Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2026 (44) 14** : 1–7.
ISSN-0037-8712
doi:10.5269/bspm.81899

On $N(k)$-Contact Metric Manifolds Admitting Conharmonic Curvature Tensor

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ABSTRACT: In this paper, we study $N(k)$-contact metric manifolds with conharmonic curvature tensor. Here, we discuss conharmonically pseudo-symmetric, conharmonically semi-symmetric $N(k)$-contact metric manifolds. Also, we study $N(k)$-contact metric manifolds satisfying $C \cdot R = 0$ and $C \cdot S = 0$, where C, R and S, respectively, denote conharmonic curvature tensor, Riemannian curvature tensor, and Ricci tensor.

Keywords: $N(k)$-contact metric manifold, conharmonic curvature tensor, η-Einstein manifold.

Contents

1. Introduction

In 1957, Ishii [12] introduced the notion of conharmonic transformation as a subgroup of the conformal transformation [19] satisfying the condition

$$\sigma,_i^i + \sigma,_i \sigma,^i = 0, \tag{1.1}$$

where σ is a real function and comma $(,)$ indicates the covariant differentiation with respect to metric g. A conharmonic curvature tensor C for $(2n+1)$-dimensional Riemannian manifold M, which remains invariant under conharmonic transformation, is given by [8]

$$C(X,Y)Z = R(X,Y)Z - \frac{1}{2n-1}\Big[g(Y,Z)QX - g(X,Z)QY + S(Y,Z)X - S(X,Z)Y\Big], \tag{1.2}$$

where R, S and Q are Riemannian curvature tensors of type (1,3), Ricci tensor of type (0,2) and Ricci operator defined by $g(QX,Y) = S(X,Y)$ respectively. The conharmonic curvature tensor C has many applications in physics and, in particular, space-time of general theory of relativity and perfect fluid cosmological models. The conharmonic curvature tensor in a Riemannian manifold with different structures has been studied by authors such as [7,2,16,11] and several others. $N(k)$-contact metric manifolds are studied in [13,9,14,17,6] and references therein.

In this paper, we study $N(k)$-contact metric manifolds satisfying certain conditions on conharmonic curvature tensor. The paper is structured as follows: In Section 2, we give some preliminaries that will be needed thereafter. In Section 3, we prove that a conharmonically semi-symmetric $N(k)$-contact metric manifold is an η-Einstein manifold. Section 4 is devoted to the study of conharmonically pseudo-symmetric $N(k)$-contact metric manifold and we prove that either $L_C = k$ or the manifold is an η-Einstein manifold. The next two sections deal with $N(k)$-contact metric manifolds satisfying $C \cdot R = 0$ and $C \cdot S = 0$ respectively. Conclusions are given in Section 7.

2020 *Mathematics Subject Classification:* 53C15, 53C25.
Submitted February 11, 2026. Published April 30, 2026.

2. Preliminaries

Let M be a $(2n+1)$-dimensional contact metric manifold with (ϕ,ξ,η,g), where ϕ is a tensor field of type (1,1), ξ is a vector field, η is a 1-form and g is a Riemannian metric on M such that

$$\phi^2(X) = -X + \eta(X)\xi \tag{2.1}$$

$$g(\phi X,\phi Y) = g(X,Y) - \eta(X)\eta(Y) \tag{2.2}$$

$$\phi\xi = 0,\ \ \eta(\phi X) = 0,\ \ \eta(X) = g(X,\xi),\ \ \eta(\xi) = 1 \tag{2.3}$$

The notion of (k,μ)-nullity distribution of a contact metric manifold M was introduced by Blair et al. [5] and is defined by

$$N(k,\mu) : p \to N_p(k,\mu) = \{U \in T_pM \mid R(X,Y)U = (kI+\mu h)[g(Y,U)X - g(X,U)Y]\},$$

for all $X,Y \in TM$, where $(k,\mu) \in R^2$. A contact metric manifold with $\xi \in N(k,\mu)$ is called a (k,μ)-contact metric manifold. If $\mu = 0$, the (k,μ)-nullity distribution reduces to k-nullity distribution. The k-nullity distribution $N(k)$ of a Riemannian manifold is defined by [21]

$$N(k) : p \to N_p(k) = \{U \in T_pM \mid R(X,Y)U = k[g(Y,U)X - g(X,U)Y]\},$$

k being a constant. If the characteristic vector field $\xi \in N(k)$, then we call a contact metric manifold as $N(k)$-contact metric manifold [4]. If $k = 1$, then the manifold is Sasakian and if $k = 0$, then the manifold is locally isometric to the product $E^{n+1}(0) \times S^n(4)$ for $n > 1$ and flat for $n = 1$ [3]. For $N(k)$-contact metric manifold, we have the following relations:

$$h^2 = (k-1)\phi^2, \tag{2.4}$$

$$R(X,Y)\xi = k[\eta(Y)X - \eta(X)Y], \tag{2.5}$$

$$S(X,Y) = [2(n-1) - n\mu]g(X,Y) + [2(n-1)+\mu]g(hX,Y), \tag{2.6}$$

$$S(\phi X,\phi Y) = S(X,Y) - 2nk\eta(X)\eta(Y) - 4(n-1)g(hX,Y), \tag{2.7}$$

$$S(X,\xi) = 2nk\eta(X), \tag{2.8}$$

$$(\nabla_X\eta)(Y) = g(X+hX,\phi Y), \tag{2.9}$$

$$r = 2n(2n-2+k), \tag{2.10}$$

$$\nabla_X\xi = -\phi X - \phi hX. \tag{2.11}$$

Definition 2.1 *A $(2n+1)$-dimensional $N(k)$-contact metric manifold M is said to be η-Einstein if its Ricci tensor S is of the form*

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y),$$

for all vector fields X and Y, where a and b are constants.

3. Conharmonically Semi-Symmetric $N(k)$-Contact Metric Manifolds

Theorem 3.1 *A conharmonically semi-symmetric $(2n+1)$-dimensional $N(k)$-contact metric manifold M is an η-Einstein manifold.*

Proof: Suppose $N(k)$-contact metric manifold M is conharmonically semi-symmetric. Then we have,

$$R(\xi,Y)\cdot C(U,V)W = 0. \tag{3.1}$$

This equation can be written as

$$R(\xi,Y)C(U,V)W - C(R(\xi,Y)U,V)W - C(\xi,R(Y,U)V)W - C(\xi,Y)R(U,V)W = 0. \tag{3.2}$$

Applying (1.2) and (2.5) in (3.2), we get

$$\begin{aligned}0 = k[&g(Y,R(U,V)W)\xi - g(\xi,R(U,V)W)Y - g(Y,U)R(\xi,V)W + \eta(U)R(Y,V)W\\
&- g(U,V)R(\xi,Y)W + g(Y,V)R(\xi,U)W - g(V,W)R(\xi,Y)U + g(U,W)R(\xi,Y)V]\\
&- \frac{k}{2n-1}[g(V,W)g(Y,QU)\xi - g(U,W)g(Y,QV)\xi + S(V,W)g(Y,U)\xi - S(U,W)g(Y,V)\xi\\
&- g(V,W)\eta(QU)Y + g(U,W)\eta(QV)Y - S(V,W)\eta(U)Y + S(U,W)\eta(V)Y]\\
&+ \frac{k}{2n-1}g(Y,U)[g(V,W)Q\xi - g(\xi,W)QV + S(V,W)\xi - S(\xi,W)V]\\
&- \frac{k}{2n-1}\eta(U)[g(V,W)QY - g(Y,W)QV + S(V,W)Y - S(Y,W)V]\\
&+ \frac{k}{2n-1}g(U,V)[g(Y,W)Q\xi - g(\xi,W)QY + S(V,W)\xi - S(\xi,W)Y]\\
&- \frac{k}{2n-1}g(Y,V)[g(U,W)Q\xi - g(\xi,W)QU + S(U,W)\xi - S(\xi,W)U]\\
&+ \frac{k}{2n-1}g(V,W)[g(Y,U)Q\xi - g(\xi,U)QY + S(Y,U)\xi - S(\xi,U)Y]\\
&- \frac{k}{2n-1}g(U,W)[g(Y,V)Q\xi - g(\xi,V)QY + S(Y,V)\xi - S(\xi,V)Y].\end{aligned} \tag{3.3}$$

Now by taking $U = W = \xi$ in (3.3), we obtain

$$0 = \frac{k}{2n-1}\Big[8nk\eta(V)\eta(Y)\xi - 2nk\eta(V)Y - 2\eta(V)QY - \eta(QV)Y] + 2k^2[\eta(V)Y - \eta(V)\eta(Y)\xi\Big]. \tag{3.4}$$

Put $V = \xi$ in (3.4) and using (2.3) and (2.8), we get

$$QY = (2n+1)k\eta(Y)\xi - kY. \tag{3.5}$$

This implies that

$$S(Y,V) = ag(Y,V) + b\eta(Y)\eta(V), \tag{3.6}$$

where $a = -k$ and $b = (2n+1)k$ □

4. Conharmonically Pseudo-Symmetric $N(k)$-Contact Metric Manifolds

A Riemannian manifold M is called locally symmetric if its curvature tensor R satisfies $\nabla R = 0$, where ∇ denotes the Levi-Civita connection. As a proper generalization of a locally symmetric manifold, the notion of a semi-symmetric manifold was defined and is given by

$$(R(X,Y)\cdot R)(U,V)W = 0,$$

where X, Y, U, V, W are differentiable vector fields on M. Locally symmetric manifolds have been studied by several authors, namely [15,20,21]. In 1992, Deszcz [10] introduced the notion of pseudo-symmetric manifolds, which are defined by the condition $(R(X,Y)\cdot R)(U,V)W = L_R[((X \wedge_A Y)\cdot R)(U,V)W]$, where L_R is some smooth function on U_R, A is a symmetric tensor field of type (0,2) on M, and $X \wedge_A Y$ is an endomorphism defined by

$$(X \wedge_A Y)Z = A(Y,Z)X - A(X,Z)Y. \tag{4.1}$$

Definition 4.1 *A $(2n+1)$-dimensional N(k)-contact metric manifold M is said to be conharmonically pseudo-symmetric if*

$$R\cdot C = L_C Q(g,C) \tag{4.2}$$

holds on the set $U_C = \{x \in M : C \neq 0\}$ at x, where L_C is some function on U_C and C is the conharmonic curvature tensor.

Theorem 4.1 *If a $(2n+1)$-dimensional $(n \geq 1)$ $N(k)$-contact metric manifold M is conharmonically pseudo-symmetric, then either $L_C = k$ or the manifold is an η-Einstein manifold.*

Proof: From (4.2), we have

$$(R(X,\xi)\cdot C)(U,V)W = L_C[((X\wedge_g \xi)\cdot C)(U,V)W]. \tag{4.3}$$

Now left hand side of (4.3) can be written as
$R(X,\xi)C(U,V)W - C(R(X,\xi)U,V)W - C(U,R(X,\xi)V)W - C(U,V)R(X,\xi)W$.
In view of (2.5), the above expression becomes

$$\begin{aligned} &k[g(\xi,C(U,V)W)X - g(X,C(U,V)W)\xi - \eta(U)C(X,V)W + g(X,U)C(\xi,V)W \\ &-\eta(V)C(U,X)W + g(X,V)C(U,\xi)W - \eta(W)C(U,V)X + g(X,W)C(U,V)\xi]. \end{aligned} \tag{4.4}$$

The right hand side of (4.3) is

$$L_C[(X\wedge_g \xi)C(U,V)W - C((X\wedge_g\xi)U,V)W - C(U,(X\wedge_g\xi)V)W - C(U,V)(X\wedge_g\xi)W]. \tag{4.5}$$

Using (4.1) in (4.5), we get

$$\begin{aligned} &L_C[g(\xi,C(U,V)W)X - g(X,C(U,V)W)\xi - \eta(U)C(X,V)W + g(X,U)C(\xi,V)W \\ &-\eta(V)C(U,X)W + g(X,V)C(U,\xi)W - \eta(W)C(U,V)X + g(X,W)C(U,V)\xi]. \end{aligned} \tag{4.6}$$

Using (4.4) and (4.6) in (4.3), we obtain

$$\begin{aligned} 0 = {}&(k - L_C)[g(\xi,C(U,V)W)X - g(X,C(U,V)W)\xi - \eta(U)C(X,V)W + g(X,U)C(\xi,V)W \\ &-\eta(V)C(U,X)W + g(X,V)C(U,\xi)W - \eta(W)C(U,V)X + g(X,W)C(U,V)\xi]. \end{aligned} \tag{4.7}$$

This implies that $L_C = k$ or

$$\begin{aligned} 0 = {}&g(\xi,C(U,V)W)X - g(X,C(U,V)W)\xi - \eta(U)C(X,V)W + g(X,U)C(\xi,V)W \\ &-\eta(V)C(U,X)W + g(X,V)C(U,\xi)W - \eta(W)C(U,V)X + g(X,W)C(U,V)\xi. \end{aligned} \tag{4.8}$$

Taking inner product of (4.8) with ξ and using (2.1), we get

$$\begin{aligned} 0 = {}&\eta(C(U,V)W)\eta(X) - g(X,C(U,V)W) - \eta(U)\eta(C(X,V)W) + g(X,U)\eta(C(\xi,V)W) \\ &-\eta(V)\eta(C(U,X)W) + g(X,V)\eta(C(U,\xi)W) - \eta(W)\eta(C(U,V)X) + g(X,W)\eta(C(U,V)\xi). \end{aligned} \tag{4.9}$$

By virtue of (1.2), we have

$$\eta(C(\xi,V)W) = \frac{1}{2n-1}\Big[-kg(V,W) + k(2n+1)\eta(V)\eta(W) - S(V,W)\Big] \text{ and} \tag{4.10}$$

$$\eta(C(U,V)\xi) = 0. \tag{4.11}$$

Using (4.10) and (4.11) in (4.9), we get

$$\begin{aligned} 0 = {}&\eta(C(U,V)W)\eta(X) - C(U,V,W,X) - \eta(U)\eta(C(X,V)W) + \left(\frac{1}{2n-1}\right)g(X,U) \\ &\Big[-kg(V,W) + k(2n+1)\eta(V)\eta(W) - S(V,W)\Big] - \eta(V)\eta(C(U,X)W) + \left(\frac{1}{2n-1}\right)g(X,V) \\ &\Big[kg(U,W) - k(2n+1)\eta(U)\eta(W) + S(U,W)\Big] - \eta(W)\eta(C(U,V)X). \end{aligned} \tag{4.12}$$

Using (1.2) and (2.5) in (4.12), we obtain

$$\begin{aligned}0=&\left(\frac{1}{2n-1}\right)\Big[-2nkg(X,V)\eta(U)\eta(W)+2nkg(X,U)\eta(V)\eta(W)-kg(X,U)g(V,W)\\&+kg(U,W)g(X,V)-g(X,U)S(V,W)+g(X,V)S(U,W)+S(V,X)\eta(U)\eta(W)-S(V,W)\\&-S(U,X)\eta(V)\eta(W)-S(U,X)\eta(V)\eta(W)+S(U,W)g(X,V)\Big]-C(U,V,W,X).\end{aligned}\tag{4.13}$$

Putting $X=U=e_i$ in (4.13), where $\{e_i\}$ $i=1,2,...,2n+1$ is an orthonormal basis of the tangent space at each point of the manifold and taking summation over i, we get

$$S(V,W)=ag(V,W)+b\eta(V)\eta(W),\tag{4.14}$$

where $a=\frac{r-2nk}{2n-1}$ and $b=\frac{2nk(2n+1)-r}{2n-1}$. □

5. $N(k)$-Contact Metric Manifolds Satisfying $C\cdot R=0$

Theorem 5.1 *If a $(2n+1)$-dimensional $(n\geq 1)$ $N(k)$-contact metric manifold M satisfies $C(\xi,X)\cdot R=0$, then the manifold is locally isometric to the product $E^{(n+1)}(0)\times S^n(4)$ or $g(X,Y)=(2n+1)\eta(X)\eta(Y)$.*

Proof: Suppose M satisfies $C(\xi,X)\cdot R(Y,Z)U=0$.
Then we have

$$C(\xi,X)R(Y,Z)U-R(C(\xi,X)Y,Z)U-R(Y,C(\xi,X)Z)U-R(Y,Z)C(\xi,X)U=0.\tag{5.1}$$

In view of (1.2), (5.1) becomes

$$\begin{aligned}0=&k[g(Z,U)R(\xi,X)Y-g(Y,U)R(\xi,X)Z-g(X,Y)R(\xi,Z)U+\eta(Y)R(X,Z)U\\&-g(X,Z)R(Y,\xi)U+\eta(Z)R(Y,X)U-g(X,U)R(Y,Z)\xi+\eta(U)R(Y,Z)X]\\&+\frac{1}{2n-1}\Big[-g(X,R(Y,Z)U)Q\xi+\eta(R(Y,Z)U)QX-S(X,R(Y,Z)U)\xi\\&+2nk\eta(R(Y,Z)U)X+g(X,Y)R(Q\xi,Z)U-\eta(Y)R(QX,Z)U+S(X,Y)R(\xi,Z)U\\&-2nk\eta(Y)R(X,Z)U+g(X,Z)R(Y,Q\xi)U-\eta(Z)R(Y,QX)U+S(X,Z)R(Y,\xi)U\\&-2nk\eta(Z)R(Y,X)U+g(X,U)R(Y,Z)Q\xi-\eta(U)R(Y,Z)QX+S(X,U)R(Y,Z)\xi\\&+2nk\eta(U)R(Y,Z)X\Big].\end{aligned}\tag{5.2}$$

Taking inner product of (5.2) with ξ, we get

$$\begin{aligned}0=&k[g(Z,U)\eta(R(\xi,X)Y)-g(Y,U)\eta(R(\xi,X)Z)-g(X,Y)\eta(R(\xi,Z)U)\\&+\eta(Y)\eta(R(X,Z)U)-g(X,Z)\eta(R(Y,\xi)U)+\eta(Z)\eta(R(Y,X)U)-g(X,U)\eta(R(Y,Z)\xi)\\&+\eta(U)\eta(R(Y,Z)X)]+\frac{1}{2n-1}\Big[-g(X,R(Y,Z)U)2nk+\eta(R(Y,Z)U)\eta(QX)\\&-S(X,R(Y,Z)U)+2nk\eta(R(Y,Z)U)\eta(X)+g(X,Y)\eta(R(Q\xi,Z)U)\\&-\eta(Y)\eta(R(QX,Z)U)+S(X,Y)\eta(R(\xi,Z)U)-2nk\eta(Y)\eta(R(X,Z)U)\\&+g(X,Z)\eta(R(Y,Q\xi)U)-\eta(Z)\eta(R(Y,QX)U)+S(X,Z)\eta(R(Y,\xi)U)\\&-2nk\eta(Z)\eta(R(Y,X)U)+g(X,U)\eta(R(Y,Z)Q\xi)-\eta(U)\eta(R(Y,Z)QX)\\&+S(X,U)\eta(R(Y,Z)\xi)+2nk\eta(U)\eta(R(Y,Z)X)\Big].\end{aligned}\tag{5.3}$$

Using (2.5) in (5.3) and on simplification, we get

$$\frac{2nk^2}{2n-1}\Big[g(X,Z)g(Y,U)+\eta(X)\eta(Y)g(Z,U)-2\eta(U)\eta(Z)g(X,Y)-\eta(Z)\eta(Y)g(X,U)+\eta(U)\eta(Y)g(Z,X)\Big]=0. \tag{5.4}$$

Putting $Z=U=e_i$ in (5.4) and taking summation over $i, 1\leq i\leq 2n+1$, we get

$$\frac{2nk^2}{2n-1}\Big[(2n+1)\eta(X)\eta(Y)-g(X,Y)\Big]=0. \tag{5.5}$$

This implies that either

$$k=0 \ \text{ or } \ g(X,Y)=(2n+1)\eta(X)\eta(Y). \tag{5.6}$$

□

6. $N(k)$-Contact Metric Manifolds Satisfying $C\cdot S=0$

Theorem 6.1 *A $(2n+1)$-dimensional $(n\geq 1)$ $N(k)$-contact metric manifold M satisfying $C\cdot S$=0 is either locally isometric to the product $E^{(n+1)}(0)\times S^n(4)$ or an Einstein manifold.*

Proof: Suppose $N(k)$-contact metric manifold M satisfies $C(\xi,X)\cdot S(Y,Z)=0$.
Then, we have

$$S(C(\xi,X)Y,Z)+S(Y,C(\xi,X)Z)=0. \tag{6.1}$$

In view of (1.2) and (2.8), (6.1) becomes

$$k[2nk\eta(Z)g(X,Y)-\eta(Y)S(X,Z)+2nk\eta(Y)g(X,Z)-\eta(Z)S(Y,X)] - \frac{2nk}{2n-1}\Big[2nk\eta(Z)g(X,Y)-\eta(Y)S(X,Z)+2nk\eta(Y)g(X,Z)-\eta(Z)S(X,Y)\Big]=0. \tag{6.2}$$

Putting $Z=\xi$ in (6.2), we obtain

$$k[2nkg(X,Y)-S(X,Y)]=0, \tag{6.3}$$

which implies that either $k=0$ or $S(X,Y)=2nkg(X,Y)$. □

7. Conclusions

In this paper, we have studied $N(k)$-contact metric manifolds considering conharmonic curvature tensor. We proved that a conharmonically semi-symmetric $N(k)$-contact metric manifold becomes an η-Einstein manifold. We observed that a conharmonically pseudo-symmetric $N(k)$-contact metric manifold also becomes an η-Einstein manifold if $L_C\neq 0$. We have discussed $N(k)$-contact metric manifolds satisfying certain conditions, such as $C\cdot R=0$ and $C\cdot S=0$, and obtained some interesting results.

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