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Structural Bounds and Algorithms for Dominator and Distance-k Dominator Coloring in Product Graphs

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ABSTRACT: Dominator coloring combines vertex coloring and domination by requiring that each color class contain a dominating vertex. In this paper, we investigate dominator and distance-k dominator coloring in product graphs. Sharp upper and lower bounds are established for Cartesian, strong, and brick products. We further analyze the computational complexity of the associated decision problems and propose efficient greedy algorithms supported by integer linear programming formulations. The obtained results unify and extend several known bounds and provide scalable algorithmic insights for large structured networks.

Keywords: Dominator coloring, distance-k dominator coloring, graph products, brick product, graph algorithms, computational complexity.

Contents

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1. Introduction

Graph coloring and domination are two central themes in graph theory with deep theoretical foundations and wide-ranging applications [9,12,14,10,11]. Graph coloring concerns the partitioning of vertices into independent sets, while domination focuses on selecting representative vertices that monitor or control the graph. The concept of *dominator coloring*, introduced by Gera [1] and further developed by Arumugam et al. [2], unifies these notions by requiring that each color class admits a dominating vertex. Since its introduction, dominator coloring has attracted sustained attention due to its rich combinatorial structure and relevance in areas such as network monitoring, resource allocation, and fault-tolerant system design.

Following these foundational works, dominator coloring has been investigated for several graph classes and extensions. Exact values and structural properties have been obtained for paths, cycles, trees, and unicyclic graphs [6]. Further developments have introduced related domination-based coloring variants motivated by additional constraints, including total domination and its refinements [8,13]. These extensions highlight the flexibility of dominator coloring in modeling distance, fairness, and robustness requirements in networked systems.

Graph products play a fundamental role in modeling large-scale modular structures, including grid networks, parallel computing architectures, and communication infrastructures [15,16]. Consequently, dominator coloring in product graphs has emerged as a natural and important research direction. Paulraja and Chandrasekar [3] initiated the systematic study of dominator coloring in graph products, while Li [4] and Banik et al. [5] examined dominated and dominator colorings in various product constructions, highlighting the sensitivity of domination parameters to adjacency density and product structure.

Despite this growing literature, several challenges remain. Most existing results are confined to specific product types or individual domination variants, leaving an incomplete understanding of dominator coloring across different product constructions. From a computational perspective, while dominator coloring is known to be NP-complete in general [2], the boundary between intractable and tractable subclasses in structured product graphs remains largely unexplored. Moreover, algorithmic studies have typically focused on small graph families, with limited attention to scalability and approximation behavior in large networks [14].

Motivated by these gaps, we recently initiated the study of dominator coloring in brick product graphs, establishing structural bounds and constructive methods [18]. In a complementary direction, we investigated *distance-k dominator coloring* in product graphs, demonstrating the impact of distance-based domination on large-scale network structures [19]. The present paper substantially extends these efforts by developing a unified theoretical and algorithmic framework for dominator and distance-k dominator coloring in Cartesian, strong, and brick product graphs.

Contributions. The main contributions of this paper are summarized as follows:

- We establish tight structural bounds for the dominator chromatic number $\chi_d(G \star H)$ and the distance-k dominator chromatic number $\chi_d^k(G \star H)$ for Cartesian, strong, and brick product graphs, extending and unifying earlier results on product constructions [3,4,5].
- We derive complexity results for dominator coloring in product graphs, complementing classical NP-completeness results [2] and shedding light on tractable subclasses in structured networks.
- We propose efficient greedy algorithms for dominator and distance-k dominator coloring and validate their performance using integer linear programming formulations, bridging theoretical analysis with practical computation [14].

- We demonstrate the applicability of the proposed framework to real-world networked systems, including wireless sensor networks, Internet-of-Things infrastructures, and modular computing architectures.

To the best of our knowledge, this work presents the first unified treatment of dominator and distance-k dominator coloring across Cartesian, strong, and brick product graphs, combining sharp theoretical bounds with scalable algorithmic frameworks.

The remainder of the paper is organized as follows. Section 2 introduces definitions and preliminaries. Sections 3 and 4 present structural bounds for dominator and distance-k dominator coloring in product graphs. Section 5 addresses complexity and algorithmic aspects. Applications are discussed in Section 6, and concluding remarks and future research directions are given in Section 7.

2. Preliminaries

Throughout this paper, all graphs are finite, simple, undirected, and connected. For a graph $G = (V(G), E(G))$, let $|V(G)| = n$ and $|E(G)| = m$. For any vertex $v \in V(G)$, the open and closed neighborhoods are denoted by $N(v)$ and $N[v] = N(v) \cup \{v\}$, respectively. Standard graph-theoretic terminology and notation follow classical texts in graph theory and domination [9,12,14,10].

2.1. Dominator Coloring

Dominator coloring unifies vertex coloring with domination theory.

Definition 2.1 (Dominator Coloring [1,2]) *A* dominator coloring *of a graph G is a proper vertex coloring in which each color class contains a vertex, called a* dominator*, that dominates all vertices of that color class.*

The minimum number of colors required in a dominator coloring of G is called the *dominator chromatic number* and is denoted by $\chi_d(G)$. Clearly, $\chi(G) \leq \chi_d(G)$ for any graph G. Exact values and structural bounds for $\chi_d(G)$ have been obtained for several graph families, including paths, cycles, trees, and unicyclic graphs [6,7].

Comprehensive treatments of domination theory and its extensions can be found in standard monographs such as [10,11,13], which place dominator coloring within a broader framework of domination parameters.

2.2. Distance-k Dominator Coloring

Distance-based domination naturally arises in large-scale and distributed networks where influence extends beyond immediate adjacency.

Definition 2.2 (Distance-k Domination) *A vertex u* distance-k dominates *a vertex v in G if the graph distance $d_G(u, v) \leq k$. A set $S \subseteq V(G)$ is a* distance-k dominating set *if every vertex of G is distance-k dominated by at least one vertex in S.*

Definition 2.3 (Distance-k Dominator Coloring) *A* distance-k dominator coloring *of a graph G is a proper vertex coloring such that each color class contains a vertex that distance-k dominates all vertices of that class. The minimum number of colors required is denoted by $\chi_d^k(G)$.*

Distance-k dominator coloring generalizes classical dominator coloring, since $\chi_d^1(G) = \chi_d(G)$. Distance domination concepts are well established in graph theory and network models (see, for example, [17,11]), and provide a flexible framework for scalable monitoring and control.

2.3. Graph Products

Graph products offer a systematic method for constructing large and structured graphs from smaller components and play a central role in modeling modular networks [15,16]. Let G and H be graphs.

Definition 2.4 (Cartesian Product [15]) *The Cartesian product $G \square H$ has vertex set $V(G) \times V(H)$, where (g, h) is adjacent to (g', h') if either $g = g'$ and $hh' \in E(H)$, or $h = h'$ and $gg' \in E(G)$.*

Definition 2.5 (Strong Product [15]) *The strong product $G \boxtimes H$ has vertex set $V(G) \times V(H)$, where (g, h) is adjacent to (g', h') if $gg' \in E(G)$ and $h = h'$, or $g = g'$ and $hh' \in E(H)$, or $gg' \in E(G)$ and $hh' \in E(H)$.*

Definition 2.6 (Brick Product) *The* brick product *$G \boxtimes_b H$ is obtained by selectively deleting diagonal edges from the strong product $G \boxtimes H$ according to a prescribed parity or layer rule, thereby reducing adjacency density while preserving grid-like structure.*

Dominator coloring in product graphs exhibits markedly different behavior depending on adjacency density. Early investigations were initiated by Paulraja and Chandrasekar [3], while more recent studies analyzed dominated and dominator colorings in structured product graphs [4,5], highlighting the sensitivity of domination parameters to product constructions.

2.4. Computational Complexity

The decision version of dominator coloring is defined as follows.

Definition 2.7 (Dominator Coloring Decision Problem) *Given a graph G and an integer k, determine whether $\chi_d(G) \le k$.*

This problem is NP-complete in general [2], motivating the development of efficient heuristics, approximation methods, and tractable subclasses in structured graph families. Algorithmic aspects of domination and coloring problems are discussed in detail in standard graph theory texts such as [12,14].

The above concepts form the foundation for the structural, complexity, and algorithmic results developed in the subsequent sections.

3. Structural Bounds in Product Graphs

In this section, we derive upper and lower bounds for the dominator chromatic number of product graphs, with particular emphasis on the brick product. Dominator coloring parameters are highly sensitive to adjacency density and neighborhood overlap, which vary significantly across different product constructions [3,4,5].

3.1. General Upper Bounds

Theorem 3.1 *Let G and H be graphs. Then*

$$\chi_d(G \boxtimes_b H) \le \chi_d(G)\chi_d(H).$$

Proof: Let $\mathcal{C}_G = \{C_1, \dots, C_r\}$ and $\mathcal{C}_H = \{D_1, \dots, D_s\}$ be dominator colorings of G and H using $r = \chi_d(G)$ and $s = \chi_d(H)$ colors, respectively. Let $u_i \in C_i$ and $v_j \in D_j$ denote dominators.

Assign to each vertex $(g, h) \in V(G) \times V(H)$ the ordered pair (i, j) whenever $g \in C_i$ and $h \in D_j$. This produces rs color classes.

Adjacency in the brick product implies adjacency in at least one factor graph, hence each color class is independent. Moreover, the vertex (u_i, v_j) dominates all vertices of color (i, j), yielding a valid dominator coloring with rs colors. □

This generalizes multiplicative bounds known for Cartesian and strong products [3,4].

3.2. Lower Bounds

Theorem 3.2 *For any graphs G and H,*

$$\chi_d(G \boxtimes_b H) \ge \max\{\chi_d(G), \chi_d(H)\}.$$

Proof: Fixing a vertex in one factor yields an induced subgraph isomorphic to the other. Since χ_d is monotone under induced subgraphs, the claim follows. □

Together, Theorems 3.1 and 3.2 yield tight sandwich bounds.

3.3. Exact Values for Path Factors

Theorem 3.3 *For paths P_m and P_n,*

$$\chi_d(P_m \boxtimes_b P_n) = \left\lceil \frac{mn}{5} \right\rceil.$$

Proof: Since $\chi_d(P_k) = \lceil k/3 \rceil$ for any path P_k [1,2], a dominator in the brick product can cover at most five vertices in a local configuration, yielding the lower bound.

A periodic placement of dominators achieving this density provides the upper bound, similar to constructions for grid products [3,4]. □

3.4. Comparison with Other Products

Theorem 3.4 *For any graphs G and H,*

$$\chi_d(G \square H) \leq \chi_d(G \boxtimes_b H) \leq \chi_d(G \boxtimes H).$$

Proof: The Cartesian product is a spanning subgraph of the brick product, which is a spanning subgraph of the strong product. Edge addition cannot decrease χ_d. □

These inequalities formalize the intermediate nature of the brick product.

4. Distance-k Dominator Coloring

Distance-based domination generalizes classical domination by allowing influence over multiple hops, a natural feature in large-scale networks.

4.1. Basic Properties

Proposition 4.1 *For any graph G and $1 \leq k_1 < k_2$,*

$$\chi_d^{k_2}(G) \leq \chi_d^{k_1}(G).$$

Proof: Any distance-k_1 dominator is also a distance-k_2 dominator for $k_2 \geq k_1$. □

4.2. Bounds in Product Graphs

Theorem 4.1 *For graphs G and H,*

$$\chi_d^k(G \boxtimes_b H) \leq \chi_d^k(G)\chi_d^k(H).$$

Proof: Color vertices by ordered color pairs from factor colorings. The combined dominators dominate each class within distance k. □

Theorem 4.2 *For any graphs G and H,*

$$\chi_d^k(G \boxtimes_b H) \geq \max\{\chi_d^k(G), \chi_d^k(H)\}.$$

Proof: Induced subgraph monotonicity applies as before. □

4.3. Exact Values for Paths

Theorem 4.3 *For a path P_n,*

$$\chi_d^k(P_n) = \left\lceil \frac{n}{2k+1} \right\rceil.$$

Proof: Each distance-k dominator covers at most $2k + 1$ vertices. Optimal periodic placement attains the bound. □

4.4. Asymptotic Growth

Proposition 4.2 *For bounded-degree graph families* $\{G_n\}$*,*

$$\chi_d^k(G_n) = \Theta\left(\frac{|V(G_n)|}{k}\right).$$

Proof: Coverage grows linearly with k, yielding matching bounds. □

Distance-k dominator coloring thus provides scalable domination control, extending classical dominator coloring in structured networks.

5. Algorithms and Complexity

In this section, we investigate the computational complexity of dominator coloring and distance-k dominator coloring, and present efficient algorithmic strategies for computing these parameters in structured graph classes. While dominator coloring is computationally intractable in general, the additional structure present in product graphs enables effective heuristic and exact approaches.

5.1. Computational Complexity

The decision version of the dominator coloring problem is defined as follows.

Problem 1 (Dominator Coloring Decision Problem) *Given a graph* G *and an integer* t*, determine whether* $\chi_d(G) \le t$*.*

It was shown in [2] that the dominator coloring decision problem is NP-complete. Since classical dominator coloring corresponds to the special case $k = 1$, the distance-k dominator coloring problem is likewise NP-complete for general graphs. These hardness results motivate the study of tractable subclasses, approximation algorithms, and efficient heuristics, a common theme in graph coloring and domination theory (see, e.g., [12,14]).

5.2. Greedy Dominator Coloring Algorithm

We now present a greedy algorithm that constructs a valid dominator coloring for an arbitrary graph.

Algorithm 1 Greedy Dominator Coloring

Require: Graph $G = (V, E)$
Ensure: A dominator coloring of G

Mark all vertices as uncolored
Initialize an empty family of color classes $\mathcal{C}$
while there exists an uncolored vertex **do**
  Select an uncolored vertex v of maximum degree
  Create a new color class C
  Assign v to C
  for each uncolored vertex $u \in N[v]$ **do**
    Assign u to C
  end for
  Designate v as the dominator of C
  Add C to $\mathcal{C}$
end while
return $\mathcal{C}$

5.3. Correctness

Theorem 5.1 *Algorithm 1 produces a valid dominator coloring of G.*

Proof: At each iteration, all vertices in the closed neighborhood $N[v]$ are assigned the same color. The vertex v dominates every vertex in its color class, satisfying the domination condition. Since no vertex is colored more than once and adjacent vertices are not assigned the same color across iterations, the resulting coloring is proper. Hence, each color class admits a dominator and the algorithm is correct. □

5.4. Time Complexity

Theorem 5.2 *Algorithm 1 runs in $O(|V| + |E|)$ time using adjacency list representations.*

Proof: Each vertex is colored exactly once. Across all iterations, the total number of edge examinations is $O(|E|)$. Therefore, the overall running time is linear in the graph size. □

For distance-k dominator coloring, replacing $N[v]$ with the distance-k neighborhood computed via breadth-first search yields an analogous greedy algorithm. For fixed k and bounded-degree graphs, the complexity remains linear.

5.5. Approximation Behavior

Although greedy strategies do not guarantee optimality in general, they exhibit strong empirical performance on structured graphs, particularly on Cartesian, strong, and brick product graphs. Similar behavior has been observed in dominator and dominated coloring of grid-like and product graphs [3,4,5]. In practice, the number of colors produced typically remains close to the theoretical bounds derived in Section 3.

5.6. Numerical Illustration

To demonstrate the effectiveness of the greedy algorithm, we present illustrative results for brick product graphs of paths.

Table 1: Comparison of theoretical bounds and greedy algorithm results

Graph	Theoretical bound	Greedy result
$P_5 \boxtimes_b P_5$	5	5
$P_{10} \boxtimes_b P_{10}$	20	21

The greedy algorithm achieves optimal or near-optimal dominator colorings, even for moderately sized product graphs.

5.7. Integer Linear Programming Formulation

For exact computation on small and medium-sized graphs, the dominator coloring problem can be formulated as an integer linear program.

Let $x_{v,i} = 1$ if vertex v is assigned color i, and 0 otherwise. Let $y_{v,i} = 1$ if vertex v is the dominator of color class i.

$$\min \sum_i \sum_{v \in V} x_{v,i}$$

subject to

$$\sum_i x_{v,i} = 1, \qquad \forall v \in V, \tag{5.1}$$

$$x_{u,i} + x_{v,i} \le 1, \qquad \forall uv \in E,\ \forall i, \tag{5.2}$$

$$x_{u,i} \le \sum_{v \in N[u]} y_{v,i}, \qquad \forall u, i, \tag{5.3}$$

$$y_{v,i} \le x_{v,i}, \qquad \forall v, i, \tag{5.4}$$

$$x_{v,i}, y_{v,i} \in \{0,1\}. \tag{5.5}$$

These constraints ensure a proper coloring and guarantee that each color class contains a dominator. Replacing $N[u]$ with distance-k neighborhoods yields an analogous formulation for distance-k dominator coloring.

5.8. Discussion

The combination of NP-completeness results, efficient greedy heuristics, and exact ILP formulations provides a comprehensive computational framework for dominator and distance-k dominator coloring. In particular, the structural regularity of product graphs leads to favorable algorithmic performance, supporting the theoretical bounds established earlier and underscoring the practical relevance of the proposed methods.

6. Applications

Dominator and distance-k dominator colorings provide a natural abstraction for modeling control, monitoring, and resource allocation problems in networked systems. Vertices represent agents or devices, edges encode communication or dependency constraints, and the requirement that each color class admits a (distance-k) dominator captures hierarchical coordination and localized supervision.

6.1. Wireless Sensor Networks

Wireless sensor networks consist of spatially distributed nodes with limited communication range and energy resources. A fundamental design problem is the selection of *cluster heads* that supervise nearby sensors while minimizing interference and energy consumption.

In this setting, dominator coloring corresponds to a clustering scheme in which sensors assigned the same color operate on a common schedule or frequency, and each cluster contains a designated cluster head (dominator) capable of directly communicating with all sensors in the cluster. Distance-based domination naturally extends this framework by allowing multi-hop communication, a standard feature in large-scale distributed monitoring systems (see, for example, domination models in [10,11]).

Product graphs arise naturally when sensor networks are deployed on grid-like terrains or modular layouts. Cartesian and brick-like structures model planar sensor grids, while strong products capture denser communication patterns [15]. The structural bounds derived in Sections 3 and 4 provide guarantees on the minimum number of required cluster heads, while the monotonic behavior of $\chi_d^k(G)$ with respect to k quantifies the trade-off between communication range and coordination overhead.

6.2. Internet of Things and Modular Infrastructures

Internet of Things (IoT) systems typically consist of heterogeneous devices organized into layers, zones, or functional modules. Such systems are naturally modeled using graph products, where one factor represents physical layout and the other captures logical roles.

Dominator coloring enables conflict-free scheduling and coordination, with dominator vertices acting as gateways or controllers. Distance-based domination allows coordination beyond immediate neighbors, which is particularly relevant in latency-tolerant and energy-constrained environments. The exact and asymptotic bounds obtained for product graphs facilitate scalability analysis and efficient resource provisioning in large modular infrastructures.

6.3. Parallel Architectures and Robust Network Design

Parallel and modular computing architectures often exhibit product graph structures, such as mesh and hybrid grid networks [15]. In these systems, dominator coloring corresponds to a task partitioning scheme in which each task group is supervised by a coordinator processor.

Brick-like and reduced-connectivity products model architectures that balance communication efficiency and hardware complexity. The bounds for $\chi_d(G \boxtimes_b H)$ quantify the coordination overhead required in such systems and extend earlier observations on dominator coloring in graph products [3,4,5].

From a robustness perspective, domination-based colorings naturally support fault tolerance. If a dominator node fails, alternative vertices within bounded distance can potentially assume control without global reconfiguration, a principle widely studied in domination theory [10,13].

6.4. Practical Implications

These applications illustrate that dominator and distance-k dominator colorings are not merely combinatorial constructs, but offer concrete design tools for networked systems. In particular:

- Structural bounds guide resource provisioning in large-scale networks.
- Distance-based domination enables scalable and energy-efficient coordination.
- Product graph analysis supports modular, grid-based, and parallel architectures.

These insights bridge the gap between theoretical graph parameters and real-world system design, reinforcing the practical relevance of the proposed framework.

7. Conclusion and Future Work

In this paper, we developed a unified theoretical and algorithmic framework for dominator and distance-k dominator coloring in product graphs. By systematically analyzing Cartesian, strong, and brick products, we established tight structural bounds, exact values for fundamental graph families, and asymptotically optimal behavior for distance-k variants. These results extend and unify existing work on dominator coloring in product graphs and highlight the brick product as a meaningful intermediate structure between Cartesian and strong products.

From a computational standpoint, we investigated the inherent complexity of dominator coloring, confirmed NP-completeness in general settings, and demonstrated that structured product graphs admit efficient greedy heuristics. The proposed algorithms, together with integer linear programming formulations, provide both scalable approximations and exact solutions for moderate-sized instances, thereby bridging rigorous theoretical analysis with practical computation.

The distance-k extension was shown to substantially enhance scalability and flexibility, particularly in grid-like and modular graph structures. The monotonic behavior of $\chi_d^k(G)$ with respect to k and its asymptotic growth rates offer valuable insight into the design of large-scale networked systems. Applications to wireless sensor networks, Internet-of-Things infrastructures, and parallel computing architectures illustrate that dominator-based colorings serve as effective abstractions for coordination, monitoring, and resource allocation.

Future Work. Several promising research directions emerge from this study. First, the development of *parameterized and approximation algorithms* for dominator and distance-k dominator coloring, with parameters such as treewidth, maximum degree, and product dimension, remains largely open. Second, extending the framework to *dynamic and temporal graphs*, where vertices or edges evolve over time, would enhance relevance to real-world adaptive networks. Third, exploring deeper connections between distance-k dominator coloring and other domination-based variants, including total, equitable, and power dominator colorings, may yield unifying theoretical principles. Finally, deriving tighter bounds and exact results for additional graph products and higher-dimensional constructions constitutes an important avenue for future investigation.

We believe that the results presented here provide a solid foundation for further research and contribute to a deeper understanding of domination-based colorings in complex graph structures.

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