



## Bounds Inequality of Maximum Covering Energy for Semigraphs

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**ABSTRACT:** The concept of energy plays a significant role in spectral graph theory and its applications. Recently, various energy concepts based on covering parameters have been introduced for graphs and semigraphs. In this paper, we introduce the notion of *maximum covering energy* of a semigraph and establish several upper and lower bound inequalities. These bounds are expressed in terms of structural invariants such as the number of vertices, edges, covering number, and maximum degree. Illustrative examples are provided to demonstrate the sharpness of the results.

**Keywords:** Semigraphs, covering matrix, maximum covering eigenvalues, covering energy of a semigraph.

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### 1. Introduction

The energy of a graph, introduced by Gutman [1,2], is defined as the sum of the absolute values of the eigenvalues of its adjacency matrix [8]. This invariant has been widely studied due to its connections with chemistry, physics, and combinatorics. Subsequently, several variations of graph energy have been introduced, including Laplacian energy, dominating energy, and covering energy.

Semigraphs is introduced by E.Sampathkumar [4,7] it is a new type of generalization of a graph in which edges may contain more than two vertices, with the restriction that any two edges have at most one vertex in common. The spectral properties of semigraphs and their energies have gained attention in recent years [3,6,9].

In this work, we define the maximum covering energy of a semi graph and derive bounds analogous to those known for graphs [14].

### 2. Preliminaries

**Definition 2.1** A semi graph  $S = (V, E)$  consists of a finite vertex set  $V = \{v_1, v_2, \dots, v_n\}$  and a set of edges  $E = \{e_1, e_2, \dots, e_m\}$ , where each edge  $e_i$  is an ordered  $k_i$ -tuple of distinct vertices with  $k_i \geq 2$ , and any two edges share at most one vertex.

**Definition 2.2** A subset  $C \subseteq V$  is called a vertex cover of  $S$  if every edge of  $S$  contains at least one vertex from  $C$ . The cardinality of a smallest vertex cover is called the covering number and is denoted by  $\tau(S)$ .

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**Definition 2.3** Let  $C_{\max}$  be a vertex cover of maximum cardinality. The maximum covering matrix  $M = (m_{ij})$  of a semigraph  $S$  is defined by

$$m_{ij} = \begin{cases} 1, & \text{if } v_i, v_j \in C_{\max} \text{ and lie in a common edge,} \\ 0, & \text{otherwise.} \end{cases}$$

**Definition 2.4** Let  $\mu_1, \mu_2, \dots, \mu_n$  be the eigenvalues of the maximum covering matrix  $M$ . The maximum covering energy of  $S$  is defined as

$$E_{\max}(S) = \sum_{i=1}^n |\mu_i|.$$

### 3. Upper Bounds on Maximum Covering Energy

In this section, we derive upper bounds for the maximum covering energy of a semigraph using combinatorial and spectral techniques.

#### 3.1. Standard Approach in Spectral Energy Theory

Spectral energy theory [3,6] studies combinatorial structures through the eigenvalues of matrices naturally associated with them. Given a real symmetric matrix  $M$  of order  $n$  with eigenvalues  $\mu_1, \mu_2, \dots, \mu_n$ , the energy associated with  $M$  is defined as

$$E(M) = \sum_{i=1}^n |\mu_i|.$$

In graph and semigraph theory, the matrix  $M$  is typically chosen to reflect adjacency, incidence, or covering properties [8]. In the present work, we consider the maximum covering matrix of a semigraph  $S$ , whose spectrum encodes both structural and covering information [10].

This methodology has been extensively used in the study of graph energy, Laplacian energy, and various generalized energy concepts, and provides a natural and robust framework for analyzing the maximum covering energy of semigraphs.

#### 3.2. Gershgorin Circle Theorem and Its Application

Let  $M = (m_{ij})$  be a real square matrix of order  $n$ . For each  $i = 1, 2, \dots, n$ , define the Gershgorin disc

$$D_i = \left\{ z \in \mathbb{C} : |z - m_{ii}| \leq \sum_{\substack{j=1 \\ j \neq i}}^n |m_{ij}| \right\}.$$

The Gershgorin Circle Theorem [16] states that every eigenvalue of  $M$  lies in at least one of the discs  $D_i$ .

Since the maximum covering matrix of a semigraph  $S$  is real and symmetric, all its eigenvalues are real and hence lie in the intervals defined by the corresponding Gershgorin discs on the real line.

Let  $\Delta$  denote the maximum vertex degree of  $S$ . For the maximum cover matrix  $M$ , the sum of the absolute values of the off-diagonal entries [5,11] in each row does not exceed  $\Delta$ . Therefore, every eigenvalue  $\mu$  of  $M$  satisfies

$$|\mu| \leq \Delta.$$

Consequently, the spectral radius of  $M$  is bounded above by  $\Delta$ , which plays a crucial role in deriving upper bounds for the maximum covering energy of semigraphs.

**Theorem 3.1** (*Upper Bound*)

Let  $S$  be a semigraph with  $n$  vertices and maximum degree  $\Delta$ . Then,

$$E_{\max}(S) \leq n\Delta.$$

**Proof:** Let  $M$  be the maximum covering matrix of the semigraph  $S$ . Since  $M$  is a real symmetric matrix, all its eigenvalues are real [5,11]. Let  $\mu_1, \mu_2, \dots, \mu_n$  denote the eigenvalues of  $M$ .

By definition, the maximum covering energy of  $S$  is

$$E_{\max}(S) = \sum_{i=1}^n |\mu_i|.$$

For any eigenvalue  $\mu_i$  of  $M$ , we have

$$|\mu_i| \leq \rho(M),$$

where  $\rho(M)$  denotes the spectral radius of  $M$ . Therefore,

$$E_{\max}(S) \leq n \rho(M).$$

Now we estimate  $\rho(M)$ . Each row of the matrix  $M$  corresponds to a vertex of the semigraph and contains at most  $\Delta$  nonzero entries, since  $\Delta$  is the maximum degree of  $S$ . Hence, the sum of the absolute values of the entries in any row of  $M$  is at most  $\Delta$ .

By Gershgorin's Circle Theorem, every eigenvalue of  $M$  lies within a disc centered at a diagonal entry of  $M$  with radius equal to the sum of the absolute values of the off-diagonal entries in the corresponding row [5,11]. Since the diagonal entries of  $M$  are zero, each eigenvalue  $\mu_i$  satisfies

$$|\mu_i| \leq \Delta.$$

Thus,  $\rho(M) \leq \Delta$ , and consequently,

$$E_{\max}(S) \leq n\Delta.$$

This completes the proof. □

**Theorem 3.2** (*Bound in Terms of Number of Edges.*)

*Let  $S$  be a semigraph with  $n$  vertices and  $m$  edges. Then*

$$E_{\max}(S) \leq 2\sqrt{mn}.$$

**Proof:** Let  $M$  be the maximum covering matrix of the semigraph  $S$ . Since  $M$  is a real symmetric matrix, all its eigenvalues are real. Let  $\mu_1, \mu_2, \dots, \mu_n$  denote the eigenvalues of  $M$ .

By definition, the maximum covering energy of  $S$  is

$$E_{\max}(S) = \sum_{i=1}^n |\mu_i|.$$

Applying the Cauchy–Schwarz inequality, we obtain

$$\left( \sum_{i=1}^n |\mu_i| \right)^2 \leq n \sum_{i=1}^n \mu_i^2.$$

The sum of squares of the eigenvalues of  $M$  is equal to the trace of  $M^2$ , that is,

$$\sum_{i=1}^n \mu_i^2 = \text{tr}(M^2).$$

Each edge of the semigraph contributes at most two nonzero entries to the maximum covering matrix. Consequently, the total contribution of all edges to  $\text{tr}(M^2)$  is at most  $4m$ . Hence,

$$\sum_{i=1}^n \mu_i^2 \leq 4m.$$

Substituting this into the earlier inequality yields

$$E_{\max}(S)^2 \leq n \cdot 4m = 4mn.$$

Taking square roots on both sides, we obtain

$$E_{\max}(S) \leq 2\sqrt{mn}.$$

This completes the proof. □

**Theorem 3.3** (*Bound Using Maximum Degree.*)

Let  $S$  be a semigraph with maximum vertex degree  $\Delta$ . Then

$$E_{\max}(S) \leq n\Delta.$$

**Proof:** Let  $M$  be the maximum covering matrix of the semigraph  $S$ . Since  $M$  is a real symmetric matrix, all its eigenvalues are real. Let  $\mu_1, \mu_2, \dots, \mu_n$  denote the eigenvalues of  $M$ .

By definition, the maximum covering energy of  $S$  is

$$E_{\max}(S) = \sum_{i=1}^n |\mu_i|.$$

For any eigenvalue  $\mu_i$  of  $M$ , we have

$$|\mu_i| \leq \rho(M),$$

where  $\rho(M)$  denotes the spectral radius of  $M$ . Hence,

$$E_{\max}(S) \leq n\rho(M).$$

We now estimate the spectral radius  $\rho(M)$ . Each row of the matrix  $M$  corresponds to a vertex of the semigraph  $S$ . Since the maximum vertex degree of  $S$  is  $\Delta$ , each row of  $M$  contains at most  $\Delta$  nonzero entries. Therefore, the sum of the absolute values of the entries in any row of  $M$  is at most  $\Delta$ .

By Gershgorin's Circle Theorem, every eigenvalue of  $M$  lies within a disc centered at a diagonal entry with radius equal to the sum of the absolute values of the off-diagonal entries in the corresponding row [5,11]. Since the diagonal entries of  $M$  are zero, it follows that

$$|\mu_i| \leq \Delta \quad \text{for all } i.$$

Thus,  $\rho(M) \leq \Delta$ , and consequently,

$$E_{\max}(S) \leq n\Delta.$$

This completes the proof. □

**Theorem 3.4** (*Bound in Terms of Covering Cardinality.*)

Let  $S$  be a semigraph and  $|C_{\max}| = k$ . Then

$$E_{\max}(S) \leq k(k-1).$$

**Proof:** Let  $C_{\max}$  be a maximum vertex cover of the semigraph  $S$  with  $|C_{\max}| = k$ . By definition, every edge of  $S$  is incident with at least one vertex in  $C_{\max}$ .

Let  $M$  denote the maximum covering matrix of  $S$ . All edges of  $S$  are represented through the vertices in  $C_{\max}$ . Consequently, all nonzero entries of  $M$  occur within the  $k \times k$  principal submatrix of  $M$  induced by the vertices of  $C_{\max}$ .

Since the diagonal entries of  $M$  are zero, each row of this  $k \times k$  submatrix contains at most  $(k-1)$  nonzero entries. Hence, the sum of the absolute values of the entries in any row is at most  $(k-1)$ .

By Gershgorin's Circle Theorem, every eigenvalue  $\mu$  of this principal submatrix satisfies

$$|\mu| \leq k - 1.$$

All remaining rows of  $M$  outside this submatrix are zero rows. Therefore, the nonzero eigenvalues of  $M$  coincide with the eigenvalues of this  $k \times k$  submatrix.

Thus, the spectral radius of  $M$  satisfies

$$\rho(M) \leq k - 1.$$

Moreover,  $M$  has at most  $k$  nonzero eigenvalues. Hence,

$$E_{\max}(S) = \sum_{i=1}^n |\mu_i| \leq k(k - 1).$$

Therefore,

$$E_{\max}(S) \leq k(k - 1).$$

This completes the proof.  $\square$

**Theorem 3.5** (A Nordhaus–Gaddum Type Inequality.) *Let  $S$  be a semigraph with  $n$  vertices and let  $\bar{S}$  denote its complement. Then*

$$E_{\max}(S) + E_{\max}(\bar{S}) \leq n(n - 1).$$

**Proof:** Let  $\Delta(S)$  and  $\Delta(\bar{S})$  denote the maximum vertex degrees of  $S$  and its complement, respectively.

By the Gershgorin Circle Theorem (see Section 2), every eigenvalue  $\mu$  of the maximum covering matrix of  $S$  satisfies [15]

$$|\mu| \leq \Delta(S).$$

Hence,

$$E_{\max}(S) \leq n\Delta(S).$$

Similarly,

$$E_{\max}(\bar{S}) \leq n\Delta(\bar{S}).$$

For any vertex  $v$  in  $S$ , we have

$$\deg_S(v) + \deg_{\bar{S}}(v) = n - 1,$$

which implies

$$\Delta(S) + \Delta(\bar{S}) \leq n - 1.$$

Combining the above inequalities, we obtain

$$\begin{aligned} E_{\max}(S) + E_{\max}(\bar{S}) &\leq n(\Delta(S) + \Delta(\bar{S})) \\ &\leq n(n - 1). \end{aligned}$$

This completes the proof.  $\square$

**Example 3.1** Consider a star semigraph with one central vertex and  $k$  edges. The maximum covering set consists of all leaf vertices. The maximum covering energy is

$$E_{\max}(S) = 2\sqrt{k},$$

which satisfies both the upper and lower bounds.

**Example 3.2** Consider a semigraph  $S$  with  $n = 6$  vertices and  $m = 5$  edges. Then,

$$E_{\max}(S) \leq 2\sqrt{30} \approx 10.95.$$

**Example 3.3** For a complete semigraph,  $\Delta = n - 1$ , and hence

$$E_{\max}(S) \leq n(n - 1),$$

which shows the sharpness of the bound.

#### 4. Lower Bounds on Maximum Covering Energy

In this section, we obtain new lower bound inequalities for the maximum covering energy of semigraphs using vertex covering number, minimum degree, and edge distribution properties.

##### 4.1. Rayleigh Quotient Technique

In this section, we briefly recall the Rayleigh quotient method, which plays a crucial role in deriving lower bounds for the maximum covering energy of semigraphs.

Let  $M$  be a real symmetric matrix of order  $n$ . The Rayleigh quotient corresponding to a nonzero vector  $x \in \mathbb{R}^n$  is defined by [12]

$$R_M(x) = \frac{x^T M x}{x^T x}.$$

It is well known that the largest eigenvalue  $\mu_1$  of  $M$  satisfies

$$\mu_1 = \max_{x \neq 0} R_M(x).$$

Similarly, the smallest eigenvalue is obtained by minimizing the Rayleigh quotient.

In the context of semigraphs, the maximum covering matrix  $M$  is real and symmetric. Hence, the Rayleigh quotient can be effectively used to obtain lower bounds for the spectral radius of  $M$  by choosing appropriate test vectors reflecting the combinatorial structure of the semigraph, such as vertex degrees or neighborhood configurations.

This technique is particularly useful in establishing lower bounds on the maximum covering energy, since

$$E_{\max}(S) = \sum_{i=1}^n |\mu_i| \geq 2|\mu_1|,$$

whenever the spectrum of  $M$  is symmetric about the origin.

**Theorem 4.1 (Lower Bound)** *For any semigraph  $S$ ,*

$$E_{\max}(S) \geq 2\tau(S),$$

where  $\tau(S)$  denotes the vertex covering number of  $S$ .

**Proof:** Let  $S$  be a semigraph with vertex set  $V(S)$  and let  $\tau(S) = k$ . Then there exists a minimum vertex cover

$$C = \{v_1, v_2, \dots, v_k\}$$

such that every edge of  $S$  is incident with at least one vertex of  $C$ .

Let  $M$  denote the maximum covering matrix of  $S$ . Since  $M$  is a real symmetric matrix, all its eigenvalues are real [12]. Let  $\mu_1, \mu_2, \dots, \mu_n$  be the eigenvalues of  $M$ .

Each vertex in the cover  $C$  corresponds to a nonzero row in  $M$ . Hence, the rank of  $M$  is at least  $k$ , implying that  $M$  has at least  $k$  nonzero eigenvalues [13].

Moreover, the trace of  $M$  is zero. Therefore,

$$\sum_{i=1}^n \mu_i = 0.$$

This implies that the positive and negative eigenvalues must balance each other. Consequently, the sum of the absolute values of the eigenvalues is at least twice the number of nonzero eigenvalues.

Hence,

$$E_{\max}(S) = \sum_{i=1}^n |\mu_i| \geq 2k = 2\tau(S).$$

This completes the proof. □

**Theorem 4.2** (*Lower Bound via Covering Number.*)

Let  $S$  be a semigraph with covering number  $\tau(S)$ . Then

$$E_{\max}(S) \geq 2\tau(S).$$

**Proof:** Let  $S$  be a semigraph on  $n$  vertices and let  $\tau(S) = k$ . Then there exists a minimum vertex cover

$$C = \{v_1, v_2, \dots, v_k\}$$

such that every edge of  $S$  is incident with at least one vertex in  $C$ .

Let  $M$  denote the maximum covering matrix of  $S$ . Since  $M$  is a real symmetric matrix, all its eigenvalues are real. Let  $\mu_1, \mu_2, \dots, \mu_n$  be the eigenvalues of  $M$ .

Each vertex belonging to the vertex cover  $C$  corresponds to a nonzero row in the matrix  $M$ . Hence, the rank of  $M$  is at least  $k$ , which implies that  $M$  has at least  $k$  nonzero eigenvalues.

Moreover, the trace of  $M$  is zero, and therefore

$$\sum_{i=1}^n \mu_i = 0.$$

This implies that the positive and negative eigenvalues must balance each other in sign. Consequently, the sum of the absolute values of the nonzero eigenvalues is at least twice their number.

Hence,

$$E_{\max}(S) = \sum_{i=1}^n |\mu_i| \geq 2k = 2\tau(S).$$

This completes the proof. □

**Theorem 4.3** (*Lower Bound Using Minimum Degree.*)

Let  $S$  be a semigraph with minimum vertex degree  $\delta$ . Then

$$E_{\max}(S) \geq 2\sqrt{\delta}.$$

**Proof:** Let  $M$  be the maximum covering matrix of the semigraph  $S$ . Since  $M$  is a real symmetric matrix, all its eigenvalues are real. Let  $\mu_1, \mu_2, \dots, \mu_n$  be the eigenvalues of  $M$ , where  $\mu_1$  denotes the largest eigenvalue [12].

By the Rayleigh quotient characterization of eigenvalues,

$$\mu_1 = \max_{x \neq 0} \frac{x^T M x}{x^T x}.$$

Let  $v$  be a vertex of  $S$  with minimum degree  $\delta$ . Define a vector  $x \in \mathbb{R}^n$  by

$$x_i = \begin{cases} 1, & \text{if } i = v \text{ or } i \text{ is adjacent to } v, \\ 0, & \text{otherwise.} \end{cases}$$

Then  $x$  has exactly  $\delta + 1$  nonzero components.

Evaluating the Rayleigh quotient for this choice of  $x$ , we obtain

$$x^T M x \geq 2\delta, \quad \text{and} \quad x^T x = \delta + 1.$$

Hence,

$$\mu_1 \geq \frac{2\delta}{\delta + 1} \geq \sqrt{\delta}.$$

Since the trace of  $M$  is zero, the sum of its eigenvalues is zero. Therefore, the positive and negative eigenvalues must balance each other. In particular, the sum of the absolute values of the eigenvalues satisfies

$$E_{\max}(S) = \sum_{i=1}^n |\mu_i| \geq 2|\mu_1|.$$

Thus,

$$E_{\max}(S) \geq 2\sqrt{\delta}.$$

This completes the proof.  $\square$

**Theorem 4.4** (*Lower Bound in Terms of Number of Edges.*)

Let  $S$  be a semigraph with  $m$  edges. Then

$$E_{\max}(S) \geq \sqrt{2m}.$$

**Proof:** Let  $M$  denote the maximum covering matrix of the semigraph  $S$ . Since  $M$  is a real symmetric matrix, all its eigenvalues are real. Let  $\mu_1, \mu_2, \dots, \mu_n$  be the eigenvalues of  $M$ .

By definition, the maximum covering energy of  $S$  is

$$E_{\max}(S) = \sum_{i=1}^n |\mu_i|.$$

The sum of the squares of the eigenvalues [5] of  $M$  is equal to the trace of  $M^2$ , that is,

$$\sum_{i=1}^n \mu_i^2 = \text{tr}(M^2).$$

Each edge of the semigraph contributes at least two nonzero entries to  $M^2$ . Therefore,

$$\text{tr}(M^2) \geq 2m.$$

Using the inequality

$$\sum_{i=1}^n |\mu_i| \geq \sqrt{\sum_{i=1}^n \mu_i^2},$$

we obtain

$$E_{\max}(S) \geq \sqrt{2m}.$$

This completes the proof.  $\square$

**Theorem 4.5** (*Lower Bound via Covering Cardinality.*)

Let  $|C_{\max}| = k$ . Then

$$E_{\max}(S) \geq k.$$

**Proof:** Let  $C_{\max} = \{v_1, v_2, \dots, v_k\}$  be a maximum vertex cover of the semigraph  $S$ . By definition, every edge of  $S$  is incident with at least one vertex in  $C_{\max}$ .

Let  $M$  denote the maximum covering matrix of  $S$ . Since  $M$  is a real symmetric matrix, all its eigenvalues are real. Let  $\mu_1, \mu_2, \dots, \mu_n$  be the eigenvalues of  $M$ .

Each vertex belonging to the vertex cover  $C_{\max}$  corresponds to a nonzero row in the matrix  $M$ . Hence, the rank of  $M$  is at least  $k$ . Therefore,  $M$  has at least  $k$  nonzero eigenvalues.

For any nonzero eigenvalue  $\mu_i$ , we have  $|\mu_i| \geq 1$ , since the entries of the maximum covering matrix are integers. Consequently, the contribution of each nonzero eigenvalue to the energy is at least one.

Thus,

$$E_{\max}(S) = \sum_{i=1}^n |\mu_i| \geq k.$$

Hence,

$$E_{\max}(S) \geq k.$$

This completes the proof.  $\square$

**Theorem 4.6** (*Lower Nordhaus–Gaddum Bound.*) *Let  $S$  be a semigraph with  $n$  vertices and let  $\bar{S}$  denote its complement. Then*

$$E_{\max}(S) + E_{\max}(\bar{S}) \geq 2(\tau(S) + \tau(\bar{S})).$$

**Proof:** Let  $\tau(S)$  and  $\tau(\bar{S})$  denote the vertex covering numbers of  $S$  and  $\bar{S}$ , respectively.

From the lower bound established in Section 3, we know that for any semigraph  $T$ ,

$$E_{\max}(T) \geq 2\tau(T).$$

Applying this inequality to  $S$  and  $\bar{S}$  separately, we obtain

$$E_{\max}(S) \geq 2\tau(S) \quad \text{and} \quad E_{\max}(\bar{S}) \geq 2\tau(\bar{S}).$$

Adding the above inequalities yields

$$E_{\max}(S) + E_{\max}(\bar{S}) \geq 2(\tau(S) + \tau(\bar{S})).$$

This completes the proof.  $\square$

**Example 4.1** Consider a semigraph with  $\tau(S) = 3$ . Then,

$$E_{\max}(S) \geq 6.$$

**Example 4.2** Let  $S$  be a path semigraph with  $\delta = 1$ . Then

$$E_{\max}(S) \geq 2,$$

showing the bound is tight.

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