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New Ranking Method for Solving Fuzzy Linear Fractional Programming Problems

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ABSTRACT: This paper presents a new ranking function method that is used as a defuzzification technique to convert any fuzzy number to a real number using mean measures such as arithmetic mean and Heronian mean for the sub-intervals that are generated according to the fuzzy numbers. This is easy to calculate and can be used for any type of fuzzy number, such as a triangular fuzzy number, a trapezoidal fuzzy number, a pentagonal fuzzy number, etc. This technique is used in fuzzy optimization programming problems when the coefficients are fuzzy numbers that are often forced by researchers when creating business and economic problems. The benefits of this technique are shown in several examples in linear fractional programming problems, and a comparison table explains its effects by comparing it with many previous methods, and it can be used for linear programming problems and quadratic programming problems.

Key Words: Linear fractional programming problems, fuzzy linear fractional programming problems, fuzzy coefficients, heronian mean, charnes and cooper's method.

Contents

1. Introduction

Nonlinear optimization is widely used in real-world applications of decision-making problems such as financial, marketing, and manufacturing planning. In traditional linear fractional programming (LFP) problems, it is presumed that the parameters are precisely known. However, due to market conditions and quick changes in measurements, scientists and researchers in the field of optimization problems frequently turn to uncertainty sets, particularly fuzzy sets, to deal with these problems because most of the time, the values of the parameters in the current problem are not exact or are obscure due to imperfect information. Sometimes problems cannot be controlled; fuzzy numbers are required in this case. This study focuses on a significant and current topic that has been of great benefit to most applied fields, this includes these problems when the coefficients of decision variables are fuzzy numbers. So it is necessary to propose a way to solve these problems scientifically and practically because it can be used in many modern business companies, such as oil companies, soft drink companies, pharmaceutical companies, etc.

In the past decades, several novel methods and techniques have emerged to solve nonlinear programming optimization such as linear fractional programming (LFP) problems and fuzzy linear fractional

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programming problems (FLFP) such as [1] introduced a method to transform LFP into linear programming (LP) problem. [2,3] suggested a new procedure and method for solving LFP problems. [4] in his book, he described LFP theory, methods, applications and software in detail. [5] proposed an iterative algorithm to solve a LFP problem. [6] proposed a new procedure to explain the multi-objective fractional programming problem using the mean deviation and point-slopes formula. [7] suggested a new approach to solving the LFP problem with rough interval coefficients in the objective function. [8] proposed new technique to solving non-linear fractional programming problems by using geometric mean. [9] suggested and derived a novel technique to solve fully fuzzy linear fractional programming under triangular fuzzy number.

The idea of fuzzy numbers was originated by [10], and fuzzy linear programming (FLP) was initially suggested by [11]. [12] introduced fully fuzzy linear programming (FFLP). Here in the literature are mentioned several of these problems that contribute to this work. [13] presented a method for solving fully fuzzy linear fractional programming (FFLFP) problems. [14] solved FLFP using a fuzzy optimization programming methodology. [15] suggested a new approach for solving FFLFP problems utilizing multi-objective linear programming. [16] presented a numerical approach to solving the LFP problem in a fuzzy environment. [17] studied a novel method to solve fully fuzzy multi-objective LFP problems without a crisp problem. [18] studied a multi-objective fuzzy inventory model and made a real case under different solution methods. [17] suggested a new method to solve a fully fuzzy multi-objective linear fractional programming problem under some limitations and using a ranking function. [19] suggested a new efficient method to solve fully fuzzy transportation problems. [20] used fuzzy harmonic mean to solve FFLP for the multi-objective function. [21] presented an efficient algorithm for explaining the FLFP problem by using a function of ranking. [22] worked on FLFP in real life using fuzzy set theory because the parameters are expressed in TrFNs that have been converted into three forms: upper, middle, and lower, with the errors of [23] are discussed. In the same way [24] made connections between fuzzy numbers and real-life problems. A different ranking method is used for trapezoidal fuzzy coefficients with score functions. [25] solved the FFLFP problem with an aspect-life problem in 2022 using efficient ranking function methods. [26] modified symmetric fuzzy technique to solve LFP problem for more than one objective function under fuzzy numbers. [9] proposed a new process for resolving FFLFP problem where the coefficients of the variables and decision variables are triangular fuzzy number using parametric approach. [27] presented an alternative approach to solve FFLFP problems under fuzzy optimization. [28] and [29] proposed a new approach to solve multi-objective fuzzy fractional programming problem. [30] suggested a novel technique to solve fuzzy linear fractional programming problem using the lexicography method. [31] suggested a new ranking function method to solve fuzzy non-linear programming problems using Kuhn Tucker condition. [32] proposed an innovative method to solve fuzzy linear fractional programming problems. [33] this work reviews all the basic concepts and previous study on fractional programming and generalized fuzzy linear fractional programming problems.

In the present study, a new technique is suggested to solve LFP problems when the fuzzy numbers are coefficients of objective function and constraints. This method is one of the most attractive and convenient methods by combining the previous two ideas, namely the Zadeh extension principle and decomposition method as a basis, with some restrictions that, according to several definitions and theorems, systematically use the Zadeh extension to convert the basic problem into multi-linear fractional programming problems. After that, the optimal solution is obtained for each LFP problem using the Charnes and Cooper approach. For decision-making and implementation of the method, an example is given.

The framework of this paper is as follows: Section 2 goes over some previous definitions and concepts of the fuzzy set and the LFP problems. Section 3 describes the forms used. In Section 4, we show the method of converting FLFP problems into the multi-objective LFP problems discussed and derive the ranking function approach to each LFP problem to obtain an optimal solution. In Section 5, the algorithm is presented. Numerical examples and table comparison are demonstrated in Section 6. The discussion is given in Section 7. The final section has a conclusion.

2. Preliminaries Notions

This section provides particular concepts and the basic ideas of fuzzy numbers.

Definition 2.1 *[34] A* ***universal set****, denoted by the symbol U, is a set that contains all possible elements or objects under consideration in a particular context.*

Definition 2.2 *[9] Consider the universal set X. The ordered pairings $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) | x \in X\}$ of A are then a subset of X called the fuzzy set, where the membership function is $\mu_{\widetilde{A}} : X \to [0, 1]$.*

Definition 2.3 *[14] The support of a fuzzy set $\widetilde{A}$ is the set of all points $x \in U$ such that $\mu_{\widetilde{A}}(x) > 0$.*

Definition 2.4 *[15] A fuzzy set $\widetilde{A}$ is said to be* ***normal*** *if $\mu_{\widetilde{A}}(x) = 1$ for at least one $x \in U$.*

Definition 2.5 *[15] A fuzzy set $\widetilde{A}$ on X is* ***convex*** *if and only if $\mu_{\widetilde{A}}(\alpha x_1 + (1 - \alpha)x_2) \geq \min(\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2))$, $\forall\, x_1, x_2 \in U$ and $\alpha \in [0, 1]$.*

Definition 2.6 *[9] A fuzzy number $\widetilde{A}$ is defined as a fuzzy subset that is convex, normal, and has bounded support of the set $\mathbb{R}$.*

Definition 2.7 *[35] The α-cut or α-level set of a fuzzy set is a* ***crisp set*** *defined by $\widetilde{A}_\alpha = \{x \in \mathbb{R} / \ \mu_{\widetilde{A}}(x) \geq \alpha\}$, where $\alpha \in [0, 1]$.*

Definition 2.8 *[36] If x^0 is feasible and there are no alternative solutions x^*, such as $Cx^0 \neq Cx^*$ and $Cx^0 \leq Cx^*$ for maximizing problems and $Cx^0 \geq Cx^*$ for minimizing problems, then x^0 is considered an efficient solution to a linear programming problem.*

Definition 2.9 *[37] In mathematical statistics, the Heronian mean (HM) of two non-negative real numbers x and y is given by the formula $HM = \frac{1}{3}(x + \sqrt{xy} + y)$.*

Definition 2.10 *[9] A fuzzy number, if the representation of a triangular fuzzy number (TrFN) is $\widetilde{A} = (a_1, a_2, a_3)$ with $a_1 \leq a_2 \leq a_3$, and the membership function is provided by*

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ \frac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3, \\ 0, & otherwise. \end{cases} \tag{2.1}$$

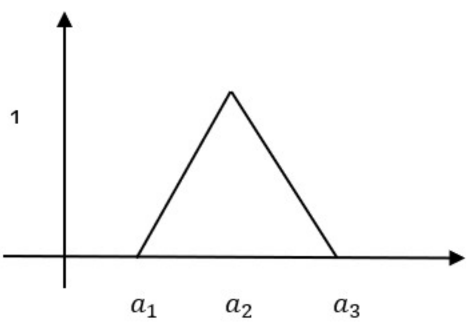

Figure 1: TrFN Membership Function.

Definition 2.11 *[38] Let $\widetilde{A} = (a_1, a_2, a_3, a_4)$ is said to be Trapezoidal fuzzy number if a fuzzy number $\widetilde{A}$ is representation $a_1 \leq a_2 \leq a_3 \leq a_4$, and with the membership function is given by*

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ 1, & a_2 \leq x \leq a_3, \\ \frac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4, \\ 0, & otherwise. \end{cases} \tag{2.2}$$

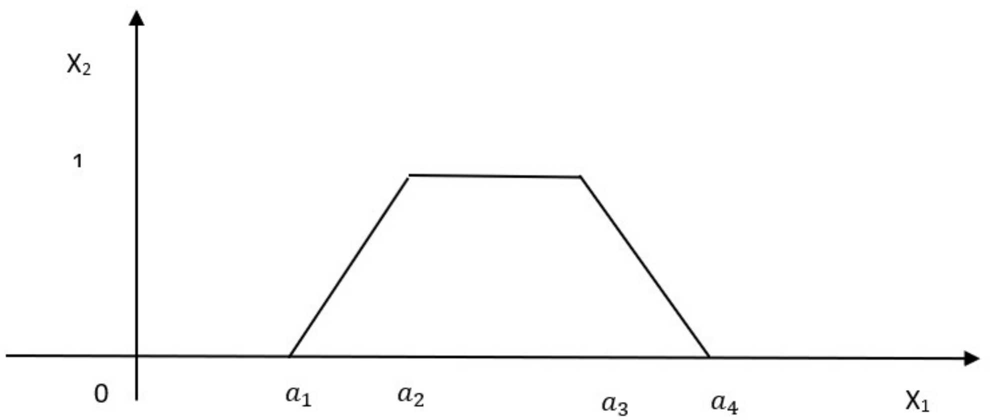

Figure 2: Trapezoidal Fuzzy Number Membership Function.

Definition 2.12 *[38] A fuzzy number $\widetilde{A}$ is called Pentagonal fuzzy number (PFN) if its representation is in the form $\widetilde{A} = (a_1, a_2, a_3, a_4, a_5)$ with $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5$, and the membership function is given by*

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ \frac{x-a_2}{a_3-a_2}, & a_2 \leq x \leq a_3, \\ 1, & x = a_3, \\ \frac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4, \\ \frac{a_5-x}{a_5-a_4}, & a_4 \leq x \leq a_5, \\ 0, & \text{otherwise.} \end{cases} \tag{2.3}$$

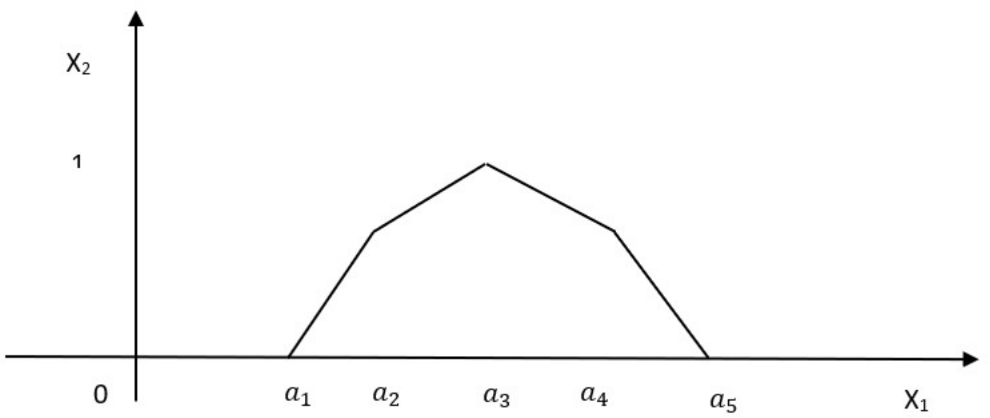

Figure 3: PFNs Membership Function.

Theorem 2.1 *[9] Let* max $F(x)$ *subject to* $x \in S$ *and* max $G(x)$ *subject to* $x \in J$ *be two optimization programming problems, then* max $F(x)$ *and* max $G(x)$ *are equivalent if and only if there is a one-to-one function of the feasible set of* max $F(x)$ *onto the feasible set of* max $G(x)$ *such that* $F(x) = G(x), \forall x \in S$.

Theorem 2.2 *[9] If there isn't an acceptable point $x^* \in S$ such that $F(x^*) \leq F(x)$, then $x^* \in S$ is an optimal solution to any optimization problem, where $S = \{x \in \mathbb{R}^n : \ Ax \leq k, x \geq 0\}$.*

3. Formulations in Mathematics

The section that follows demonstrates the mathematical forms related to this study as follows:

Linear Fractional Programming (LFP) Problem [41]:

The LFP problem is usually expressed as follows:

$$\max \ z = \frac{aX + \alpha}{dX + \beta} \tag{3.1}$$

Subject to:

$$AX \leq b, \ X \geq 0.$$

Where $a, d, X \in \mathbb{R}^m$, $b \in \mathbb{R}^n$, A is an $n \times m$ real matrix and $\alpha, \beta \in \mathbb{R}$.

Fuzzy Linear Fractional Programming (FLFP) problems [39,40]:

We will model the LFP problem using FNs for all coefficients. The FLFP problem has the following general form:

$$\max \ z = \frac{\widetilde{a}X + \widetilde{\alpha}}{\widetilde{d}X + \widetilde{\beta}} \tag{3.2}$$

Subject to:

$$\widetilde{A}X \leq \widetilde{b}, \ X \geq 0.$$

Where $\widetilde{A}, \widetilde{a}, \widetilde{d}, \widetilde{b}, \widetilde{\alpha}, \widetilde{\beta} \in$ fuzzy numbers (FN)s.

4. Ranking Function Procedure

To explain the proposed new ranking function method, it is necessary to know some facts: that the x-axis or real line has a separate real point called a_i, $i = 1, 2, \ldots, n$, and between the separate points there is I_n in the number of sub intervals (a_i, a_j) $i < j$, where $1 \leq i \leq j \leq n$; of course, these intervals consist of upper and lower limits. According to the sub intervals, the proposed new ranking function method can be implemented using the Heronian Mean, which is a statistical measure. However, it is suggested that this rule $\frac{n(n+1)}{2}$ should be taken into account in the creation of sub intervals.

Let $A^2 = (a_1, a_2)$ the sub intervals in the interval fuzzy number A^2 by above rule as (a_1, a_1), (a_1, a_2) and (a_2, a_2) the heronian mean of the lower and upper limit of each sub interval is as follow; $[\frac{1}{3}(a_1 + \sqrt{a_1 a_1} + a_1)]$, $[\frac{1}{3}(a_1 + \sqrt{a_1 a_2} + a_2)]$ and $[\frac{1}{3}(a_2 + \sqrt{a_2 a_2} + a_2)]$ since $a_i \geq 0$, $i = 1, 2, \ldots, n$ then the values of heronian mean are $(a_1$, $[\frac{1}{3}(a_1 + \sqrt{a_1 a_2} + a_2)]$, and $a_2)$. The arithmetic mean of the above values of heronian mean

$$\frac{[\frac{1}{3}(a_1 + \sqrt{a_1 a_1} + a_1)] + [\frac{1}{3}(a_1 + \sqrt{a_1 a_2} + a_2)] + [\frac{1}{3}(a_2 + \sqrt{a_2 a_2} + a_2)]}{3}$$

In consequence

$$R(A^2) = \frac{4a_1 + 4a_2 + \sqrt{a_1 a_2}}{9}. \tag{4.1}$$

Consider a TrFN $A^3 = (a_1, a_2, a_3)$. The intervals in the TrFN A^3 according to the above rule then (a_1, a_1), (a_1, a_2), (a_1, a_3), (a_2, a_2), (a_2, a_3) and (a_3, a_3).

The heronian mean of each sub intervals $[\frac{1}{3}(a_1 + \sqrt{a_1 a_1} + a_1)]$, $[\frac{1}{3}(a_1 + \sqrt{a_1 a_2} + a_2)]$, $[\frac{1}{3}(a_1 + \sqrt{a_1 a_3} + a_3)]$, $[\frac{1}{3}(a_2 + \sqrt{a_2 a_2} + a_2)]$, $[\frac{1}{3}(a_2 + \sqrt{a_2 a_3} + a_3)]$ and $[\frac{1}{3}(a_3 + \sqrt{a_3 a_3} + a_3)]$. The arithmetic mean is

$$\frac{[\frac{1}{3}(a_1 + \sqrt{a_1 a_1} + a_1)] + [\frac{1}{3}(a_1 + \sqrt{a_1 a_2} + a_2)] + [\frac{1}{3}(a_1 + \sqrt{a_1 a_3} + a_3)]}{6}$$
$$+ \frac{[\frac{1}{3}(a_2 + \sqrt{a_2 a_2} + a_2)] + [\frac{1}{3}(a_2 + \sqrt{a_2 a_3} + a_3)] + [\frac{1}{3}(a_3 + \sqrt{a_3 a_3} + a_3)]}{6}.$$

Then the ranking function is

$$R(A^3) = \frac{5a_1 + 5a_2 + 5a_3 + \sqrt{a_1 a_2} + \sqrt{a_1 a_3} + \sqrt{a_2 a_3}}{18}. \tag{4.2}$$

In the same way, consider trapezoidal fuzzy number $A^4 = (a_1, a_2, a_3, a_4)$. Then the ranking function is

$$R(A^4) = \frac{6a_1 + 6a_2 + 6a_3 + 6a_4 + \sqrt{a_1a_2} + \sqrt{a_1a_3} + \sqrt{a_1a_4} + \sqrt{a_2a_3} + \sqrt{a_2a_4} + \sqrt{a_3a_4}}{30}. \tag{4.3}$$

Let $A^n = (a_1, \ldots, a_n)$ be a polygon fuzzy number where $a_1, \ldots, a_n$ are real numbers. In the similar way the ranking function, in general is

$$R(A^n) = \frac{\sum_{i=1}^{n} (n+2)a_i + \sum_{i=1}^{n} \sum_{j>i}^{n} \sqrt{a_ia_j}}{3 \cdot \frac{n(n+1)}{2}}. \tag{4.4}$$

4.1. Note

Given two fuzzy numbers $\widetilde{A}_1$, and $\widetilde{A}_2$ then

1. $\mathcal{R}(\widetilde{A}_1 + \widetilde{A}_2) = \mathcal{R}(\widetilde{A}_1) + \mathcal{R}(\widetilde{A}_2)$
2. $\mathcal{R}(\widetilde{A}_1 - \widetilde{A}_2) = \mathcal{R}(\widetilde{A}_1) - \mathcal{R}(\widetilde{A}_2)$
3. $\mathcal{R}(\widetilde{A}_1) \leq \mathcal{R}(\widetilde{A}_2) \Longrightarrow \widetilde{A}_1 \leq \widetilde{A}_2$
4. $\mathcal{R}(\widetilde{A}_1) \geq \mathcal{R}(\widetilde{A}_2) \Longrightarrow \widetilde{A}_1 \geq \widetilde{A}_2$
5. $\mathcal{R}(\widetilde{A}_1) = \mathcal{R}(\widetilde{A}_2) \Longrightarrow \widetilde{A}_1 \approx \widetilde{A}_2$
6. $\mathcal{R}(\sum_{i=1}^{n} \widetilde{A}_i) = \sum_{i=1}^{n} \mathcal{R}(\widetilde{A}_i)$

5. Algorithm

In the following section, we present the stages of our algorithm to solve the fuzzy linear fractional programming problems and use the ranking function method to convert fuzzy coefficients into crisp coefficients.

1. Consider FFLFP problem (3.2).
2. Use the ranking function method (4.4) to convert fuzzy coefficients into crisp coefficients, and then LFP problem is obtained.
3. Convert LFP problem into LP problem using charnes and cooper's method.
4. Solve the LP problems by using simplex method.
5. Finally, the optimal solution is transform according to $[x_i = \frac{y_i}{t}$ and $z = \frac{z^*}{t}]$, where $i = 1, \ldots, n$.

Testing Steps Approach [13]:

The use of LFP problems in scenarios where uncertainty and imprecision are represented by fuzzy numbers is the subject of the numerical case study, finding the best solution while accounting for the inherent ambiguity of these fuzzy values is the main goal. This optimization procedure results in a solution that is fuzzy or ambiguous; this technique is referred to as a "fuzzy optimal solution." This method is useful for making decisions when dealing with issues impacted by unclear circumstances or in situations where exact numerical data may be absent.

6. Numerical Examples

Example 6.1

$$Max.z = \frac{\widetilde{2}x_1 + \widetilde{1}x_2 + \widetilde{1}}{\widetilde{1}x_1 + \widetilde{1}x_2 + \widetilde{2}}$$

Subject to:

$$\widetilde{1}x_1 + \widetilde{2}x_2 \leq \widetilde{2}$$
$$\widetilde{1}x_1 + \widetilde{1}x_2 \leq \widetilde{1}$$
$$x_1, x_2 \geq 0$$

Proof: The following is a formula for the problem (Example 6.1):

$$Max.z = \frac{(1,2,3)x_1 + (0,1,2)x_2 + (0,1,2)}{(0,1,2)x_1 + (0,1,2)x_2 + (1,2,3)}$$

Subject to:

$$(0,1,2)x_1 + (1,2,3)x_2 \leq (1,2,3)$$
$$(0,1,2)x_1 + (0,1,2)x_2 \leq (0,1,2)$$
$$x_1, x_2 \geq 0$$

Using Eq. (4.2):

$$R(0,1,2) = \frac{5(0) + 5(1) + 5(2) + \sqrt{0 \cdot 1} + \sqrt{0 \cdot 2} + \sqrt{1 \cdot 2}}{18} = \frac{15 + \sqrt{2}}{18} = 0.91$$

$$R(1,2,3) = \frac{5(1) + 5(2) + 5(3) + \sqrt{1 \cdot 2} + \sqrt{1 \cdot 3} + \sqrt{2 \cdot 3}}{18} = \frac{30 + \sqrt{2} + \sqrt{3} + \sqrt{6}}{18} = 1.98$$

Then the crisp problem is:

$$Max.z = \frac{1.98x_1 + 0.91x_2 + 0.91}{0.91x_1 + 0.91x_2 + 1.98}$$

Subject to:

$$0.91x_1 + 1.98x_2 \leq 1.98$$
$$0.91x_1 + 0.91x_2 \leq 0.91$$
$$x_1, x_2 \geq 0$$

Apply Charnes Cooper's technique to the aforementioned situation.

$$Max.z^* = 1.98y_1 + 0.91y_2 + 0.91t$$

Subject to

$$0.91y_1 + 0.91y_2 + 1.98t = 1$$
$$0.91y_1 + 1.98y_2 - 1.98t \leq 0$$
$$0.91y_1 + 0.91y_2 - 0.91t \leq 0$$
$$y_1, y_2, t \geq 0$$

Solve the above LP problem by simplex method: $y_1 = 0.346021$, $y_2 = 0$, $t = 0.346021$ and $z^* = 1$. Therefore $x_1 = \frac{y_1}{t} = \frac{0.346021}{0.346021} = 1$, $x_2 = \frac{y_2}{t} = \frac{0}{0.346021} = 0$ and $z = \frac{z^*}{t} = \frac{1}{0.346021} = 2.89$. □

Example 6.2

$$Max.z = \frac{(2,4,6,8)x_1 + (1,2,3,4)x_2 + (0,1,2,3)}{(0,1,2,3)x_1 + (1,2,3,4)x_2 + (0,1,2,3)}$$

Subject to:

$$\begin{aligned} (1,2,3,4)x_1 + (2,4,6,8)x_2 &\leq (2,4,6,8) \\ (0,1,2,3)x_1 + (1,2,3,4)x_2 &\leq (1,2,3,4) \\ x_1, x_2 &\geq 0 \end{aligned}$$

Proof: Using Eq. (4.3):

$$\begin{aligned} R(0,1,2,3) =& \frac{6(0)+6(1)+6(2)+6(3)+\sqrt{0}+\sqrt{0}+\sqrt{0}+\sqrt{2}+\sqrt{3}+\sqrt{6}}{30} \\ =& \frac{36+\sqrt{2}+\sqrt{3}+\sqrt{6}}{30} = 1.39 \\ R(1,2,3,4) =& \frac{6(1)+6(2)+6(3)+6(4)+\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}+\sqrt{8}+\sqrt{12}}{30} \\ =& \frac{62+3\sqrt{2}+3\sqrt{3}+\sqrt{6}}{30} = 2.46 \\ R(2,4,6,8) =& \frac{6(2)+6(4)+6(6)+6(8)+\sqrt{8}+\sqrt{12}+\sqrt{16}+\sqrt{24}+\sqrt{32}+\sqrt{48}}{30} \\ =& \frac{124+6\sqrt{2}+6\sqrt{3}+2\sqrt{6}}{30} = 4.92 \end{aligned}$$

Then:

$$Max.z = \frac{4.92x_1 + 2.46x_2 + 1.39}{1.39x_1 + 2.46x_2 + 1.39}$$

Subject to:

$$\begin{aligned} 2.46x_1 + 4.92x_2 &\leq 4.92 \\ 1.39x_1 + 2.46x_2 &\leq 2.46 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Apply Charnes Cooper's technique:

$$Max.z^* = 4.92y_1 + 2.46y_2 + 1.39t$$

Subject to

$$\begin{aligned} 1.39y_1 + 2.46y_2 + 1.39t &= 1 \\ 2.46y_1 + 4.92y_2 - 4.92t &\leq 0 \\ 1.39y_1 + 4.92y_2 - 2.46t &\leq 0 \\ y_1, y_2, t &\geq 0 \end{aligned}$$

Solve by simplex method: $y_1 = 0.459684$, $y_2 = 0$, $t = 0.25974$ and $z^* = 2.6226$. Therefore $x_1 = \frac{0.459684}{0.25974} = 1.7697$, $x_2 = 0$ and $z = \frac{2.6226}{0.25974} = 10.097$. □

Example 6.3

$$Max.z = \frac{(0,1,2,3,4)x_1 + (1,2,3,4,5)x_2 + (0,1,2,3,4)}{(0,1,2,3,4)x_1 + (0,1,2,3,4)x_2 + (1,2,3,4,5)}$$

Subject to:

$$\begin{aligned} (0,1,2,3,4)x_1 + (1,2,3,4,5)x_2 &\leq (1,2,3,4,5) \\ (0,1,2,3,4)x_1 + (0,1,2,3,4)x_2 &\leq (0,1,2,3,4) \\ x_1, x_2 &\geq 0 \end{aligned}$$

Proof: Using the general ranking formula (4.4) with $n = 5$:

$$R(0,1,2,3,4) = \frac{7(0+1+2+3+4) + (\sqrt{0}+\sqrt{0}+\sqrt{0}+\sqrt{0}+\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}+\sqrt{8}+\sqrt{12})}{45}$$
$$= \frac{70+3\sqrt{2}+3\sqrt{3}+\sqrt{6}}{45} = 1.84$$
$$R(1,2,3,4,5) = \frac{7(1+2+3+4+5)+\sum\sqrt{a_i a_j}}{45}$$
$$= \frac{105+3\sqrt{2}+3\sqrt{3}+3\sqrt{5}+\sqrt{4}+\sqrt{6}+\sqrt{10}+\sqrt{15}}{45} = 2.95$$

Then:

$$Max.z = \frac{1.84x_1 + 2.95x_2 + 1.84}{1.84x_1 + 1.84x_2 + 2.95}$$

Subject to:

$$1.84x_1 + 2.95x_2 \leq 2.95$$
$$1.84x_1 + 1.84x_2 \leq 1.84$$
$$x_1, x_2 \geq 0$$

Apply Charnes Cooper's technique:

$$Max.z^* = 1.84y_1 + 2.95y_2 + 1.84t$$

Subject to

$$1.84y_1 + 1.84y_2 + 2.95t = 1$$
$$1.84y_1 + 2.95y_2 - 2.95t \leq 0$$
$$1.84y_1 + 1.84y_2 - 1.84t \leq 0$$
$$y_1, y_2, t \geq 0$$

Solve by simplex method: $y_1 = 0$, $y_2 = 0.208768$, $t = 0.208768$ and $z^* = 1$.
Therefore $x_1 = 0$, $x_2 = 1$ and $z = \frac{1}{0.208768} = 4.79$. □

Table 1: Comparison with other Methods

Problems	Existence Methods	Our Method
1	[41] $x_1 = 1$, $x_2 = 0$, $z = 1.43$	$x_1 = 1$, $x_2 = 0$, $z = 2.89$
2	[42] $x_1 = 1$, $x_2 = 0$, $z = 4.174$	$x_1 = 1.7697$, $x_2 = 0$, $z = 10.097$
3	[43] $x_1 = 0$, $x_2 = 1$, $z = 2.2857$	$x_1 = 0$, $x_2 = 1$, $z = 4.79$

7. Discussion

This section presents the findings and analysis. In general, the effects of the proposed method are clear, and the implications are typically obvious. I have solved the above examples with the previous methods and compared them with our method. The results are clear in Table 1, and very reasonable and acceptable in terms of the solution and the values of the objective functions, using the algorithm in Section 5, which makes the study easier to decide. However, the proposed method is not able to solve some problems, such as fully fuzzy optimization programming problems. This paper outlines and explains several assumptions in detail.

8. Conclusion

We have tackled the FLFP problems in the current research and presented an efficient method that can be used to fuzzy number coefficient situations. It has been tried and solved by scientists in a number of ways. The ranking function approach is one of the methods. We have presented a unique ranking function approach in this work that is easy to compute. In addition, it is highly practical and effective, as it can be applied to solve problems in real life and facilitate getting the desired outcome. Other problems, such as those involving linear and quadratic programming and transportation in uncertain environments, can be solved using an extension of the ranking approach. Specifically, the FLFP problem may be solved by extending the ranking approach to fuzzy numbers that are hexagonal, octagonal, heptagonal, etc.

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