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# Applications of Fuzzy Maximal $\mu$-Open sets in Generalized Fuzzy Topology and Fuzzy Quasi Topology

A. Swaminathan and Margaret Sheela

ABSTRACT: This work explores key characteristics of fuzzy maximal $\mu$-open sets in the setting of a generalized fuzzy topological space. A decomposition theorem for fuzzy maximal $\mu$-open sets is presented along with fundamental results regarding their intersections. In addition, a formulation consistent with the fuzzy $\mu$-radical $\mu$-closure is established in the framework of quasi-fuzzy topological spaces other related findings.

Keywords: Fuzzy maximal $\mu$-open, fuzzy $\mu$-radical, fuzzy quasi topology.

## Contents



## 1. Introduction

Chang [1] presented fuzzy topology after the discovery of fuzzy sets by Zadeh [7]. The notion of generalized topology proposed by Cs´asz´ar in [3]. Let $I^X$ denote fuzzy space. A fuzzy subcollection $\mu$ of $I^X$ is called a generalized fuzzy topology on $X$ if $0 \in \mu$ and $\bigvee\{\xi_\alpha, \alpha \in \Delta\} \in \mu$ . A fuzzy set $\xi \in \mu$ is called fuzzy $\mu$-open [2]. The complement of fuzzy $\mu$-open set is called fuzzy $\mu$-closed of $(X, \mu)$. A fuzzy set $\xi$ of $X$, the intersection of all fuzzy $\mu$-closed sets containing $\xi$ is the generalized fuzzy closure of $\lambda$ and is denoted by $c_\mu(\xi)$. Also for a fuzzy set $\xi$ of $X$, the union of all fuzzy $\mu$-open sets contained in $\xi$ is the generalized fuzzy interior of $\xi$ and is denoted by $i_\mu(\xi)$. For our better understanding, a fuzzy set $\xi$ of $X$ is fuzzy $\mu$-open (resp. fuzzy $\mu$-closed) if and only if $\xi = i_\mu(\xi)$ (resp. fuzzy $\xi = c_\mu(\xi)$). Also for a fuzzy set $\xi$ of $X$, we have $c_\mu(\xi) = 1 - i_\mu(\xi)$ [2]. It can be noted that the operators $i_\mu$ and $c_\mu$ possess both the *idempotence* and *monotonicity* properties. Let $\gamma : I^X \to I^X$ be an operator on the collection $I^X$ of all fuzzy subsets of $X$. The operator $\gamma$ is called idempotent if for every $\xi \in I^X$, $\gamma\big(\gamma(\xi)\big) = \gamma(\xi)$ and is called monotonic if for all $\xi, \zeta \in I^X$ with $\xi \leq \zeta$, it holds that $\gamma(\xi) \leq \gamma(\zeta)$. It is known that if $\mu$ is a generalized fuzzy topology on $X$, $\xi \subseteq X$ and $x_\alpha \in X$, then $x_\alpha \in c_\mu(\xi)$ iff $x_\alpha \in \vartheta \in \mu \Rightarrow \vartheta \cap \xi \neq 0$ and also $c_\mu(X \setminus \xi) = X \setminus i_\mu(\xi)$. Moreover $x_\alpha \in i_\mu(\xi)$ if and only if there exists a fuzzy $\mu$-open set $\lambda$ such that $x_\alpha \in \lambda$ and $\lambda \subseteq \xi$ [2].

The purpose of this study is to explore various properties of fuzzy $\mu$-radicals corresponding to fuzzy maximal $\mu$-open sets. A decomposition theorem for fuzzy maximal $\mu$-open sets is established which is then applied to obtain a sufficient condition for the existence of such sets. In addition, the fuzzy $\mu$-closures of the fuzzy $\mu$-radicals of fuzzy maximal $\mu$-open sets are investigated, leading to the formulation of the law of fuzzy $\mu$-radical $\mu$-closure.

## 2. Characteristics of Fuzzy $\mu$-Radicals

**Definition 2.1** [6] A proper nonempty fuzzy $\mu$-open set $\xi \subseteq X$ is said to be a fuzzy maximal $\mu$-open set if there does not exist any fuzzy $\mu$-open set $\zeta$ such that $\xi \subsetneq \zeta \subsetneq X$.

**Theorem 2.1** [6] Let $\xi$ and $\zeta$ be two fuzzy maximal $\mu$-open sets in $X$. Then $\xi = \zeta$ or $\xi \cup \zeta = X$.

**Definition 2.2** Let $\mathscr{G} = \{G_m : m \in \Omega\}$ be a family of fuzzy maximal $\mu$-open sets in a generalized fuzzy topological space $(X, \mu)$. Then $\bigcap \mathscr{G} = \bigcap_{m \in \Omega} G_m$ is called the fuzzy $\mu$-radical of $\mathscr{G}$.

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**Theorem 2.2** Let $(X,\mu)$ be a GFTS and let $G_m$ be a fuzzy maximal $\mu$-open set for each $m \in \Omega$ where $|\Omega| \geq 2$ and $G_m \neq G_n$ for $m \neq n$. Then
(i)$X \setminus (\bigcap_{m\in\Omega\setminus\{n\}} G_m) \subseteq G_n$ for any $n \in \Omega$.
(ii)$\bigcap_{m\in\Omega\setminus\{n\}} G_m \neq \emptyset$ for any $n \in \Omega$.

**Proof:** (i) Let n be any element of $\Omega$. Then by Theorem 2.1, we have $X\setminus G_n \subseteqq G_m$ for any element $m$ of $\Omega$ with $m \neq n$. Hence $X\setminus G_n \subseteqq \bigcap_{m\in\Omega\setminus\{n\}} G_m$. Thus $X\setminus \bigcap_{m\in\Omega\setminus\{n\}} G_m \subseteq G_n$.

(ii) If $\bigcap_{m\in\Omega\setminus\{n\}} G_m = 0$, then by (i) above $X = G_n$. But this contradicts the fact that $G_n$ is a fuzzy maximal $\mu$-open set. Therefore $\bigcap_{m\in\Omega\setminus\{n\}} G_m \neq 0$. □

**Corollary 2.1** Let $X$ be a GFTS and let $G_m$ be a fuzzy maximal $\mu$-open set for each element $m$ of $\Omega$ and $G_m \neq G_n$ for any elements $m,n \in \Omega$ with $m \neq n$. Then $G_m \cap G_n \neq 0$ for any two elements $m,n \in \Omega$ with $m \neq n$.

**Proof:** This assertion is an immediate consequence of Theorem 2.2$(ii)$. □

**Theorem 2.3** Let $X$ be a GFTS and let $G_m$ be a fuzzy maximal $\mu$-open set for every $m \in \Omega$ with $|\Omega| \geq 2$ and $G_m \neq G_n$ for any distinct $m,n \in \Omega$. Then $\bigcap_{m\in\Omega\setminus\{n\}} G_m \nsubseteq G_n$ and $G_n \nsubseteq \bigcap_{m\in\Omega\setminus\{n\}} G_m$ for any $n \in \Omega$.

**Proof:** Let $n$ be any element of $\Omega$. If $\cap_{m\in\Omega\setminus\{n\}} G_m \subseteqq G_n$, then
$X = \left(X\setminus \bigcap_{m\in\Omega\setminus\{n\}} G_m\right) \bigcup \left(\bigcap_{m\in\Omega\setminus\{n\}} G_m\right) \subseteqq G_n$ (by Theorem 2.2). Hence $G_m = X$. This contradicts the fact that $G_n$ is a fuzzy maximal $\mu$-open. Now if $G_n \subseteqq \bigcap_{m\in\Omega\setminus\{n\}} G_m$ , then we have $G_n \subseteqq G_m$ for each $m \in \Omega\setminus\{n\}$ and hence $G_n = G_m$ for any $m \in \Omega\setminus\{n\}$ (as $G_n$ is fuzzy maximal $\mu$-open). Thus, we obtain a contradiction with the fact that $G_m \neq G_n$ for $m \neq n$. □

**Corollary 2.2** Let $(X,\mu)$ be a GFTS and $G_m$ be a fuzzy maximal $\mu$-open set for each $m \in \Omega$ and $G_m \neq G_n$ for any distinct $m,n$ in $\Omega$. If $\Gamma \subsetneq \Omega$, then $\bigcap_{m\in(\Omega\setminus\Gamma)} G_m \nsubseteq \bigcap_{n\in\Gamma} G_n$ and $\bigcap_{n\in\Gamma} G_n \nsubseteq \bigcap_{m\in(\Omega\setminus\Gamma)} G_m$.

**Proof:** For $n \in \Gamma$, we have $\bigcap_{m\in\Omega\setminus\Gamma} G_m = \bigcap_{m\in(\Omega\setminus\Gamma)\cup\{n\}\setminus\{n\}} G_m \nsubseteqq G_n$ (by Theorem 2.3). Thus $\bigcap_{m\in\Omega\setminus\Gamma} G_m \nsubseteqq \bigcap_{n\in\Gamma} G_n$. Now $\bigcap_{n\in\Gamma} G_n = \bigcap_{n\in\Omega\setminus(\Omega\setminus\Gamma)} G_m \nsubseteqq \bigcap_{m\in\Omega\setminus\Gamma} G_m$ and hence $\bigcap_{n\in\Gamma} G_n \nsubseteqq \bigcap_{m\in\Omega\setminus\Gamma} G_m$ . □

**Theorem 2.4** Let $(X,\mu)$ be a GFTS and $G_m$ be a fuzzy maximal $\mu$-open set for each $m \in \Omega$ and $G_m \neq G_n$ for any $m,n \in \Omega$ where $m \neq n$. If $\Gamma \subsetneq \Omega$, then $\bigcap_{m\in\Omega} G_m \subsetneq \bigcap_{n\in\Gamma} G_n$.

**Proof:** It is obvious that $\bigcap_{m\in\Omega} G_m= (\bigcap_{m\in\Omega\setminus\Gamma} G_m)\bigcap(\bigcap_{n\in\Gamma} G_n) \subsetneqq \bigcap_{n\in\Gamma} G_n$ (by Corollary 2.2). □

**Theorem 2.5** (Decomposition Theorem): Let $(X,\mu)$ be a GFTS and $G_m$ be a fuzzy maximal $\mu$-open set for each $m \in \Omega$ where $|\Omega| \geq 2$ and $G_m \neq G_n$ for all $m \neq n \in \Omega$. Then $G_n = (\bigcap_{m\in\Omega} G_m) \cup (X\setminus \bigcap_{m\in\{n\}} G_m)$ for any $n \in \Omega$.

**Proof:** Now $(\bigcap_{m\in\Omega} G_m)\bigcup(X\setminus \bigcap_{m\in\Omega\setminus\{n\}} G_m) = ((\bigcap_{m\in\Omega\setminus\{n\}} G_m)\bigcap G_n)\bigcup(X\setminus \bigcap_{m\in\Omega\setminus\{n\}} G_m)$
$= ((\bigcap_{m\in\Omega\setminus\{n\}} G_m)\bigcup(X\setminus \bigcap_{m\in\Omega\setminus\{n\}} G_m))\bigcap(G_m\cup(X\setminus \bigcap_{m\in\Omega\setminus\{n\}} G_m)) = X\bigcap(G_n\cup(X\setminus \bigcap_{m\in\Omega\setminus\{n\}} G_m))$
$= G_n \cup (X\setminus \bigcap_{m\in\Omega\setminus\{n\}} G_m) = G_n$ (by Theorem 2.2.) □

As an explanation of Theorem 2.5, an alternate proof of Theorem 2.4 as follows:

**Proof:** As $0 \neq \Gamma \subsetneq \Omega$, there exists an element $\xi$ of $\Omega$ such that $\xi \notin \Gamma$ and an element $n$ of $\Gamma$. If $\mid \Gamma \mid = 1$, then we have $\bigcap_{m\in\Omega} G_m \subseteqq G_n$. If $\bigcap_{m\in\Omega} G_m = G_n$, then we have $G_n \subseteqq G_m$ for any element $m$ of $\Omega$. Since $G_m$ is a fuzzy maximal $\mu$-open set for any element $m$ of $\Omega$ , we have $G_n = G_m$ which contradicts our assumption.
Hence, we have $\bigcap_{m\in\Omega} G_m \subsetneqq G_n$. If $|\Gamma| \geq 2$, then by Theorem 2.5, we have

$$G_\xi = (\bigcap_{m\in\Omega} G_m) \cup (X \setminus \bigcap_{m\in\Omega\setminus\{\xi\}} G_m)$$
$$G_n = (\bigcap_{\zeta\in\Gamma} G_m) \cup (X \setminus \bigcap_{\zeta\in\Gamma\setminus\{n\}} G_\zeta).$$

If $\bigcap_{m\in\Omega} G_m = \bigcap_{\zeta\in\Gamma} G_\zeta$ , then $\bigcap_{\zeta\in\Gamma} G_\zeta = \bigcap_{m\in\Omega} G_m \subsetneqq \bigcap_{m\in\Omega\setminus\{\xi\}} G_m \subsetneqq \bigcap_{\zeta\in\Gamma} G_\zeta$ . Now $\bigcap_{m\in\Omega\setminus\{\xi\}} G_m = \bigcap_{\zeta\in\Gamma} G_\zeta$ . So $\bigcap_{m\in\Omega\setminus\{\xi\}} G_m = \bigcap_{\zeta\in\Gamma} G_\zeta \subsetneqq \bigcap_{\zeta\in\Gamma\setminus\{n\}} G_\zeta$. Therefore $G_\xi \supsetneqq G_n$ which gives $G_\xi = G_n$ with $\xi \neq n$. This contradicts our assumption.

□

The next Theorem gives a description of fuzzy maximal $\mu$-open sets.

**Theorem 2.6** Let $(X, \mu)$ be a GFTS and $G_m$ be a fuzzy maximal $\mu$-open set for each $m \in \Omega$ with $\mid \Omega \mid \geq 2$, $G_m \neq G_n$ for any $m, n \in \Omega$ with $m \neq n$. If $\bigcap_{m\in\Omega} G_m = 0$, then $\{G_m : m \in \Omega\}$ is the set of all fuzzy maximal $\mu$-open sets of $X$.

**Proof:** Assume if possible $G_m$ be another fuzzy maximal $\mu$-open set of $X$ which is not equal to $G_m$ for any $m \in \Omega$. Then $0 = \bigcap_{m\in\Omega} G_m = \bigcap_{m\in(\Omega\cup\{\xi\})\setminus\{\xi\}} G_m$. By Theorem 2.2, we look $\bigcap_{m\in(\Omega\cup\xi)} G_m \neq 0$. This contradicts our assumption. □

**Example 2.1** Let $X = \{a, b, c, d\}$ and the fuzzy sets are $\gamma_1 = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{0}{d}$; $\gamma_2 = \frac{0}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}$; $\gamma_3 = \frac{1}{a} + \frac{0}{b} + \frac{0}{c} + \frac{1}{d}$; We define GFT $\mu = \{0, \gamma_1, \gamma_2, \gamma_3, 1\}$ on $X$. Now $(X, \mu)$ is a GFTS. As $\bigcap\{\gamma_k : k \in \Delta\} = 0$, we conclude from Theorem 2.6 that all $\gamma_k$ are fuzzy maximal $\mu$-open sets for any $k$.

## 3. Fuzzy $\mu$-Radical and $\mu$-Closure in Fuzzy Quasi Topology

**Definition 3.1** *A GFT $\mu$ is said to be a fuzzy quasi topology (briefly FQT) if for $\xi, \xi' \in \mu \Rightarrow \xi \cap \xi' \in \mu$. The pair $(X, \mu)$ is said to be a FQTS if $\mu$ is a FQT on $X$.*

**Proposition 3.1** Let $\mu$ be a fuzzy quasi topology (FQT) and $\xi, \zeta$ be two fuzzy subsets of $X$. If $\xi \cup \zeta = X$, $\xi \cap \zeta$ is a fuzzy $\mu$-closed set and $\xi$ is a fuzzy $\mu$-open set, then $\zeta$ is a fuzzy $\mu$-closed set.

**Proof:** As $X \setminus \xi \subseteq \zeta$, it follows that $(\xi \cap \zeta) \cup (X \setminus \xi) = (\xi \cup (X \setminus \zeta)) \cap (\zeta \cup (X \setminus \xi)) = \zeta \cup (X \setminus \xi) = \zeta$. Moreover, both $\xi \cap \zeta$ and $X \setminus \xi$ are fuzzy $\mu$-closed. As $(X, \mu)$ is a FQTS, we conclude that $\zeta$ is fuzzy $\mu$-closed. □

**Proposition 3.2** Let $(X, \mu)$ be a GFTS and let for each $m \in \Omega$, $G_m$ be a fuzzy $\mu$-open set for all $m \neq n \in \Omega$, we have $G_m \cup G_n = X$. If $\bigcap_{m\in\Omega} G_m$ is a fuzzy $\mu$-closed set, then $\bigcap_{m\in\Omega\setminus\{n\}} G_m$ is a fuzzy $\mu$-closed set for any $n \in \Omega$.

**Proof:** Since $G_m \cup G_n = X$ for any element $m$ of $\Omega$ with $m \neq n$,
$G_n \bigcup (\bigcap_{m\in\Omega\setminus\{n\}} G_m) = \bigcap_{m\in\Omega\setminus\{n\}} (G_n \cup G_m) = X$, As by Proposition 3.1, $G_n \bigcap (\bigcap_{m\in\{n\}\setminus\{n\}} G_m) = \bigcap_{m\in\Omega} G_m$ is a fuzzy $\mu$-closed set. Hence $\bigcap_{m\in\Omega\setminus\{n\}} G_m$ is fuzzy $\mu$-closed for any element $n$ of $\Omega$ . □

**Theorem 3.1** Let $\mu$ be a FQTS on $X$ and let for each $m \in \Omega$, $G_m$ be a fuzzy maximal $\mu$-open set. If $G_m \neq G_n$ for $m \neq n \in \Omega$ and $\bigcap_{m\in\Omega} G_m$ is a fuzzy $\mu$-closed set, then $\bigcap_{m\in\Omega} G_m$ is a fuzzy $\mu$-closed set.

**Proof:** By Theorem 2.2, we obtain $G_m \cup G_n = X$ for any $m, n \in \Omega$ with $m \neq n$. Hence, by Proposition 3.2, the set $\bigcap_{m\in\Omega\setminus\{n\}} G_m$ is fuzzy $\mu$-closed. However, the preceding three results do not hold in a GFTS, as demonstrated in the following Example. □

**Example 3.1** Let $X = \{a, b, c, d\}$ and the fuzzy sets are $\gamma_1 = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{0}{d}$; $\gamma_2 = \frac{0}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}$; $\gamma_3 = \frac{0}{a} + \frac{1}{b} + \frac{1}{c} + \frac{0}{d}$; $\gamma_4 = \frac{1}{a} + \frac{0}{b} + \frac{0}{c} + \frac{1}{d}$; We define GFT $\mu = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ on $X$. Now $(X, \mu)$ is a GFTS. Now $\gamma_1$ and $\gamma_4$ are two fuzzy maximal $\mu$-open sets in $X$ and $\gamma_1 \wedge \gamma_4 = \frac{1}{a} + \frac{0}{b} + \frac{0}{c} + \frac{0}{d}$ is fuzzy $\mu$-closed but $\gamma_1$ is not so.

**Theorem 3.2** If $\mu$ is a FQT on $X$, then $c_\mu$ is fuzzy $\mu$-friendly. In otherwords for $\xi \subseteq X$ and $\beta \in \mu$, $c_\mu(\xi) \cap \beta \subseteq c_\mu(\xi \cap \beta)$.

We also recall from [6] that for any fuzzy maximal $\mu$-open set $\xi$ in GFTS $(X, \mu)$, either $c_\mu(\xi) = X$ or $c_\mu(\xi) = \xi$.

**Theorem 3.3** Let $G_m$ denote the fuzzy maximal $\mu$-open set in a FQTS $(X, \mu)$ for each element $m$ of a finite set $\Omega$. If $c_\mu(\bigcap_{m\in\Omega} G_m) \neq X$, then there exists some $m \in \Omega$ such that $c_\mu(G_m) = G_m$.

**Proof:** Consider $c_\mu(G_m) = X$ for every $m \in \Omega$. Let $n$ be an arbitrary element of $\Omega$. Since $\bigcap_{m\in\Omega\setminus\{n\}} G_m$ is fuzzy $\mu$-open, we obtain $c_\mu(\bigcap_{m\in\Omega} G_m) = c_\mu\left(\left(\bigcap_{m\in\Omega\setminus\{n\}} G_m\right) \bigcap G_n\right) \supseteq \left(\bigcap_{m\in\Omega\setminus\{n\}} G_m\right) \bigcap c_\mu(G_n)$, by Theorem 3.2. Therefore, $c_\mu(\bigcap_{m\in\Omega} G_m) \supseteq \bigcap_{m\in\Omega\setminus\{n\}} G_m$. Hence, $c_\mu\left(\bigcap_{m\in\Omega\setminus\{n\}} G_m\right) \subseteq c_\mu(\bigcap_{m\in\Omega} G_m)$. On the other hand, since $c_\mu$ is monotone, we also have $c_\mu\left(\bigcap_{m\in\Omega\setminus\{n\}} G_m\right) \supseteq c_\mu(\bigcap_{m\in\Omega} G_m)$. Thus, $c_\mu\left(\bigcap_{m\in\Omega\setminus\{n\}} G_m\right) = c_\mu(\bigcap_{m\in\Omega} G_m)$. By induction on the elements of $\Omega$, it follows that $c_\mu(\bigcap_{m\in\Omega} G_m) = c_\mu(G_m) = X$ for all $m \in \Omega$ which contradicts the initial assumption. Hence $c_\mu(\bigcap_{m\in\Omega} G_m) \neq X$. Consequently, there exists some $m \in \Omega$ such that $c_\mu(G_m) = G_m$. □

This theorem does not necessarily apply when $\Omega$ is infinite as seen in the following example. Furthermore, an additional example illustrates that the outcome is not true in a GFTS.

**Example 3.2** Let $X = \{a, b, c\}$. The fuzzy sets are $\beta_1 = \frac{1}{a} + \frac{1}{b} + \frac{0}{c}$; $\beta_2 = \frac{0}{a} + \frac{1}{b} + \frac{1}{c}$ and $\beta_3 = \frac{1}{a} + \frac{0}{b} + \frac{1}{c}$. We define GFT $\mu = \{0, \beta_1, \beta_2, \beta_3, 1\}$ on $X$ and $(X, \mu)$ is a GFTS. It is easy to see that for fuzzy collection $\{\beta_i : i \in I\}$, we have $c_\mu(\bigcap_{i\in I} \beta_i) \neq X$ for any $i$.

The fuzzy $\mu$-radical of fuzzy maximal $\mu$-open sets have the following significance property.

**Theorem 3.4** (Law of fuzzy $\mu$-radical and fuzzy $\mu$-closure) Let $\Omega$ be a finite set and let $G_m$ is a fuzzy maximal $\mu$-open set for each $m \in \Omega$ in FQTS. Let $\Gamma \subseteq \Omega$ be a fuzzy subset such that $c_\mu(G_m) = G_m$ for all $m \in \Gamma$ and $c_\mu(G_m) = X$ for all $m \in \Omega\setminus\Gamma$. Then $c_\mu\left(\bigcap_{m\in\Omega} G_m\right) = \bigcap_{m\in\Gamma} G_m$, and also if $\Gamma = \emptyset$, then the right-hand side is $X$.

**Proof:** If $\Gamma = 0$, then it holds from Theorem 3.3. If $\Gamma \neq 0$, $c_\mu(\cap_{m\in\Omega} G_m) = c_\mu((\bigcap_{m\in\Omega} G_m) \bigcap (\bigcap_{m\in\Omega\setminus\Gamma} G_m) \supseteqq (\bigcap_{m\in\Gamma} G_m) \bigcap c_\mu(\bigcap_{m\in\Omega\setminus\Gamma} G_m)$ $= \bigcap_{m\in\Gamma} G_m \bigcap X = \bigcap_{m\in\Gamma} G_m$ by Theorem 3.2 and since $\bigcap_{m\in\Gamma} G_m$ is a fuzzy $\mu$-open set. So $c_\mu(\bigcap_{m\in\Omega} G_m = c_\mu(c_\mu(\bigcap_{m\in\Gamma} G_m)) \supseteqq c_\mu(\bigcap_{m\in\Gamma} G_m)$. Again since $(\bigcap_{m\in\Omega} G_m) \subseteqq \bigcap_{m\in\Omega} G_m$, then $c_\mu(\bigcap_{m\in\Omega} G_m) \subseteqq c_\mu(\bigcap_{m\in\Gamma} G_m)$. Thus $c_\mu(\bigcap_{m\in\Omega} G_m) = c_\mu(\bigcap_{m\in\Gamma} G_m)$. The fuzzy $\mu$-radical $\bigcap_{m\in\Gamma} G_m$ is a fuzzy $\mu$-closed set since $G_m$ is fuzzy $\mu$-closed for any $m \in \Gamma$ by our assumption. Hence we conclude that $c_\mu(\bigcap_{m\in\Omega} G_m) = \bigcap_{m\in\Gamma} G_m$. □

As a consequence of Theorem 3.4, we establish the succeeding Theorem.

**Theorem 3.5** Let $\mu$ be a FQT on $X$ and $G_m$ be a fuzzy maximal $\mu$-open set for each element $m \in \Omega$ and if $G_m \neq G_n$ for any distinct elements $m, n \in \Omega$. If $\bigcap_{m\in\Omega} G_m$ is a fuzzy $\mu$-closed set, then $G_m$ is fuzzy $\mu$-closed for each $m \in \Omega$.

**Proof:** Assume that $\Gamma$ be a fuzzy subset of $\Omega$. Then $c_\mu(G_m) = G_m$ for any $m \in \Gamma$ and $c_\mu(G_m) = X$ for any $m \in \Omega\setminus\Gamma$. By assumption, the fuzzy $\mu$-radical $\bigcap_{m\in\Omega} G_m$ is a fuzzy $\mu$-closed set. It is evident from Theorem 3.4 that $\Gamma \neq 0$. Then for $\Gamma \subseteqq \Omega$. $\bigcap_{m\in\Omega} G_m = c_\mu(\bigcap_{m\in\Omega} G_m) = \bigcap_{m\in\Gamma} G_m$ by Theorem 3.4. Hence by Theorem 2.4, we conclude that $\Omega = \Gamma$. □

The subsequent Example depicts that the above Theorem is not true in GFTS.

**Example 3.3** We observe that from Example 3.1, $\gamma_1$, $\gamma_2$ and $\gamma_4$ are three fuzzy maximal $\mu$-open sets in $X$. It is easy to see that $\bigcap\{\gamma_k : k \in \Delta\} = 0$ is fuzzy $\mu$-closed but $\gamma_k$ is not fuzzy $\mu$-closed for any $k$.

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*A. Swaminathan*
*Department of Mathematics*
*Government Arts College(Autonomous)*
*Kumbakonam, Tamil Nadu-612 002, India.*
*E-mail address:* asnathanway@gmail.com

*and*

*Margaret Sheela*
*Research Scholar*
*Department of Mathematics*
*Annamalai University, Annamalainagar*
*Tamil Nadu-608 002, India.*
*E-mail address:* mshee1836@gmail.com