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Bianchi Type V String Cosmological Models with Late Time Acceleration

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ABSTRACT: In the current work, the possibility of late time acceleration with string fluid as the matter source in Bianchi type V is considered. By assuming the functional form of the Hubble parameter, we explore cosmological hypotheses. The resulting models begin in the early stages of evolution as highly anisotropic and gradually move closer to an isotropic universe. Additionally, we have calculated cosmological parameters such as the string tension density, the particle density, the average Hubble parameter, shear scalar, scalar expansion and the energy density of the string. Physical and geometrical behavior of the universe has been discussed in detail.

Keywords: Bianchi type V, deceleration parameter q, Hubble parameter H, string tension density λ.

Contents

1. Introduction

Theoretically and experimentally, the well-known, spatially homogeneous, isotropic FRW model adequately describes the modern universe. Nonetheless, there has been evidence for the presence of a small magnetic field across cosmic distant scales and small anisotropy. Hence, studies of anisotropic models, which are different from FRW models, are necessary. Anisotropic, spatially homogeneous Bianchi-type models will do for this. In order to understand the early phases of the evolution of the universe, Bianchi models are essential. There are nine types of Bianchi cosmological models that are necessarily anisotropic and spatially homogeneous. The generalized open FRW model is represented by the Bianchi type-V model, which is of interest to us in this case. Furthermore, the Bianchi type-V model allows for the formation of galaxies and intelligent life while tending towards isotropy at arbitrarily large times.It is a difficult task to pinpoint the precise physical and geometrical characteristics at the very beginning of the creation of our universe. Strings are among the many topological defects that emerged in the early cosmos during the phase transition and prior to the birth of particles. These defects have intriguing cosmological ramifications, and they have been thoroughly studied [1]. It is thought that cosmic strings cause density disruption, which results in the formation of galaxies [2]. These strings are connected to the gravitational field and have stress energy.

Vilenkin[3] has researched the gravitational impact of such strings. Large structures like galaxies and clusters of galaxies are created from the seeds of massive closed loops of string. As matter is added to loops, they oscillate ferociously, lose energy through gravitational radiation, and eventually vanish. These cosmic strings are bound to the gravitational field and have energy associated with stress. The gravitational effects that strings have are therefore interesting to explore.Strings have been treated relativistically by Stachel[4] and Letelier[5,6]. Yadav[7] was studied Bianchi-I string cosmological models with variable deceleration parameter. Cosmological strings and gravitational lens effects has been studied by M. V. Sazhin and M . Y. Khlopov[8]. M.V. Sazhin, et al. [9] was investigated possible observation

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of a cosmic string. General relativity among the Bianchi types-I, III and IX string cosmological models was put forth by Bali et al.[10–15]. Wang[16–19] has investigated the LRS Bianchi type–I and type–III cosmological models for cloud strings with bulk viscosity. An investigation into LRS Bianchi I Universe in Brans-Dicke Theory have been studied by Çağlar et al.[[20]. Several authors, including Pawar et al.[21], Singh [22], Reddy[23], Singh et al.[24], Rao et al.[25, 26], Pradhan[27], Meitei et al. [28], Pawar et al.[29] Kumawat et al.[30] offered string cosmological models in various contexts.

In this study, we demonstrate that string cosmological models with functional Hubble parameter forms exist in LRS Bianchi type I cosmological models. By assuming the functional form of Hubble parameter$H = a(R^{-n}+1)$, where $a>0, n>0$ are constants and R is the scale factor, the exact answer has been obtained. The effects of these models on the cosmos have been discussed.

2. Metric and Field Equations

The orthogonal form of the Bianchi Type V space-time is represented by the line-element

$$ds^2 = -dt^2 + A^2dx^2 + e^{2\alpha x}\left\{B^2dy^2 + C^2dz^2\right\}, \tag{2.1}$$

where $A(t), B(t)$ and $C(t)$ are scale factor in different special directions and α is a constant.
We suppose that the energy-momentum tensor describes a cloud of massive strings as the source field's matter content

$$T_i^j = \rho v_i v^j - \lambda x_i x^j. \tag{2.2}$$

Here, ρ represents the matter energy density of a cloud of particle-attached cloud strings; λ is the density of the string tension, $v^i = (0,0,0,1)$ is the four –velocity of the particles and x^i is a unit-space-like vector that denotes the direction of the string. The vector v^i and x^i satisfy the conditions

$$v_i v^i = -x_i x^i = -1, v^i x_i = 0. \tag{2.3}$$

Choosing x^i parallel to $\frac{\partial}{\partial x}$, we obtain

$$x^i = (A^{-1}, 0, 0, 0). \tag{2.4}$$

The particle density of configuration is denoted and defined as

$$\rho_p = \rho - \lambda. \tag{2.5}$$

Einstein field equations (in gravitational unit $8\pi G = c = 1$)

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j. \tag{2.6}$$

for the metric (1), leads to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = \lambda, \tag{2.7}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} - \frac{\alpha^2}{A^2} = 0, \tag{2.8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = 0 \tag{2.9}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3\alpha^2}{A^2} = \rho, \tag{2.10}$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0, \tag{2.11}$$

where an overhead dot (.) signify ordinary time derivatives and expansion scalar denote the rate of expansion of the fluid.
From equations (2.7)-(2.11), we obtain

$$\dot{\rho} + \rho\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) - \lambda\frac{\dot{A}}{A} = 0. \tag{2.12}$$

We define average scale factor R as

$$R^3 = ABC \tag{2.13}$$

The spatial volume V is given by

$$V = R^3. \tag{2.14}$$

Defining a generalized Hubble parameter H and a generalized deceleration parameter q as analogous to the FRW universe

$$H = \frac{\dot{R}}{R} = \frac{1}{3}(H_1 + H_2 + H_3), \tag{2.15}$$

$$q = -1 - \frac{\dot{H}}{H}, \tag{2.16}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble's factors along x, y and z directions respectively. Expansion scalar θ and mean anisotropy parameter $\bar{A}$ are given by

$$\theta = 3H, \tag{2.17}$$

$$\bar{A} = \frac{1}{9H^2}\left[\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)^2 + \left(\frac{\dot{C}}{C} - \frac{\dot{A}}{A}\right)^2\right], \tag{2.18}$$

Shear scalar σ is given by

$$\sigma^2 = \frac{3}{2}\bar{A}H^2, \tag{2.19}$$

From equation (2.8), (2.9) and (2.13), we get

$$\frac{\dot{C}}{C} - \frac{\dot{B}}{B} = \frac{k_1}{R^3}, \tag{2.20}$$

$k_1$being constant of integration. From equations (2.11) and (2.13), we get

$$\frac{\dot{A}}{A} = \frac{\dot{R}}{R}. \tag{2.21}$$

From equations (2.11) and (2.20), we get

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{2R^3}, \tag{2.22}$$

3. Result and Discussion

We presumptively have a functional relationship between the Hubble parameter H and the average scale factor R provided by Singh[31]

$$H = a(R^{-n} + 1), a > 0, n > 0 \tag{3.1}$$

which yields a model begin with early decelerating expansion and enter into accelerating phase at late times.
Integrating equation (1), we get

$$R^n = e^{na(t-t_1)} - 1, \tag{3.2}$$

where t_1 is constant of integration. Since $R = 0$ for $t = 0$, we get $t_1 = 0$. Therefore

$$R = (e^{nat} - 1)^{\frac{1}{n}}, \tag{3.3}$$

The proper volume V, volume expansionθ, shear σ and deceleration parameter are defined as

$$V = (e^{nat} - 1)^{\frac{3}{n}}, \tag{3.4}$$

$$\theta = \frac{3a}{1 - e^{-nat}}, \tag{3.5}$$

$$\sigma = \frac{1}{2}\frac{3a}{1 - e^{nat}}, \tag{3.6}$$

$$q = 1 + ne^{-nat}, \tag{3.7}$$

Matter density ρ, particle density ρ_p and string tension density λ for the model are given by

$$\rho = \frac{3a^2}{(1 - e^{-nat})^2} + \frac{k_1^2}{4(e^{nat} - 1)^{\frac{6}{n}}} - \frac{3\alpha^2}{(e^{nat} - 1)^{\frac{2}{n}}}, \tag{3.8}$$

$$\rho_p = \frac{2a^2ne^{-nat}}{(1 - e^{-nat})^2} - \frac{2\alpha^2}{(e^{nat} - 1)^{\frac{2}{n}}}, \tag{3.9}$$

$$\lambda = -\frac{2a^2ne^{-nat}}{(1 - e^{-nat})^2} + \frac{3a^2}{(1 - e^{-nat})^2} + \frac{k_1^2}{4(e^{nat} - 1)^{\frac{6}{n}}} - \frac{\alpha^2}{(e^{nat} - 1)^{\frac{2}{n}}} \tag{3.10}$$

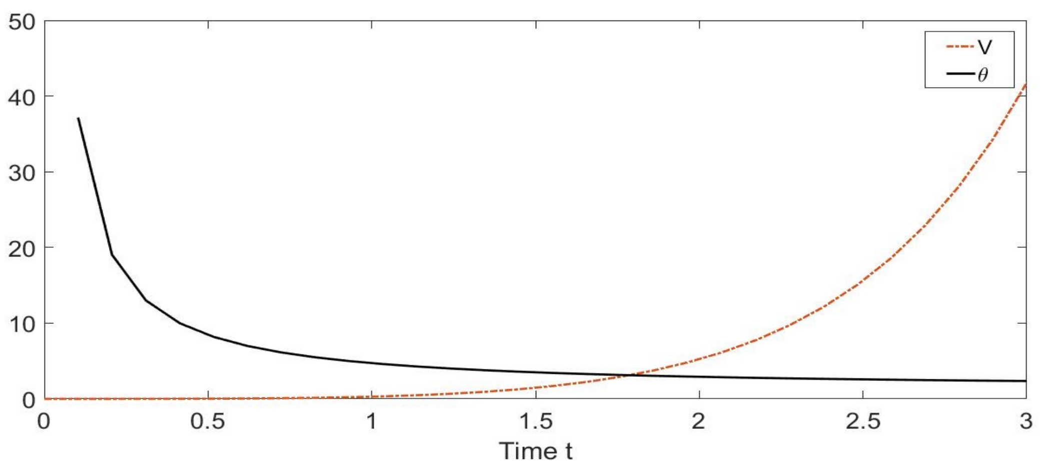

Figure 1: Variation of special volume V and volume expansion θ with cosmic time t

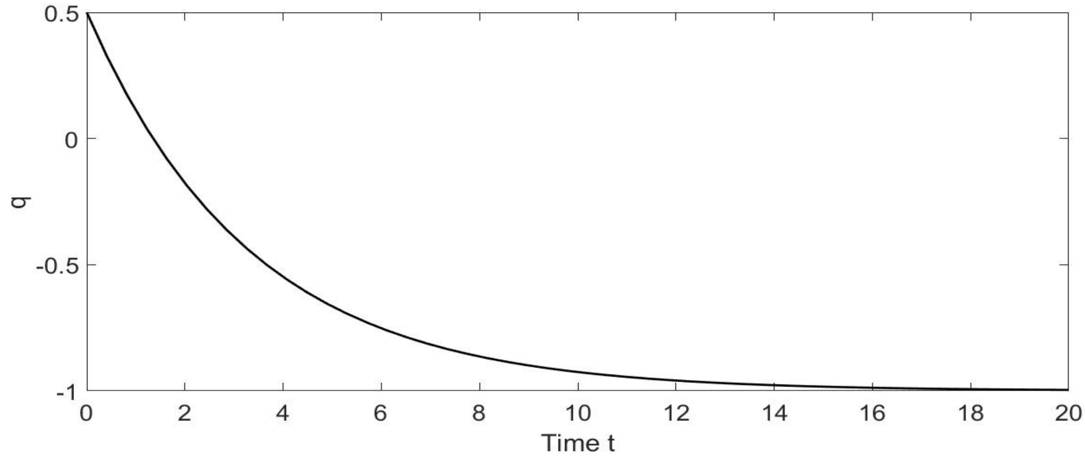

Figure 2: Variation of deceleration parameter q with cosmic time t

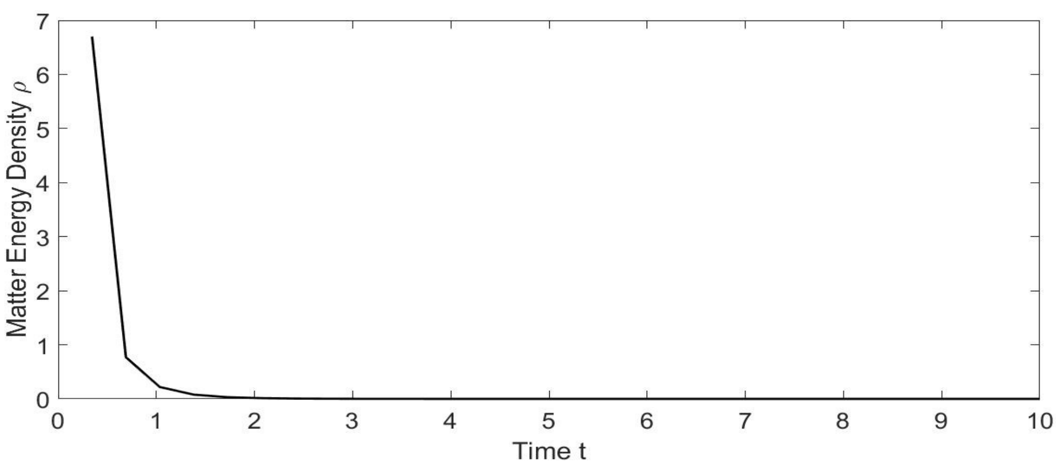

Figure 3: Variation of matter energy density ρ with cosmic time t

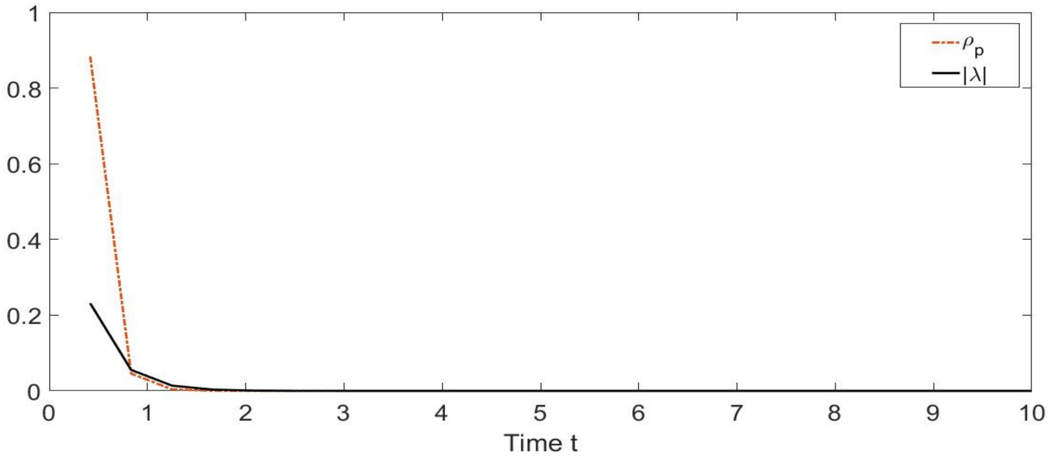

Figure 4: Plots of particle density ρ_p and string tension density λ with cosmic time t

As predicted, all of the above solutions satisfy the equation (2.12). We observe that the model has singularity at $t = 0$. The model starts with big-bang from its singular state at $t = 0$. The universe starts evolving with zero volume at $t = 0$. As t increases, the spatial volume increases whereas the expansion scalar decreases, which the graph in Fig. 1 validates. The parameters σ, ρ, ρ_p and λ start off with extremely large values. In particular, the large values of ρ_p and λ in the beginning suggest that strings dominate the early universe. In the limit of large times (i.e.$t \to \infty$),$\rho \to 3a^2, \lambda \to 3a^2, \rho_p \to 0$, and $\sigma \to 0$. Therefore, the strings disappear from universe for large time that is why, the strings are not observable in the present universe. It can be shown from equation (3.8), that the energy density ρ decreases over time. At $t = 0, q = n - 1 > 0$, and $q = -1$ for $t = \infty$. As a result, the model evolves with a decelerating expansion at first and an accelerating one afterwards.

From equations (3.9) and (3.10), we also notice that $\rho_p > |\lambda|$. This indicates that during the cosmic expansion, especially in the early stages of the universe, particle density stays higher than string tension density. The radiation decoupling and matter-dominated eras may have something to do with this. Letelier [32] and Krori et al [33] state that, when $\frac{\rho_p}{|\lambda|} > 1$, during the evolution process, the universe is dominated by massive strings, and when $\frac{\rho_p}{|\lambda|} < 1$, the universe is dominated by strings. We observe that $\frac{\rho_p}{|\lambda|} > 1$. In the derived scenario, massive string therefore dominates the early universe. We are generally interested in building models of the universe that evolve solely from the era dominated by either geometric strings or massive strings and end up in the particle-dominated era with or without remnants of strings, according to Ref. [34], since there is no direct evidence of strings in the current universe. Thus, the model mentioned above describes the evolution of the universe and is consistent with recent findings. Thus, the string disappears from the universe for later times and hence it is not observable for today. The evolution of cosmological parameters over cosmic time t is shown in Figs. 2, 3 and 4.

For the model

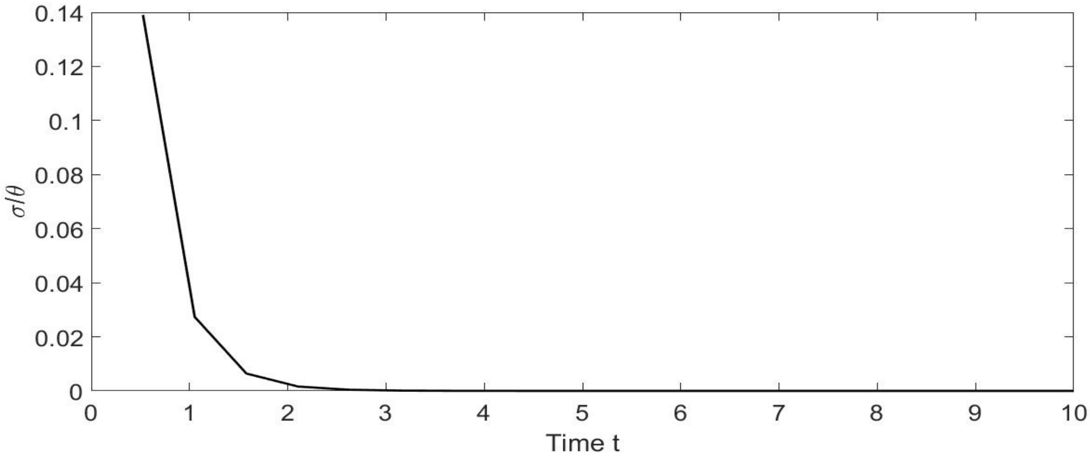

Figure 5: Variation of expansion anisotropy $\frac{\sigma}{\theta}$ with cosmic time t

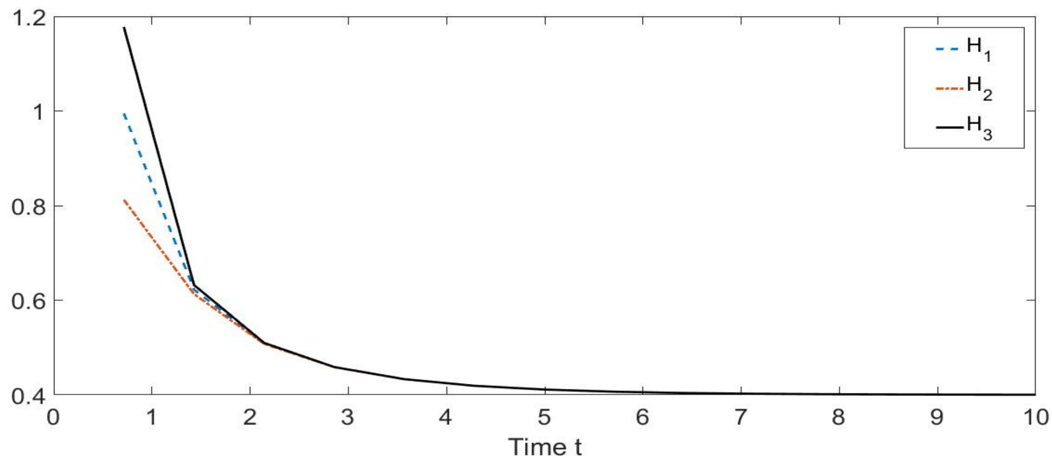

Figure 6: Variation of rate of expansion in the direction of x, y and z with cosmic time t

$$\frac{\sigma}{\theta} = \frac{k_1 e^{-at}}{6a(1-e^{-nat})^{\frac{3}{n-1}}} \tag{3.11}$$

The rate of expansion in the direction of and are given by

$$H_1 = \frac{a}{1-e^{-nat}}, \tag{3.12}$$

$$H_2 = \frac{a}{1-e^{-nat}} - \frac{k_1}{2(e^{nat}-1)^{\frac{3}{n}}}, \tag{3.13}$$

$$H_3 = \frac{a}{1-e^{-nat}} + \frac{k_1}{2(e^{nat}-1)^{\frac{3}{n}}}, \tag{3.14}$$

At the initial moment, we see that $\frac{\sigma}{\theta}$ is infinitely large. When cosmic time is sufficiently long, it decreases until it reaches zero. Because of this, the model displays isotropic behaviour in its later stages of evolution, which are validated through graph in Fig. 5. When the initial singularity occurs, the parameters H_1, H_2, H_3 and θ diverge. As the universe expands, these parameters get smaller and smaller until they eventually become constant, which are verified by graph in Fig. 6.

4. Conclusion

In this research, we investigate string cosmological models of the Bianchi type V universe with a variable deceleration parameter. A variation law for the Hubble parameter H has been used to precisely solve Einstein field equations.The resulting model transitions from a decelerating phase to an accelerating one. This cosmological scenario is in agreement with S Ne Ia astronomical observations and it presents a unified description of the evolution of the universe. The particle density and string tension density are comparable; however, the string tension density vanishes more rapidly than the particle density. As a result, at late times, our model represents a matter-dominated universe, consistent with present-day observational data. The solutions provided in this paper may be helpful in the investigation of the dynamical and physical behaviour of homogeneous and anisotropic cosmological models with late time acceleration. Expansion anisotropy $\frac{\sigma}{\theta}$ in the model decreases with cosmic time and tends to zero for sufficiently large times. As a result, the model is isotropic for large values of t.

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