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Fixed Point Theorems for Generalized S-Contraction and Kannan Type Contraction in Weighted ψ-b Metric Spaces

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ABSTRACT: In this paper, we define contraction mapping with their generalization name as s-contraction mapping and kannan type contraction. Also, introduced the b-metric space and their generalization name as weighted ψ-b metric space and supported some examples for weighted ψ-b metric space with their application in fixed point theory. Furthermore, we introduced the new contraction mappings like modified s-contraction mapping and modified kannan type contraction mapping with multiplicative coefficients α, β, γ. Further, we have proved some fixed point theorems for above modified contractions with weighted ψ- b metric spaces. Also, we supported some numerical applications related to modified s-contraction and modified Kannan type contraction with weighted ψ-b metric space.

Key Words: S-contraction, Kannan type contraction, weighted ψ-b metric space, fixed point, multiplicative coefficients.

Contents

1. Introduction and Definitions

In 1906, the great mathematician Frechet [8] introduced the concept of metric space. After this, many mathematician introduced different type of metric spaces like partial metric space, b-metric space, extended b-metric space, complex valued metric space, etc which are basic extension of metric space.

In similar way, Natthaphon et al. [11] introduced the concept of weighted ψ - b-metric space and proved many fixed point theorems with the help of modified Rus-Reich-Ćirić [4,12] type contraction via interpolation.

Here, we are generalized S-contraction [2,16] and Kannan type contraction [13,14,15] by multiplicative coefficients α, β, γ and we prove many fixed point results for self mapping on weighted ψ-b-metric space.

Now, we define some basic definitions and results which are already proved in following papers [1,10,7].

Definition 1.1 [5,6,17,19] : Let X be a non-empty set and $s \geq 1$ be a given real number. A mapping $d : X \times X \to [0, \infty)$ is called b-metric space if it satisfies the following properties for each $x, y, z \in X$.

(b1) Identity: $d(x, y) = 0 \Leftrightarrow x = y$

(b2) Symmetry: $d(x, y) = d(y, x)$

(b3) Triangle type inequality: $d(x, z) \leq s[d(x, y) + d(y, z)]$.

Then the pair (X, b) is called b-metric space.

Example [18,19,20] : Let $X = L_p[0, 1]$ be the space of all real functions $x(t)$, $t \in [0, 1]$ such that $\int_0^1 |x(t)|^p dt < \infty$ with $0 < p < 1$. Define $d : X \times X \to \mathbb{R}^+$ as:

$$d(x, y) = \Big(\int_0^1 |x(t) - y(t)|^p dt \Big)^{\frac{1}{p}}. \tag{1.1}$$

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Then d is b-metric space with coefficient $s = 2^{\frac{1}{p}}$.

The above example show that the class of b-metric space is larger than the class of metric spaces. When $s = 1$, the concept of b-metric space coincides with the concept of metric space.

Definition 1.2 [11]**:** Let X be a non-empty set and $d : X \times X \to [0, \infty)$ be a function. Let $\psi : [0, \infty)^2 \to [0, \infty)$ be a continuous and increasing in both arguments, and let $w : X \times X \to [0, \infty)$. The pair (X, d) is called a weighted ψ-b-metric space, if the following conditions are satisfied for all $x, y, z \in X$:

(wb1) Identity: $d(x, x) = 0$

(wb2) Symmetry: $d(x, y) = d(y, x)$

(wb3) Triangle type inequality: $d(x, y) \leq \psi(d(x, z), d(z, y)).[w(x, z)d(x, z) + w(z, y)d(z, y)]$.

Example [11]**:** Let $X = \mathbb{R}$, define

$$d(x, y) = \frac{|x - y|}{1 + x^2 + y^2}, \tag{1.2}$$

$$w(x, y) = 1 + sin^2(x + y), \tag{1.3}$$

$$\psi(s, t) = 1 + \frac{st}{1 + s + t}, \tag{1.4}$$

then d satisfied all conditions (wb1)-(wb3), therefore $(\mathbb{R}, d)$ is a weighted ψ-b-metric space.

We present the following key definitions as

Definition 1.3 [11]**:** Let $\{\rho_n\}$ be a sequence in a weighted ψ-b-metric space (X, d). Then

(i) The sequence $\{\rho_n\}$ is said to be convergent in (X, d), if there exist $\rho \in X$ such that $lim_{n\to\infty}(\rho_n, \rho) = 0$.

(ii) The sequence $\{\rho_n\}$ is said to be Cauchy in (X, d), if $lim_{n,m\to\infty}(\rho_n, \rho_m) = 0$.

(iii) (X, d) is called a complete weighted ψ-b-metric space if every Cauchy sequence in X is convergent.

Now, we introduce a new class of contraction mappings which are generalizes the S-contraction [2,16] and Kannan type contraction [9,13,14] names as modified S-contraction and modified Kannan type contraction with multiplicative coefficients α, β, γ.

Definition 1.4 (Modified Kannan type contraction with multiplicative coefficients α, β, γ): Let (X, d) be a complete metric space and a self mapping $T : X \to X$ such that

$$d(Tx, Ty) \leq \alpha d(x, y) + \beta d(x, Tx) + \gamma d(y, Ty) \tag{1.5}$$

where $\alpha, \beta, \gamma \in (0, 1)$ with $0 < \alpha + \beta + \gamma < 1$ and for all $x, y \in X/F(x)$, where $F(x) = \{x \in X | T(x) = x$. Then T is called modified Kannan type contraction with multiplicative coefficients α, β, γ.

Definition 1.5 (Modified S-contraction with multiplicative coefficients α, β, γ) : Let (X, d) be a complete metric space and a self mapping $T : X \to X$ such that

$$d(Tx, Ty) \leq \alpha d(x, y) + \beta d(x, Ty) + \gamma d(y, Tx) \tag{1.6}$$

where $\alpha, \beta, \gamma \in (0, 1)$ with $0 < \alpha + \beta + \gamma < 1$ and for all $x, y \in X/F(x)$, where $F(x) = \{x \in X | T(x) = x$. Then T is called modified S-contraction with multiplicative coefficients α, β, γ.

2. Main Results

Theorem 2.1 *Let (X, d) be a complete weighted ψ-b-metric space and mapping $T : X \to X$ be a modified Kannan type contraction. Then T has a unique fixed point in X.*

Proof: Let $x_0 \in (X, d)$, we will set a constructive sequence $\{x_n\}$ in X such that $x_{n+1} = T(x_n) = T^n(x_0$ for all positive integer n. Indeed, if there exist a non negative integer n_0 such that $x_{n_0} = x_{n_0+1} = T(x_{n_0})$, then x_{n_0} forms a fixed point. Thus prove is trivial. So, we assume that $x_n \neq x_{n+1}$ and setting $x_{n+1} = Tx_n$ for all $n \in N$.
Taking $x = x_n$, $y = x_{n+1}$ in modified Kannan type contraction. we derive that

$$\begin{aligned} d(x_{n+1}, x_{n+2}) &= d(Tx_n, Tx_{n+1}) \\ &\leq \alpha d(x_n, x_{n+1}) + \beta d(x_n, Tx_n) + \gamma d(x_{n+1}, Tx_{n+1}) \\ &= \alpha d(x_n, x_{n+1}) + \beta d(x_n, x_{n+1}) + \gamma d(x_{n+1}, x_{n+2}) \\ &= (\alpha+\beta) d(x_n, x_{n+1}) + \gamma d(x_{n+1}, x_{n+2}) \\ \Rightarrow (1-\gamma) d(x_{n+1}, x_{n+2}) &\leq (\alpha+\beta) d(x_n, x_{n+1}) \\ \Rightarrow d(x_{n+1}, x_{n+2}) &\leq \Big(\frac{\alpha+\beta}{1-\gamma}\Big) d(x_n, x_{n+1}). \end{aligned}$$

Thus, we deduce that the sequence $\{d(x_n, x_{n+1})\}$ is non-increasing and non-negative (monotone decreasing sequence). Similarly, we obtain

$$d(x_{n+1}, x_{n+2}) \leq \Big(\frac{\alpha+\beta}{1-\gamma}\Big)^n d(x_0, x_1). \tag{2.1}$$

Since $\alpha + \beta + \gamma < 1 \Rightarrow \alpha + \beta < 1 - \gamma$ then $\frac{\alpha+\beta}{1-\gamma} < 1$. Letting $n \to \infty$ in the above, we observe that $\lim_{n\to\infty} \Big(\frac{\alpha+\beta}{1-\gamma}\Big)^n = 0$, so from above inequality, we obtain

$$\lim_{n\to\infty} d(x_{n+1}, x_{n+2}) = 0. \tag{2.2}$$

We show that $\{x_n\}$ is a Cauchy sequence. Let $m > n$, we use the triangle inequality recursively (by b-metric space)

$$d(x_n, x_m) \leq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + ... + d(x_{m-1}, x_m). \tag{2.3}$$

Since $d(x_n, x_{n+1}) \to 0$ as $\lim_{n\to\infty}$, so $\lim_{n\to\infty} d(x_n, x_m) = 0$. Therefore $\{x_n\}$ is a Cauchy sequence. Since (X, d) is a complete metric space, then there exist $x \in X$ such that

$$\lim_{n\to\infty} x_n = x. \tag{2.4}$$

Now, we prove that $Tx = x$ i.e., x is a fixed point of T. Since $x_n \to x$ and $x_{n+1} = T(x_n)$ so,

$$\lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} T(x_n) = T(x)$$

but also $\lim_{n\to\infty} x_{n+1} = x$, therefore

$$T(x) = x. \tag{2.5}$$

Hence x is a fixed point of T. Now, we prove that x is a unique fixed point. On contrary, suppose two distinct fixed points $x, y \in X$ such that $Tx = x$ and $Ty = y$. Now, we apply the modified Kannan type contraction as

$$\begin{aligned} d(x, y) = d(Tx, Ty) &\leq \alpha d(x, y) + \beta d(x, Tx) + \gamma d(y, Ty) \\ \Rightarrow d(x, y) &\leq \alpha d(x, y) + \beta d(x, x) + \gamma d(y, y) \\ \Rightarrow (1-\alpha) d(x, y) &\leq 0. \end{aligned}$$

Since $\alpha \in (0, 1)$, so $1 - \alpha > 0$, therefore $d(x, y) \leq 0$, which is contradiction to our assumption. This is only possible, when $d(x, y) = 0$ that is $x = y$. Thus fixed point is unique. Hence T has a unique fixed point in X.

□

Theorem 2.2 *Let (X, d) be a complete weighted ψ-b-metric space and mapping $T : X \to X$ be a modified S-contraction. Then T has a unique fixed point in X.*

Proof: Let us define a sequence x_n in X by choosing an arbitrary $x_0 \in X$ and setting $x_{n+1} = T(x_n) = T^n(x_0$ for all positive integer n. Now, if we choose a non negative integer n_0 such that $x_{n_0} = x_{n_0+1} = T(x_{n_0})$, then x_{n_0} forms a fixed point. That is prove is trivial. So, we assume that $x_n \neq x_{n+1}$ and setting $x_{n+1} = Tx_n$ for all $n \in N$.
Taking $x = x_n$, $y = x_{n+1}$ in modified S-contraction. we derive that

$$d(x_{n+1}, x_{n+2}) = d(Tx_n, Tx_{n+1})$$
$$\leq \alpha d(x_n, x_{n+1}) + \beta d(x_n, Tx_{n+1}) + \gamma d(x_{n+1}, Tx_n)$$
$$= \alpha d(x_n, x_{n+1}) + \beta d(x_n, x_{n+2}) + \gamma d(x_{n+1}, x_{n+1})$$

by triangle inequality and by definition of metric space $d(x, x) = 0$, we obtain

$$\Rightarrow d(x_{n+1}, x_{n+2}) \leq \alpha d(x_n, x_{n+1}) + \beta d(x_n, x_{n+1}) + \beta d(x_{n+1}, x_{n+2}) + 0$$
$$\Rightarrow (1 - \beta)d(x_{n+1}, x_{n+2}) \leq (\alpha + \beta)d(x_n, x_{n+1})$$
$$\Rightarrow d(x_{n+1}, x_{n+2}) \leq \Big(\frac{\alpha + \beta}{1 - \beta}\Big)d(x_n, x_{n+1}).$$

Thus, we deduce that the sequence $\{d(x_n, x_{n+1})\}$ is non-increasing and non-negative (monotone decreasing sequence). Similarly, we obtain

$$d(x_{n+1}, x_{n+2}) \leq \Big(\frac{\alpha + \beta}{1 - \beta}\Big)^n d(x_0, x_1). \tag{2.6}$$

Since $\alpha, \beta, \gamma \in (0, 1)$ with $\alpha + \beta + \gamma < 1 \Rightarrow \alpha + \beta < 1 - \gamma \approx 1 - \beta$ i.e., $\frac{\alpha+\beta}{1-\beta} < 1$. Taking $n \to \infty$ in the above, we observe that $\lim_{n\to\infty} \Big(\frac{\alpha+\beta}{1-\beta}\Big)^n = 0$, so from above inequality, we obtain

$$\lim_{n\to\infty} d(x_{n+1}, x_{n+2}) = 0. \tag{2.7}$$

We show that $\{x_n\}$ is a Cauchy sequence. Let $m > n$, we use the triangle inequality recursively (by b-metric space)

$$d(x_n, x_m) \leq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + ... + d(x_{m-1}, x_m). \tag{2.8}$$

Since $d(x_n, x_{n+1}) \to 0$ as $\lim_{n\to\infty}$, so $\lim_{n\to\infty} d(x_n, x_m) = 0$. Therefore $\{x_n\}$ is a Cauchy sequence. Since (X, d) is a complete metric space, then there exist $x \in X$ such that

$$\lim_{n\to\infty} x_n = x. \tag{2.9}$$

Now, we prove that $Tx = x$ i.e., x is a fixed point of T. Since $x_n \to x$ and $x_{n+1} = T(x_n)$ so,

$$\lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} T(x_n) = T(x)$$

but also $\lim_{n\to\infty} x_{n+1} = x$, therefore

$$T(x) = x. \tag{2.10}$$

Hence x is a fixed point of T. Now, we prove that x is a unique fixed point. On contrary, suppose two distinct fixed points $x, y \in X$ such that $Tx = x$ and $Ty = y$. Now, we apply the modified S-contraction as

$$d(x, y) = d(Tx, Ty) \leq \alpha d(x, y) + \beta d(x, Ty) + \gamma d(y, Tx)$$
$$\Rightarrow d(x, y) \leq \alpha d(x, y) + \beta d(x, y) + \gamma d(y, x)$$
$$\Rightarrow [1 - (\alpha + \beta + \gamma)]d(x, y) \leq 0.$$

Since $0 < \alpha + \beta + \gamma < 1$, so $[1 - (\alpha + \beta + \gamma)] > 0$, therefore $d(x, y) \leq 0$, which is contradiction to our assumption. This is only possible, when $d(x, y) = 0$ that is $x = y$. Thus fixed point is unique. Hence T has a unique fixed point in X.

□

Example : Let $X = [0,1]$ and define the function $d : X \times X \to \mathbb{R}^+$ by $d(x,y) = \frac{(x-y)^2}{1+x^2+y^2}$, control function $\psi : \mathbb{R}^{+^2} \to \mathbb{R}^+$ by $\psi(x,y) = 2+x+y$ and weight function $w : X \times X \to \mathbb{R}^+$ by $w(x,y) = \frac{xy}{1+x+y}$. Verifying the conditions of a weighted ψ-b-metric space .

(wb1) $d(x,x) = \frac{(x-x)^2}{1+x^2+x^2} = 0, \forall x \in X$.

(wb2) $d(x,y) = \frac{(x-y)^2}{1+x^2+y^2} = \frac{(y-x)^2}{1+y^2+x^2} = d(y,x)$.

(wb3) Numerically we can show (wb3). Consider $x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{4}$, then

$$d(1/2,1/3) = \frac{(1/2-1/3)^2}{1+1/2^2+1/3^2} = 1/49 = 0.02040816326$$

$$d(1/2,1/4) = \frac{(1/2-1/4)^2}{1+1/2^2+1/4^2} = 1/21 = 0.047619047$$

$$d(1/3,1/4) = \frac{(1/3-1/4)^2}{1+1/3^2+1/4^2} = 1/169 = 0.0059171597$$

$$w(1/2,1/3) = 1 + \frac{1/2.1/3}{1+1/2+1/3} = 12/11 = 1.0909$$

$$w(1/2,1/4) = 1 + \frac{1/2.1/4}{1+1/2+1/4} = 15/14 = 1.07142857$$

$$w(1/3,1/4) = 1 + \frac{1/3.1/4}{1+1/3+1/4} = 20/19 = 1.05263.$$

Now, we will check for triangle type inequality (wb3)

$$d(x,y) \leq \psi(d(x,z),d(z,y)).[w(x,z)d(x,z) + w(z,y)d(z,y)]$$

such that

$$\begin{aligned} d(1/2,1/4) \leq \psi(d(1/2,1/3),d(1/3,1/4)).[&w(1/2,1/3)d(1/2,1/3) \\ &+w(1/3,1/4)d(1/3,1/4)] \end{aligned}$$

$$\begin{aligned} 0.047619041 = 1/21 &\leq \psi(1/49,1/169).[1/4912/11 + 1/169.20/19] \\ &= [2+1/49+1/169][1/4912/11 + 1/169.20/19] \\ &= (2.0263253)(0.028492) = 0.05773426644476. \end{aligned}$$

Hence (wb3) satisfied. Similarly, we can show for all $x,y,z \in X$. Thus (X,d) is a weighted ψ-b-metric space.
Now, consider the mapping $T : X \to X$ such that $T(x) = \frac{x^2+x}{3x^2+4}$. We will check that the mapping defined above satisfy the modified kannan type contraction

$$d(Tx,Ty) \leq \alpha d(x,y) + \beta d(x,Tx) + \gamma d(y,Ty)$$

and modified S-contraction

$$d(Tx,Ty) \leq \alpha d(x,y) + \beta d(x,Ty) + \gamma d(y,Tx).$$

Choose $x = \frac{1}{5}$ and $y = \frac{1}{6}$, then

$$T(x) = \frac{1/25+1/5}{3/25+4} = \frac{6}{103} = 0.058252427$$

$$T(y) = \frac{1/36 + 1/6}{3/36 + 4} = \frac{7}{147} = 0.047619047.$$

and

$$d(Tx, Ty) = \frac{(Tx - Ty)^2}{1 + Tx^2 + Ty^2} = \frac{(6/103 - 7/147)^2}{1 + (6/103)^2 + (7/147)^2}$$
$$= 0.000112432290894004 \approx 0.00011$$

$$d(x, y) = \frac{(x - y)^2}{1 + x^2 + y^2} = \frac{(1/5 - 1/6)^2}{1 + (1/5)^2 + (1/6)^2}$$
$$= 0.001040582726326742 \approx 0.00104$$

$$d(x, Ty) = \frac{(x - Ty)^2}{1 + x^2 + Ty^2} = \frac{(1/5 - 7/147)^2}{1 + (1/5)^2 + (7/147)^2}$$
$$= 0.022278304760247149 \approx 0.02228$$

$$d(y, Tx) = \frac{(y - Tx)^2}{1 + y^2 + Tx^2} = \frac{(1/6 - 6/103)^2}{1 + (1/6)^2 + (6/103)^2}$$
$$= 0.011398348013985764 \approx 0.01140$$

$$d(x, Tx) = \frac{(x - Tx)^2}{1 + x^2 + Tx^2} = \frac{(1/5 - 6/103)^2}{1 + (1/5)^2 + (6/103)^2}$$
$$= 0.01925675919836377 \approx 0.01926$$

$$d(y, Ty) = \frac{(y - Ty)^2}{1 + y^2 + Ty^2} = \frac{(1/6 - 7/147)^2}{1 + (1/6)^2 + (7/147)^2}$$
$$= 0.013758943313153549 \approx 0.01376.$$

By definitions of both above contractions, choose $\alpha, \beta, \gamma \in (0, 1)$ such that $\alpha + \beta + \gamma < 1$. Consider $\alpha = 0.1$, $\beta = 0.2$, $\gamma = 0.3$.
For modified kannan type contraction

$$\alpha d(x, y) + \beta d(x, Tx) + \gamma d(y, Ty)$$
$$= (0.1)(0.00104) + (0.2)(0.01926) + (0.3)(0.01376)$$
$$= 0.008084 \geq d(Tx, Ty) = 0.00011.$$

For modified s-contraction

$$\alpha d(x, y) + \beta d(x, Ty) + \gamma d(y, Tx)$$
$$= (0.1)(0.00104) + (0.2)(0.02228) + (0.3)(0.01140)$$
$$= 0.00798 \geq d(Tx, Ty) = 0.00011.$$

Hence T satisfied both contractions. Now, we find the fixed point of $T(x)$ by $T(x) = x$, we obtain $x = 0$ and $x = \frac{1 \pm \sqrt{37}}{6}$. Since, $x = \frac{1 \pm \sqrt{37}}{6} \notin X$, so $x = 0$ is unique fixed point of T in X.
Thus $x = 0$ is a unique fixed point of T. Now, we represent the graph between $y = T(x)$ and $y = x$.

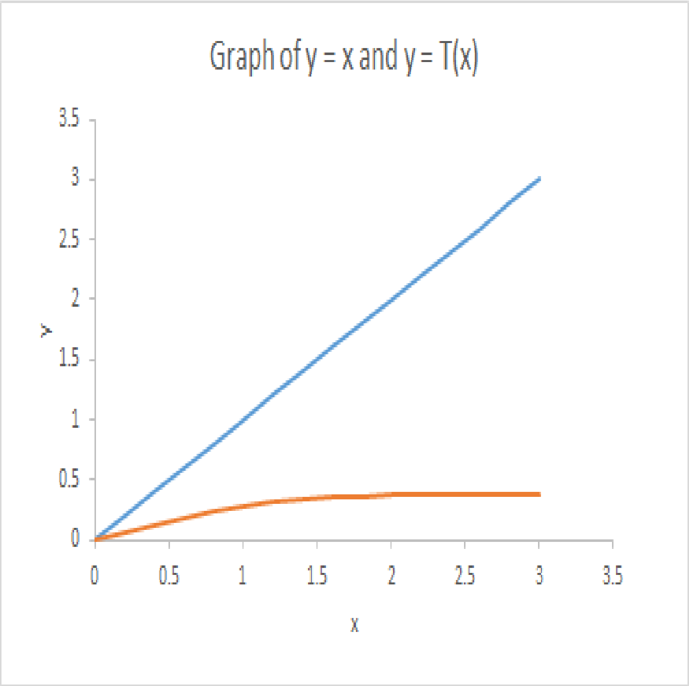

Figure 1: Graph between y=x and y=f(x)

□

Now, if we replace the coefficients α, β, γ in modified Kannan type contraction as interpolative form, we obtain the following results.

Proposition 2.1 *[3] Let (X, d) be a complete metric space and mapping $T : X \to X$ be a Kannan type contraction via interpolation. That is, if there exist constants $\lambda \in (0, 1)$ and $\alpha \in (0, 1)$ such that*

$$d(Tx, Ty) \leq \lambda[d(x, Tx)]^{\alpha}[d(y, Ty)]^{1-\alpha} \tag{2.11}$$

for all $x, y \in X$ with $Tx \neq x$. Then T has a unique fixed point in X.

□

Proposition 2.2 *[11] Let (X, d) be a complete weighted ψ-b-metric space and mapping $T : X \to X$ be an interpolative Kannan type contraction. That is, if there exist constants $k \in (0, 1)$ and $\alpha, \beta, \gamma \in (0, 1)$ with $\alpha + \beta + \gamma < 1$ such that*

$$d(Tx, Ty) \leq k[d(x, y)]^{\alpha}[d(x, Tx)]^{\beta}[d(y, Ty)]^{\gamma} \tag{2.12}$$

for all $x, y \in X$ with $Tx \neq x$. Then T has a unique fixed point in X.

□

Now, if we assume $\alpha = \beta = \gamma = k$ (say) in modified s-contraction, then we obtain the following result in different metric spaces as:

Proposition 2.3 *[16] Let (X, d) be a complete partial b-metric space with coefficient $k \geq 1$ and mapping $T : X \to X$ be a self mapping on X satisfying the following conditions*

$$d(Tx, Ty) \leq k[d(x, y) + d(x, Ty) + d(y, Tx)] \tag{2.13}$$

for all $x, y \in X$ and $k \in [0, \frac{1}{3})$. Then T has a unique fixed point in X.

□

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