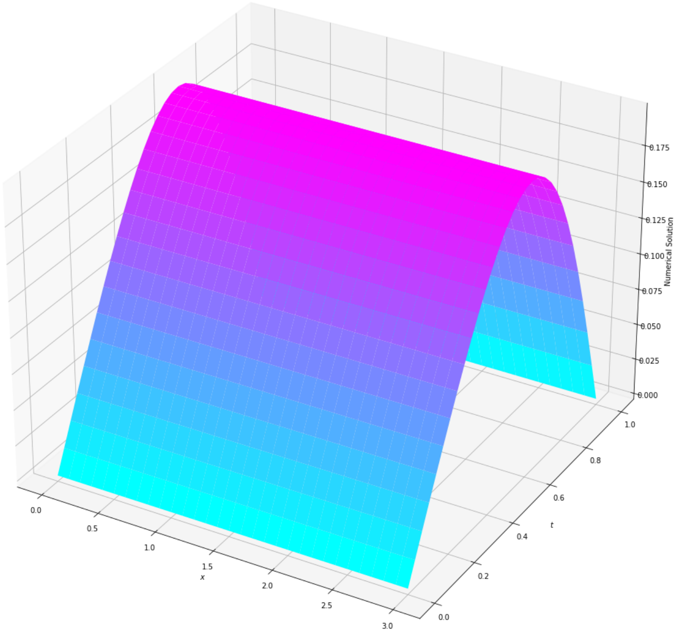

Figure 5: Physical characteristics of the predicted numerical solution of Problem 3 for $\nu = 0.1$, $a = 2$ with $\Delta t = 0.0001$ at time $t = 0.5$.

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