



Laceability Properties in Edge-Tolerant Total Transformation Graphs $G^{\alpha\beta\gamma}$

Nagarathnamma K. G. and Leena N. Shenoy

ABSTRACT: A bipartite graph G is hamiltonian laceable if there is a hamiltonian path between any two vertices of G from distinct vertex bipartite sets. A bipartite graph G is k -edge fault-tolerant hamiltonian laceable if $G - F$ is hamiltonian laceable for every $F \subseteq E(G)$ with $|F| \leq k$. A graph G is k -edge fault-tolerant conditional hamiltonian if $G - F$ is hamiltonian for every $F \subseteq E(G)$ with $|F| \leq k$ and $\delta(G - F) \geq 2$. In this paper, we establish laceability properties in the edge tolerant total transformation graphs $G^{\alpha\beta\gamma}$.

Keywords: Hamiltonian laceable, fault-tolerant, total transformation graph.

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1. Introduction

An interconnection network links the processing units in parallel computing systems. Its structure can be modeled as a graph, where vertices represent processors and edges represent communication links. As a result, the terms graph and network are often used synonymously. Designing an efficient network topology involves balancing multiple conflicting requirements. Among the most widely used topologies is the n -dimensional hypercube, Q_n [14]. However, the hypercube does not achieve the minimal possible diameter given its number of connections. To address this limitation, researchers have developed various modified networks by reconfiguring certain links in hypercubes [1, 8-10].

For graph definitions and notations, we follow [25]. A graph $G = (V, E)$ consists of a finite vertex set V and an edge set $E \subseteq \{\{u, v\} \mid u, v \in V\}$. The neighborhood of a vertex $u \in G$, denoted by $N_G(u)$, is the set $\{v \mid \{u, v\} \in E\}$. The neighbor edges of u are denoted by $NE_G(u) = \{\{u, v\} \mid v \in N_G(u)\}$. The degree of u , written as $\deg_G(u)$, equals $|N_G(u)|$. A graph is k -regular if $\deg_G(u) = k$ for all $u \in V$, and we denote the minimum degree by $\delta(G) = \min\{\deg_G(x) \mid x \in V\}$.

A *Hamiltonian path* spans all vertices of G , and a *cycle* is a closed path with no repeated vertices except the start/end vertex. A *Hamiltonian cycle* traverses every vertex exactly once, and a graph possessing such a cycle is *Hamiltonian*.

A graph is *bipartite* if its vertex set partitions into V_0 and V_1 such that every edge connects V_0 and V_1 . A bipartite graph is *Hamiltonian laceable* if there exists a Hamiltonian path between any two vertices from different partitions.

Hsieh et al.[11] introduced *edge fault-tolerant Hamiltonicity* to assess Hamiltonian properties in faulty networks. A graph G is *k -edge fault-tolerant Hamiltonian* if $G - F$ remains Hamiltonian for any $F \subseteq E(G)$ with $|F| \leq k$. For bipartite graphs, G is *k -edge fault-tolerant Hamiltonian laceable* if $G - F$ remains Hamiltonian laceable under the same conditions. Latifi et al. [13] proved that the n -dimensional hypercube Q_n is $(n - 2)$ -edge fault-tolerant Hamiltonian, while Li et al. [15] showed the n -dimensional star graph S_n is $(n - 3)$ -edge fault-tolerant Hamiltonian. Edge fault tolerance has been widely examined in networks [10,11,13,15,18,20].

2020 *Mathematics Subject Classification:* 05C40, 05C90.

Submitted February 03, 2026. Published April 30, 2026.

1.1. Total transformation graphs

Wu and Meng [28] have generalized the total graph as follows:

Definition 1.1 Let $G = (V, E)$ be a graph and α, β, γ be three variables taking values $+$ or $-$. The total transformation graph $G^{\alpha\beta\gamma}$ is a graph having $V(G) \cup E(G)$ as a vertex set, and for $\rho, \mathcal{U} \in V(G) \cup E(G)$, ρ and \mathcal{U} are adjacent in $G^{\alpha\beta\gamma}$ if and only if

1. $\rho, \mathcal{U} \in V(G)$, ρ, \mathcal{U} are adjacent in G if $\alpha = +$ and ρ and \mathcal{U} are not adjacent in G if $\alpha = -$.
2. $\rho, \mathcal{U} \in E(G)$, ρ, \mathcal{U} are adjacent in G if $\beta = +$ and ρ and \mathcal{U} are not adjacent in G if $\beta = -$.
3. $\rho \in V(G)$ and $\mathcal{U} \in E(G)$, ρ, \mathcal{U} are incident in G if $\gamma = +$ and ρ and \mathcal{U} are not incident in G if $\gamma = -$.

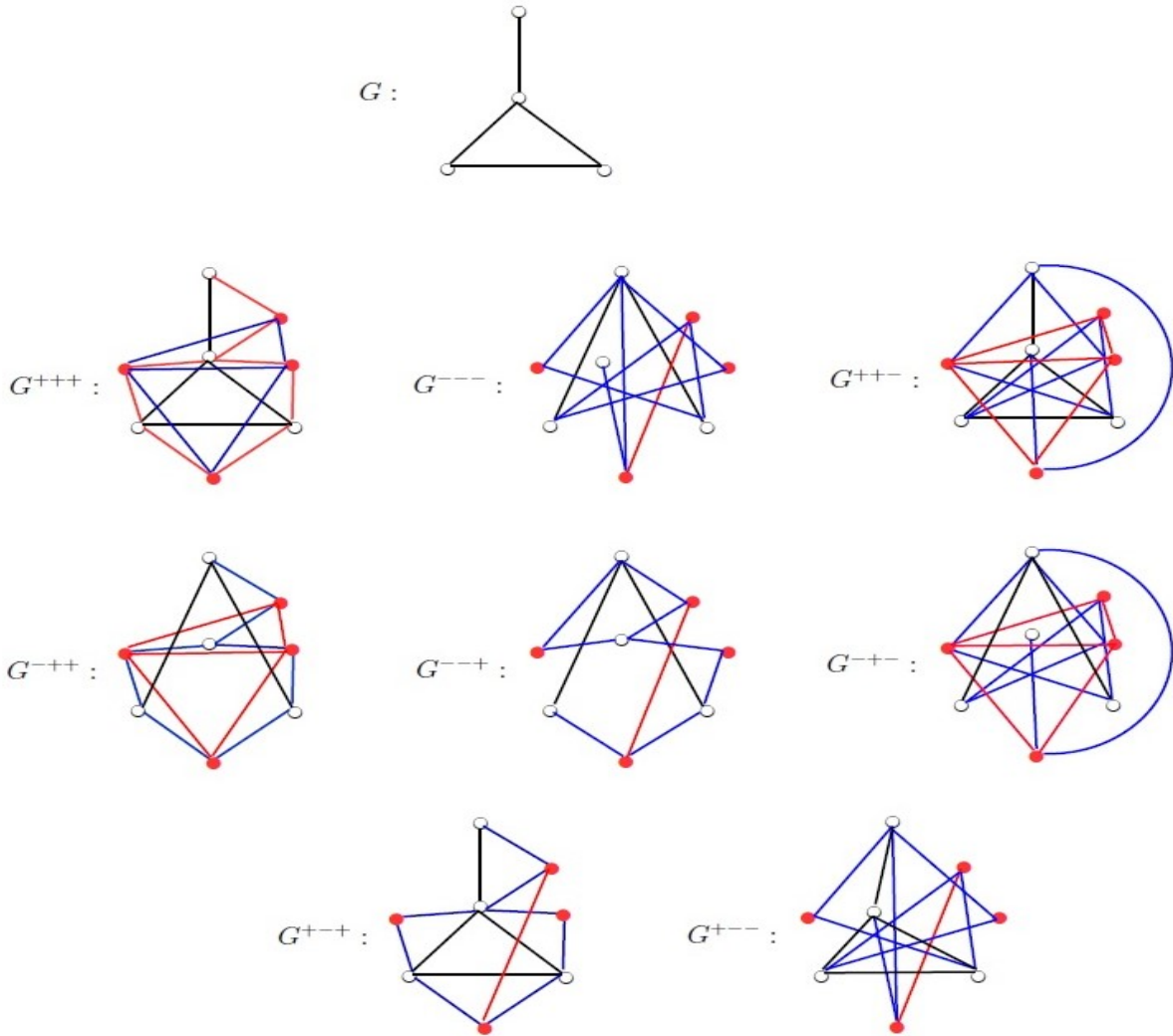


Figure 1. A graph G and its total transformation graph G^{xyz} .

The basic properties of these total transformation can be seen in [28,29].

2. Results

Theorem 2.1 For $n \geq 2$, the $m + \frac{\sum_{i=1}^n d_G(v_i)^2}{2}$ edge fault tolerant graph G^{+++} is Hamiltonian laceable if and only if G is laceable.

Proof: Let G be a laceable graph of order n and size m such that $V(G) = \{v_1, v_2, v_3, \dots, v_n\}$ and $E(G) = \{e_1, e_2, e_3, \dots, e_m\}$. Then by the definition of G^{+++} we have $V(G^{+++}) = V(G) \cup E(G)$. That is $V(G^{+++}) = \{v_1, v_2, v_3, \dots, v_n, e_1, e_2, e_3, \dots, e_m\}$. Further, $|V(G^{+++})| = n + m$ and $|E(G^{+++})| = \frac{1}{2}[4m + \sum_{i=1}^n d_G(v_i)^2]$. Since G is laceable, we need to show that G^{+++} is laceable after removal of

$m + \frac{\sum_{i=1}^n d_G(v_i)^2}{2}$ edges from $E(G^{+++})$. It is easy to observe that for every edge $e = uv \in G$, uve will be a cycle of length three in G^{+++} . A graph is bipartite if and only if it does not contain odd cycle, therefore, G^{+++} is non-bipartite graph and therefore non-laceable. By the definition of G^{+++} , G is an induced subgraph, since G is laceable therefore, G^{+++} is $E(G^{+++}) - E(G)$ edge fault tolerant graph. That is, $m + \frac{\sum_{i=1}^n d_G(v_i)^2}{2}$ edge fault tolerant graph G^{+++} is Hamiltonian laceable.

Conversely, assume that G^{+++} is Hamiltonian laceable and G is non-laceable graph. Then G^{+++} does not contain any odd cycle, since by the definition and structural analysis it is easy to observe that C_3 is a subgraph of G^{+++} , a contradiction to our assumption. \square

Example 1. The shadow graph is an edge-fault-tolerant t^* -laceable. Therefore, consider the shadow graph of $B(C_4)$ and its total transformation graph B^{+++} is also an edge-fault-tolerant t^* -laceable.

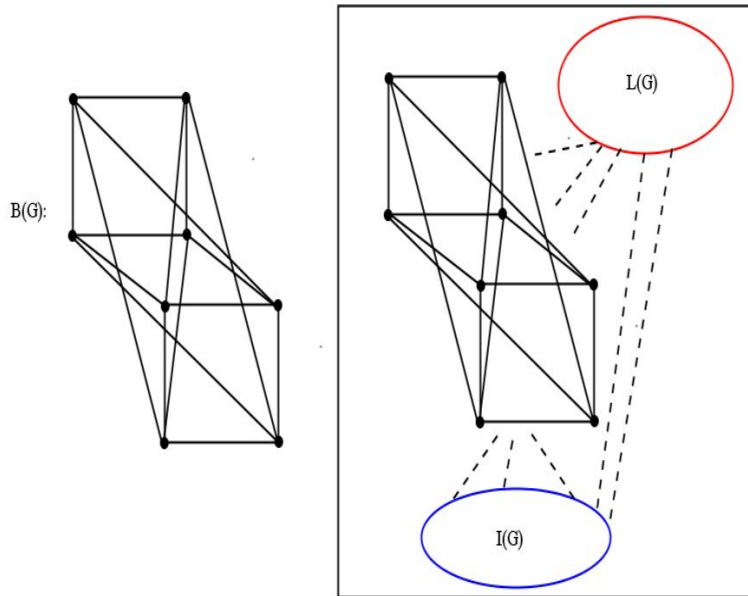


Figure 2. The shadow graph of C_4 and its total transformation graph G^{+++} .

Theorem 2.2 For $n \geq 2$, the $m(n-2) - m + \frac{\sum_{i=1}^n d_G(v_i)^2}{2}$ edge fault tolerant graph G^{+++} is Hamiltonian laceable if and only if G is laceable.

Proof: Let G be a laceable graph of order n and size m such that $V(G) = \{v_1, v_2, v_3, \dots, v_n\}$ and $E(G) = \{e_1, e_2, e_3, \dots, e_m\}$. Then by the definition of G^{+++} we have $E(G^{+++}) = E(G) \cup E(L(G)) \cup (n-2) \cdot E(G)$, $|V(G^{+++})| = m + n$, $|E(G^{+++})| = m + \frac{1}{2}[2m(n-2) + \sum_{i=1}^n d_G(v_i)^2]$. Since G is laceable, we need to show

that G^{+++} is laceable after removal of $m(n-2) - m + \frac{\sum_{i=1}^n d_G(v_i)^2}{2}$ edges from $E(G^{+++})$. It is easy to observe that for every edge $e = uv \in G$, uve will be a cycle of length three in G^{+++} . A graph is bipartite if and only if it does not contain odd cycle, therefore, G^{+++} is non-bipartite graph and therefore non-laceable. By the definition of G^{+++} , G and its line graph (G) are subgraphs, since G is laceable therefore, G^{+++} is

$E(G^{+++}) - E(G)$ edge fault tolerant graph. That is, $m(n-2) - m + \frac{\sum_{i=1}^n d_G(v_i)^2}{2}$ edge fault tolerant graph G^{+++} is Hamiltonian laceable.

Conversely, assume that G^{+++} is Hamiltonian laceable and G is non-laceable graph. Then G^{+++} does not contain any odd cycle, since by the definition and structural analysis it is easy to observe that C_3 is a subgraph of G^{+++} , a contradiction to our assumption. \square

Example 2. The shadow graph is an edge-fault-tolerant t^* -laceable. Therefore, consider the shadow graph of $B(C_4)$ and its total transformation graph B^{+++} is also an edge-fault-tolerant t^* -laceable due to the fact that $B(G)$ is a subgraph of B^{+++} .

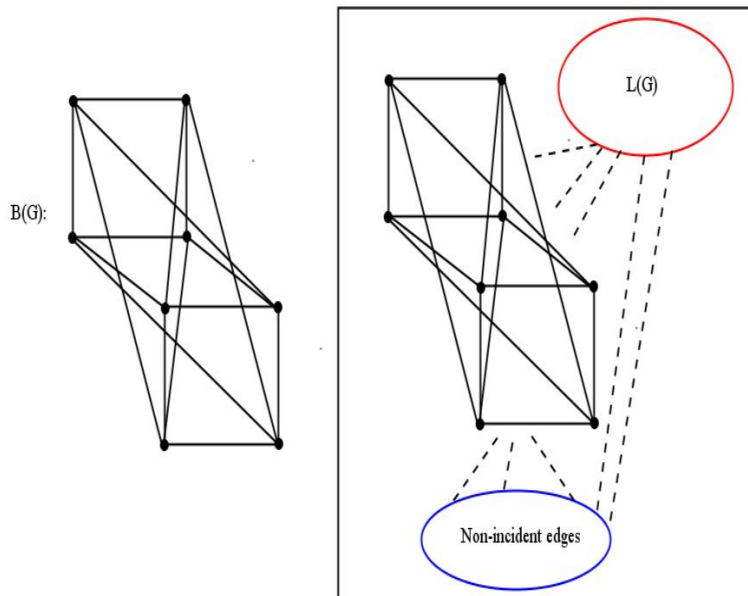


Figure 3. The shadow graph of C_4 and its total transformation graph G^{+++} .

Theorem 2.3 For $n \geq 2$, the $2m + \bar{q}_l$ edge fault tolerant graph G^{+++} is Hamiltonian laceable if and only if G is laceable, where \bar{q}_l the number of edges in $L(G)$.

Proof: Let G be a laceable graph of order n and size m such that $V(G) = \{v_1, v_2, v_3, \dots, v_n\}$ and $E(G) = \{e_1, e_2, e_3, \dots, e_m\}$. Then by the definition of G^{+++} we have $E(G^{+++}) = E(G) \cup E(L(G)) \cup 2E(G)$, $|V(G^{+++})| = m + n$ $|E(G^{+++})| = m = \frac{1}{2}[m(m+7) - \sum_{i=1}^n d_G(v_i)^2]$. Since G is laceable, we need to show that G^{+++} is laceable after removal of $2m + \bar{q}_l$ edges from $E(G^{+++})$. It is easy to observe that for every edge $e = uv \in G$, uve will be a cycle of length three in G^{+++} . A graph is bipartite if and only if it does not contain odd cycle, therefore, G^{+++} is non-bipartite graph and therefore non-laceable. By the definition of G^{+++} , G and its line graph (G) are subgraphs, since G is laceable therefore, G^{+++} is $E(G^{+++}) - E(G)$ edge fault tolerant graph. That is, $2m + \bar{q}_l$ edge fault tolerant graph G^{+++} is Hamiltonian laceable.

Conversely, assume that G^{+++} is Hamiltonian laceable and G is non-laceable graph. Then G^{+++} does not contain any odd cycle, since by the definition and structural analysis it is easy to observe that C_3 is a subgraph of G^{+++} , a contradiction to our assumption. \square

Example 3. The shadow graph is an edge-fault-tolerant t^* -laceable. Therefore, consider the shadow graph of $B(C_4)$ and its total transformation graph B^{+++} is also an edge-fault-tolerant t^* -laceable due to the fact that $B(G)$ is a subgraph of B^{+++} .

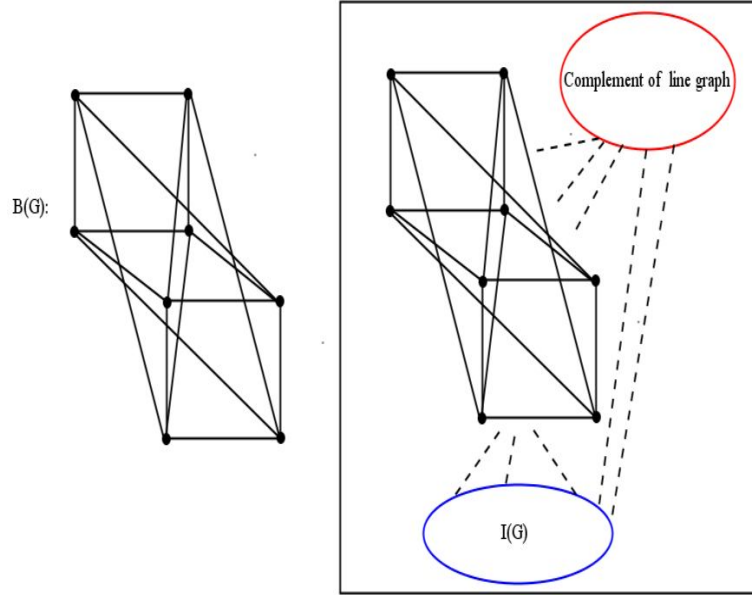


Figure 4. The shadow graph of C_4 and its total transformation graph G^{+--} .

Theorem 2.4 For $n \geq 2$, the $2m(n-2) + \bar{q}_l$ edge fault tolerant graph G^{+--} is Hamiltonian laceable if and only if G is laceable, where \bar{q}_l the number of edges in $\overline{L(G)}$.

Proof: Let G be a laceable graph of order n and size m such that $V(G) = \{v_1, v_2, v_3, \dots, v_n\}$ and $E(G) = \{e_1, e_2, e_3, \dots, e_m\}$. Then by the definition of G^{+--} we have $E(G^{+--}) = E(G) \cup E(L(G)) \cup (n-2) \cdot E(G)$, $|V(G^{+--})| = m + n$, $|E(G^{+--})| = m = \frac{1}{2}[m(m+2n-1) - \sum_{i=1}^n d_G(v_i)^2]$. Since G is laceable, we need to show that G^{+--} is laceable after removal of $2m(n-2) + \bar{q}_l$ edges from $E(G^{+--})$. It is easy to observe that for every edge $e = uv \in G$, uve will be a cycle of length three in G^{+--} . A graph is bipartite if and only if it does not contain odd cycle, therefore, G^{+--} is non-bipartite graph and therefore non-laceable. By the definition of G^{+--} , G and its line graph $L(G)$ are subgraphs, since G is laceable therefore, G^{+--} is $E(G^{+--}) - E(G)$ edge fault tolerant graph. That is, $2m(n-2) + \bar{q}_l$ edge fault tolerant graph G^{+--} is Hamiltonian laceable.

Conversely, assume that G^{+--} is Hamiltonian laceable and G is non-laceable graph. Then G^{+--} does not contain any odd cycle, since by the definition and structural analysis it is easy to observe that C_3 is a subgraph of G^{+--} , a contradiction to our assumption. \square

Example 4. The shadow graph is an edge-fault-tolerant t^* -laceable. Therefore, consider the shadow graph of $B(C_4)$ and its total transformation graph B^{+--} is also an edge-fault-tolerant t^* -laceable due to the fact that $B(G)$ is a subgraph of B^{+--} .

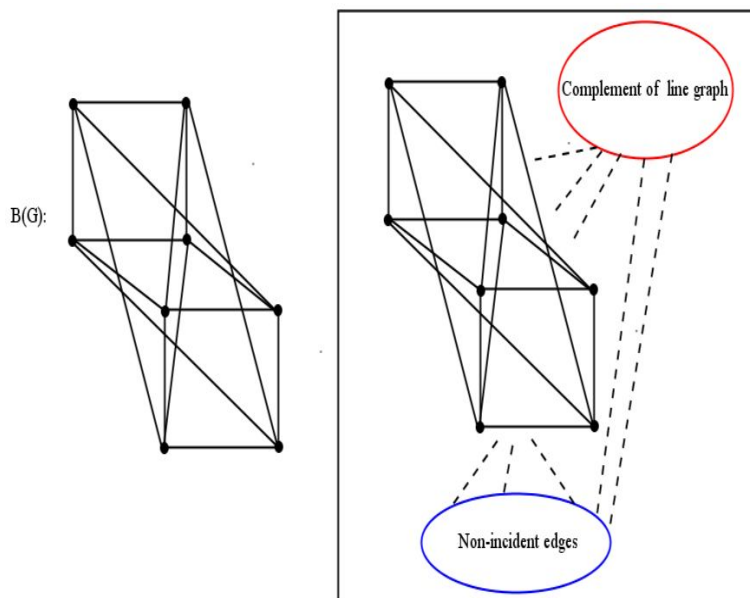


Figure 5. The shadow graph of C_4 and its total transformation graph G^{+-+} .

Theorem 2.5 For $n \geq 2$, the $m + \frac{\sum_{i=1}^n d_G(v_i)^2}{2}$ edge fault tolerant graph G^{-++} is Hamiltonian laceable if and only if \overline{G} is laceable.

Proof: Let G be a laceable graph of order n and size m such that $V(G) = \{v_1, v_2, v_3, \dots, v_n\}$ and $E(G) = \{e_1, e_2, e_3, \dots, e_m\}$. Then by the definition of G^{-++} we have $E(G^{-++}) = E(\overline{G}) \cup E(\overline{L(G)}) \cup 2E(G)$, $|V(G^{-++})| = m + n$, $|E(G^{-++})| = m + \frac{1}{2}[n(n-1) + \sum_{i=1}^n d_G(v_i)^2]$. Since G is laceable, we need to show

that G^{-++} is laceable after removal of $m + \frac{\sum_{i=1}^n d_G(v_i)^2}{2}$ edges from $E(G^{-++})$. It is easy to observe that for every edge $e = uv \in G$, uve will be a cycle of length three in G^{-++} . A graph is bipartite if and only if it does not contain odd cycle, therefore, G^{-++} is non-bipartite graph and therefore non-laceable. By the definition of G^{-++} , G and its line graph (G) are subgraphs, since G is laceable therefore, G^{-++} is $E(G^{-++}) - E(G)$ edge fault tolerant graph. That is, $m + \frac{\sum_{i=1}^n d_G(v_i)^2}{2}$ edge fault tolerant graph G^{-++} is Hamiltonian laceable.

Conversely, assume that G^{-++} is Hamiltonian laceable and G is non-laceable graph. Then G^{-++} does not contain any odd cycle, since by the definition and structural analysis it is easy to observe that C_3 is a subgraph of G^{-++} , a contradiction to our assumption. \square

Example 5. The shadow graph is an edge-fault-tolerant t^* -laceable. Therefore, consider the complement of the shadow graph of $B(C_4)$ and its total transformation graph B^{-++} is also an edge-fault-tolerant t^* -laceable due to the fact that $\overline{B(G)}$ is a subgraph of B^{-++} .

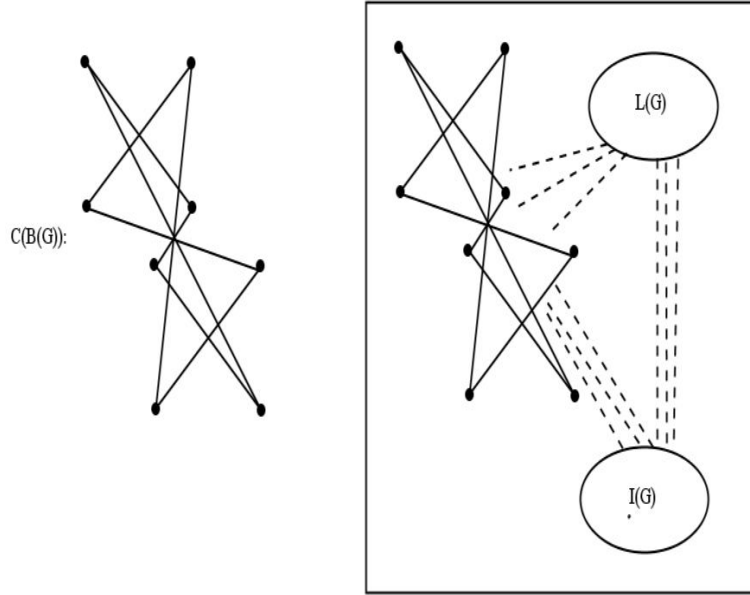


Figure 6. The complement of shadow graph of C_4 and its total transformation graph G^{--+} .

Theorem 2.6 For $n \geq 2$, the $2m(n-2) + \bar{q}_l$ edge fault tolerant graph G^{--+} is Hamiltonian laceable if and only if \bar{G} is laceable, where \bar{q}_l the number of edges in $\bar{L}(G)$.

Proof: Let G be a laceable graph of order n and size m such that $V(G) = \{v_1, v_2, v_3, \dots, v_n\}$ and $E(G) = \{e_1, e_2, e_3, \dots, e_m\}$. Then by the definition of G^{--+} we have $E(G^{--+}) = E(\bar{G}) \cup E(\bar{L}(G)) \cup 2E(G)$, $|V(G^{--+})| = m+n$, $|E(G^{--+})| = m = \frac{1}{2}[m(m+n) + n(n+1) - \sum_{i=1}^n d_G(v_i)^2]$. Since G is laceable, we need to show that G^{--+} is laceable after removal of $2m(n-2) + \bar{q}_l$ edges from $E(G^{--+})$. It is easy to observe that for every edge $e = uv \in G$, uve will be a cycle of length three in G^{--+} . A graph is bipartite if and only if it does not contain odd cycle, therefore, G^{--+} is non-bipartite graph and therefore non-laceable. By the definition of G^{--+} , G and its line graph (G) are subgraphs, since G is laceable therefore, G^{--+} is $E(G^{--+}) - E(G)$ edge fault tolerant graph. That is, $2m(n-2) + \bar{q}_l$ edge fault tolerant graph G^{--+} is Hamiltonian laceable.

Conversely, assume that G^{--+} is Hamiltonian laceable and G is non-laceable graph. Then G^{--+} does not contain any odd cycle, since by the definition and structural analysis it is easy to observe that C_3 is a subgraph of G^{--+} , a contradiction to our assumption. \square

Example 6. The shadow graph is an edge-fault-tolerant t^* -laceable. Therefore, consider the complement of the shadow graph of $\bar{B}(C_4)$ and its total transformation graph B^{--+} is also an edge-fault-tolerant t^* -laceable due to the fact that $\bar{B}(G)$ is a subgraph of B^{--+} .

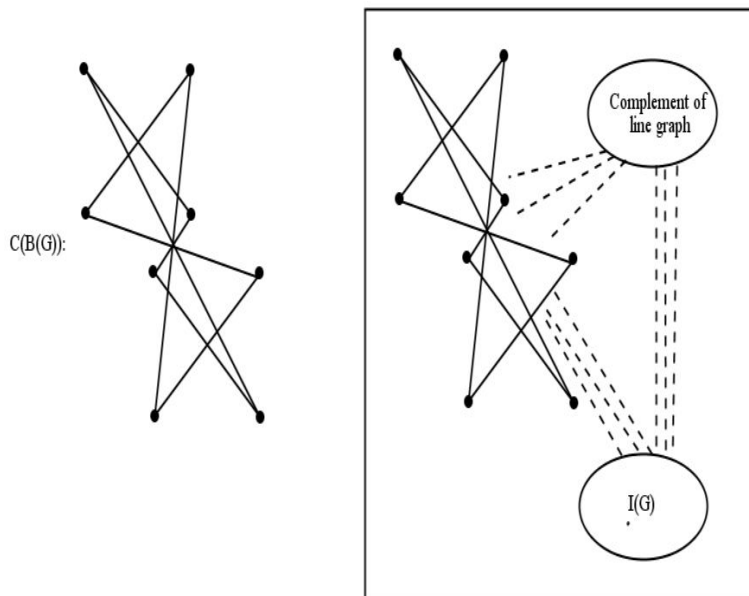


Figure 7. The complement of shadow graph of C_4 and its total transformation graph G^{--} .

Theorem 2.7 For $n \geq 2$, the $2m(n-2) - m + \frac{\sum_{i=1}^n d_G(v_i)^2}{2}$ edge fault tolerant graph G^{--} is Hamiltonian laceable if and only if \overline{G} is laceable.

Proof: Let G be a laceable graph of order n and size m such that $V(G) = \{v_1, v_2, v_3, \dots, v_n\}$ and $E(G) = \{e_1, e_2, e_3, \dots, e_m\}$. Then by the definition of G^{--} we have $E(G^{--}) = E(\overline{G}) \cup E(L(G)) \cup (n-2) \cdot E(G)$, $|V(G^{--})| = m+n$, $|E(G^{--})| = m + \frac{1}{2}[n(m+n-1) + m(n-8) + \sum_{i=1}^n d_G(v_i)^2]$. Since G is laceable, we

need to show that G^{--} is laceable after removal of $2m(n-2) - m + \frac{\sum_{i=1}^n d_G(v_i)^2}{2}$ edges from $E(G^{--})$. It is easy to observe that for every edge $e = uv \in G$, uve will be a cycle of length three in G^{--} . A graph is bipartite if and only if it does not contain odd cycle, therefore, G^{--} is non-bipartite graph and therefore non-laceable. By the definition of G^{--} , G and its line graph (G) are subgraphs, since G is laceable therefore, G^{--} is $E(G^{--}) - E(G)$ edge fault tolerant graph. That is, $2m(n-2) - m + \frac{\sum_{i=1}^n d_G(v_i)^2}{2}$ edge fault tolerant graph G^{--} is Hamiltonian laceable.

Conversely, assume that G^{--} is Hamiltonian laceable and G is non-laceable graph. Then G^{--} does not contain any odd cycle, since by the definition and structural analysis it is easy to observe that C_3 is a subgraph of G^{--} , a contradiction to our assumption. \square

Example 7. The shadow graph is an edge-fault-tolerant t^* -laceable. Therefore, consider the complement of the shadow graph of $B(C_4)$ and its total transformation graph B^{--} is also an edge-fault-tolerant t^* -laceable due to the fact that $\overline{B(G)}$ is a subgraph of B^{--} .

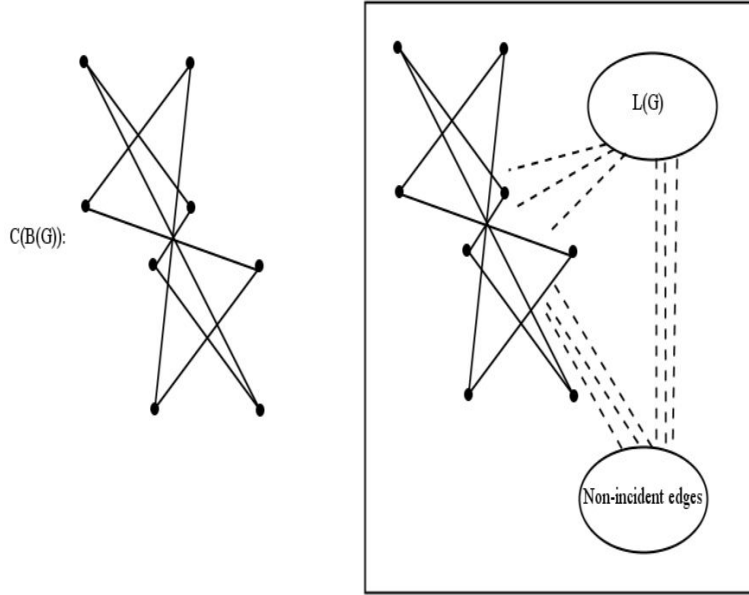


Figure 7. The complement of shadow graph of C_4 and its total transformation graph G^{-+-} .

Theorem 2.8 For $n \geq 2$, the $2m(n-2) + \bar{q}_l$ edge fault tolerant graph G^{---} is Hamiltonian laceable if and only if \bar{G} is laceable, where \bar{q}_l the number of edges in $\bar{L}(G)$.

Proof: Let G be a laceable graph of order n and size m such that $V(G) = \{v_1, v_2, v_3, \dots, v_n\}$ and $E(G) = \{e_1, e_2, e_3, \dots, e_m\}$. Then by the definition of G^{---} we have $E(G^{---}) = E(G) \cup E(L(G)) \cup (n-2) \cdot E(G)$, $|V(G^{---})| = m+n$, $|E(G^{---})| = m = \frac{1}{2}[(m+n-1)(m+n) - 4m - \sum_{i=1}^n d_G(v_i)^2]$. Since G is laceable, we need to show that G^{---} is laceable after removal of $2m(n-2) + \bar{q}_l$ edges from $E(G^{---})$. It is easy to observe that for every edge $e = uv \in G$, uve will be a cycle of length three in G^{---} . A graph is bipartite if and only if it does not contain odd cycle, therefore, G^{---} is non-bipartite graph and therefore non-laceable. By the definition of G^{---} , G and its line graph (G) are subgraphs, since G is laceable therefore, G^{---} is $E(G^{---}) - E(G)$ edge fault tolerant graph. That is, $2m(n-2) + \bar{q}_l$ edge fault tolerant graph G^{---} is Hamiltonian laceable.

Conversely, assume that G^{---} is Hamiltonian laceable and G is non-laceable graph. Then G^{---} does not contain any odd cycle, since by the definition and structural analysis it is easy to observe that C_3 is a subgraph of G^{---} , a contradiction to our assumption. \square

Example 8. The shadow graph is an edge-fault-tolerant t^* -laceable. Therefore, consider the complement of the shadow graph of $B(C_4)$ and its total transformation graph B^{---} is also an edge-fault-tolerant t^* -laceable due to the fact that $\bar{B}(G)$ is a subgraph of B^{---} .

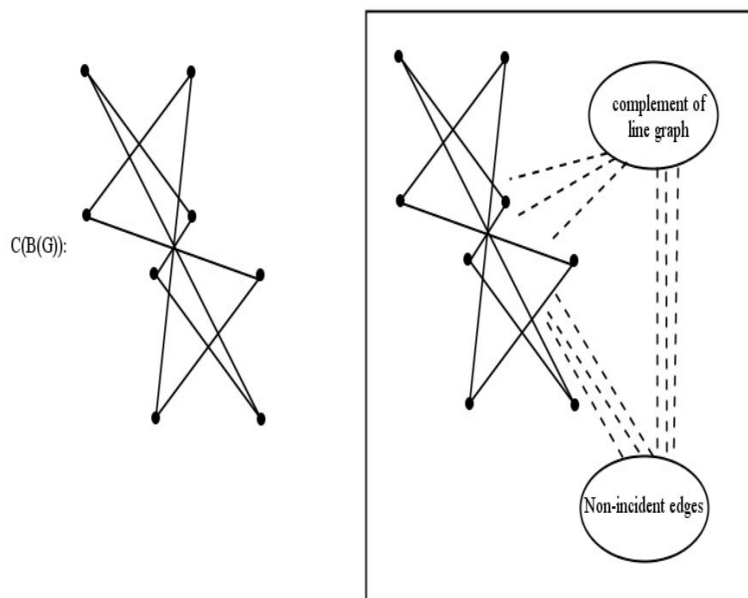


Figure 8. The complement of shadow graph of C_4 and its total transformation graph G^{---} .

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Nagarathnamma K. G.,
Department of Mathematics,
Bapuji Institute of Engineering and Technology, Davanagere -577 004
India.
and
Research Scholar
Visveswaraya Technological University, Belagavi-590018
E-mail address: nagarathnammakg777@gmail.com

and

Leena N. Shenoy,
Department of Mathematics,
B.N.M. Institute of Technology, Bengaluru-560070
India.
and
Visveswaraya Technological University, Belagavi-590018
E-mail address: leenanshenoy@bnmit.in