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Intuitionistic Fuzzification Applied to ⋆-Ternary Semihypergroups

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ABSTRACT: In this paper, we make a new approach to ⋆-ternary semihypergroup(ternary semihypergroup with involution ′⋆′) via intuitionistic fuzzification, with the concept of intutionistic fuzzy ternary subsemihypergroup(I.F.T.sub-S.), intutionistic fuzzy right hyperideals(I.F.R.H.), intutionistic fuzzy lateral hyperideals(I.F.Lt.H.), intutionistic fuzzy left hyperideals(I.F.L.H.) and intutionistic fuzzy hyperfilters(I.F.H.F.) and, explore some properties using involution theoretic concepts in ⋆-ternary semihypergroups for intutionistic fuzzy hyperideals and intutionistic fuzzy hyperfilters. Particularly, we obtain the relationship between the complement of intuitionistic fuzzy hyperideals and intuitionistic fuzzy hyperfilters.

Keywords: ⋆-Ternary semihypergroups, hyperideals, hyperfilters, intuitionistic fuzzy hyperideals, intuitionistic fuzzy hyperfilters

Contents

1. Introduction and Preliminaries

Marty [15] introduced the notion of algebraic hyperstructures as a natural extension of classical algebraic structures, where the composition of two elements is a non-empty set. In [6], Corsini and Leoreanu presented some of the numerous applications of hyperstructures. A ternary semihypergroup is a particular case of an n-ary semihypergroup (n-semihypergroup) for n = 3. Davvaz and Leoreanu [9] studied binary relations on ternary semihypergroups. Hila et al. [13] gave some properties of left (right) and lateral hyperideals in ternary semihypergroups.

Zadeh [21] proposed the theory of fuzzy sets and fuzzy logic to address vagueness in classical set theory. Fuzzy sets are mathematical means to model uncertainty and imprecise information across various applications including fuzzy logic systems, pattern recognition, decision-making, risk assessment, and more, making them a valuable tool in many fields of mathematics, engineering, computer science, economics, and artificial intelligence. Rosenfeld [17] initiated the study of fuzzy algebraic structures in groups and groupoids.

Attansov [4,5] expanded fuzzy theory by introducing the concept of intutionistic fuzzy sets which include both membership and non-membership degrees making it more suitable for dealing uncertainity. It has wide usage in various fields, including decision-making, pattern recognition, medical diagnosis, data mining, control systems, information retrieval, and risk assessment, enabling the creation of more refined and precise models when faced with uncertainty. The concept of intuitionistic fuzzy hyperstructures is an interesting research idea for intutionistic fuzzy set theory. We have observed that the relationship between the intuitionistic fuzzy set and algebraic hyperstructures have been already considered by Davvaz, Hila, Abdulmula, Hila and Abdullah and others, for instance, the reader can refer to [3,8,12,14].

After the introduction of ⋆-ternary semihypergroup by Abbasi et al. [2], it is natural to investigate intuitionistic fuzzification of ⋆-ternary semihypergroups. In [20], Tang et al. defined hyperfilters and

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fuzzy hyperfilters of ordered semihypergroups. In [19], Tang and Yaqoob investigated fuzzy hyperideals of ordered $\star$-semihypergroups. Davvaz [8] established the intuitionistic fuzzification of the concept of hyperideals in a semihypergroup. Hila et al. [12], introduced the concepts of intuitionistic fuzzy hyperideal extension of semihypergroups and intuitionistic fuzzy prime (semiprime) hyperideal of semihypergroup. As a further study of semihypergroup and ternary semihypergroup theory, we attempt in the present paper to study the intuitionistic fuzzification of hyperideals of $\star$-ternary semihypergroups in detail.

Definition 1.1 [9] A function $f : \mathbf{T} \times \mathbf{T} \times \mathbf{T} \rightarrow \wp^*(\mathbf{T})$ is referred to as a ternary hyperoperation on the set $\mathbf{T} \neq \emptyset$, where $\wp^*(\mathbf{T})$ denotes the power set of $\mathbf{T}$ except empty-set. The pair $(\mathbf{T}, f)$ is called the ternary hypergroupoid.

If $P \neq \emptyset$, $Q \neq \emptyset$, and $R \neq \emptyset$ are subsets of $\mathbf{T}$, then we define

$$f(P, Q, R) = \bigcup_{p \in P, q \in Q, r \in R} f(p, q, r).$$

Definition 1.2 [9] A ternary hypergroupoid $(\mathbf{T}, f)$ is called a ternary semihypergroup(briefly, T.S.), if for all $\gamma_1, \gamma_2, ..., \gamma_5 \in \mathbf{T}$, we have

$$f(f(\gamma_1, \gamma_2, \gamma_3), \gamma_4, \gamma_5) = f(\gamma_1, f(\gamma_2, \gamma_3, \gamma_4), \gamma_5) = f(\gamma_1, \gamma_2, f(\gamma_3, \gamma_4, \gamma_5)).$$

Because of associative law in T.S. $\mathbf{T}$, for any component $\gamma_1, \gamma_2, ..., \gamma_{2n+1} \in \mathbf{T}$ and positive integers m, n with $m \leq n$, one may write,

$$\begin{aligned} f(\gamma_1, \gamma_2, ..., \gamma_{2n+1}) &= f(\gamma_1, ..., \gamma_m, \gamma_{m+1}, \gamma_{m+2}, ..., \gamma_{2n+1}) \\ &= f(\gamma_1, ..., f(f(\gamma_m, \gamma_{m+1}, \gamma_{m+2}), \gamma_{m+3}, \gamma_{m+4}), ..., \gamma_{2n+1}). \end{aligned}$$

2. $\star$-Ternary Semihypergroup

Definition 2.1 [2] A ternary semihypergroup $\mathbf{T}$ is said to be a ternary semihypergroup with involution(briefly, '$\star$-ternary semihypergroup'), if there is a unary operation $\star$: $\mathbf{T} \rightarrow \mathbf{T}$, which satisfies the following conditions:

$$(f(\gamma_1, \gamma_2, \gamma_3))^\star = f(\gamma_3^\star, \gamma_2^\star, \gamma_1^\star) \text{ and } (\gamma_1^\star)^\star = \gamma_1 \ \forall \ \gamma_1, \gamma_2, \gamma_3 \in \mathbf{T}.$$

In this paper we refer to $\mathbf{T}$ as a $\star$-ternary semihypergroup.

For a subset $P(\neq \emptyset)$ of $\mathbf{T}$,

$$P^\star = \{p^\star \in \mathbf{T} : p \in P\}.$$

Example 2.1 [2] Let $L = \{0, j, -j\}$, then L is a ternary semigroup under multiplication of complex numbers. Consider $\mathbf{T} = \left\{ \begin{pmatrix} 0 & 0 & 0 & \gamma \\ 0 & 0 & \delta & 0 \\ 0 & \gamma & 0 & 0 \\ \delta & 0 & 0 & 0 \end{pmatrix} : \gamma, \delta \in L \right\}$.

Now, let

$$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

.
Then $\mathbf{T}$ is a ternary semihypergroup under ternary hyperoperation $'f'$ defined as follows:

$f(P,Q,R) = \{P\cdot P_1\cdot Q\cdot P_1\cdot R, P\cdot P_1\cdot Q\cdot P_2\cdot R, P\cdot P_2\cdot Q\cdot P_1\cdot R, P\cdot P_2\cdot Q\cdot P_2\cdot R\}\ \forall\ P,Q,R\in\mathbf{T}$, where $'\cdot'$ denotes the multiplication of matrices over complex numbers. A unary operation $\star$: $\mathbf{T}\to\mathbf{T}$ is specified by mapping $P^\star$ to P^T, for every $P\in\mathbf{T}$, where P^T represents the transpose of P. It is straightforward to confirm that $\mathbf{T}$ forms a $\star$-ternary semihypergroup. For an overview of the definitions and results, we refer [1,7,10,11,16,18].

Definition 2.2 [2] Any subset $(S\neq\emptyset)$ of $\mathbf{T}$ is called a ternary sub-semihypergroup (briefly, T.sub-S.) of $\mathbf{T}$ if $f(S,S,S)\subseteq S$.

Proposition 2.1 *[2] If T is a T.sub-S. of $\mathbf{T}$. Then $T^\star$ is also a T.sub-S. of $\mathbf{T}$.*

Definition 2.3 [2] A subset $(I\neq\emptyset)$ of $\mathbf{T}$ is called a left hyperideal of $\mathbf{T}$ if $f(I,\mathbf{T},\mathbf{T})\subseteq I$ (resp., right if $f(\mathbf{T},\mathbf{T},I)\subseteq I$, resp., lateral if $f(\mathbf{T},I,\mathbf{T})\subseteq I$).

Example 2.2 Consider the Example 2.1

Let $M=\left\{\begin{pmatrix}0&0&0&0\\0&0&\delta&0\\0&\gamma&0&0\\0&0&0&0\end{pmatrix}:\gamma,\delta\in L\right\}$ s.t. $M\subseteq\mathbf{T}$. Then M is a lateral hyperideal of $\mathbf{T}$.

Definition 2.4 [4] An intuitionistic fuzzy set(briefly, I.F.S.) I in a non-empty set $\mathbf{T}$ is an object having the form $I=\{(z,\mu_I(z),\lambda_I(z))\mid z\in\mathbf{T}\}$, where the functions $\mu_I:\mathbf{T}\to[0,1]$ and $\lambda_I:\mathbf{T}\to[0,1]$ denote the degree of membership and degree of non-membership of each element $z\in\mathbf{T}$ to the set I respectively and $0\leq\mu_I(z)+\lambda_I(z)\leq 1$ for all $z\in\mathbf{T}$. We will use the symbol $I=(\mu_I,\lambda_I)$ for the IFS $I=\{(z,\mu_I(z),\lambda_I(z))\mid z\in\mathbf{T}\}$.

Definition 2.5 [4] Let I_1, I_2 and I_3 be three I.F.S. of $\mathbf{T}$, then the union and intersection of I.F.S. are defined as follows:

1. $I_1\cup I_2\cup I_3=\{(z,\mu_{I_1\cup I_2\cup I_3}(z),\lambda_{I_1\cup I_2\cup I_3}(z))\mid z\in\mathbf{T}\}$ and $\mu_{I_1\cup I_2\cup I_3}(z)=max\{\mu_{I_1}(z),\mu_{I_2}(z),\mu_{I_3}(z)\}$, $\lambda_{I_1\cup I_2\cup I_3}(z)=min\{\lambda_{I_1}(z),\lambda_{I_2}(z),\lambda_{I_3}(z)\}$.
2. $I_1\cap I_2\cap I_3=\{(z,\mu_{I_1\cap I_2\cap I_3}(z),\lambda_{I_1\cap I_2\cap I_3}(z))\mid z\in\mathbf{T}\}$ and $\mu_{I_1\cap I_2\cap I_3}(z)=min\{\mu_{I_1}(z),\mu_{I_2}(z),\mu_{I_3}(z)\}$, $\lambda_{I_1\cap I_2\cap I_3}(z)=max\{\lambda_{I_1}(z),\lambda_{I_2}(z),\lambda_{I_3}(z)\}$.

Definition 2.6 Let $\{I_{\mathbf{T}_j}\}_{j\in\mathcal{J}}$ is a collection of I.F.S. of $\mathbf{T}$, where $\mathcal{J}$ is the index set, then $\bigcap_{j\in\mathcal{J}} I_{\mathbf{T}_j}$ is defined as $\bigcap_{j\in\mathcal{J}} I_{\mathbf{T}_j}=(min\{\mu_{I_j}\},max\{\lambda_{I_j}\})$.

3. Intuitionistic Fuzzy Hyperideals in ⋆-Ternary Semihypergroup

The current section discusses intuitiontistic fuzzy hyperideals in $\star$-ternary semihypergroup(ternary semihypergroups equipped with involution $'\star'$). Moreover, intuitiontistic fuzzy hyperideals help in characterization of intra-regular $\star$-ternary semihypergroups.
To proceed further, we need the following:

Definition 3.1 Let I be an I.F.S. in $\mathbf{T}$ and let $\xi,\zeta\in[0,1]$ such that $\xi+\zeta\leq 1$. Then the upper ξ-level cut of $\mathbf{T}$ is defined as

$$U(\mu_I,\xi)=\{z\in\mathbf{T}:\mu_I(z)\geq\xi\}$$

and the lower ζ-level cut of $\mathbf{T}$ is defined as

$$L(\lambda_I,\zeta)=\{z\in\mathbf{T}:\lambda_I(z)\leq\zeta\}$$

Definition 3.2 Let I be an I.F.S. of $\mathbf{T}$. Assume $\xi, \zeta \in [0,1]$ such that $\xi + \zeta \leq 1$, then the set

$$I_{\langle \xi, \zeta \rangle} = \{z \in \mathbf{T} : \mu_I(z) \geq \xi \ and \ \lambda_I(z) \leq \zeta\}$$

is referred as (ξ, ζ)-level cut of I.

We define $\mathbf{T}_z$ as follows, where $z \in \mathbf{T}$,

$$\mathbf{T}_z = \{(v, w, x) \in \mathbf{T} \times \mathbf{T} \times \mathbf{T} \mid z \in f(v, w, x)\}$$

Definition 3.3 A collection of all intuitionistic fuzzy subsets of $\mathbf{T}$ is denoted by $J(\mathbf{T})$. For any three intuitionistic fuzzy sets $U = (\mu_U, \lambda_U), V = (\mu_V, \lambda_V)$ and $W = (\mu_W, \lambda_W)$ and of $\mathbf{T}$, we define

$$U \diamond V \diamond W = \{(z, (\mu_{U\diamond V\diamond W}), (\lambda_{U\diamond V\diamond W})) : z \in \mathbf{T}\},$$

where

$$(\mu_{U\diamond V\diamond W})(z) = \begin{cases} \sup\limits_{z\in f(u,v,w)} \{min\{\mu_U(u), \mu_V(v), \mu_W(w)\}\}, & \text{if } z \in f(u,v,w) \\ 0, & \text{otherwise.} \end{cases}$$

and

$$(\lambda_{U\diamond V\diamond W})(z) = \begin{cases} \inf\limits_{z\in f(u,v,w)} \{max\{\lambda_U(v), \lambda_V(v), \lambda_W(w)\}\}, & \text{if } z \in f(u,v,w) \\ 1, & \text{otherwise.} \end{cases}$$

Definition 3.4 An I.F.S. $I = (\mu_I, \lambda_I)$ of $\mathbf{T}$ is referred as an intuitionistic fuzzy ternary sub-semihypergroup (briefly, I.F.T.sub-S.) of $\mathbf{T}$ if for every $t_1, t_2, t_3 \in \mathbf{T}$, $\inf\limits_{\vartheta\in f(t_1,t_2,t_3)} \mu_I(\vartheta) \geq \min \{\mu_I(t_1), \mu_I(t_2), \mu_I(t_3)\}$ and $\sup\limits_{\vartheta\in f(t_1,t_2,t_3)} \lambda_I(\vartheta) \leq \max\{\lambda_I(t_1), \lambda_I(t_2), \lambda_I(t_3)\}$ holds simultaneously.

Definition 3.5 An I.F.S. I of $\mathbf{T}$ is referred as an intuitionistic fuzzy right hyperideal(briefly, I.F.R.H.) intuitionistic fuzzy left hyperideal(briefly, I.F.L.H.), intuitionistic fuzzy lateral hyperideal(briefly, I.F.Lt.H.) of $\mathbf{T}$ if for every $r, m, l \in \mathbf{T}$,
$\inf\limits_{\vartheta\in f(r,m,l)} \mu_I(\vartheta) \geq \mu_I(r)$ ($\inf\limits_{\vartheta\in f(r,m,l)} \mu_I(\vartheta) \geq \mu_I(l)$, $\inf\limits_{\vartheta\in f(r,m,l)} \mu_I(\vartheta) \geq \mu_I(m)$ resp.) and
$\sup\limits_{\vartheta\in f(r,m,l)} \lambda_I(\vartheta) \leq \lambda_I(r)$ ($\sup\limits_{\vartheta\in f(r,m,l)} \lambda_I(\vartheta) \leq \lambda_I(l)$, $\sup\limits_{\vartheta\in f(r,m,l)} \lambda_I(\vartheta) \leq \lambda_I(m)$ resp.) holds simultaneously.

I is referred as an I.F. hyperideal of $\mathbf{T}$ if for every $r, m, l \in \mathbf{T}$, if
$\inf\limits_{\vartheta\in f(r,m,l)} \mu_I(\vartheta) \geq max\{\mu_I(r), \mu_I(m), \mu_I(l)\}$ and $\sup\limits_{\vartheta\in f(r,m,l)} \lambda_I(\vartheta) \leq min\{\lambda_I(r), \lambda_I(m), \lambda_I(l)\}$ holds simultaneously.

Example 3.1 Consider the Example 2.1. Let $I = (\mu_I, \lambda_I)$ be the I.F.S. defined on $\mathbf{T}$, where $I : \mathbf{T} \longrightarrow [0,1]$

$\mu_I(X_i) = 0.5$ and $\lambda_I(X_i) = 0.5$, where $X_i \in \left\{ \begin{pmatrix} 0 & 0 & 0 & \gamma \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \delta & 0 & 0 & 0 \end{pmatrix} : \gamma \in \{0, j, -j\} \right\}$

$\mu_I(X_j) = 1$ and $\lambda_I(X_j) = 0$, where $X_j \in \left\{ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma & 0 \\ 0 & \delta & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} : \gamma \in \{j, -j\} \right\}$ and

$\mu_I(X_k) = 0$ and $\lambda_I(X_k) = 1$, where $X_k \in \left\{ \begin{pmatrix} 0 & 0 & 0 & \gamma \\ 0 & 0 & \delta & 0 \\ 0 & \gamma & 0 & 0 \\ \delta & 0 & 0 & 0 \end{pmatrix} : \gamma, \delta \in \{j, -j\} \right\}$.

Then, we can verify that $\inf\limits_{\vartheta \in f(r,m,l)} \mu_I(\vartheta) \geq \mu_I(m)$ and $\sup\limits_{\vartheta \in f(r,m,l)} \lambda_I(\vartheta) \leq \lambda_I(m)$. Therefore, $I = (\mu_I, \lambda_I)$ is an I.F.Lt.H. of $\mathbf{T}$.

Theorem 3.1 *Suppose $\{I_j\}_{j\in\mathcal{J}}$ is a family of I.F.T.sub-S. of $\mathbf{T}$, where $\mathcal{J}$ is the index set. Then $\bigcap\limits_{j\in\mathcal{J}} I_j$ is an I.F.T.sub-S. of $\mathbf{T}$.*

Proof: Let $I = \bigcap\limits_{j\in\mathcal{J}} I_j = (\inf\limits_{j\in\mathcal{J}} \mu_{I_j}, \sup\limits_{j\in\mathcal{J}} \lambda_{I_j})$. Now, we have

$$\begin{aligned}
\inf_{\vartheta \in f(t_1,t_2,t_3)} \mu_I(\vartheta) &= \inf_{\vartheta \in f(t_1,t_2,t_3)} \{\inf_{j\in\mathcal{J}} \mu_{I_j}(\vartheta)\} \\
&= \inf_{j\in\mathcal{J}} [\inf_{\vartheta \in f(t_1,t_2,t_3)} \mu_{I_j}(\vartheta)] \\
&\geq \inf_{j\in\mathcal{J}} [min\{\mu_{I_j}(t_1), \mu_{I_j}(t_2), \mu_{I_j}(t_3)\}] \\
&= min\{\inf_{j\in\mathcal{J}} \mu_{I_j}(t_1), \inf_{j\in\mathcal{J}} \mu_{I_j}(t_2), \inf_{j\in\mathcal{J}} \mu_{I_j}(t_3)\}] \\
&= min\{\mu_I(t_1), \mu_I(t_2), \mu_I(t_3)\}].
\end{aligned}$$

and

$$\begin{aligned}
\sup_{\vartheta \in f(t_1,t_2,t_3)} \lambda_I(\vartheta) &= \sup_{\vartheta \in f(t_1,t_2,t_3)} \{\sup_{j\in\mathcal{J}} \lambda_{I_j}(\vartheta)\} \\
&= \sup_{j\in\mathcal{J}} [\sup_{\vartheta \in f(t_1,t_2,t_3)} \lambda_{I_j}(\vartheta)] \\
&\leq \sup_{j\in\mathcal{J}} [max\{\lambda_{I_j}(t_1), \lambda_{I_j}(t_2), \lambda_{I_j}(t_3)\}] \\
&= max\{\sup_{j\in\mathcal{J}} \lambda_{I_j}(t_1), \sup_{j\in\mathcal{J}} \lambda_{I_j}(t_2), \sup_{j\in\mathcal{J}} \lambda_{I_j}(t_3)\}] \\
&= max\{\lambda_I(t_1), \lambda_I(t_2), \lambda_I(t_3)\}].
\end{aligned}$$

Hence, $I = \bigcap\limits_{j\in\mathcal{J}} I_j$ is an I.F.T.sub-S. of $\mathbf{T}$. □

Proposition 3.1 *A subset $I (\neq \emptyset)$ of $\mathbf{T}$, is an T.sub-S. of $\mathbf{T}$ if and only if $(\varkappa_I, \bar{\varkappa}_I)$ is an I.F.T.sub-S. of $\mathbf{T}$.*

Proof: Assume I is an T.sub-S. of $\mathbf{T}$, then $f(I, I, I) \subseteq I$.
The following cases are required to prove that I is an I.F.T.sub-S. of $\mathbf{T}$.

Case 1: If $t_1, t_2, t_3 \in I$, it implies $f(t_1, t_2, t_3) \subseteq I$. Then $\inf\limits_{\vartheta \in f(t_1,t_2,t_3)} \varkappa_I(\vartheta) = 1 \geq \min\{\varkappa_I(t_1), \varkappa_I(t_2), \varkappa_I(t_3)\}$ and $\sup\limits_{\vartheta \in f(t_1,t_2,t_3)} \bar{\varkappa}_I(\vartheta) = 0 \leq \max\{\bar{\varkappa}_I(t_1), \bar{\varkappa}_I(t_2), \bar{\varkappa}_I(t_3)\}$.

Case 2 : If $t_1 \notin I$ or $t_2 \notin I$ or $t_3 \notin I$ then $\varkappa_I(t_1) = 0$ and $\bar{\varkappa}_I(t_1) = 1$ or $\varkappa_I(t_2) = 0$ and $\bar{\varkappa}_I(t_2) = 1$ or $\varkappa_I(t_3) = 0$ and $\bar{\varkappa}_I(t_3) = 1$. Thus, we get, $\inf\limits_{\vartheta \in f(t_1,t_2,t_3)} \varkappa_I(\vartheta) \geq 0 = \min\{\varkappa_I(t_1), \varkappa_I(t_2), \varkappa_I(t_3)\}$ and $\sup\limits_{\vartheta \in f(t_1,t_2,t_3)} \bar{\varkappa}_I(\vartheta) \leq 1 = \max\{\bar{\varkappa}_I(t_1), \bar{\varkappa}_I(t_2), \bar{\varkappa}_I(t_3)\}$.

Thus, $\inf\limits_{\vartheta \in f(t_1,t_2,t_3)} \varkappa_I(\vartheta) \geq \min\{\varkappa_I(t_1), \varkappa_I(t_2), \varkappa_I(t_3)\}$ and $\sup\limits_{\vartheta \in f(t_1,t_2,t_3)} \bar{\varkappa}_I(\vartheta) \leq \max\{\bar{\varkappa}_I(t_1), \bar{\varkappa}_I(t_2), \bar{\varkappa}_I(t_3)\}$ holds true in preceding cases. Therefore, $(\varkappa_I, \bar{\varkappa}_I)$ is an I.F.T.sub-S. of $\mathbf{T}$.
Conversely, assume $(\varkappa_I, \bar{\varkappa}_I)$ is an I.F.T.sub-S. of $\mathbf{T}$ and $t_1, t_2, t_3 \in I$ implying $\varkappa_I(t_1) = \varkappa_I(t_2) = \varkappa_I(t_3) = 1$ and $\bar{\varkappa}_I(t_1) = \bar{\varkappa}_I(t_2) = \bar{\varkappa}_I(t_3) = 0$. Since $\inf\limits_{\vartheta \in f(t_1,t_2,t_3)} \varkappa_I(\vartheta) \geq \min\{\varkappa_I(t_1), \varkappa_I(t_2), \varkappa_I(t_3)\} = 1$ and $\sup\limits_{\vartheta \in f(t_1,t_2,t_3)} \bar{\varkappa}_I(\vartheta) \leq \max\{\bar{\varkappa}_I(t_1), \bar{\varkappa}_I(t_2), \bar{\varkappa}_I(t_3)\} = 0$. Thus, we have $\inf\limits_{\vartheta \in f(t_1,t_2,t_3)} \varkappa_I(\vartheta) \geq 1$ and

$\sup_{\vartheta\in f(t_1,t_2,t_3)} \bar{\varkappa}_I(\vartheta) \leq 0$ which implies $\vartheta \in f(t_1,t_2,t_3) \subseteq I$ and thus $f(I,I,I) \subseteq I$. Hence, I is an T.sub-S. of $\mathbf{T}$. □

Proposition 3.2 *Assume $J(\neq \emptyset)$ to be a subset of $\mathbf{T}$. Then J is a R.H.(L.H., Lt.H. resp.) of T if and only if $(\varkappa_J, \bar{\varkappa}_J)$ is an I.F.R.H.(I.F.L.H., I.F.Lt.H. resp.) of T.*

This is similar to the proof of Proposition 3.1.

Proposition 3.3 *Let $I = (\mu_I, \lambda_I)$ be an I.F.S. of $\mathbf{T}$, then I is an I.F.T.sub-S. of $\mathbf{T}$ if and only if $I_{\langle\xi,\zeta\rangle} \neq \emptyset$ is an T.sub-S. of $\mathbf{T}$.*

Proof: Let $I = (\mu_I, \lambda_I)$ be an I.F.T.sub-S. of $\mathbf{T}$ and suppose $I_{\langle\xi,\zeta\rangle} \neq \emptyset$ is the level subset of I with $t_1,t_2,t_3 \in I_{\langle\xi,\zeta\rangle}$ for any $t_1,t_2,t_3 \in \mathbf{T}$. Thus, $\mu_I(t_1) \geq \xi$, $\mu_I(t_2) \geq \xi$ and $\mu_I(t_3) \geq \xi$ and $\lambda_I(t_1) \leq \zeta$, $\lambda_I(t_2) \leq \zeta$ and $\lambda_I(t_3) \leq \zeta$. Since I is an I.F.T.sub-S. of $\mathbf{T}$, we have $\inf_{\vartheta\in f(t_1,t_2,t_3)} \mu_I(\vartheta) \geq \min\{\mu_I(t_1), \mu_I(t_2), \mu_I(t_3)\} \geq \xi$ and $\sup_{\vartheta\in f(t_1,t_2,t_3)} \lambda_I(\vartheta) \leq \max\{\lambda_I(t_1), \lambda_I(t_2), \lambda_I(t_3)\} \leq \zeta$. It would imply $\inf_{\vartheta\in f(t_1,t_2,t_3)} \mu_I(\vartheta) \geq \xi$ and $\sup_{\vartheta\in f(t_1,t_2,t_3)} \lambda_I(\vartheta) \leq \zeta$ which means $f(t_1,t_2,t_3) \subseteq I_{\langle\xi,\zeta\rangle}$. Hence, $I_{\langle\xi,\zeta\rangle} \neq \emptyset$ is an T.sub-S. of $\mathbf{T}$.
Conversely, assume a level subset $I_{\langle\xi,\zeta\rangle} \neq \emptyset$ to be an T.sub-S. of $\mathbf{T}$. For any $t_1,t_2,t_3 \in I_{\langle\xi,\zeta\rangle}$, we have $f(t_1,t_2,t_3) \subseteq I_{\langle\xi,\zeta\rangle}$. Suppose that $\inf_{\vartheta\in f(t_1,t_2,t_3)} \mu_I(\vartheta) \leq \min\{\mu_I(t_1), \mu_I(t_2), \mu_I(t_3)\}$ and $\sup_{\vartheta\in f(t_1,t_2,t_3)} \lambda_I(\vartheta) \geq \max\{\lambda_I(t_1), \lambda_I(t_2), \lambda_I(t_3)\}$ holds for some $t_1,t_2,t_3 \in \mathbf{T}$. Now, assume that there exists $\gamma,\delta \in [0,1]$ satisfying $\inf_{\vartheta\in f(t_1,t_2,t_3)} \mu_I(\vartheta) \leq \gamma \leq \min\{\mu_I(t_1), \mu_I(t_2), \mu_I(t_3)\}$ and $\sup_{\vartheta\in f(t_1,t_2,t_3)} \lambda_I(\vartheta) \geq \delta \geq \max\{\mu_I(t_1), \mu_I(t_2), \mu_I(t_3)\}$. It infers that $t_1,t_2,t_3 \in I_{\langle\gamma,\delta\rangle}$ but $f(t_1,t_2,t_3) \nsubseteq I_{\langle\gamma,\delta\rangle}$ which contradicts that $I_{\langle\gamma,\delta\rangle}$ is an T.sub-S.. It is concluded that our assumption is inaccurate. Therefore, $I = (\mu_I, \lambda_I)$ be an I.F.S. of $\mathbf{T}$, then I is an I.F.T.sub-S. of $\mathbf{T}$. □

Theorem 3.2 *Every I.F.R.H.(I.F.L.H., I.F.Lt.H. resp.) of $\mathbf{T}$ is an I.F.T.sub-S. of $\mathbf{T}$.*

Proof: Suppose $I = (\mu_I, \lambda_I)$ is an I.F.R.H. of $\mathbf{T}$, therefore $\inf_{\vartheta\in f(r,m,l)} \mu_I(\vartheta) \geq \mu_I(r)$ and $\sup_{\vartheta\in f(r,m,l)} \lambda_I(\vartheta) \leq \lambda_I(r)$ holds for all $r,m,l \in \mathbf{T}$. Subsequently, $\inf_{\vartheta\in f(r,m,l)} \mu_I(\vartheta) \geq \min\{\mu_I(r), \mu_I(m), \mu_I(l)\}$ and $\sup_{\vartheta\in f(r,m,l)} \lambda_I(\vartheta) \leq \max\{\lambda_I(r), \lambda_I(m), \lambda_I(l)\}$. It is concluded that $I = (\mu_I, \lambda_I)$ is an I.F.T.sub-S. of $\mathbf{T}$. □

Assume $I(\mathbf{T})$ is the set of all the I.F. subsets of $\mathbf{T}$ where $I = (\mu_I, \lambda_I)$. We define $\mu_I^\star : \mathbf{T} \longrightarrow [0,1]$ as $\mu_I^\star(u) = \mu_I(u^\star)$ and $\lambda_I^\star : \mathbf{T} \to [0,1]$ as $\lambda_I^\star(u) = \lambda_I(u^\star)$. Here $\mu_I^\star$ and $\lambda_I^\star$ are fuzzy membership and non-membership of I.F. subset of $\mathbf{T}$. Let $\widetilde{\mathcal{V}}, \widetilde{\mathcal{W}}, \widetilde{\mathcal{X}} \in I(\mathbf{T})$ where $\widetilde{\mathcal{V}} = (\mu_{\widetilde{\mathcal{V}}}, \lambda_{\widetilde{\mathcal{V}}})$, $\widetilde{\mathcal{W}} = (\mu_{\widetilde{\mathcal{W}}}, \lambda_{\widetilde{\mathcal{W}}})$ and $\widetilde{\mathcal{X}} = (\mu_{\widetilde{\mathcal{X}}}, \lambda_{\widetilde{\mathcal{X}}})$ and for all $u \in \mathbf{T}$, the unary operation is defined as :

1. $(\widetilde{\mathcal{V}}^\star)^\star(u) \Rightarrow (\mu_{\widetilde{\mathcal{V}}}^\star)^\star(u) = \mu_{\widetilde{\mathcal{V}}}(u)$ *and* $(\lambda_{\widetilde{\mathcal{V}}}^\star)^\star(u) = \lambda_{\widetilde{\mathcal{V}}}(u)$
2. $(\widetilde{\mathcal{V}} \diamond \widetilde{\mathcal{W}} \diamond \widetilde{\mathcal{X}})^\star(u) = (\widetilde{\mathcal{X}}^\star \diamond \widetilde{\mathcal{W}}^\star \diamond \widetilde{\mathcal{V}}^\star)(u) \Rightarrow$
$(\mu_{\widetilde{\mathcal{V}}} \diamond \mu_{\widetilde{\mathcal{W}}} \diamond \mu_{\widetilde{\mathcal{X}}})^\star(u) = (\mu_{\widetilde{\mathcal{X}}}^\star \diamond \mu_{\widetilde{\mathcal{W}}}^\star \diamond \mu_{\widetilde{\mathcal{V}}}^\star)(u)$ and $(\lambda_{\widetilde{\mathcal{V}}} \diamond \lambda_{\widetilde{\mathcal{W}}} \diamond \lambda_{\widetilde{\mathcal{X}}})^\star(u) = (\lambda_{\widetilde{\mathcal{X}}}^\star \diamond \lambda_{\widetilde{\mathcal{W}}}^\star \diamond \lambda_{\widetilde{\mathcal{V}}}^\star)(u)$
3. $\widetilde{\mathcal{V}} \subseteq \widetilde{\mathcal{W}} \Rightarrow \widetilde{\mathcal{V}}^\star \subseteq \widetilde{\mathcal{W}}^\star$ implies $\mu_{\widetilde{\mathcal{V}}}^\star(u) \leq \mu_{\widetilde{\mathcal{W}}}^\star(u)$ and $\lambda_{\widetilde{\mathcal{V}}}^\star(u) \geq \lambda_{\widetilde{\mathcal{W}}}^\star(u)$.

Proposition 3.4 *In $\mathbf{T}$, let $\widetilde{\mathcal{V}}, \widetilde{\mathcal{W}}, \widetilde{\mathcal{X}} \in I(\mathbf{T})$ be I.F. subsets. Then the following assertions hold:*

1. $(\widetilde{\mathcal{V}}\cup\widetilde{\mathcal{W}}\cup\widetilde{\mathcal{X}})^{\star}=\widetilde{\mathcal{V}}^{\star}\cup\widetilde{\mathcal{W}}^{\star}\cup\widetilde{\mathcal{X}}^{\star}$
$\Rightarrow (max\{\mu_{\widetilde{\mathcal{V}}}(u),\mu_{\widetilde{\mathcal{W}}}(u),\mu_{\widetilde{\mathcal{X}}}(u)\})^{\star}=max\{\mu^{\star}_{\widetilde{\mathcal{V}}}(u),\mu^{\star}_{\widetilde{\mathcal{W}}}(u),\mu^{\star}_{\widetilde{\mathcal{X}}}(u)\}$ *and*
$(min\{\lambda_{\widetilde{\mathcal{V}}}(u),\lambda_{\widetilde{\mathcal{W}}}(u),\lambda_{\widetilde{\mathcal{X}}}(u)\})^{\star}=min\{\lambda^{\star}_{\widetilde{\mathcal{V}}}(u),\lambda^{\star}_{\widetilde{\mathcal{W}}}(u),\lambda^{\star}_{\widetilde{\mathcal{X}}}(u)\}$, *for all* $u\in\mathbf{T}$.

2. $(\widetilde{\mathcal{V}}\cap\widetilde{\mathcal{W}}\cap\widetilde{\mathcal{X}})^{\star}=\widetilde{\mathcal{V}}^{\star}\cap\widetilde{\mathcal{W}}^{\star}\cap\widetilde{\mathcal{X}}^{\star}$
$\Rightarrow (min\{\mu_{\widetilde{\mathcal{V}}}(u),\mu_{\widetilde{\mathcal{W}}}(u),\mu_{\widetilde{\mathcal{X}}}(u)\})^{\star}=min\{\mu^{\star}_{\widetilde{\mathcal{V}}}(u),\mu^{\star}_{\widetilde{\mathcal{W}}}(u),\mu^{\star}_{\widetilde{\mathcal{X}}}(u)\}$ *and*
$(max\{\lambda_{\widetilde{\mathcal{V}}}(u),\lambda_{\widetilde{\mathcal{W}}}(u),\lambda_{\widetilde{\mathcal{X}}}(u)\})^{\star}=max\{\lambda^{\star}_{\widetilde{\mathcal{V}}}(u),\lambda^{\star}_{\widetilde{\mathcal{W}}}(u),\lambda^{\star}_{\widetilde{\mathcal{X}}}(u)\}$, *for all* $u\in\mathbf{T}$.

Proof: The proof is omitted due to its triviality. □

Proposition 3.5 *An I.F.S.* $I^{\star}=(\mu_I^{\star},\lambda_I^{\star})$ *is an I.F.L.H.(I.F.Lt.H., I.F.R.H. resp.) of* $\mathbf{T}$ *for any I.F.R.H.(I.F.Lt.H., I.F.L.H. resp.)* $I=(\mu_I,\lambda_I)$ *of* $\mathbf{T}$.

Proof: Assume $I=(\mu_I,\lambda_I)$ is an I.F.R.H. of $\mathbf{T}$ implying $\inf\limits_{\vartheta\in f(r,m,l)}\mu_I(\vartheta)\geq\mu_I(r)$ and $\sup\limits_{\vartheta\in f(r,m,l)}\lambda_I(\vartheta)\leq\lambda_I(r)$ for every $r,m,l\in\mathbf{T}$. To proof $I^{\star}$ to be an I.F.L.H. of $\mathbf{T}$, we proceed as

$$\begin{aligned}\inf_{\vartheta\in f(r,m,l)}\mu_I^{\star}(\vartheta)&=\inf_{\vartheta^{\star}\in f(r,m,l)}\mu_I(\vartheta^{\star})\\&=\inf_{\vartheta\in f(r,m,l)^{\star}}\mu_I(\vartheta)\\&=\inf_{\vartheta\in f(l^{\star},m^{\star},r^{\star})}\mu_I(\vartheta)\\&\geq\mu_I(l^{\star})\\&=\mu_I^{\star}(l)\end{aligned}$$

and

$$\begin{aligned}\sup_{\vartheta\in f(r,m,l)}\lambda_I^{\star}(\vartheta)&=\sup_{\vartheta^{\star}\in f(r,m,l)}\lambda_I(\vartheta^{\star})\\&=\sup_{\vartheta\in f(r,m,l)^{\star}}\lambda_I(\vartheta)\\&=\sup_{\vartheta\in f(l^{\star},m^{\star},r^{\star})}\lambda_I(\vartheta)\\&\leq\lambda_I(l^{\star})\\&=\lambda_I^{\star}(l).\end{aligned}$$

Hence, $I^{\star}=(\mu_I^{\star},\lambda_I^{\star})$ is an I.F.L.H. of $\mathbf{T}$. □

Proposition 3.6 *In* $\mathbf{T}$, *an I.F.S.* $I^{\star}=(\mu_I^{\star},\lambda_I^{\star})$ *is an I.F. hyperideal if* $I=(\mu_I,\lambda_I)$ *is an I.F. hyperideal.*

This is similar to the proof of Proposition 3.5 .

Proposition 3.7 *Assume* $\mathbf{T}$ *is equipped with involution* $'\star'$ *satisfying* $\mathbf{T}=f(\mathbf{T},k^{\star},\mathbf{T},k^{\star},\mathbf{T},k^{\star},\mathbf{T})$. *Then* $I^{\star}\diamond I^{\star}\diamond I^{\star}\supseteq I$ *for any I.F. hyperideal* $I=(\mu_I,\lambda_I)$ *of* $\mathbf{T}$.

Proof: Suppose $\mathbf{T}=f(\mathbf{T},k^{\star},\mathbf{T},k^{\star},\mathbf{T},k^{\star},\mathbf{T})$ and $I=(\mu_I,\lambda_I)$ is an I.F. hyperideal of $\mathbf{T}$. For any element $k\in\mathbf{T}$, $k\in f(f(x,k^{\star},y),k^{\star},f(z,k^{\star},w))$ for every $x,y,z,w\in\mathbf{T}$. Then,

$$\begin{aligned}(\mu_I^{\star}\diamond\mu_I^{\star}\diamond\mu_I^{\star})(k)&=\sup_{(l,m,n)\in A_k}[min(\mu_I^{\star}(l),\mu_I^{\star}(m),\mu_I^{\star}(n)]\\&\geq min\,\{\inf_{\vartheta\in f(x,k^{\star},y)}\mu_I^{\star}(\vartheta),\mu_I^{\star}(k^{\star}),\inf_{\vartheta\in f(z,k^{\star},w)}\mu_I^{\star}(\vartheta)\}\\&=min\{\inf_{\vartheta^{\star}\in f(x,k^{\star},y)}\mu_I(\vartheta^{\star}),\mu_I^{\star}(k^{\star}),\inf_{\vartheta^{\star}\in f(z,k^{\star},w)}\mu_I(\vartheta^{\star})\}\\&=min\{\inf_{\vartheta\in f(x,k^{\star},y)^{\star}}\mu_I(\vartheta),\mu_I(k),\inf_{\vartheta\in f(z,k^{\star},w)^{\star}}\mu_I(\vartheta)\}\\&=min\{\inf_{\vartheta\in f(y^{\star},k,x^{\star})}\mu_I(\vartheta),\mu_I(k),\inf_{\vartheta\in f(w^{\star},k,l^{\star})}\mu_I(\vartheta)\}\\&\geq min\{\mu_I(k),\mu_I(k),\mu_I(k)\}\\&=\mu_I(k).\end{aligned}$$

and

$$\begin{aligned}(\lambda_I^\star \diamond \lambda_I^\star \diamond \lambda_I^\star)(k) &= \inf_{(l,m,n)\in A_k}[max\{\lambda_I^\star(l), \lambda_I^\star(m), \lambda_I^\star(n)\}] \\ &\leq max\{\sup_{\vartheta\in f(x,k^\star,y)} \lambda_I^\star(\vartheta), \lambda_I^\star(k^\star), \sup_{\vartheta\in f(z,k^\star,w)} \lambda_I^\star(\vartheta)\} \\ &= max\{\sup_{\vartheta^\star\in f(x,k^\star,y)} \lambda_I(\vartheta^\star), \lambda_I^\star(k^\star), \sup_{\vartheta^\star\in f(z,k^\star,w)} \lambda_I(\vartheta^\star)\} \\ &= max\{\sup_{\vartheta\in f(x,k^\star,y)^\star} \lambda_I(\vartheta), \lambda_I(k), \sup_{\vartheta\in f(z,k^\star,w)^\star} \lambda_I(\vartheta)\} \\ &= max\{\sup_{\vartheta\in f(y^\star,k,x^\star)} \lambda_I(\vartheta), \lambda_I(k), \sup_{\vartheta\in f(w^\star,k,l^\star)} \lambda_I(\vartheta)\} \\ &\leq max\{\lambda_I(k), \lambda_I(k), \lambda_I(k)\} \\ &= \lambda_I(k).\end{aligned}$$

Hence, $I^\star \diamond I^\star \diamond I^\star \supseteq I$. □

Definition 3.6 A $\star$-ternary semihypergroup $\mathbf{T}$ is called intra-regular if $a \in f(\mathbf{T}, (a^\star)^3, \mathbf{T})$, for any $a \in \mathbf{T}$.

Theorem 3.3 *[2] Let $\mathbf{T}$ be a $\star$-ternary semihypergroup such that $f(a^\star, a^\star, a^\star) = f(a, a, a)$, for any $a \in \mathbf{T}$. Then the following statements are equivalent:*

1. *$\mathbf{T}$ is intra-regular;*
2. *$R^\star \cap M^\star \cap L^\star \subseteq f(L, M, R)$ for every right hyperideal R, every lateral hyperideal M and every left hyperideal L of $\mathbf{T}$.*

Theorem 3.4 *Assume $\mathbf{T}$ is a ternary semihypergroup with involution $'\star'$ and $\widetilde{R} = (\mu_{\widetilde{R}}, \lambda_{\widetilde{R}})$ be any I.F.R.H., $\widetilde{M} = (\mu_{\widetilde{M}}, \lambda_{\widetilde{M}})$ be any I.F.Lt.H. and $\widetilde{L} = (\mu_{\widetilde{L}}, \lambda_{\widetilde{L}})$ be any I.F.L.H. hyperideal of $\mathbf{T}$. Then the following statements hold the same implication:*

1. *$\mathbf{T}$ is intra-regular;*
2. *$\widetilde{R}^\star \cap \widetilde{M}^\star \cap \widetilde{L}^\star \subseteq \widetilde{L} \diamond \widetilde{M} \diamond \widetilde{R}$.*

Proof: Assume $\mathbf{T}$ is an intra-regular and $\widetilde{R}$ to be an I.F.R.H., $\widetilde{M}$ to be a lateral hyperideal and $\widetilde{L}$ to be an I.F.L.H. hyperideal of $\mathbf{T}$. For any $a \in \mathbf{T}$, there exist $x, y \in \mathbf{T}$ such that

$$\begin{aligned}a &\in f(x, f(a^\star, a^\star, a^\star), y) \\ &= f(x, f(f(x, f(a^\star, a^\star, a^\star), y), a^\star, a^\star), y) \\ &= f(f(x, x, a^\star), f(a^\star, a^\star, y), f(a^\star, a^\star, y)).\end{aligned}$$

Then there exists $x_1 \in f(x, x, a^\star)$, $x_2 \in f(a^\star, a^\star, y)$ and $x_3 \in f(a^\star, a^\star, y)$ such that $a \in f(x_1, x_2, x_3)$. So, $(x_1, x_2, x_3) \in A_a$. Thus, we have $(\widetilde{R}^\star \cap \widetilde{M}^\star \cap \widetilde{L}^\star)(a) = \widetilde{R}^\star(a) \cap \widetilde{M}^\star(a) \cap \widetilde{L}^\star(a)$, it implies that

$$\begin{aligned}(\mu_{\widetilde{L}} \diamond \mu_{\widetilde{M}} \diamond \mu_{\widetilde{R}})(a) &= \sup_{(l,m,n)\in A_a} min\{\mu_{\widetilde{L}}(l), \mu_{\widetilde{M}}(m), \mu_{\widetilde{R}}(n)\} \\ &\geq min\{\mu_{\widetilde{L}}(x_1), \mu_{\widetilde{M}}(x_2), \mu_{\widetilde{R}}(x_3)\}.\end{aligned}$$

and

$$\begin{aligned}(\lambda_{\widetilde{L}} \diamond \lambda_{\widetilde{M}} \diamond \lambda_{\widetilde{R}})(a) &= \inf_{(l,m,n)\in A_a} max\{\lambda_{\widetilde{L}}(l), \lambda_{\widetilde{M}}(m), \lambda_{\widetilde{R}}(n)\} \\ &\leq max\{\lambda_{\widetilde{L}}(x_1), \lambda_{\widetilde{M}}(x_2), \lambda_{\widetilde{R}}(x_3)\}.\end{aligned}$$

As $\widetilde{L}$ is an I.F.L.H., we have $\inf_{\vartheta\in f(x,x,a^\star)} \mu_{\widetilde{L}}(\vartheta) \geq \mu_{\widetilde{L}}(a^\star)$ and $\sup_{\vartheta\in f(x,x,a^\star)} \lambda_{\widetilde{L}}(\vartheta) \leq \lambda_{\widetilde{L}}(a^\star)$. Since $x_1 \in f(x, x, a^\star)$, it implies $\mu_{\widetilde{L}}(x_1) \geq \mu_{\widetilde{L}}(a^\star)$ and $\lambda_{\widetilde{L}}(x_1) \leq \lambda_{\widetilde{L}}(a^\star)$. $\widetilde{M}$ is an I.F.Lt.H., we have $\inf_{\vartheta\in f(a^\star,a^\star,y)} \mu_{\widetilde{M}}(\vartheta) \geq \mu_{\widetilde{M}}(a^\star)$ and $\sup_{\vartheta\in f(a^\star,a^\star,y)} \lambda_{\widetilde{M}}(\vartheta) \leq \lambda_{\widetilde{M}}(a^\star)$. Since $x_2 \in f(a^\star, a^\star, y)$, it implies

$\mu_{\widetilde{M}}(x_2) \geq \mu_{\widetilde{M}}(a^\star)$ and $\lambda_{\widetilde{M}}(x_2) \leq \lambda_{\widetilde{M}}(a^\star)$. $\widetilde{R}$ is an I.F.R.H., we have $\inf_{\vartheta \in f(a^\star, a^\star, y)} \mu_{\widetilde{R}}(\vartheta) \geq \mu_{\widetilde{R}}(a^\star)$ and $\sup_{\vartheta \in f(a^\star, a^\star, y)} \lambda_{\widetilde{R}}(\vartheta) \leq \lambda_{\widetilde{R}}(a^\star)$. Since $x_3 \in f(a^\star, a^\star, y)$, it implies $\mu_{\widetilde{R}}(x_3) \geq \mu_{\widetilde{R}}(a^\star)$ and $\lambda_{\widetilde{R}}(x_3) \leq \lambda_{\widetilde{R}}(a^\star)$.

It is concluded that

$$\begin{aligned}
(\mu_{\widetilde{L}} \diamond \mu_{\widetilde{M}} \diamond \mu_{\widetilde{R}})(a) &\geq min\{\mu_{\widetilde{L}}(x_1), \mu_{\widetilde{M}}(x_2), \mu_{\widetilde{R}}(x_3)\} \\
&\geq min\{\mu_{\widetilde{L}}(a^\star), \mu_{\widetilde{M}}(a^\star), \mu_{\widetilde{R}}(a^\star)\} \\
&= min\{\mu^\star_{\widetilde{L}}(a), \mu^\star_{\widetilde{M}}(a), \mu^\star_{\widetilde{R}}(a)\} \\
&= min\{\mu^\star_{\widetilde{R}}(a), \mu^\star_{\widetilde{M}}(a), \mu^\star_{\widetilde{L}}(a)\}.
\end{aligned}$$

and

$$\begin{aligned}
(\lambda_{\widetilde{L}} \diamond \lambda_{\widetilde{M}} \diamond \lambda_{\widetilde{R}})(a) &\leq max\{\lambda_{\widetilde{L}}(x_1), \lambda_{\widetilde{M}}(x_2), \lambda_{\widetilde{R}}(x_3)\} \\
&\leq max\{\lambda_{\widetilde{L}}(a^\star), \lambda_{\widetilde{M}}(a^\star), \lambda_{\widetilde{R}}(a^\star)\} \\
&= max\{\lambda^\star_{\widetilde{L}}(a), \lambda^\star_{\widetilde{M}}(a), \lambda^\star_{\widetilde{R}}(a)\} \\
&= max\{\lambda^\star_{\widetilde{R}}(a), \lambda^\star_{\widetilde{M}}(a), \lambda^\star_{\widetilde{L}}(a)\}.
\end{aligned}$$

Hence, $\widetilde{\mathcal{R}}^\star \cap \widetilde{\mathcal{M}}^\star \cap \widetilde{\mathcal{L}}^\star \subseteq \widetilde{\mathcal{L}} \diamond \widetilde{\mathcal{M}} \diamond \widetilde{\mathcal{R}}$.

Taking (2) implies (1), suppose $\mathcal{R}$, $\mathcal{M}$ and $\mathcal{L}$ be an R.H., an Lt.H. and an L.H. of $\mathbf{T}$. Consequently, $(\varkappa_{\mathcal{R}^\star}, \bar{\varkappa}_{\mathcal{R}^\star})$ be an I.F.L.H. hyperideal, $(\varkappa_{\mathcal{M}^\star}, \bar{\varkappa}_{\mathcal{M}^\star})$ be an I.F.Lt.H. and $(\varkappa_{\mathcal{L}^\star}, \bar{\varkappa}_{\mathcal{L}^\star})$ be an I.F.R.H. of $\mathbf{T}$ as $\mathcal{R}^\star, \mathcal{M}^\star$ and $\mathcal{L}^\star$ be an L.H., an Lt.H. and an R.H. of $\mathbf{T}$. Subsequently, $(\varkappa_{\mathcal{R}^\star}, \bar{\varkappa}_{\mathcal{R}^\star}) = (\varkappa^\star_{\mathcal{R}}, \bar{\varkappa}^\star_{\mathcal{R}})$, $(\varkappa_{\mathcal{M}^\star}, \bar{\varkappa}_{\mathcal{M}^\star}) =(\varkappa^\star_{\mathcal{M}}, \bar{\varkappa}^\star_{\mathcal{M}})$, $(\varkappa_{\mathcal{L}^\star}, \bar{\varkappa}_{\mathcal{L}^\star}) = (\varkappa^\star_{\mathcal{L}}, \bar{\varkappa}^\star_{\mathcal{L}})$. Let $a^\star \in \mathcal{R}^\star \cap \mathcal{M}^\star \cap \mathcal{L}^\star$, then for $(\varkappa_{\mathcal{R}^\star}, \bar{\varkappa}_{\mathcal{R}^\star}) \diamond (\varkappa_{\mathcal{M}^\star}, \bar{\varkappa}_{\mathcal{M}^\star}) \diamond (\varkappa_{\mathcal{L}^\star}, \bar{\varkappa}_{\mathcal{L}^\star})$, we have

$$\begin{aligned}
(\varkappa^\star_{\mathcal{R}} \diamond \varkappa^\star_{\mathcal{M}} \diamond \varkappa^\star_{\mathcal{L}})^\star(a^\star) &\geq min(\varkappa^\star_{\mathcal{L}^\star}, \varkappa^\star_{\mathcal{M}^\star}, \varkappa^\star_{\mathcal{R}^\star})^\star(a^\star) \\
&= min(\varkappa_{\mathcal{L}^\star}, \varkappa_{\mathcal{M}^\star}, \varkappa_{\mathcal{R}^\star})(a^\star) \\
&= min(\varkappa_{\mathcal{L}^\star}(a^\star), \varkappa_{\mathcal{M}^\star}(a^\star), \varkappa_{\mathcal{R}^\star}(a^\star) \\
&= 1
\end{aligned}$$

and

$$\begin{aligned}
(\bar{\varkappa}^\star_{\mathcal{R}} \diamond \bar{\varkappa}^\star_{\mathcal{M}} \diamond \bar{\varkappa}^\star_{\mathcal{L}})(a^\star) &\leq max(\bar{\varkappa}^\star_{\mathcal{L}^\star}, \bar{\varkappa}^\star_{\mathcal{J}^\star}, \bar{\varkappa}^\star_{\mathcal{R}^\star})^\star(a^\star) \\
&= max(\bar{\varkappa}_{\mathcal{L}^\star}, \bar{\varkappa}_{\mathcal{J}^\star}, \bar{\varkappa}_{\mathcal{R}^\star})(a^\star) \\
&= max(\bar{\varkappa}_{\mathcal{L}^\star}(a^\star), \bar{\varkappa}_{\mathcal{J}^\star}(a^\star), \bar{\varkappa}_{\mathcal{R}^\star}(a^\star)) \\
&= 0.
\end{aligned}$$

Since $(\varkappa^\star_{\mathcal{R}} \diamond \varkappa^\star_{\mathcal{M}} \diamond \varkappa^\star_{\mathcal{L}})^\star$ and $(\bar{\varkappa}^\star_{\mathcal{R}} \diamond \bar{\varkappa}^\star_{\mathcal{M}} \diamond \bar{\varkappa}^\star_{\mathcal{L}})^\star$ are fuzzy sets, we get $(\varkappa^\star_{\mathcal{R}} \diamond \varkappa^\star_{\mathcal{M}} \diamond \varkappa^\star_{\mathcal{L}})^\star(a^\star) \leq 1$ and $(\bar{\varkappa}^\star_{\mathcal{R}} \diamond \bar{\varkappa}^\star_{\mathcal{M}} \diamond \bar{\varkappa}^\star_{\mathcal{L}})^\star(a^\star) \geq 0$ for any $a^\star \in \mathbf{T}$. Thus,

$$\begin{aligned}
\sup_{(l,m,n) \in A^\star_a} min[\varkappa_{\mathcal{L}}(l), \varkappa_{\mathcal{M}}(m), \varkappa_{\mathcal{R}}(n)] &= min(\varkappa^\star_{\mathcal{L}^\star}, \varkappa^\star_{\mathcal{M}^\star}, \varkappa^\star_{\mathcal{R}^\star})^\star(a^\star) \\
&= (\varkappa_{\mathcal{R}^\star} \diamond \varkappa_{\mathcal{M}^\star} \diamond \varkappa_{\mathcal{L}^\star})^\star(a^\star).
\end{aligned}$$

and

$$\begin{aligned}
\inf_{(l,m,n) \in A^\star_a} max[\bar{\varkappa}_{\mathcal{L}}(l), \bar{\varkappa}_{\mathcal{M}}(m), \bar{\varkappa}_{\mathcal{R}}(n)] &= max(\bar{\varkappa}^\star_{\mathcal{L}^\star}, \bar{\varkappa}^\star_{\mathcal{M}^\star}, \bar{\varkappa}^\star_{\mathcal{R}^\star})^\star(a^\star) \\
&= (\bar{\varkappa}_{\mathcal{R}^\star} \diamond \bar{\varkappa}_{\mathcal{M}^\star} \diamond \bar{\varkappa}_{\mathcal{L}^\star})^\star(a^\star).
\end{aligned}$$

Thus, $(\varkappa^\star_{\mathcal{R}} \diamond \varkappa^\star_{\mathcal{M}} \diamond \varkappa^\star_{\mathcal{L}})^\star(a^\star) = 1$ and $(\bar{\varkappa}^\star_{\mathcal{R}} \diamond \bar{\varkappa}^\star_{\mathcal{M}} \diamond \bar{\varkappa}^\star_{\mathcal{L}})^\star(a^\star) = 0$. It implies the existence of $l, m, r \in \mathbf{T}$ such that $a^\star \in f(l, m, r)$ and $\varkappa_{\mathcal{L}}(l) = \varkappa_{\mathcal{M}}(m) = \varkappa_{\mathcal{R}}(r) = 1$ and $\bar{\varkappa}_{\mathcal{L}}(l) = \bar{\varkappa}_{\mathcal{M}}(m) = \bar{\varkappa}_{\mathcal{R}}(r) = 0$. Therefore, $a^\star \in f(l, m, r) \subseteq f(\mathcal{L}, \mathcal{M}, \mathcal{R})$ and hence $\mathcal{R}^\star \cap \mathcal{M}^\star \cap \mathcal{L}^\star \subseteq f(\mathcal{L}, \mathcal{M}, \mathcal{R})$. By Theorem 3.3, it is concluded that $\mathbf{T}$ proves to be intra-regular. □

4. Intuitionistic Fuzzy Hyperfilters in $\star$-Ternary Semihypergroups

Here, we discusses intuitionistic fuzzy hyperfilters in $\star$-ternary semihypergroups(ternary semihypergroups equipped with involution $'\star'$). Additionally, with the assistance of intuitionistic fuzzy prime hyperideals and intuitionistic fuzzy hyperfilters, we have associated intuitionistic fuzzy sets with their complements.

Definition 4.1 A T.sub-S. $\mathcal{F}$ of $\mathbf{T}$ is called a hyperfilter(briefly, H.F.) of $\mathbf{T}$ if for every $h_1, h_2, h_3 \in \mathbf{T}$, $f(h_1, h_2, h_3) \cap \mathcal{F} \neq \emptyset$ implies $h_1^\star \in \mathcal{F}$, $h_2^\star \in \mathcal{F}$ and $h_3^\star \in \mathcal{F}$.

Example 4.1 Consider the Example 2.1. In Example 2.1 if we replace

$$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \text{ by } \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and

$$P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ by } \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

Then $\mathbf{T}$ will be a $\star$-ternary semihypergroup.

Now, let $\mathcal{F} = \left\{ \begin{pmatrix} 0 & 0 & 0 & a \\ 0 & 0 & b & 0 \\ 0 & a & 0 & 0 \\ b & 0 & 0 & 0 \end{pmatrix} : a, b \in \{j, -j\} \right\}$ s.t. $\mathcal{F} \subseteq \mathbf{T}$. Then by some small calculations $\mathcal{F}$ will be a hyperfilter of $\mathbf{T}$.

Definition 4.2 An I.F.T.sub-S. I of $\mathbf{T}$ is referred as an intuitionistic fuzzy hyperfilter(briefly, I.F.H.F.) of $\mathbf{T}$, if

$$\sup_{\vartheta \in f(h_1,h_2,h_3)} \mu_I(\vartheta) \leq \min\{\mu_I^\star(h_1), \mu_I^\star(h_2), \mu_I^\star(h_3)\} \text{ and } \inf_{\vartheta \in f(h_1,h_2,h_3)} \lambda_I(\vartheta) \geq \max\{\lambda_I^\star(h_1), \lambda_I^\star(h_2), \lambda_I^\star(h_3)\}.$$

Example 4.2 Consider the Example 4.1. Let $I = (\mu_I, \lambda_I)$ be the I.F.S. defined on $\mathbf{T}$, where $I : \mathbf{T} \longrightarrow [0, 1]$ by

$$\mu_I(X_i) = 1 \text{ and } \lambda_I(X_i) = 0, \text{ where } X_i \in \left\{ \begin{pmatrix} 0 & 0 & 0 & a \\ 0 & 0 & b & 0 \\ 0 & a & 0 & 0 \\ b & 0 & 0 & 0 \end{pmatrix} : a, b \in \{j, -j\} \right\}$$

$$\mu_I(X_j) = 0 \text{ and } \lambda_I(X_j) = 1, \text{ where } X_j \in \left\{ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & 0 \\ a & 0 & 0 & 0 \end{pmatrix} : a \in \{j, -j\} \right\} \text{ and}$$

$$\mu_I(X_k) = 0 \text{ and } \lambda_I(X_k) = 1, \text{ where } X_k \in \left\{ \begin{pmatrix} 0 & 0 & 0 & a \\ 0 & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} : a \in \{0, j, -j\} \right\}.$$

Then, we can verify that $\sup_{\vartheta \in f(h_1,h_2,h_3)} \mu_I(\vartheta) \leq \min\{\mu_I^\star(h_1), \mu_I^\star(h_2), \mu_I^\star(h_3)\}$ and $\inf_{\vartheta \in f(h_1,h_2,h_3)} \lambda_I(\vartheta) \geq \max\{\lambda_I^\star(h_1), \lambda_I^\star(h_2), \lambda_I^\star(h_3)\}$. Hence, $I = (\mu_I, \lambda_I)$ is an I.F.H.F. of $\mathbf{T}$.

Theorem 4.1 *A subset $\mathcal{F}(\neq \emptyset)$ of $\mathbf{T}$ is a H.F. of $\mathbf{T}$ if and only if $(\varkappa_{\mathcal{F}}, \bar{\varkappa}_{\mathcal{F}})$ is an I.F.H.F. of $\mathbf{T}$.*

Proof: Assume $\mathcal{F}$ is the H.F. of $\mathbf{T}$. Let $h_1, h_2, h_3 \in \mathbf{T}$. Then $\sup_{\vartheta \in f(h_1,h_2,h_3)} \varkappa_{\mathcal{F}}(\vartheta) \leq \min\{\varkappa_{\mathcal{F}}^\star(h_1), \varkappa_{\mathcal{F}}^\star(h_2), \varkappa_{\mathcal{F}}^\star(h_3)\}$ and $\inf_{\vartheta \in f(h_1,h_2,h_3)} \bar{\varkappa}_{\mathcal{F}}(\vartheta) \geq \max\{\bar{\varkappa}_{\mathcal{F}}^\star(h_1), \bar{\varkappa}_{\mathcal{F}}^\star(h_2), \bar{\varkappa}_{\mathcal{F}}^\star(h_3)\}$. Indeed, if $f(h_1, h_2, h_3) \cap \mathcal{F} = \emptyset$, then ϑ

$\in f(h_1,h_2,h_3)$ implying $\vartheta \notin \mathcal{F}$, so $\varkappa_{\mathcal{F}}(\vartheta)=0$ and $\bar{\varkappa}_{\mathcal{F}}(\vartheta)=1$. In this case, we have $\sup_{\vartheta\in f(h_1,h_2,h_3)} \varkappa_{\mathcal{F}}(\vartheta)=0$ and $\inf_{\vartheta\in f(h_1,h_2,h_3)} \bar{\varkappa}_{\mathcal{F}}(\vartheta)=1$. Thus for any $h_1\in \mathbf{T}$, $\varkappa^{\star}_{\mathcal{F}}(h_1)=\varkappa_{\mathcal{F}}(h_1^{\star})\geq 0$ and $\bar{\varkappa}^{\star}_{\mathcal{F}}(h_1)=\bar{\varkappa}_{\mathcal{F}}(h_1^{\star})\leq 1$. Thus, we get $\sup_{\vartheta\in f(h_1,h_2,h_3)} \varkappa_{\mathcal{F}}(\vartheta)\leq \min\{\varkappa^{\star}_{\mathcal{F}}(h_1),\varkappa^{\star}_{\mathcal{F}}(h_2),\varkappa^{\star}_{\mathcal{F}}(h_3)\}$ and $\inf_{\vartheta\in f(h_1,h_2,h_3)} \bar{\varkappa}_{\mathcal{F}}(\vartheta)\geq \max\{\bar{\varkappa}^{\star}_{\mathcal{F}}(h_1), \bar{\varkappa}^{\star}_{\mathcal{F}}(h_2),\bar{\varkappa}^{\star}_{\mathcal{F}}(h_3)\}$. On the other side if $f(h_1,h_2,h_3)\cap\mathcal{F}\neq\emptyset$, then there exists $\vartheta\in f(h_1,h_2,h_3)$ such that $\vartheta\in\mathcal{F}$, it implies $\sup_{\vartheta\in f(h_1,h_2,h_3)} \varkappa_{\mathcal{F}}(\vartheta)=1$ and $\inf_{\vartheta\in f(h_1,h_2,h_3)} \bar{\varkappa}_{\mathcal{F}}(\vartheta)=0$. Also, by the definition of H.F., we have $h_1^{\star}\in\mathcal{F}$, $h_2^{\star}\in\mathcal{F}$ and $h_3^{\star}\in\mathcal{F}$. Then $\varkappa^{\star}_{\mathcal{F}}(h_1)=\varkappa_{\mathcal{F}}(h_1^{\star})=1$, $\varkappa^{\star}_{\mathcal{F}}(h_2)=\varkappa_{\mathcal{F}}(h_2^{\star})=1$, $\varkappa^{\star}_{\mathcal{F}}(h_3)=\varkappa_{\mathcal{F}}(h_3^{\star})=1$ and, $\bar{\varkappa}^{\star}_{\mathcal{F}}(h_1)=\bar{\varkappa}_{\mathcal{F}}(h_1^{\star})=0$, $\bar{\varkappa}^{\star}_{\mathcal{F}}(h_2)=\bar{\varkappa}_{\mathcal{F}}(h_2^{\star})=0$, $\bar{\varkappa}^{\star}_{\mathcal{F}}(h_3)=\bar{\varkappa}_{\mathcal{F}}(h_3^{\star})=0$, which implies that $\sup_{\vartheta\in f(h_1,h_2,h_3)} \varkappa_{\mathcal{F}}(\vartheta)=1\leq\min\{\varkappa^{\star}_{\mathcal{F}}(h_1),\varkappa^{\star}_{\mathcal{F}}(h_2),\varkappa^{\star}_{\mathcal{F}}(h_3)\}$ and $\inf_{\vartheta\in f(h_1,h_2,h_3)} \bar{\varkappa}_{\mathcal{F}}(\vartheta)=0\geq \max\{\bar{\varkappa}^{\star}_{\mathcal{F}}(h_1),\bar{\varkappa}^{\star}_{\mathcal{F}}(h_2),\bar{\varkappa}^{\star}_{\mathcal{F}}(h_3)\}$. By Proposition 3.1, in $\mathbf{T}$, $(\varkappa_{\mathcal{F}},\bar{\varkappa}_{\mathcal{F}})$ is concluded to be an I.F.T.sub-S.. Therefore, $(\varkappa_{\mathcal{F}},\bar{\varkappa}_{\mathcal{F}})$ proves to be a I.F.H.F. of $\mathbf{T}$.
Conversely, assume $(\varkappa_{\mathcal{F}},\bar{\varkappa}_{\mathcal{F}})$ is an I.F.H.F. of $\mathbf{T}$. Let $h_1,h_2,h_3\in\mathbf{T}$ such that $f(h_1,h_2,h_3)\cap\mathcal{F}\neq\emptyset$. We have to show that $h_1^{\star}\in\mathcal{F}$, $h_2^{\star}\in\mathcal{F}$ and $h_3^{\star}\in\mathcal{F}$. If $h_1^{\star}\notin\mathcal{F}$ or $h_2^{\star}\notin\mathcal{F}$ or $h_3^{\star}\notin\mathcal{F}$. Then $\varkappa^{\star}_{\mathcal{F}}(h_1)=\varkappa_{\mathcal{F}}(h_1^{\star})=0$ or $\varkappa^{\star}_{\mathcal{F}}(h_2)=\varkappa_{\mathcal{F}}(h_2^{\star})=0$ or $\varkappa^{\star}_{\mathcal{F}}(h_3)=\varkappa_{\mathcal{F}}(h_3^{\star})=0$ and, $\bar{\varkappa}^{\star}_{\mathcal{F}}(h_1)=\bar{\varkappa}_{\mathcal{F}}(h_1^{\star})=1$ or $\bar{\varkappa}^{\star}_{\mathcal{F}}(h_2)=\bar{\varkappa}_{\mathcal{F}}(h_2^{\star})=1$ or $\bar{\varkappa}^{\star}_{\mathcal{F}}(h_3)=\bar{\varkappa}_{\mathcal{F}}(h_3^{\star})=1$. By hypothesis $\sup_{\vartheta\in f(h_1,h_2,h_3)} \varkappa_{\mathcal{F}}(\vartheta)\leq\min\{\varkappa^{\star}_{\mathcal{F}}(h_1),\varkappa^{\star}_{\mathcal{F}}(h_2),\varkappa^{\star}_{\mathcal{F}}(h_3)\}=0$ and $\inf_{\vartheta\in f(h_1,h_2,h_3)} \bar{\varkappa}_{\mathcal{F}}(\vartheta)\geq\max\{\bar{\varkappa}^{\star}_{\mathcal{F}}(h_1),\bar{\varkappa}^{\star}_{\mathcal{F}}(h_2),\bar{\varkappa}^{\star}_{\mathcal{F}}(h_3)\}=1$. It means that $\sup_{\vartheta\in f(h_1,h_2,h_3)} \varkappa_{\mathcal{F}}(\vartheta)=0$ and $\inf_{\vartheta\in f(h_1,h_2,h_3)} \bar{\varkappa}_{\mathcal{F}}(\vartheta)=1$. Thus for any $\vartheta\in f(h_1,h_2,h_3)$, $\varkappa_{\mathcal{F}}(\vartheta)=0$ and $\bar{\varkappa}_{\mathcal{F}}(\vartheta)=1$, which contradict the fact that $f(h_1,h_2,h_3)\cap\mathcal{F}\neq\emptyset$. Hence, $h_1^{\star}\in\mathcal{F}$, $h_2^{\star}\in\mathcal{F}$ and $h_3^{\star}\in\mathcal{F}$. Therefore, $\mathcal{F}$ is the H.F. of $\mathbf{T}$. □

Proposition 4.1 *An I.F.S.* $I=(\mu_I,\lambda_I)$ *is an I.F.H.F. of* $\mathbf{T}$ *if and only if* $I_{\langle\xi,\zeta\rangle}\neq\emptyset$ *is a H.F. of* $\mathbf{T}$.

Proof: Assume $I=(\mu_I,\lambda_I)$ is an I.F.H.F. of $\mathbf{T}$. Then $I=(\mu_I,\lambda_I)$ is an I.F.T.sub-S. of $\mathbf{T}$. Using Proposition 3.3, in $\mathbf{T}$, $I_{\langle\xi,\zeta\rangle}$ is a T.sub-S. of $\mathbf{T}$. Let $h_1,h_2,h_3\in\mathbf{T}$ such that $f(h_1,h_2,h_3)\cap I_{\langle\xi,\zeta\rangle}\neq\emptyset$. then there exists $\vartheta\in f(h_1,h_2,h_3)$ such that $\vartheta\in I_{\langle\xi,\zeta\rangle}$, so we have $\mu_I(\vartheta)\geq\xi$ and $\lambda_I(\vartheta)\leq\zeta$. Since I is a I.F.H.F. of $\mathbf{T}$, we have $\xi\leq\sup_{\vartheta\in f(h_1,h_2,h_3)} \mu_I(\vartheta)\leq\min\{\mu_I^{\star}(h_1),\mu_I^{\star}(h_2),\mu_I^{\star}(h_3)\}$ and $\zeta\geq\inf_{\vartheta\in f(h_1,h_2,h_3)} \lambda_I(\vartheta)\geq \max\{\lambda_I^{\star}(h_1),\lambda_I^{\star}(h_2),\lambda_I^{\star}(h_3)\}$. It implies that $\mu_I^{\star}(h_1)=\mu_I(h_1^{\star})\geq\xi$, $\mu_I^{\star}(h_2)=\mu_I(h_2^{\star})\geq\xi$, $\mu_I^{\star}(h_3)=\mu_I(h_3^{\star})\geq\xi$ and $\lambda_I^{\star}(h_1)=\lambda_I(h_1^{\star})\leq\zeta$, $\lambda_I^{\star}(h_2)=\lambda_I(h_2^{\star})\leq\zeta$, $\lambda_I^{\star}(h_3)=\lambda_I(h_3^{\star})\leq\zeta$. This shows that $h_1^{\star}\in I_{\langle\xi,\zeta\rangle}$, $h_2^{\star}\in I_{\langle\xi,\zeta\rangle}$ and $h_3^{\star}\in I_{\langle\xi,\zeta\rangle}$. Therefore, $I_{\langle\xi,\zeta\rangle}$ is the H.F. of $\mathbf{T}$.

On the converse, assume $I_{\langle\xi,\zeta\rangle}\neq\emptyset$ is a H.F. of $\mathbf{T}$. Let $h_1,h_2,h_3\in\mathbf{T}$ be arbitrary elements. We claim that $I=(\mu_I,\lambda_I)$ is an I.F.H.F. of $\mathbf{T}$ i.e. $\sup_{\vartheta\in f(h_1,h_2,h_3)} \mu_I(\vartheta)\leq\min\{\mu_I^{\star}(h_1),\mu_I^{\star}(h_2),\mu_I^{\star}(h_3)\}$ and $\inf_{\vartheta\in f(h_1,h_2,h_3)} \lambda_I(\vartheta)\geq\max\{\lambda_I^{\star}(h_1),\lambda_I^{\star}(h_2),\lambda_I^{\star}(h_3)\}$. If $\sup_{\vartheta\in f(h_1,h_2,h_3)} \mu_I(\vartheta)>\min\{\mu_I^{\star}(h_1),\mu_I^{\star}(h_2),\mu_I^{\star}(h_3)\}$ and $\inf_{\vartheta\in f(h_1,h_2,h_3)} \lambda_I(\vartheta)<\max\{\lambda_I^{\star}(h_1),\lambda_I^{\star}(h_2),\lambda_I^{\star}(h_3)\}$ for some $h_1,h_2,h_3\in\mathbf{T}$. Then there exists $t,s\in(0,1]$ such that $\sup_{\vartheta\in f(h_1,h_2,h_3)} \mu_I(\vartheta)\geq t>\min\{\mu_I^{\star}(h_1),\mu_I^{\star}(h_2),\mu_I^{\star}(h_3)\}$ and $\inf_{\vartheta\in f(h_1,h_2,h_3)} \lambda_I(\vartheta)\leq s< \max\{\lambda_I^{\star}(h_1),\lambda_I^{\star}(h_2),\lambda_I^{\star}(h_3)\}$. It follows that μ_I has supremum property and λ_I has infimum property that $\mu_I(\vartheta)>t$ and $\lambda_I(\vartheta)<s$, for some $\vartheta\in f(h_1,h_2,h_3)$. It implies $\vartheta\in I_{\langle\xi,\zeta\rangle}$ and $f(h_1,h_2,h_3)\cap I_{\langle\xi,\zeta\rangle}\neq\emptyset$. Since $I_{\langle\xi,\zeta\rangle}$ is a H.F. of $\mathbf{T}$, it implies $h_1^{\star}\in I_{\langle\xi,\zeta\rangle}$, $h_2^{\star}\in I_{\langle\xi,\zeta\rangle}$ and $h_3^{\star}\in I_{\langle\xi,\zeta\rangle}$. Then $\min\{\mu_I^{\star}(h_1),\mu_I^{\star}(h_2),\mu_I^{\star}(h_3)\}\geq t$ and $\max\{\lambda_I^{\star}(h_1),\lambda_I^{\star}(h_2),\lambda_I^{\star}(h_3)\}\leq s$, which is a contradiction. Hence, $\sup_{\vartheta\in f(h_1,h_2,h_3)} \mu_I(\vartheta)\leq\min\{\mu_I^{\star}(h_1),\mu_I^{\star}(h_2),\mu_I^{\star}(h_3)\}$ and $\inf_{\vartheta\in f(h_1,h_2,h_3)} \lambda_I(\vartheta)\geq\max\{\lambda_I^{\star}(h_1),\lambda_I^{\star}(h_2),\lambda_I^{\star}(h_3)\}$ for all $h_1,h_2,h_3\in\mathbf{T}$. By Proposition 3.3, I is an I.F.T.sub-S. of $\mathbf{T}$. Therefore, $I=(\mu_I,\lambda_I)$ is an I.F.H.F. of $\mathbf{T}$. □

Definition 4.3 In $\mathbf{T}$, the complement of an I.F.S. $I=(\mu_I,\lambda_I)$ is also an I.F.S., represented by $I^c=(\mu_I^c,\lambda_I^c)$ and defined as :

$$I^c_{\mathbf{T}} : \mathbf{T} \to [0,1] \ s.t\ I^c = \langle w, 1-\mu_I(w), 1-\lambda_I(w) : w \in \mathbf{T}\rangle$$

Lemma 4.1 *Suppose $I = (\mu_I, \lambda_I)$ is an I.F.S. of $\mathbf{T}$. Then the following statements hold with the same implication:*

1. $\sup\limits_{\vartheta\in f(h_1,h_2,h_3)} \mu_I(\vartheta) \leq min\{\mu^\star_I(h_1), \mu^\star_I(h_2), \mu^\star_I(h_3)\}$ *and* $\inf\limits_{\vartheta\in f(h_1,h_2,h_3)} \lambda_I(\vartheta) \geq max\{\lambda^\star_I(h_1), \lambda^\star_I(h_2), \lambda^\star_I(h_3)\}$.
2. $\inf\limits_{\vartheta\in f(h_1,h_2,h_3)} \mu^c_I(\vartheta) \geq max\{\mu^{\star^c}_I(h_1), \mu^{\star^c}_I(h_2), \mu^{\star^c}_I(h_3)\}$ *and* $\sup\limits_{\vartheta\in f(h_1,h_2,h_3)} \lambda^c_I(\vartheta) \leq min\{\lambda^{\star^c}_I(h_1), \lambda^{\star^c}_I(h_2), \lambda^{\star^c}_I(h_3)\}$.

Proof: (1) $\Longrightarrow$ (2): Suppose that $\sup\limits_{\vartheta\in f(h_1,h_2,h_3)} \mu_I(\vartheta) \leq \min\{\mu^\star_I(h_1), \mu^\star_I(h_2), \mu^\star_I(h_3)\}$ and $\inf\limits_{\vartheta\in f(h_1,h_2,h_3)} \lambda_I(\vartheta) \geq \max\{\lambda^\star_I(h_1), \lambda^\star_I(h_2), \lambda^\star_I(h_3)\}$, for all $h_1, h_2, h_3 \in \mathbf{T}$. Then for any $\vartheta \in f(h_1,h_2,h_3)$, $\mu_I(\vartheta) \leq \min\{\mu^\star_I(h_1), \mu^\star_I(h_2), \mu^\star_I(h_3)\}$ and $\lambda_I(\vartheta) \geq \max\{\lambda^\star_I(h_1), \lambda^\star_I(h_2), \lambda^\star_I(h_3)\}$. Then,

$$\begin{aligned}
\mu^c_I(\vartheta) &= 1-\mu_I(\vartheta)\\
&\geq 1 - min\{\mu^\star_I(h_1), \mu^\star_I(h_2), \mu^\star_I(h_3)\}\\
&= max\{(1-\mu^\star_I(h_1)), (1-\mu^\star_I(h_2)), (1-\mu^\star_I(h_3))\}\\
&= max\{\mu^{\star^c}_I(h_1), \mu^{\star^c}_I(h_2), \mu^{\star^c}_I(h_3)\}
\end{aligned}$$

and

$$\begin{aligned}
\lambda^c_I(\vartheta) &= 1-\lambda_I(\vartheta)\\
&\leq 1 - max\{(\lambda^\star_I(h_1), \lambda^\star_I(h_2), \lambda^\star_I(h_3)\}\\
&= min\{(1-\lambda^\star_I(h_1)), (1-\lambda^\star_I(h_2)), (1-\lambda^\star_I(h_3))\}\\
&= min\{\lambda^{\star^c}_I(h_1), \lambda^{\star^c}_I(h_2), \lambda^{\star^c}_I(h_3)\}.
\end{aligned}$$

It implies that $\inf\limits_{\vartheta\in f(h_1,h_2,h_3)} \mu^c_I(\vartheta) \geq \max\{\mu^{\star^c}_I(h_1), \mu^{\star^c}_I(h_2), \mu^{\star^c}_I(h_3)\}$ and $\sup\limits_{\vartheta\in f(h_1,h_2,h_3)} \lambda^c_I(\vartheta) \leq \min\{\lambda^{\star^c}_I(h_1), \lambda^{\star^c}_I(h_2), \lambda^{\star^c}_I(h_3)\}$.

(2) $\Longrightarrow$ (1): Now, suppose $\inf\limits_{\vartheta\in f(h_1,h_2,h_3)} \mu^c_I(\vartheta) \geq \max\{\mu^{\star^c}_I(h_1), \mu^{\star^c}_I(h_2), \mu^{\star^c}_I(h_3)\}$ and $\sup\limits_{\vartheta\in f(h_1,h_2,h_3)} \lambda^c_I(\vartheta) \leq \min\{\lambda^{\star^c}_I(h_1), \lambda^{\star^c}_I(h_2), \lambda^{\star^c}_I(h_3)\}$, for all $h_1, h_2, h_3 \in \mathbf{T}$. Then for any $\vartheta \in f(h_1,h_2,h_3)$, $\mu^c_I(\vartheta) \geq \max\{\mu^{\star^c}_I(h_1), \mu^{\star^c}_I(h_2), \mu^{\star^c}_I(h_3)\}$ and $\lambda^c_I(\vartheta) \leq \min\{\lambda^{\star^c}_I(h_1), \lambda^{\star^c}_I(h_2), \lambda^{\star^c}_I(h_3)\}$ then,

$$\begin{aligned}
\mu^c_I(\vartheta) &\geq max\{\mu^{\star^c}_I(h_1), \mu^{\star^c}_I(h_2), \mu^{\star^c}_I(h_3)\}\\
1-\mu_I(\vartheta) &\geq max\{(1-\mu^\star_I(h_1)), (1-\mu^\star_I(h_2)), (1-\mu^\star_I(h_3))\}\\
&= 1 - min\{\mu^\star_I(h_1), \mu^\star_I(h_2), \mu^\star_I(h_3)\}\\
\mu_I(\vartheta) &\leq min\{\mu^\star_I(h_1), \mu^\star_I(h_2), \mu^\star_I(h_3)\}.
\end{aligned}$$

and

$$\begin{aligned}
\lambda^c_I(\vartheta) &\leq min\{\lambda^{\star^c}_I(h_1), \lambda^{\star^c}_I(h_2), \lambda^{\star^c}_I(h_3)\}\\
1-\lambda_I(\vartheta) &\leq \min\{(1-(\lambda^\star_I(h_1)), (1-\lambda^\star_I(h_2)), (1-\lambda^\star_I(h_3))\}\\
&= 1 - max\{(\lambda^\star_I(h_1), \lambda^\star_I(h_2), \lambda^\star_I(h_3)\}\\
\lambda_I(\vartheta) &\geq max\{(\lambda^\star_I(h_1), \lambda^\star_I(h_2), \lambda^\star_I(h_3)\}.
\end{aligned}$$

It follows that $\sup\limits_{\vartheta\in f(h_1,h_2,h_3)} \mu_I(\vartheta) \leq \min\{\mu^\star_I(h_1), \mu^\star_I(h_2), \mu^\star_I(h_3)\}$ and $\inf\limits_{\vartheta\in f(h_1,h_2,h_3)} \lambda_I(\vartheta) \geq \max\{\lambda^\star_I(h_1), \lambda^\star_I(h_2), \lambda^\star_I(h_3)\}$. □

Lemma 4.2 *Suppose I is an I.F.S. of $\mathbf{T}$. Then, the following statements hold with the same implication:*

1. $\inf\limits_{\vartheta\in f(h_1,h_2,h_3)} \mu_I(\vartheta) \geq min\{\mu_I^\star(h_1), \mu_I^\star(h_2), \mu_I^\star(h_3)\}$ *and* $\sup\limits_{\vartheta\in f(h_1,h_2,h_3)} \lambda_I(\vartheta) \leq max\{\lambda_I^\star(h_1), \lambda_I^\star(h_2), \lambda_I^\star(h_3)\}$, *for all* $h_1, h_2, h_3 \in \mathbf{T}$.

2. $\sup\limits_{\vartheta\in f(h_1,h_2,h_3)} \mu_I^c(\vartheta) \leq max\{\mu_I^{\star^c}(h_1), \mu_I^{\star^c}(h_2), \mu_I^{\star^c}(h_3)\}$ *and* $\inf\limits_{\vartheta\in f(h_1,h_2,h_3)} \lambda_I^c(\vartheta) \geq min\{\lambda_I^{\star^c}(h_1), \lambda_I^{\star^c}(h_2), \lambda_I^{\star^c}(h_3)\}$, *for all* $h_1, h_2, h_3 \in \mathbf{T}$.

Proof: (1) $\Longrightarrow$ (2): Suppose that $\inf\limits_{\vartheta\in f(h_1,h_2,h_3)} \mu_I(\vartheta) \geq \min\{\mu_I^\star(h_1), \mu_I^\star(h_2), \mu_I^\star(h_3)\}$ and $\sup\limits_{\vartheta\in f(h_1,h_2,h_3)} \lambda_I(\vartheta) \leq \max\{\lambda_I^\star(h_1), \lambda_I^\star(h_2), \lambda_I^\star(h_3)\}$, for all $h_1, h_2, h_3 \in \mathbf{T}$. Then for any $\vartheta \in f(h_1,h_2,h_3)$, $\mu_I(\vartheta) \geq \min\{\mu_I^\star(h_1), \mu_I^\star(h_2), \mu_I^\star(h_3)\}$ and $\lambda_I(\vartheta) \leq \max\{\lambda_I^\star(h_1), \lambda_I^\star(h_2), \lambda_I^\star(h_3)\}$. Then, we have

$$\begin{aligned}
\mu_I^c(\vartheta) &= 1 - \mu_I(\vartheta) \\
&\leq 1 - (min\{\mu_I^\star(h_1), \mu_I^\star(h_2), \mu_I^\star(h_3)\}) \\
&= max\{(1-\mu_I^\star(h_1)), (1-\mu_I^\star(h_2)), (1-\mu_I^\star(h_3))\} \\
&= max\{\mu_I^{\star^c}(h_1), \mu_I^{\star^c}(h_2), \mu_I^{\star^c}(h_3)\}
\end{aligned}$$

and

$$\begin{aligned}
\lambda_I^c(\vartheta) &= 1 - \lambda_I(\vartheta) \\
&\geq 1 - max\{\lambda_I^\star(h_1), \lambda_I^\star(h_2), \lambda_I^\star(h_3)\} \\
&= min\{(1-\lambda_I^\star(h_1)), (1-\lambda_I^\star(h_2)), (1-\lambda_I^\star(h_3))\} \\
&= min\{\lambda_I^{\star^c}(h_1), \lambda_I^{\star^c}(h_2), \lambda_I^{\star^c}(h_3)\}.
\end{aligned}$$

It implies that $\sup\limits_{\vartheta\in f(h_1,h_2,h_3)} \mu_I^c(\vartheta) \leq \max\{\mu_I^{\star^c}(h_1), \mu_I^{\star^c}(h_2), \mu_I^{\star^c}(h_3)\}$ and $\inf\limits_{\vartheta\in f(h_1,h_2,h_3)} \lambda_I^c(\vartheta) \geq \min\{\lambda_I^{\star^c}(h_1), \lambda_I^{\star^c}(h_2), \lambda_I^{\star^c}(h_3)\}$, for all $h_1, h_2, h_3 \in \mathbf{T}$.
(2) $\Longrightarrow$ (1): Suppose that $\sup\limits_{\vartheta\in f(h_1,h_2,h_3)} \mu_I^c(\vartheta) \leq \max\{\mu_I^{\star^c}(h_1), \mu_I^{\star^c}(h_2), \mu_I^{\star^c}(h_3)\}$ and $\inf\limits_{\vartheta\in f(h_1,h_2,h_3)} \lambda_I^c(\vartheta) \geq \min\{\lambda_I^{\star^c}(h_1), \lambda_I^{\star^c}(h_2), \lambda_I^{\star^c}(h_3)\}$, for all $h_1, h_2, h_3 \in \mathbf{T}$. Then for any $\vartheta \in f(h_1,h_2,h_3)$, $\mu_I^c(\vartheta) \leq \max\{\mu_I^{\star^c}(h_1), \mu_I^{\star^c}(h_2), \mu_I^{\star^c}(h_3)\}$ and $\lambda_I^c(\vartheta) \geq \min\{\lambda_I^{\star^c}(h_1), \lambda_I^{\star^c}(h_2), \lambda_I^{\star^c}(h_3)\}$. Thus,

$$\begin{aligned}
\mu_I(\vartheta) &= 1 - \mu_I^c(\vartheta) \\
&\geq 1 - max\{\mu_I^{\star^c}(h_1), \mu_I^{\star^c}(h_2), \mu_I^{\star^c}(h_3)\} \\
&= min\{(1-\mu_I^{\star^c}(h_1)), (1-\mu_I^{\star^c}(h_2)), (1-\mu_I^{\star^c}(h_3))\} \\
&= min\{\mu_I^\star(h_1), \mu_I^\star(h_2), \mu_I^\star(h_3)\}.
\end{aligned}$$

and

$$\begin{aligned}
\lambda_I(\vartheta) &= 1 - \lambda_I^c(\vartheta) \\
&\leq 1 - min\{\lambda_I^{\star^c}(h_1), \lambda_I^{\star^c}(h_2), \lambda_I^{\star^c}(h_3)\} \\
&= max\{(1-\lambda_I^{\star^c}(h_1)), (1-\lambda_I^{\star^c}(h_2)), (1-\lambda_I^{\star^c}(h_3))\} \\
&= max\{\lambda_I^\star(h_1), \lambda_I^\star(h_2), \lambda_I^\star(h_3)\}
\end{aligned}$$

Hence, $\inf\limits_{\vartheta\in f(h_1,h_2,h_3)} \mu_I(\vartheta) \geq \min\{\mu_I^\star(h_1), \mu_I^\star(h_2), \mu_I^\star(h_3)\}$ and $\sup\limits_{\vartheta\in f(h_1,h_2,h_3)} \lambda_I(\vartheta) \leq \max\{\lambda_I^\star(h_1), \lambda_I^\star(h_2), \lambda_I^\star(h_3)\}$, for all $h_1, h_2, h_3 \in \mathbf{T}$. □

Lemma 4.3 *Let I be an I.F.S. of $\mathbf{T}$. Then the following statements hold with the same implications:*

1. $\inf\limits_{\vartheta \in f(h_1,h_2,h_3)} \mu_I(\vartheta) \geq min\{\mu_I(h_1), \mu_I(h_2), \mu_I(h_3)\}$ *and* $\sup\limits_{\vartheta \in f(h_1,h_2,h_3)} \lambda_I(\vartheta) \leq max\{\lambda_I(h_1), \lambda_I(h_2), \lambda_I(h_3)\}$, *for all* $h_1, h_2, h_3 \in \mathbf{T}$.

2. $\sup\limits_{\vartheta \in f(h_1,h_2,h_3)} \mu_I^c(\vartheta) \leq max\{\mu_I^c(h_1), \mu_I^c(h_2), \mu_I^c(h_3)\}$ *and* $\inf\limits_{\vartheta \in f(h_1,h_2,h_3)} \lambda_I^c(\vartheta) \geq min\{\lambda_I^c(h_1), \lambda_I^c(h_2), \lambda_I^c(h_3)\}$, *for all* $h_1, h_2, h_3 \in \mathbf{T}$.

This proof is similar to the Lemma 4.2.

Lemma 4.4 *Let I be an I.F.S. of $\mathbf{T}$. Then the following statements hold with the same implications:*

1. $\inf\limits_{\vartheta \in f(h_1,h_2,h_3)} \mu_I^c(\vartheta) \geq min\{\mu_I^c(h_1), \mu_I^c(h_2), \mu_I^c(h_3)\}$ *and* $\sup\limits_{\vartheta \in f(h_1,h_2,h_3)} \lambda_I^c(\vartheta) \leq max\{\lambda_I^c(h_1), \lambda_I^c(h_2), \lambda_I^c(h_3)\}$, *for all* $h_1, h_2, h_3 \in \mathbf{T}$.

2. $\sup\limits_{\vartheta \in f(h_1,h_2,h_3)} \mu_I(\vartheta) \leq max\{\mu_I(h_1), \mu_I(h_2), \mu_I(h_3)\}$ *and* $\inf\limits_{\vartheta \in f(h_1,h_2,h_3)} \lambda_I(\vartheta) \geq min\{\lambda_I(h_1), \lambda_I(h_2), \lambda_I(h_3)\}$, *for all* $h_1, h_2, h_3 \in \mathbf{T}$.

This proof is similar to the Lemma 4.2.

Definition 4.4 A subset $\mathcal{P}_{\mathbf{T}}(\neq \emptyset)$ of $\mathbf{T}$ is defined as a prime subset $\mathcal{P}_{\mathbf{T}}$ if for $p_1, p_2, p_3 \in \mathbf{T}$, $f(p_1, p_2, p_3) \cap \mathcal{P}_{\mathbf{T}} \neq \emptyset$ implies $p_1^\star \in \mathcal{P}_{\mathbf{T}}$ or $p_2^\star \in \mathcal{P}_{\mathbf{T}}$ or $p_3^\star \in \mathcal{P}_{\mathbf{T}}$.
An hyperideal I that is also a prime subset of $\mathbf{T}$ is referred as a prime hyperideal.

Definition 4.5 An I.F.S. $I = (\mu_I, \lambda_I)$ of $\mathbf{T}$ is referred as completely prime if $\sup\limits_{\vartheta \in f(p_1,p_2,p_3)} \mu_I(\vartheta) \leq \max\{\mu_I^\star(p_1), \mu_I^\star(p_2), \mu_I^\star(p_3)\}$ and $\inf\limits_{\vartheta \in f(p_1,p_2,p_3)} \lambda_I(\vartheta) \geq \min\{\lambda_I^\star(p_1), \lambda_I^\star(p_2), \lambda_I^\star(p_3)\}$ satisfies for every $p_1, p_2, p_3 \in \mathbf{T}$.
An I.F. hyperideal I of $\mathbf{T}$ is referred as completely prime intuitionistic fuzzy hyperideal(briefly, C.P.I.F.H.) whenever I is a completely prime I.F.S. of $\mathbf{T}$.

Lemma 4.5 *Suppose that $I = (\mu_I, \lambda_I)$ is an I.F.S. of $\mathbf{T}$ satisfying $I_{\mathbf{T}}(u^\star) \subseteq I_{\mathbf{T}}(u)$ $\forall u \in \mathbf{T}$. Assuming $I_{\mathbf{T}}$ to be an I.F.H.F. of $\mathbf{T}$, then the complement $I_{\mathbf{T}}^c$ should be a C.P.I.F.H. of $\mathbf{T}$.*

Proof: Assume $I = (\mu_I, \lambda_I)$ is an I.F.H.F. of $\mathbf{T}$ implying $\sup\limits_{\vartheta \in f(p_1,p_2,p_3)} \mu_I(\vartheta) \leq \min\{\mu_I^\star(p_1), \mu_I^\star(p_2), \mu_I^\star(p_3)\}$ and $\inf\limits_{\vartheta \in f(p_1,p_2,p_3)} \lambda_I(\vartheta) \geq \max\{\lambda_I^\star(p_1), \lambda_I^\star(p_2), \lambda_I^\star(p_3)\}$ for all $p_1, p_2, p_3 \in \mathbf{T}$ and assume $I^c = (\mu_I^c, \lambda_I^c) = (1 - \mu_I, 1 - \lambda_I)$ to be its complement of I of $\mathbf{T}$. Given that $I(u^\star) \subseteq I(u)$, it implies $\mu_I(u^\star) \leq \mu_I(u)$ and $\lambda_I(u^\star) \geq \lambda_I(u)$, it is concluded $\mu_I^c(u^\star) \geq \mu_I^c(u)$ and $\lambda_I^c(u^\star) \leq \lambda_I^c(u)$. By Lemma 4.1, $\inf\limits_{\vartheta \in f(p_1,p_2,p_3)} \mu_I^c(\vartheta) \geq \max\{\mu_I^{\star^c}(p_1), \mu_I^{\star^c}(p_2), \mu_I^{\star^c}(p_3)\}$ and $\sup\limits_{\vartheta \in f(p_1,p_2,p_3)} \lambda_I^c(\vartheta) \leq \min\{\lambda_I^{\star^c}(p_1), \lambda_I^{\star^c}(p_2), \lambda_I^{\star^c}(p_3)\}$. So, we have

$$\begin{aligned}
\inf_{\vartheta \in f(p_1,p_2,p_3)} \mu_I^c(\vartheta) &\geq max\{\mu_I^{\star^c}(p_1), \mu_I^{\star^c}(p_2), \mu_I^{\star^c}(p_3)\} \\
&= max\{\mu_I^c(p_1^\star), \mu_I^c(p_2^\star), \mu_I^c(p_3^\star)\} \\
&\geq max\{\mu_I^c(p_1), \mu_I^c(p_2), \mu_I^c(p_3)\}
\end{aligned}$$

and

$$\begin{aligned}
\sup_{\vartheta \in f(p_1,p_2,p_3)} \lambda_I^c(\vartheta) &\leq min\{\lambda_I^{\star^c}(p_1), \lambda_I^{\star^c}(p_2), \lambda_I^{\star^c}(p_3)\} \\
&= min\{\lambda_I^c(p_1^\star), \lambda_I^c(p_2^\star), \lambda_I^c(p_3^\star)\} \\
&\leq min\{\lambda_I^c(p_1), \lambda_I^c(p_2), \lambda_I^c(p_3)\}.
\end{aligned}$$

Therefore, I^c of I of $\mathbf{T}$ is an I.F. hyperideal of $\mathbf{T}$. As every I.F.H.F. is an I.F.T.sub-S. implying for every $p_1, p_2, p_3 \in \mathbf{T}$, $\inf\limits_{\vartheta \in f(p_1,p_2,p_3)} \mu_I(\vartheta) \geq \min\ \{\mu_I(p_1), \mu_I(p_2), \mu_I(p_3)\}$ and $\sup\limits_{\vartheta \in f(p_1,p_2,p_3)} \lambda_I(\vartheta) \leq \max\{\lambda_I(p_1), \lambda_I(p_2), \lambda_I(p_3)\}$. By Lemma 4.3, we have $\sup\limits_{\vartheta \in f(p_1,p_2,p_3)} \mu_I^c(\vartheta) \leq \max\{\mu_I^c(p_1), \mu_I^c(p_2), \mu_I^c(p_3)\}$ and $\inf\limits_{\vartheta \in f(p_1,p_2,p_3)} \lambda_I^c(\vartheta) \geq \min\{\lambda_I^c(p_1), \lambda_I^c(p_2), \lambda_I^c(p_3)\}$, for all $p_1, p_2, p_3 \in \mathbf{T}$. Now,

$$\begin{aligned}\sup_{\vartheta \in f(p_1,p_2,p_3)} \mu_I^c(\vartheta) &\leq max\{\mu_I^c(p_1), \mu_I^c(p_2), \mu_I^c(p_3)\} \\ &\leq max\{\mu_I^c(p_1^\star), \mu_I^c(p_2^\star), \mu_I^c(p_3^\star)\} \\ &= max\{\mu_I^{\star^c}(p_1), \mu_I^{\star^c}(p_2), \mu_I^{\star^c}(p_3)\}.\end{aligned}$$

and

$$\begin{aligned}\inf_{\vartheta \in f(p_1,p_2,p_3)} \lambda_I^c(\vartheta) &\geq min\{\lambda_I^c(p_1), \lambda_I^c(p_2), \lambda_I^c(p_3)\} \\ &\geq min\{\lambda_I^c(p_1^\star), \lambda_I^c(p_2^\star), \lambda_I^c(p_3^\star)\} \\ &= min\{\lambda_I^{\star^c}(p_1), \lambda_I^{\star^c}(p_2), \lambda_I^{\star^c}(p_3)\}.\end{aligned}$$

Hence, I^c is proved to be an C.P.I.F.H. of $\mathbf{T}$. □

5. Conclusion

Throughout this paper, we have studied ternary semihypergroup with involution(⋆-ternary semihypergroup) in aspect of intuitionistic fuzzification of a set. Also, we have studied different kind of intuitionistic fuzzy hyperideals and intuitionistic fuzzy hyperfilters in ⋆-ternary semihypergroup with some interesting examples. Finally, we have studied and proved some results on complement of a intuitionistic fuzzy set. It is expected that the intuitionistic fuzzy hyperideals and intuitionistic fuzzy hyperfilters may motivate for further research in other ternary algebraic structures.

References

1. M. Y. Abbasi, S. A. Khan and A. Ali, *A note on soft ideals and soft filters in ternary semigroups with involution*, Discussiones Mathematicae-General Algebra and Applications, **40**, 119-133, (2020).
2. M. Y. Abbasi, S. A. Khan, K. Hila and A. Raza, *Soft hyperideals and soft hyperfilters in ∗-ternary semihypergroups*, Journal of Multiple-Valued Logic and Soft Computing, **41**, 169-186, (2023).
3. K. S. Abdulmula and A. R. Salleh, *Intuitionistic fuzzy hyperideals in intuitionistic fuzzy semi-hypergroups*, International Journal of Algebra, **6(3)**, 617-636, (2012).
4. K.T. Atanassov, *Intuitionistic fuzzy sets*, VII ITKR Session, Sofia, 20-23, (1983).
5. K.T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems , **20(1)**, 87-96, (1986).
6. P. Corsini, V. Leoreanu, *Applications of hyperstructure theory*, Kluwer, Dordrecht, (2003).
7. B. Davvaz, *Fuzzy hyperideals in semihypergroups*, Italian Journal of Pure and Applied Mathematics, **8** , 67-74, (2000).
8. B. Davvaz, *Intuitionistic hyperideals of semihypergroups*, Bulletin of the Malaysian Mathematical Sciences Society, **29(1)**, 203-207, (2006).
9. B. Davvaz and V. Leoreanu, *Binary relations on ternary semihypergroups*, Communications in Algebra, **38(10)**, 3621-3636, (2010).
10. B. Davvaz and A. Khan, *Generalized fuzzy filters in ordered semigroups*, Iranian Journal of Science and Technology, **A1**, 77-86, (2012).
11. X. Feng, J. Tang, B. Davvaz and Y. Luo, *A novel study on fuzzy ideals and fuzzy filters of ordered ⋆-semigroups*, Journal of Intelligent and Fuzzy Systems, **33**, 423-431, (2017).
12. K. Hila and S. Abdullah, *A study on intuitionistic fuzzy sets in Γ-semihypergroups*, Journal of Intelligent and Fuzzy Systems: Applications in Engineering and Technology, **26(4)**, 1695-1710, (2014).
13. K. Hila, B. Davvaz and K. Naka, *On hyperideal structure of ternary semihypergroups*, Iranian Journal of Mathematical Sciences and Informatics, **9(1)**, 81-98, (2014).

14. K. Hila, L. Kikina and B. Davvaz, *Intuitionistic fuzzy hyperideal extensions of semihypergroups*, Thai Journal of Mathematics, **13(2)**, 291-305, (2015).

15. F. Marty, *Sur une generalization de la notion de group*, 8th Congres Math. Scandinaves Stockholm, 45-49, (1934).

16. T. E. Nordahl and H. E. Scheiblich, *Regular $\star$-semigroups*, Semigroup Forum, **16**, 369-377, (1978).

17. Rosenfeld, *Fuzzy groups*, J. Math. Anal. Appl., **35**, 512-517, (1971).

18. M. Shabir and A. Khan, *Fuzzy filters in ordered semigroups*, Lobachevskii Journal of Mathematics, **29(2)**, 82-89, (2008).

19. J. Tang and N. Yaqoob, *A novel investigation on fuzzy hyperideals in ordered $\star$-semihypergroups*, Computational and Applied Mathematics 40:43, (2021).

20. J. Tang, B. Davvaz and Y. Luoa, *Hyperfilters and fuzzy hyperfilters of ordered semihypergroups*, Journal of Intelligent and Fuzzy Systems **29**, 75-84, (2015).

21. L. A. Zadeh, *Fuzzy sets*, Information and Control, **8**, 338-353, (1965).

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