Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2026 (44) 2** : 1–18.
ISSN-0037-8712
doi:10.5269/bspm.81799

Numerical Investigation of Unsteady Mixed Convective Hydromagnetic Flow and Heat Transfer over a Vertical Stretching Surface with Variable Viscosity and Dissipation Effects

K. Periyadurai, T. Gowpriyanka, M. Kousalya

ABSTRACT: An unsteady two-dimensional laminar mixed convection flow of an incompressible, viscous and electrically conducting fluid over a vertically stretching surface is numerically analyzed. The study accounts for the combined effects of temperature-dependent viscosity, viscous dissipation and Joule heating under the influence of an applied magnetic field. The governing nonlinear partial differential equations are transformed into a system of nonlinear ordinary differential equations through appropriate similarity transformations. These equations are solved numerically using the Nachtsheim-Swigert shooting technique coupled with a fourth-order Runge-Kutta integration scheme to satisfy the asymptotic boundary conditions. Computations are performed for various values of the controlling physical parameters, and the influences of the variable viscosity parameter, unsteadiness parameter, Eckert number, Prandtl number, mixed convection parameter and Hartmann number on the velocity and temperature fields are illustrated graphically. In addition, numerical values of the skin-friction coefficient and Nusselt number are evaluated and presented in tabular form. Overall, the study demonstrates that the combined effects of variable viscosity, Joule heating, and viscous dissipation play a crucial role in controlling flow resistance and heat transfer characteristics over a vertical stretching surface. The findings provide useful physical insights and may serve as a reference for the design and optimization of MHD-based thermal systems encountered in industrial and engineering applications.

Keywords: Viscous Dissipation, mixed convection, boundary layer flow, magnetic field.

Contents

1 Introduction 1

2 Mathematical Formulation 3

3 Parameters of Engineering Interest 6

4 Numerical Solutions 7

5 Results and Discussion 8

6 Conclusion 15

1. Introduction

The study of magnetohydrodynamic (MHD) flow of electrically conducting fluids has attracted considerable attention due to its significant role in modern metallurgical and metal-forming processes. Hydromagnetic flow and heat transfer problems have gained increasing industrial relevance, particularly following the development of liquid metal heat exchangers. Moreover, investigations of thermal instability in MHD flows have direct applications in geophysical and astrophysical phenomena. In recent decades, hydromagnetic flows induced by stretching surfaces, coupled with heat transfer, have received growing interest due to their wide-ranging applications in manufacturing and industrial processes, including polymer processing and material fabrication technologies. Furthermore, thermal boundary-layer flows involving temperature-dependent viscosity over isothermal heated surfaces are of both theoretical and practical importance, as they effectively model numerous fluid transport mechanisms encountered in engineering systems. Typical applications include hot rolling, wire drawing, glass fiber production, paper manufacturing, adhesive coating processes, and the extrusion of plastic films. Motivated by these practical considerations, the present study focuses on unsteady MHD flow and mixed convection heat

2020 *Mathematics Subject Classification:* 76W05.
Submitted February 07, 2026. Published March 14, 2026

transfer over a vertical stretching surface, accounting for the combined effects of variable viscosity, viscous dissipation, and Joule heating.

Various aspects of flow and heat transfer over stretching surfaces moving in an infinite fluid medium have been extensively investigated in the literature. The pioneering work on two-dimensional boundary-layer flow induced by a moving rigid surface was carried out by Sakiadis [1]. Subsequently, Crane [2] analyzed the convection boundary-layer flow over a linearly stretching sheet and obtained an exact similarity solution for the steady two-dimensional case. Since then, numerous studies have examined boundary-layer flows over stretching surfaces under diverse physical conditions. Later investigations focused on unsteady flows over stretching surfaces, where unsteadiness may arise due to time-dependent stretching rates or sudden changes in surface temperature. Eldabe [3] examined the unsteady free convection flow of an incompressible, electrically conducting viscous fluid through a porous medium over a porous stretching sheet. The problem of unsteady flow past a stretching surface was further analyzed by Ioan Pop and Tsung-Yen Na [4].

In recent years, research on fluid flow induced by stretching boundaries coupled with heat transfer has expanded rapidly. Problems involving temperature-dependent viscosity are particularly important due to their wide range of applications in geophysical processes, such as geothermal energy extraction and underground storage systems. Pop et al. [5] investigated the influence of variable viscosity on flow and heat transfer over a continuously moving flat plate. Kafoussias and Williams [6] analyzed Dufour and Soret effects in mixed convection flows with temperature-dependent viscosity, while Barletta [7] studied mixed convection flow in a vertical channel with isothermal–isoflux boundary conditions, accounting for viscous dissipation. Furthermore, Anjali Devi and Ganga [8] examined viscous dissipation effects on nonlinear MHD flow through a porous medium over a stretching porous surface. The influence of variable viscosity on MHD flow and heat transfer over an unsteady stretching surface in the presence of Hall current was explored by Shateyi et al. [9]. Additionally, Kishore and Bhanumathi [10] investigated the combined effects of chemical reaction and viscous dissipation on unsteady MHD free convection flow past an exponentially accelerated vertical plate with variable surface conditions.

The effect of radiation and variable viscosity on unsteady MHD flow of a rotating fluid from stretching surface in porous medium was explored by Rashad [11]. Vajravelu and Prasad [12] studied on MHD flow and heat transfer of an Ostwald-de Waele fluid over an unsteady stretching surface. Aurang Zaib and Sharidan Shafie [13] discussed on thermal diffusion and diffusion thermo effects on unsteady MHD free convection flow over a stretching surface considering Joule heating and viscous dissipation with thermal stratification, chemical reaction and Hall current. Recently, Mohamed Abd El-Aziz [14] studied an Unsteady mixed convection heat transfer along a vertical stretching surface with variable viscosity and viscous dissipation. Hayat et al. [16] studied the combined effects of Joule heating and viscous dissipation in MHD flow of Powell-Eyring fluid over a stretching surface, showing that these dissipative mechanisms substantially increase fluid temperature. Mabood et al. [17] explored the effects of variable fluid properties and convective boundary conditions on MHD flow and heat transfer over a stretching surface, demonstrating that variable thermal conductivity significantly impacts the Nusselt number. Khan et al. [18] reported on the unsteady MHD flow of a Maxwell fluid over a stretching sheet with temperature-dependent viscosity and thermal stratification, concluding that stratification reduces the thermal boundary layer thickness. Srinivasacharya and Beg [19] studied the influence of variable viscosity and thermal conductivity on MHD natural convection flow in a porous medium with stretching walls, finding that variable properties significantly affect the flow and heat transfer characteristics.

In the most recently, Khan et al. [20] investigated the combined effects of variable thermal conductivity and radiative heat transfer on unsteady MHD nanofluid flow over an exponentially stretching surface, highlighting the significance of dissipation and Joule heating in energy transport, and they concluded that variable thermal conductivity notably enhances the rate of heat transfer. Shaw et al. [21] analyzed entropy generation in mixed convective MHD flow of a hybrid nanofluid over a vertical stretching sheet with variable viscosity and thermal radiation, providing insights into thermodynamic optimization and reporting that entropy generation is significantly influenced by the magnetic field and variable viscosity. Mondal et al. [22] examined the unsteady MHD flow and heat transfer of a Maxwell fluid over a stretching surface with temperature-dependent viscosity and viscous dissipation using a fractional derivative approach, finding that the fractional parameter effectively captures the memory effects on velocity and temperature fields. Very recently, Patil et al. [23] explored the influence of Hall current and ion slip on unsteady mixed convective MHD flow past a vertical stretching surface with variable fluid properties and Joule heating, demonstrating that Hall and ion slip parameters substantially reduce the skin friction coefficient while enhancing the Nusselt number.

To the best of the authors' knowledge, no comprehensive study has been reported on unsteady mixed convective hydromagnetic flow and heat transfer over a vertical stretching surface in the presence of variable viscosity, viscous dissipation, and Joule heating under an applied magnetic field. Motivated by its relevance to several industrial applications, the present work addresses this research gap. The governing boundary-layer equations for momentum and energy, which are nonlinear partial differential equations, are transformed into a system of coupled nonlinear ordinary differential equations using appropriate similarity transformations. The resulting boundary-value problem is solved numerically using the Nachtsheim–Swigert shooting iteration technique to satisfy the asymptotic boundary conditions, in combination with a fourth-order Runge–Kutta integration scheme. Numerical computations are performed for a range of governing physical parameters to examine their effects on the velocity and temperature distributions. The influence of these parameters is illustrated graphically, while numerical values of the skin-friction coefficient and the non-dimensional rate of heat transfer are presented in tabular form.

2. Mathematical Formulation

A two-dimensional, mixed convective, boundary layer flow of an incompressible, viscous, electrically conducting fluid along an unsteady stretching sheet, which issues vertically in the upward direction from a slot with velocity u_w is considered. The Cartesian coordinate system (x, y) is chosen with the positive x-axis is measured along the stretching sheet with the slot as the origin and the positive y-axis is measured normal to the sheet in the outward direction toward the fluid. The configurations of the flow and coordinate systems are shown in Fig. 1. A uniform magnetic field of strength $\vec{B}_0$ is applied normal to the sheet. The stretching velocity of the plate is given by

$$u_w(x,t) = \frac{bx}{1-\alpha t} \tag{2.1}$$

Here b is the initial stretching rate, whereas the effective stretching rate $\frac{b}{1-\alpha t}$ is increasing with time. The equation 2.1 is valid only for time $t < \alpha^{-1}$ unless $\alpha = 0$. The surface temperature T_w of the stretching sheet varies with the distance x from the slot and time t. It is assumed that

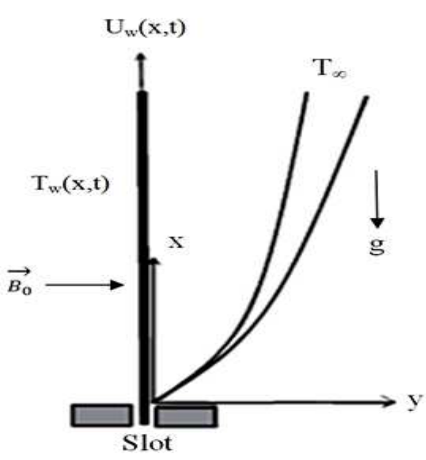

Figure 1: Schematic representation of the physical model and coordinate system

$$T_w(x,t) = T_\infty + T_0 \frac{bx^2}{2\nu_\infty} (1-\alpha t)^{-2} \tag{2.2}$$

The following assumptions are made in the analysis of the problem.

- The magnetic Reynolds number is assumed as so small so that the induced magnetic field is negligible. Since $\vec{B}_0$ is independent of time, $curl\vec{E} = 0$. Also $div\vec{E} = 0$ in the absence of surface charge density. Hence $\vec{E} = 0$.
- Viscosity and Joule's dissipation effects are considered.
- The fluid has variable viscosity.
- Boussinesq approximation is employed.

For a viscous fluid, Lai and Kulacki [15] suggested a viscosity dependence on the temperature of the form

$$\frac{1}{\mu} = \frac{1}{\mu_\infty} [1 + \gamma(T - T_\infty)] \tag{2.3}$$

which implies that the viscosity is an inverse linear function of temperature T. One can rewrite equation (2.3) as,

$$\frac{1}{\mu} = a(T - T_r) \tag{2.4}$$

where $a = \frac{\gamma}{\mu_\infty}$ and $T_r = T_\infty - \frac{1}{\gamma}$

In the above equation (2.4), both a and T_r are constants and their values depend on the reference state and γ, a thermal property of the fluid. For liquids $a > 0$ and for gases $a < 0$ in general. Typical values of γ and a for air are $\gamma = 0.026240$ and a = -123.2.

Under the above assumptions, two-dimensional hydromagnetic boundary layer equations governing the problem in the presence of viscous and Joules dissipation are given as

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.5}$$

Momentum equation:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + g\beta(T - T_\infty) - \frac{\sigma B_0^2}{\rho}u \tag{2.6}$$

Energy equation:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B_0^2}{\rho c_p}u^2 \tag{2.7}$$

The appropriate boundary conditions for the problem are given by

$u = u_w, \quad v = 0, \quad T = TW, \quad$ at $y = 0$

$u \to 0, \quad T \to \infty \quad$ as $y = \infty$

Similarity Transformations

In order to solve the continuity, momentum and energy equations (2.5-2.7) with boundary conditions, the following similarity transformations are introduced. The suitable transformations are

$$\eta = y\sqrt{\frac{b}{\nu_\infty(1-\alpha t)}} \tag{2.8}$$

$$\psi = \sqrt{\frac{\nu_\infty b}{1-\alpha t}} f(\eta) \tag{2.9}$$

$$T = T_\infty + T_0 \left[\frac{bx^2}{2v_\infty}\right](1-\alpha t)^{-2}\theta(\eta) \tag{2.10}$$

The stream function ψ is introduced such that

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x} \tag{2.11}$$

Substituting equations (2.8), (2.9) in (2.11) we get

$$u = u_w f'(n), \qquad v = -\sqrt{\frac{\nu_\infty b}{1-\alpha t}}\, f(\eta) \tag{2.12}$$

where f is non-dimensional stream function with respect to η.

An increase in temperature reduces fluid viscosity within the momentum boundary layer, thereby enhancing transport phenomena and significantly influencing the wall heat transfer rate. Consequently, accurate prediction of flow behavior and heat transfer characteristics requires the incorporation of temperature-dependent viscosity effects.

The dimensionless temperature θ can also be written as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{2.13}$$

Substituting equations (2.7) to (2.13) in equations (2.6) and (2.7), the governing equations are transformed into

Momentum equation:

$$\left(\frac{\theta_r}{\theta_r - \theta}\right) f''' - \left(\frac{\theta_r}{(\theta_r - \theta)^2}\right) \theta' f'' + f f'' - (f')^2 - A\left(f' + \frac{\eta}{2} f''\right) + \lambda\theta - M^2 f' = 0 \tag{2.14}$$

Energy equation:

$$\theta'' + Pr\left(f\theta' - 2f'\theta - \frac{A}{2}(4\theta + \eta\theta')\right) + PrEc\left(\frac{\theta_r}{\theta_r - \theta}\right)(f'')^2 + PrEcM^2(f')^2 = 0 \tag{2.15}$$

The transformed boundary conditions are:

$$\begin{aligned} f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1 \ \text{ at } \ \eta \to 0 \\ f'(\infty) = 0, \quad \theta(\infty) = 0 \ \text{ as } \ \eta \to \infty \end{aligned} \tag{2.16}$$

where a prime denotes ordinary differentiation with respect to η. Also given by

$$\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = \frac{1}{\gamma(T_w - T_\infty)}, \tag{2.17}$$

$$A = \frac{\alpha}{b}, \tag{2.18}$$

$$Ec = \frac{2b\nu_\infty}{T_0 c_p} = \frac{u_w^2}{c_p(T_w - T_\infty)}, \tag{2.19}$$

$$Pr = \frac{\gamma_\infty \rho_\infty c_p}{k}, \tag{2.20}$$

$$\lambda = \frac{g\beta T_0 x}{2\nu_\infty b} = \frac{Gr_x}{Re_x^2}, \tag{2.21}$$

$$Re_x = \frac{u_w x}{\nu_\infty}, \tag{2.22}$$

$$Gr_x = \frac{g\beta(T_w - T_\infty)x^3}{\nu_\infty^2}, \tag{2.23}$$

$$M^2 = \frac{\sigma B_0^2(1-\alpha t)}{b\rho_\infty}. \tag{2.24}$$

The case in which $\lambda = 0$ corresponds to the forced convection regime while that in which λ is large corresponds to the reduction to the free convection regime. For $t = 0(A = 0)$ and equations (2.14) and (2.15) reduce to those of steady flow and for $t > 0(A \neq 0)$ it applies to unsteady flow.

3. Parameters of Engineering Interest

The physical quantities of interest such as Skin friction coefficient Cf and non-dimensional rate of heat transfer are also obtained utilizing

Skin friction coefficient:

$$C_f = \frac{\tau_w}{\rho u_w^2} \tag{3.1}$$

where the wall shear stress τ_w is given below

$$\tau_w = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

Nusselt number:

$$Nu_x = \frac{x q_w}{k_f(T_w - T_0)} \tag{3.2}$$

where the rate of heat transfer q_w is given by

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

The dimensionless form of the skin friction coefficient and non-dimensional rate of heat transfer are respectively obtained as

$$C_f = \frac{2\mu(\partial u/\partial y)_{y=0}}{\rho u_w^2} = \frac{2\theta_r}{\theta_r - 1} Re_x^{-1/2} f''(0) \tag{3.3}$$

$$Nu_x = -\frac{x}{T_0}(\partial T/\partial y)_{y=0} = -\frac{Re_x^{3/2}}{2(1-\alpha t)}\theta'(0) \tag{3.4}$$

4. Numerical Solutions

In this study, the effects of variable viscosity, viscous dissipation, and Joule heating on unsteady two-dimensional MHD flow and mixed convective heat transfer of a viscous, incompressible, and electrically conducting fluid over a vertically stretching surface are investigated numerically. The resulting coupled nonlinear ordinary differential equations, obtained from equations (2.14) and (2.15) and subject to the appropriate boundary conditions, are solved using the Nachtsheim-Swigert shooting iteration technique in combination with a fourth-order Runge-Kutta integration scheme.

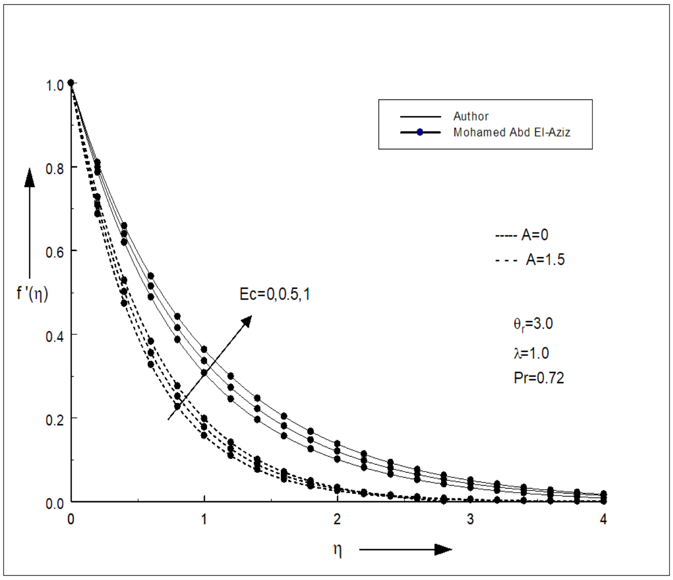

Figure 2: Comparative study of dimensionless velocity profiles for various values of Ec and A.

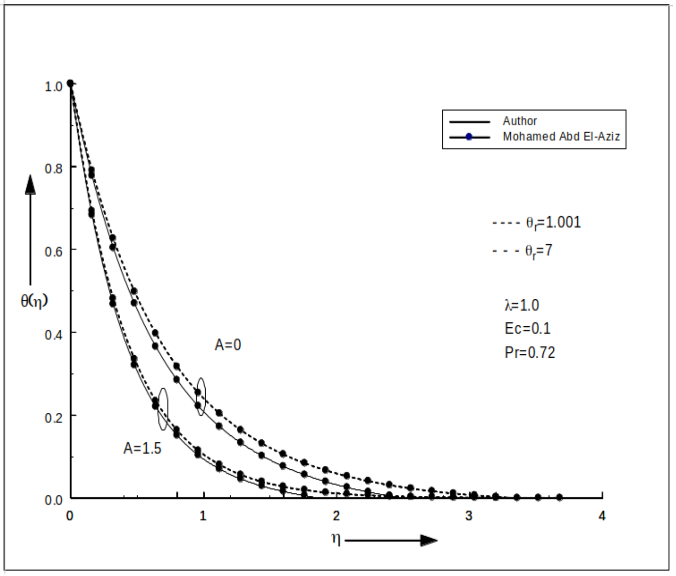

Figure 3: Comparative study of dimensionless temperature profiles for various values of λ and θ_r.

In order to solve the above equations, five initial conditions are needed while only three initial conditions $f(0)$, $f'(0)$ on f and $\theta(0)$ on θ are available. The remaining initial conditions $f''(0)$ and $\theta'(0)$ that are not prescribed are found using the Nachtsheim-Sweigert shooting iteration scheme. Once these values are found, the system is solved by utilizing the Fourth Order Runge Kutta Integration method with step size $h = 0.01$ and the numerical values of dimensionless velocity and temperature are obtained. The level of accuracy for convergence is taken up to 10^{-5}.

5. Results and Discussion

The transformed non-linear differential equations (2.14) and (2.15) were solved numerically using he Nachtsheim-Swigert shooting technique combined with the fourth-order Runge-Kutta method. To study the effects of various physical parameters such as the magnetic interaction parameter M^2, mixed convection parameter λ, variable viscosity θ_r, mixed convection parameter λ, Eckert number E_c and Prandlt number Pr on the non-dimensional velocity $f'(\eta)$ and temperature $\theta(\eta)$. In order to have a clear insight of the problem, numerical solutions are obtained for various values of physical parameters and are represented through graphs. In the absence of magnetic field effects, the present numerical results reduce to those reported by Mohamed Abd El-Aziz [14], thereby confirming the accuracy of the adopted numerical scheme. A comparison of the results is presented in Figures 2 and 3, which demonstrate excellent agreement with the previously published data and validate the reliability of the computational approach used in this study.

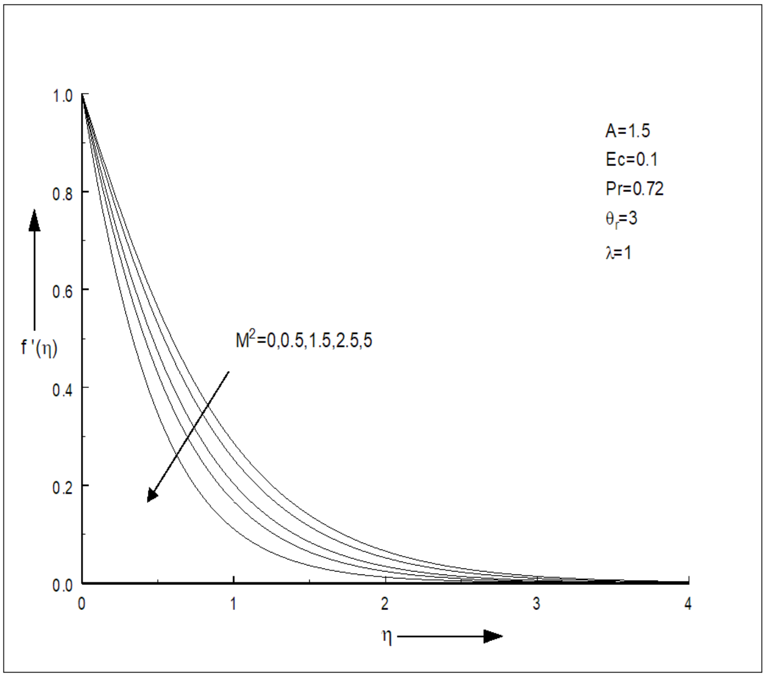

Figure 4: Dimensionless velocity profiles for various values of M^2.

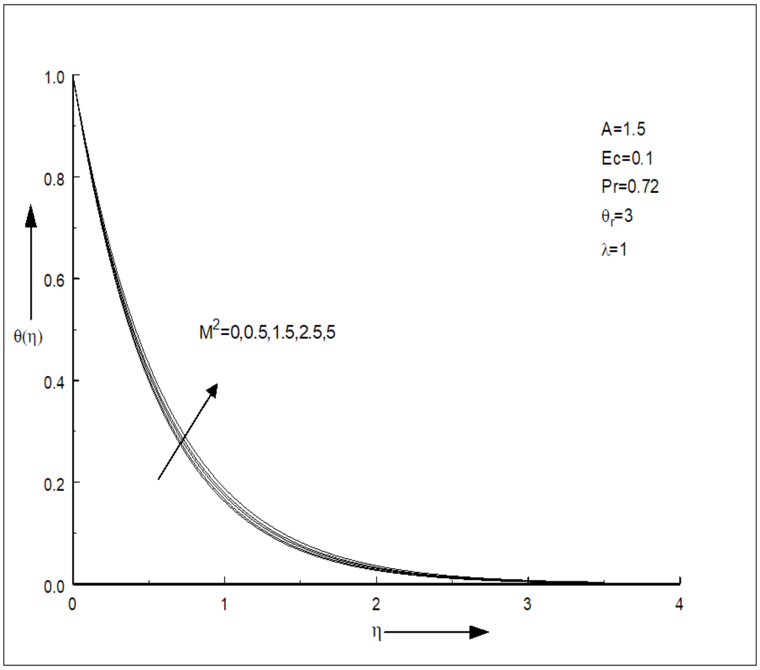

Figure 5: Dimensionless temperature profiles for various values of M^2

Figures 4 and 5 illustrate the influence of the magnetic interaction parameter M^2 of the velocity and temperature distributions. Figure 4 shows that increasing the magnetic parameter M^2 reduces the dimensionless velocity, leading to a reduction in the momentum boundary-layer thickness. Physically, the applied magnetic field to an electrically conducting fluids generates a Lorentz force that acts opposite to the direction of motion. This resistive force significantly suppresses fluid motion and enhance the momentum diffusion, thereby reducing the flow velocity and results in a thinner momentum boundary layer. Figure 5 illustrates the dimensionless temperature distribution for various values of the magnetic interaction parameter M^2. The magnetic field produces Joule heating due to the interaction between induced electric currents and magnectic field. This internal heat generation induced the thermal energy of the fluid, which results in a thicker boundary layer in flow. Also, it is observed that increasing M^2 leads to a rise in fluid temperature, indicating that the applied magnetic field enhances thermal energy within the boundary layer.

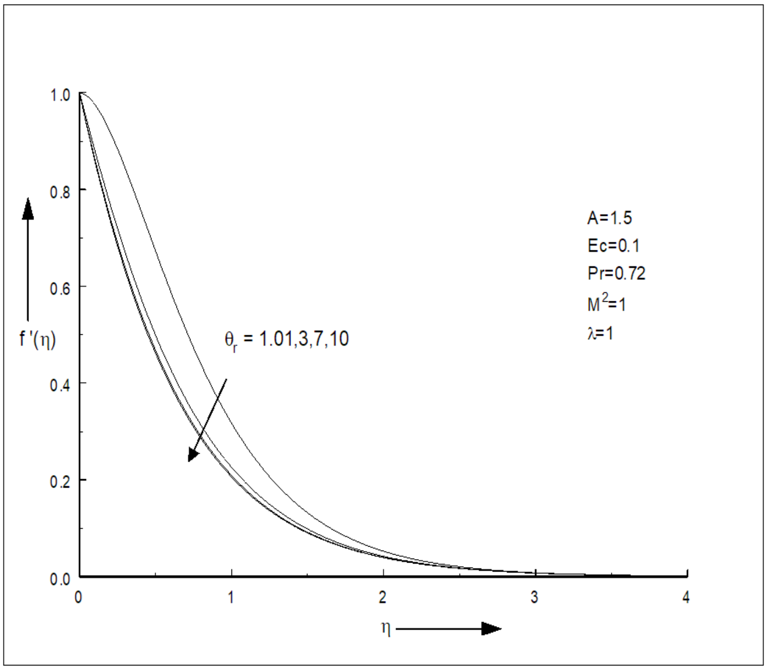

Figure 6: Dimensionless velocity profiles for various values of θ_r.

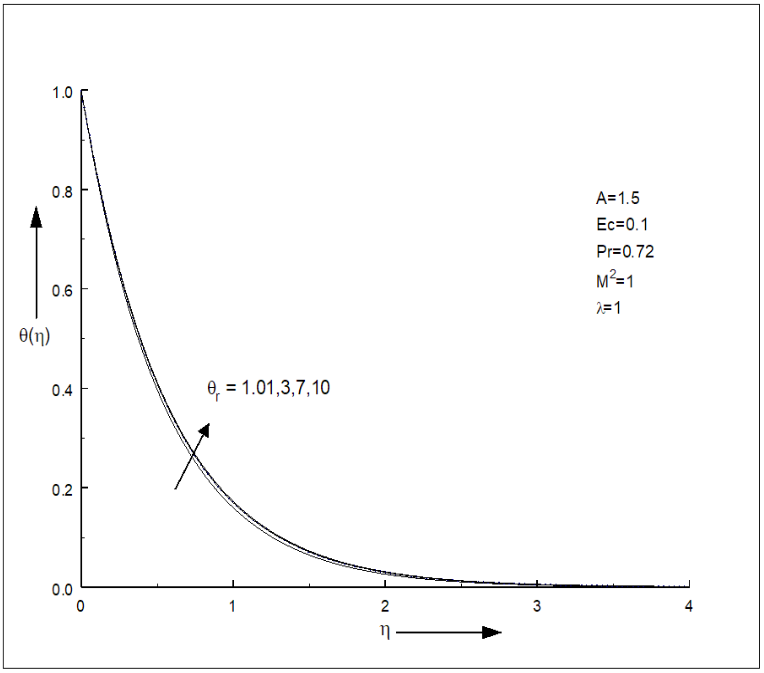

Figure 7: Dimensionless temperature profiles for various values of θ_r.

Figure 6 and 7 present the effect of variable viscosity parameter θ_r on the flow field. It is noted that an increase in the variable viscosity parameter θ_r leads to a decrease in the fluid velocity. Since viscosity is temperature-dependent, so that higher θ_r value gives more stronger sensitivity of viscosity to temperature variations. An increase in viscosity enhances internal fluid resistance, thereby slowing down the fluid motion and reducing the momentum boundary layer thickness. However, the temperature distribution increases with θ_r. As viscosity increases, fluid motion weakens, reducing convective heat transport away from the wall. This leads to higher temperature accumulation near the surface and thickening of the thermal boundary layer. It is noteworthy that the effect of θ_r is more pronounced in the momentum equation than in the energy equation, indicating stronger influence on velocity than temperature.

The effect of mixed convection parameter λ over the velocity profile is illustrated in Figure 8. The results show that increasing λ enhances the velocity distribution. Physically, higher λ implies stronger buoyancy forces due to thermal gradients. These buoyancy forces accelerate the fluid in the upward direction, strengthening the flow and increasing the momentum boundary layer thickness. Figure 9 portrays the effect of mixed convection parameter over the temperature distribution. It is clear that the temperature decreases with increasing λ. Enhanced buoyancy-induced motion improves convective heat transport, carrying thermal energy away from the surface more effectively. As a result, the thermal boundary layer thickness decreases. Therefore, buoyancy assists fluid motion but promotes cooling of the boundary layer. However, its effect is not so prominent.

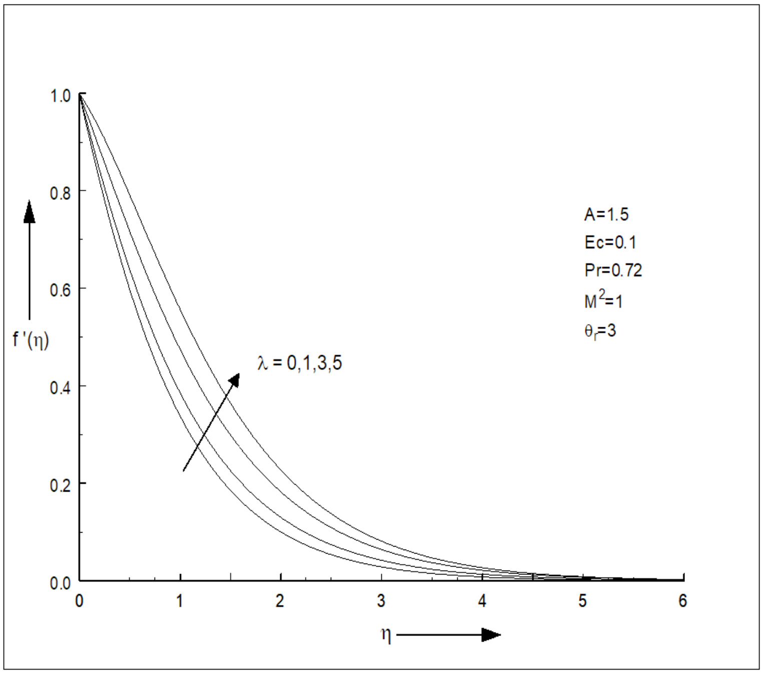

Figure 8: Dimensionless velocity profiles for various values of λ.

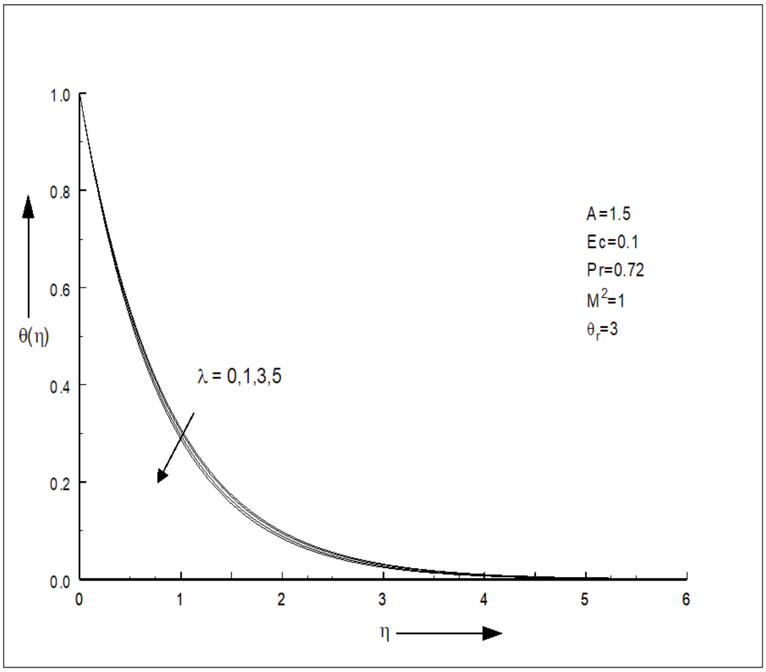

Figure 9: Dimensionless temperature profiles for various values of λ.

Figures 10 shows the effect of Prandtl number on dimensionless velocity field. when Pr increases, non-dimensional velocity decreases and the momentum boundary layer becomes thinner. Higher Pr fluids (such as oils) have lower thermal diffusivity, which indirectly influences the momentum field through coupling in the governing equations. Hence it is also noticed that the effect of Prandtl number is to reduce the thickness of the momentum boundary layer. The dimensionless temperature distribution for various values of Prandtl number is highlighted in Figure 11. As Prandtl number increases, the thermal diffusivity decreases which reduces the energy transfer ability. Hence an increase in Prandtl number values lead to the decrease of temperature and the thickness of the thermal boundary layer becomes thinner. Physically, it is observed that a larger Pr values correspond to weaker thermal diffusion, so that it slow down the heat diffusions when it compared to momentum. Consequently, the thermal boundary layer thickness decreases sharply and indicates that fluids with high Prandtl numbers cool more rapidly near the surface.

The effect of Eckert number Ec on the velocity and temperature distributions is shown through Figures 12 and 13 for both the steady and unsteady cases. It measures the conversion of kinetic energy into internal energy due to viscous dissipation. The fluid velocity and the temperature are observed to be increased for increasing Eckert number. The reason behind that the viscous dissipation converts mechanical energy into thermal energy, thereby it increasing fluid temperature and hence thickens the thermal boundary layer. The increase in temperature also reduces fluid viscosity, thereby slightly accelerating the flow and increasing the velocity distribution. The influence of Ec is more significant in steady flow compared to unsteady flow because, in steady state conditions, the dissipative heating accumulates continuously without temporal modulation.

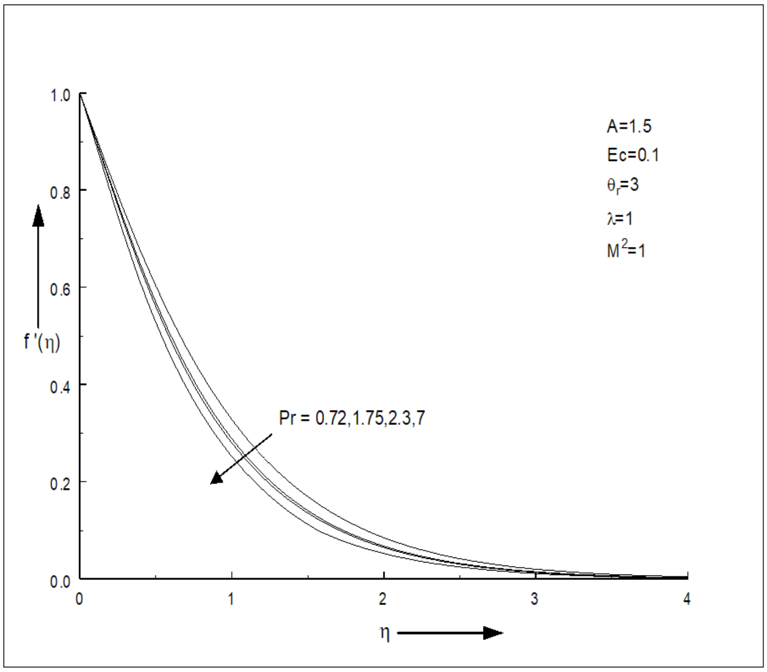

Figure 10: Dimensionless velocity profiles for various values of Pr.

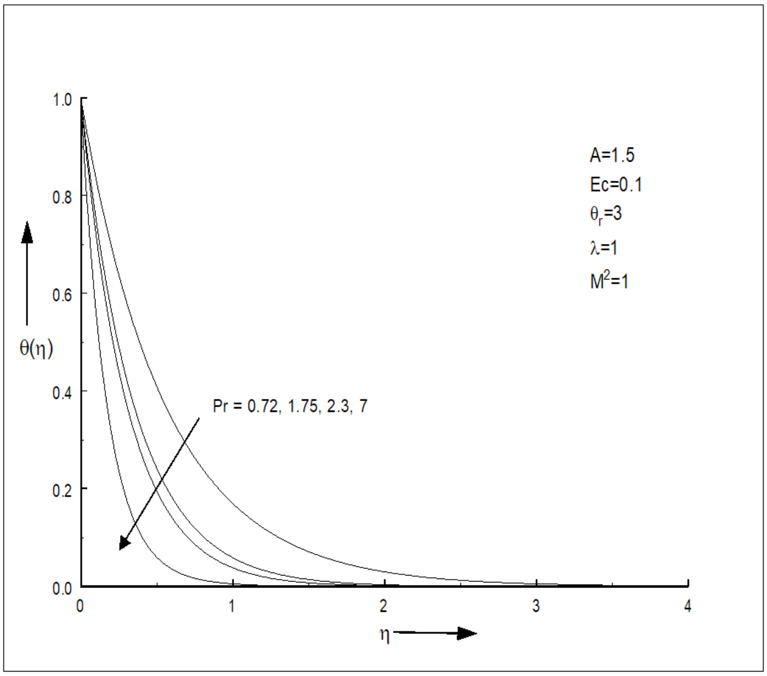

Figure 11: Dimensionless temperature profiles for various values of Pr.

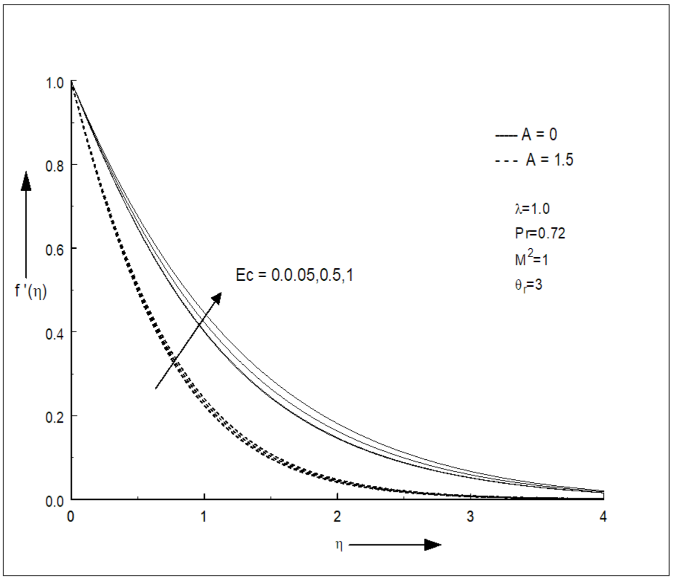

Figure 12: Dimensionless velocity profiles for various values of Ec.

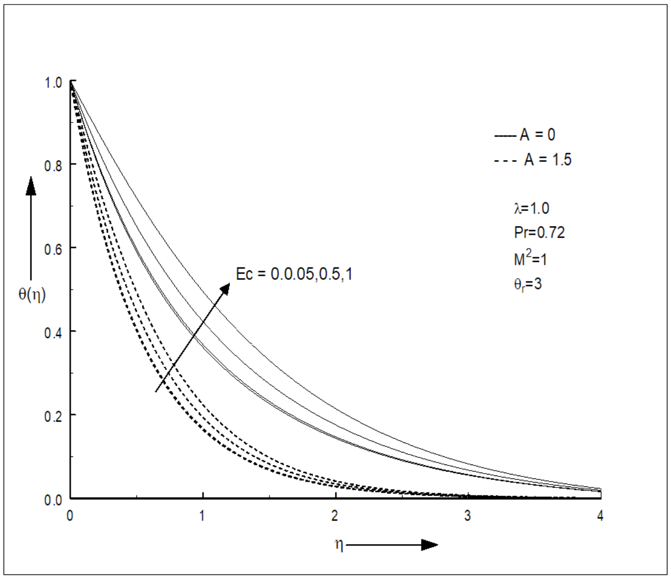

Figure 13: Dimensionless temperature profiles for various values of Ec.

Table 1 provides the values of skin friction coefficient and non-dimensional rate of heat transfer for various values of M^2, λ, A, Ec, Pr and θ_r. It is observed that with increase in parameter M^2 and θ_r, both the skin friction coefficient and the non-dimensional rate of heat transfer decrease. It is also shown that with increase in λ, both the skin friction coefficient and the non-dimensional rate of heat transfer increase. As Ec increases, the skin friction coefficient increases while the non-dimensional rate of heat transfer decreases. On increasing the values of A and Pr, the Skin friction coefficient decreases where as the non-dimensional rate of heat transfer enhances.

Table 1: Variation of $f''(0)$ and $-\theta'(0)$ for different values of M^2, A, θ_r, λ, Pr and Ec.

M^2	A	θ_r	λ	Pr	Ec	$f''(0)$	$-\theta'(0)$
0.0	1.5	3.0	1.0	0.72	0.1	-0.97716	1.85307
0.5	1.5	3.0	1.0	0.72	0.1	-1.10124	1.82900
1.5	1.5	3.0	1.0	0.72	0.1	-1.32372	1.78706
2.5	1.5	3.0	1.0	0.72	0.1	-1.52057	1.75117
1.0	0.0	3.0	1.0	0.72	0.1	-0.79534	1.11177
1.0	0.6	3.0	1.0	0.72	0.1	-0.98309	1.42069
1.0	1.5	3.0	1.0	0.72	0.1	-1.21625	1.80713
1.0	3.0	3.0	1.0	0.72	0.1	-1.52579	2.31444
1.0	1.5	1.01	1.0	0.72	0.1	-0.02615	1.86428
1.0	1.5	3.00	1.0	0.72	0.1	-1.21625	1.80713
1.0	1.5	7.00	1.0	0.72	0.1	-1.44285	1.79769
1.0	1.5	10.0	1.0	0.72	0.1	-1.48954	1.79581
1.0	1.5	3.0	0	0.72	0.1	-1.41544	1.78425
1.0	1.5	3.0	1	0.72	0.1	-1.21625	1.80713
1.0	1.5	3.0	3	0.72	0.1	-0.83257	1.84696
1.0	1.5	3.0	5	0.72	0.1	-0.46541	1.87964
1.0	1.5	3.0	1.0	0.72	0.1	-1.21625	1.80713
1.0	1.75	3.0	1.0	0.72	0.1	-1.23557	2.85109
1.0	2.30	3.0	1.0	0.72	0.1	-1.23990	3.27657
1.0	7.00	3.0	1.0	0.72	0.1	-1.25040	5.74878
1.0	1.5	3.0	1.0	0.72	0.00	-1.21727	1.86227
1.0	1.5	3.0	1.0	0.72	0.05	-1.21676	1.83468
1.0	1.5	3.0	1.0	0.72	0.50	-1.21218	1.58821
1.0	1.5	3.0	1.0	0.72	1.00	-1.20704	1.31825

6. Conclusion

Numerical investigation of Unsteady mixed convective MHD boundary layer flow and heat transfer over a vertical stretching surface in the presence of variable viscosity, viscous dissipation and Joule heating has been carried out. Numerical solutions are obtained and graphical results are displayed to illustrate the details of flow and heat transfer characteristics and their dependence on some physical parameters. In the absence of MHD flow, these results are identical to that of Mohamed Abd El-Aziz [14] justifying the adopted numerical scheme.

From the results obtained, the following conclusions are drawn.

- The effect of the physical parameters such as unsteadiness parameter, magnetic interaction parameter, variable viscosity parameter and Prandtl number is to decelerate the velocity with an accompanying decrease in the momentum boundary layer thickness as they increase respectively. However, the mixed convection parameter and Eckert number enhance the velocity along with a increase in the momentum boundary layer thickness.

- The temperature and as well as thermal boundary layer thickness are enhanced due to the individual influence of magnetic field, variable viscosity parameter and Eckert number. On the other hand, the thermal boundary layer thickness is suppressed due to an increase in unsteadiness parameter, mixed convection parameter and Prandtl number respectively.

- The Skin friction coefficient gets decreased for an increase in unsteadiness parameter, the magnetic interaction parameter, variable viscosity parameter and Prandtl number respectively. However, it is interesting to note that for an increase in the mixed convection parameter and Eckert number, the skin friction coefficient gets enhanced.

- The non-dimensional rate of heat transfer deceases with an increase in magnetic interaction parameter, variable viscosity parameter and Eckert number whereas it increases due to the individual effect of unsteadiness parameter, mixed convection parameter and Prandtl number.

Table 2: **Nomenclature**

Symbol	Description	Units
A	Unsteadiness parameter	–
b	Stretching rate parameter	s^{-1}
B_0	Applied magnetic field strength	T
C_p	Specific heat at constant pressure	$J\ kg^{-1}\ K^{-1}$
Ec	Eckert number	–
g	Acceleration due to gravity	$m\ s^{-2}$
Gr_x	Local Grashof number	–
k	Thermal conductivity	$W\ m^{-1}\ K^{-1}$
M	Magnetic parameter	–
Pr	Prandtl number	–
Re_x	Local Reynolds number	–
T	Fluid temperature	K
T_w	Wall temperature	K
T_∞	Ambient temperature	K
u_w	Stretching velocity	$m\ s^{-1}$
x	Distance along the sheet	m

Symbol	Description	Units
α	Thermal diffusivity	$m^2\ s^{-1}$
β	Thermal expansion coefficient	K^{-1}
γ	Variable viscosity parameter	–
η	Similarity variable	–
λ	Mixed convection parameter	–
μ	Dynamic viscosity	$kg\ m^{-1}\ s^{-1}$
ν	Kinematic viscosity	$m^2\ s^{-1}$
ρ	Fluid density	$kg\ m^{-3}$
σ	Electrical conductivity	$S\ m^{-1}$
θ	Dimensionless temperature	–
θ_r	Viscosity variation parameter	–

References

1. Sakiadis B.C., *Boundary-layer behavior on continuous solid surfaces: I. Boundary-layer equations for two-dimensional and axisymmetric flow*, A.I.Ch.E. J. 7(1): 26-28, (1961)

2. Crane L.J., *Flow past a stretching plate*, Zeitschrift fur Angewandte Mathematik und Physik, 21(4): 645-647, (1970)

3. Eldabe N. T., *Magnetohydrodynamic unsteady free convective flow through a porous medium bounded by an infinite vertical porous plate*, Can. J. Phys., 64(1): 84-89, (1986).

4. Ioan Pop, Tsung - Yen Na, *Unsteady flow past a stretching sheet*, Mech. Res. Comun., 23(4): 413-422, (1996).

5. Ioan Pop, *The effect of variable viscosity on flow and heat transfer to a continuous moving flat plate*, International Journal of Engineering Science 30(1): 1-6, (1992).

6. Kafoussias N. G., *Williams E. W., Thermal-diffusion and diffusion-thermo effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity*, International Journal of Engineering Science 33(9): 1369-1384, (1995).

7. Barletta A., *Analysis of Combined Forced and Free Flow in a Vertical Channel With Viscous Dissipation and Isothermal-Isoflux Boundary Conditions*, ASME J. Heat Transfer 121(2): 349-356, (19929.

8. Anjali Devi S.P., Ganga B., *Viscous dissipation effects on nonlinear MHD flow in a porous medium over a stretching porous surface*, Int. J. Appl. Math. Mech 5(7): 45-49, (2009).

9. Shateyi S., Motsa S. S., Sibanda P., *The Effects of Thermal Radiation, Hall Currents, Soret, and Dufour on MHD Flow by Mixed Convection over a Vertical Surface in Porous Media*, Mathl. Probs. Engg. (2010).

10. Kishore P.M., Bhanumathi D., Verma Vijayakumar S., *The influence of chemical reaction and viscous dissipation on unsteady MHD free convection flow past an exponentially accelerated vertical plate with variable surface conditions*, Chemical Industry and Chemical Engineering Quarterly 19(2): 181-193, (2013)

11. Rashad A.M., *Effects of radiation and variable viscosity on unsteady MHD flow of a rotating fluid from stretching surface in porous medium*, Journal of the Egyptian Mathematical Society 22(1): 134-142, (2014).

12. Vajravelu K., Prasad K.V, Datti P.S., Raju B.T., *MHD flow and heat transfer of an Ostwald-de Waele fluid over an unsteady stretching surface*, Ain Shams Engineering Journal 5(1): 157-167, (2014).

13. Aurang Zaib, Sharidan Shafie, *Thermal diffusion and diffusion thermo effects on unsteady MHD free convection flow over a stretching surface considering Joule heating and viscous dissipation with thermal stratification, chemical reaction and Hall current*, Journal of the Franklin Institute 351(3): 1268-1287, (2014).

14. Mohamed Abd El-Aziz, *Unsteady mixed convection heat transfer along a vertical stretching surface with variable viscosity and viscous dissipation*, Journal of the Egyptian Mathematical Society 22(3): 529-537, (2014).

15. Lai F. C., Kulacki F. A., *The effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium*, Int J Heat and Mass Transfer 33(5): 1028-1031, (1990).

16. Hayat, T., Qayyum, S., Alsaedi, A., Ahmad, B., *Joule heating and viscous dissipation in flow of Powell-Eyring fluid with convective conditions*, Int J of Heat and Mass Transfer, 115, 1007–1014, (2017.)

17. Mabood, F., Ibrahim, S. M., Rashidi, M. M., Shadloo, M. S., Lorenzini, G., *Non-uniform heat source/sink and Soret effects on MHD non-Darcian convective flow past a stretching sheet in a micropolar fluid with radiation*, Inl J Heat and Mass Transfer, 93, 674-682, (2019).

18. Khan, M. I., Shah, F., Hayat, T., *Unsteady MHD flow of Maxwell fluid over a stretching sheet with temperature-dependent viscosity and thermal stratification*, J Molecular Liquids, 312, 113393, (2020).

19. Srinivasacharya, D., Beg, O. A., *Influence of variable viscosity and thermal conductivity on MHD natural convection flow in a porous medium with stretching walls*, J Applied and Comp Mechanics, 8(1), 323–336, (2022).

20. Khan, M. I., Alzahrani, F., Hobiny, A., *Variable thermal conductivity and radiative heat transfer in unsteady MHD nanofluid flow over an exponential stretching surface with viscous dissipation*, Int Comm Heat and Mass Transfer, 142, 106845, (2023).

21. Shaw, S., Kumbhakar, B., Makinde, O. D., *Entropy generation analysis in mixed convective MHD flow of hybrid nanofluid over a vertical stretching sheet with variable viscosity and thermal radiation*, J Thermal Analysis and Calorimetry, 149(5), 2107–2122, (2024).

22. Mondal, H., Pal, D., Sibanda, P., *Unsteady MHD flow and heat transfer of Maxwell fluid over a stretching surface with variable viscosity and viscous dissipation: a fractional derivative approach*, Chinese J Physics, 87, 478–493, (2024).

23. Patil, V. S., Warke, A. S., Kumbhar, A. R., *Hall and ion slip effects on unsteady mixed convective MHD flow past a vertical stretching surface with variable fluid properties and Joule heating*, Int J Applied and Comp Mathematics, 11(1), 45, (2025).

K. Periyadurai, T. Gowpriyanka and M. Kousalya

Department of Mathematics,

College of Engineering and Technology, SRM Institute of Science and Technology,

Chennai - 603 203, India.

E-mail address: kperiyadurai@gmail.com

and

T. Gowpriyanka,

Department of Mathematics,
College of Engineering and Technology, SRM Institute of Science and Technology,
Chennai - 603 203, India.
E-mail address: gowpriyankat@gmail.com

and

M. Kousalya,
Department of Mathematics,
College of Engineering and Technology, SRM Institute of Science and Technology,
Chennai - 603 203, India.
E-mail address: kousimaths@gmail.com