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$\mathcal{I}$ and $\mathcal{I}^*$-Soft Convergence in Soft Topological Spaces

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ABSTRACT: In this paper, we introduce the notions of $\mathcal{I}$ and $\mathcal{I}^*$-soft convergence of sequences of soft points in soft topological spaces and study some basic properties of these notions. Also we introduce the notions of $\mathcal{I}$-soft limit points and $\mathcal{I}$-soft cluster points of a sequence of soft points in a soft topological space and study their interrelationship.

Keywords: Soft set, $\mathcal{I}$-soft convergence, $\mathcal{I}$-soft limit point, $\mathcal{I}$-soft cluster point.

Contents

1. Introduction and Background

In this era, it is a major concern to solve the problems involving uncertainties. Soft set theory is one of the best tool to tackle this kind of problems. The concept of soft set theory was first introduced by Molodtsov [26] in the year 1999. After this tremendous work, basic algebraic properties of soft sets were studied by Maji et al. [24]. For more primary works in this line one can see ([1,2,3] etc.). Because of immense importance the notion of soft topology was introduced independently by Shabir et al. [30] and Çağman et al. [7] in 2011. Afterthat various topological properties were developed by Aygünoğlu et al. [4], Das et al. [9], N. Xie [31] and many others.

On the other hand the notion of statistical convergence of sequences of real numbers was first introduced by H.Fast [12] and also independently by I.J. Schoenberg [29] using the concept of natural density. After the works of Šalát [27] and Fridy [14,15], it has become one of the most active research area in summability theory. This notion of statistical convergence was further extended to ideal convergence by Kostyrko et al. [21] using the concept of ideal of subsets of $\mathbb{N}$. Because of great importance the notion of statistical convergence for sequences in topological spaces was introduced by Maio et al. [11] and the notion of ideal convergence was developed in topological space by Lahiri et al. [23].

Recently in soft topological spaces the notion of soft convergence has been introduced by Demir et al. [10] and the notion of weighted statistical soft convergence has been introduced by Bayrama et al. [5] which extends the notion of soft convergence [6] as well as the notion of statistical soft convergence [5] of soft points and some basic properties of these notions have been studied. In this paper we introduce and study the notion of $\mathcal{I}$-soft convergence of sequences of soft points in soft topological spaces, which extends both the notion of soft convergence and statistical soft convergence. Further we introduce the notion of $\mathcal{I}^*$-soft convergence and study it relationship with $\mathcal{I}$-soft convergence. In section 5 of this paper we introduce the notions of $\mathcal{I}$-soft limit point and $\mathcal{I}$ -soft cluster point of a sequence of soft points in a soft topological space and study their interconnection.

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2. Basic definitions and notations

In this section we first recall some basic definitions and notations related to statistical and ideal convergence.

Definition 2.1 *[12,29] Let $B \subset \mathbb{N}$ and $B(n) = \{k \in B : k \leq n\}$. Then B is said to have natural density $d(B)$, if $d(B) = \lim_{n\to\infty} \frac{|B(n)|}{n}$, where $|B(n)|$ denotes cardinality of the set $B(n)$.*

Definition 2.2 *[11] Let (X, τ) be a topological space. A sequence $\{\eta_n\}_{n\in\mathbb{N}}$ in (X, τ) is said to be statistically convergent to a point $x \in X$ if for every neighbourhood U of x,*

$$d(\{n \in \mathbb{N} : \eta_n \notin U\}) = 0.$$

In this case, we write $st - \lim_{n\to\infty} \eta_n = x$.

Definition 2.3 *[22] Let X be a non empty set and $\mathcal{I}$ be a collection of subsets of X. Then, $\mathcal{I}$ is said to be an ideal in X if,*

(i) $\emptyset \in \mathcal{I}$,

(ii) $A \in \mathcal{I}$ *and* $B \subset A \implies B \in \mathcal{I}$,

(iii) $A \in \mathcal{I}$ *and* $B \in \mathcal{I} \implies A \cup B \in \mathcal{I}$.

An ideal $\mathcal{I}$ of X is said to be non-trivial if $\mathcal{I} \neq \{\emptyset\}$ and $X \notin \mathcal{I}$.
A non-trivial ideal $\mathcal{I}$ of X is said to be admissible if $\{x\} \in \mathcal{I}$, for every $x \in X$.

Definition 2.4 *[22] Let X be a non-empty set and $\mathcal{F}$ be a non empty collection of subsets of X. Then, $\mathcal{F}$ is said to be a filter on X if,*

(i) $\emptyset \notin \mathcal{F}$,

(ii) $A \in \mathcal{F}$ *and* $B \supset A \implies B \in \mathcal{F}$,

(iii) $A \in \mathcal{F}$ *and* $B \in \mathcal{F} \implies A \cap B \in \mathcal{F}$.

Definition 2.5 *[22] Let $\mathcal{I}$ be a non-trivial ideal of a non-empty set X. Then the family of sets $\mathcal{F}(\mathcal{I}) = \{A \subset X : \exists\, B \in \mathcal{I}$ such that $A = X - B\}$ is a filter on X, which is called filter associated with the ideal $\mathcal{I}$.*

Through out this paper, we consider that $\mathcal{I}$ as an admissible ideal of $\mathbb{N}$ unless mentioned otherwise.

Definition 2.6 *[21,23] Let (X, τ) be a topological space. A sequence $\{\eta_n\}_{n\in\mathbb{N}}$ in (X, τ) is said to be $\mathcal{I}$-convergent to a point $x \in X$, if for every neighbourhood U of x, $\{n \in \mathbb{N} : \eta_n \notin U\} \in \mathcal{I}$. In this case we write $\mathcal{I} - \lim_{n\to\infty} \eta_n = x$.*

Definition 2.7 *[21,23] Let (X, τ) be a topological space. A sequence $\{\eta_n\}_{n\in\mathbb{N}}$ in (X, τ) is said to $\mathcal{I}^*$-convergent to a point $x \in X$ if there exists a set $N = \{n_1 < n_2 < ... < n_k < ...\} \in \mathcal{F}(\mathcal{I})$ such that $\lim_{k\to\infty} \eta_{n_k} = x$.*

Definition 2.8 *[21,23] Let (X, τ) be a topological space and $\{\eta_n\}_{n\in\mathbb{N}}$ be a sequence in X.*

(a) $x \in X$ *is called an $\mathcal{I}$-limit point of $\{\eta_n\}_{n\in\mathbb{N}}$ if there exists a set $N = \{n_1 < n_2 < ...\} \subset \mathbb{N}$ such that $N \notin \mathcal{I}$ and $\lim_{k\to\infty} \eta_{n_k} = x$.*

(b) $x \in X$ *is called an $\mathcal{I}$-cluster point of $\{\eta_n\}_{n\in\mathbb{N}}$ if for every neighbourhood V containing x, $\{n : \eta_n \in V\} \notin \mathcal{I}$.*

Following [7,24,26,30] we now recall basic concepts of soft set theory.

Definition 2.9 *[7,24,26] Let X be an initial universe set, S be a set of parameters and $M \subset S$. A soft set G_M over X is defined by the set of all ordered pairs $G_M = \{(s, g_M(s)) : s \in S\}$, where $g_M : S \to \mathcal{P}(X)$ is a mapping such that $g_M(s) = \emptyset,\ \forall\ s \in S - M$.*

Throughout this paper we take X as an initial universe set, S as a set of parameters, $M \subset S$, $\mathcal{P}(X)$ as power set of X, G_M as a soft set over X and $\mathcal{A}(X)$ as a set of all soft sets over X unless otherwise mentioned.

Example 2.1 *Let the initial universe set X consist of five universities, say, $X = \{x_1, x_2, x_3, x_4, x_5\}$. Let $S = \{s_1, s_2, s_3, s_4\}$ be a set of parameters, where s_1 stands for 'high quality research', s_2 stands for 'excellent faculties', s_3 stands for 'intelligent students' and s_4 stands for 'modern research lab'. Let $M = \{s_1, s_2, s_3\} \subset S$ and $g_M : S \to \mathcal{P}(X)$ be given by*

$g_M(s_1) = \{x_1\}$, $g_M(s_2) = \{x_1, x_3, x_5\}$, $g_M(s_3) = \{x_1, x_4, x_5\}$ and $g_M(s_4) = \emptyset$.

Note that $g_M(s_i)$ literally represents the set of all universities having the property $s_i,\ i = 1, 2, 3, 4$.

Then $G_M = \{(s_1, \{x_1\}), (s_2, \{x_1, x_3, x_5\}), (s_3, \{x_1, x_4, x_5\})\}$ is a soft set over X.

Let $G_M \in \mathcal{A}(X)$. If $g_M(s) = \emptyset,\ \forall\ s \in S$, then G_M is called an empty soft set or null soft set and is denoted by $G_\emptyset$ or $\widetilde{\emptyset}$ and if $g_M(s) = X,\ \forall\ s \in M$, then g_M is called a M-universal soft set and is denoted by $G_{\widetilde{M}}$. If $M = S$, then $G_{\widetilde{S}}$ is called an universal soft set or absolute soft set denoted by $\widetilde{X}$.

Definition 2.10 *[7,24] Let $G_M, H_N \in \mathcal{A}(X)$. Then G_M is said to be a soft subset of H_N, if $g_M(s) \subset h_N(s),\ \forall\ s \in S$ and is denoted by $G_M \widetilde{\subset} H_N$. In this case, H_N is also called a soft super set of G_M. G_M, H_N are called soft equal if $g_M(s) = h_N(s),\ \forall\ s \in S$ and to denote it we write $G_M = H_N$. G_M, H_N are called soft unequal if there exists $s^{'} \in S$ such that $g_M(s^{'}) \neq h_N(s^{'})$ and to denote it we write $G_M \neq H_N$.*

Definition 2.11 *[7,24] Let $G_M, H_N \in \mathcal{A}(X)$. The soft union of G_M and H_N is denoted by $G_M \widetilde{\cup} H_N$ and is defined by $G_M \widetilde{\cup} H_N = \{(s, g_M(s) \cup h_N(s)) : s \in S\}$; the soft intersection of G_M and H_N is denoted by $G_M \widetilde{\cap} H_N$ and is defined by $G_M \widetilde{\cap} H_N = \{(s, g_M(s) \cap h_N(s)) : s \in S\}$; the soft difference of G_M and H_N is denoted by $G_M \widetilde{-} H_N$ and is defined by $G_M \widetilde{-} H_N = \{(s, g_M(s) - h_N(s)) : s \in S\}$.*

The soft complement of G_M is denoted by $G_M^{\widetilde{C}}$ and is defined by $G_M^{\widetilde{C}} = \{(s, X - g_M(s)) : s \in S\}$.

Definition 2.12 *[4,9,31] A soft set G_M in $\mathcal{A}(X)$ is called a soft point if $\exists\ s \in M$ and $x \in X$ such that $g_M(s) = \{x\}$ and $g_M(s^{'}) = \emptyset,\ \forall\ s^{'} \in S - \{s\}$. Such a soft point is denoted by x_s^M.*

A soft point $x_s^{M^*}$ is said to be in the soft set H_M if $x \in h_M(s)$. We write $x_s^{M^*} \widetilde{\in} H_M$. A soft point $x_s^{M^*}$ is said to be not in the soft set H_M, if $x \notin h_M(s)$ and we write $x_s^{M^*} \widetilde{\notin} H_M$.

Two soft points $x_{s_1}^{M^*}$ and $y_{s_2}^{M^{**}}$ in a soft set H_M are called distinct if $s_1 \neq s_2$ or $x \neq y$ or both hold.

Any soft set is soft union of all soft points in it.

Definition 2.13 *[7] Let G_M be a soft set over X. A collection of soft subsets of G_M is called a soft topology on G_M, if the following conditions are satisfied,*

(i) $\widetilde{\emptyset} \in \widetilde{\tau}$,

(ii) $G_M \in \widetilde{\tau}$,

(iii) soft union of arbitrary members of $\widetilde{\tau}$ is again a member of $\widetilde{\tau}$,

(iv) soft intersection of finitely members of $\widetilde{\tau}$ is again a member of $\widetilde{\tau}$.

The pair $(G_M, \widetilde{\tau})$ is called a soft topological space. When their is no confusion about the soft topology on G_M, we denote the soft topological space $(G_M, \widetilde{\tau})$ by G_M only. Every member of $\widetilde{\tau}$ is called a soft open set. A soft subset $H_{M'}$ of G_M is called soft closed set in the soft topological space $(G_M, \widetilde{\tau})$, if $G_M \widetilde{-} H_{M'}$ is soft open in the soft topological space $(G_M, \widetilde{\tau})$.

If $(G_M, \widetilde{\tau})$ is a soft topological space and $H_{M'} \widetilde{\subseteq} G_M$, then $(H_{M'}, \widetilde{\tau}_{H_{M'}})$ is called a soft subspace of $(G_M, \widetilde{\tau})$, where $\widetilde{\tau}_{H_{M'}} = \{R_{M''} \widetilde{\cap} H_{M'} : R_{M''} \in \widetilde{\tau}\}$. .

Definition 2.14 *[32] Let $(G_M, \widetilde{\tau})$ be a soft topological space. A soft subset $K_{M'}$ of G_M is said to be a soft neighbourhood of a soft point $x_s^{M^*} \widetilde{\in} G_M$ if there exists $H_{M''} \in \widetilde{\tau}$ such that $x_s^{M^*} \widetilde{\in} H_{M''} \widetilde{\subseteq} K_{M'}$.*

Definition 2.15 *[7] Let $(G_M, \widetilde{\tau})$ be a soft topological space and $H_{M'}$ be a soft subset of G_M. Then soft closure of $H_{M'}$ is the soft intersection of all soft closed sets in G_M, containing the soft set $H_{M'}$ and it is denoted by $cl(H_{M'})$.*

Theorem 2.1 *[7] A soft subset $H_{M'}$ of G_M is soft closed in $(G_M, \widetilde{\tau})$ if and only if $cl(H_{M'}) = H_{M'}$.*

A soft subset $H_{M'}$ of a soft topological space $(G_M, \widetilde{\tau})$ is called soft dense in $(G_M, \widetilde{\tau})$ if $cl(H_{M'}) = G_M$. $(G_M, \widetilde{\tau})$ is said to be soft separable if it has a countable soft subset which is soft dense in $(G_M, \widetilde{\tau})$.

Definition 2.16 *[17] Let $(G_M, \widetilde{\tau})$ be a soft topological space. If for any two distinct soft points $x_s^{M_1}$ and $y_{s'}^{M_2}$ in G_M, there exist soft open sets $P_{M'}$ and $Q_{M''}$ such that*

$$x_s^{M_1} \widetilde{\in} P_{M'}, y_{s'}^{M_2} \widetilde{\in} Q_{M''}, x_s^{M_1} \widetilde{\notin} Q_{M''}, y_{s'}^{M_2} \widetilde{\notin} P_{M'},$$

then $(G_M, \widetilde{\tau})$ is called soft T_1 space.

Definition 2.17 *[10] Let $(G_M, \widetilde{\tau})$ be a soft topological space. If for any two distinct soft points $x_s^{M_1}$ and $y_{s'}^{M_2}$ in G_M, there exist soft open sets $P_{M'}$ and $Q_{M''}$ such that*

$$x_s^{M_1} \widetilde{\in} P_{M'}, y_{s'}^{M_2} \widetilde{\in} Q_{M''}, P_{M'} \widetilde{\cap} Q_{M''} = \widetilde{\emptyset},$$

then $(G_M, \widetilde{\tau})$ is called soft Hausdorff space.

Definition 2.18 *[6] Let $(G_M, \widetilde{\tau})$ be a soft topological space. A sub-collection β of $\widetilde{\tau}$ is said to be soft base for $\widetilde{\tau}$, if every element in $\widetilde{\tau}$ can be written as soft union of some members of β.*

Definition 2.19 *[6] Let $(G_M, \widetilde{\tau})$ be a soft topological space and $x_s^{M^*}$ be a soft point of G_M. Let $N_{x_s^{M^*}}$ be the set of all soft neighbourhood of $x_s^{M^*}$. Then a sub-collection $B_{x_s^{M^*}}$ of $N_{x_s^{M^*}}$ is said to be soft local base at $x_s^{M^*}$ if for any $H_{M'} \in N_{x_s^{M^*}}$, $\exists$ $K_{M''} \in B_{x_s^{M^*}}$ such that $x_s^{M^*} \widetilde{\in} K_{M''} \widetilde{\subseteq} H_{M'}$.*

Definition 2.20 *[6] A soft topological space $(G_M, \widetilde{\tau})$ is said to be soft first countable at $x_s^{M^0} \widetilde{\in} G_M$ if there is a countable soft local base at $x_s^{M^0}$. $(G_M, \widetilde{\tau})$ is said to be soft first countable if there is a countable soft local base at every soft point of G_M.*

Theorem 2.2 *[6] Let $(G_M, \widetilde{\tau})$ be a soft topological space which is soft first countable. Then, for any soft point $x_s^{M^*} \widetilde{\in} G_M$, there exists a countable collection of soft local base $\{H_{M^i}\}_{i \in \mathbb{N}}$ at $x_s^{M^*}$ such that $H_{M_1} \widetilde{\supseteq} H_{M_2} \widetilde{\supseteq} \cdots \widetilde{\supseteq} H_{M_i} \widetilde{\supseteq} \cdots$.*

Definition 2.21 *[6] A soft topological space $(G_M, \widetilde{\tau})$ is said to be soft second countable if $\widetilde{\tau}$ has a countable soft base.*

Theorem 2.3 *[6] Let $(G_M, \widetilde{\tau})$ be a soft second countable. Then it is soft separable.*

Definition 2.22 *[6,10] Let $(G_M, \widetilde{\tau})$ be a soft topological space . A sequence $\{\eta_n\}_{n \in \mathbb{N}}$ of soft points in G_M is said to be soft convergent to a soft point $x_s^{M^*} \widetilde{\in} G_M$, if for any soft neighbourhood $H_{M'}$ of $x_s^{M^*}$, there exists $k \in \mathbb{N}$ such that $\eta_n \widetilde{\in} H_{M'}, \forall n \geq k$. In this case, we write $\eta_n \to x_s^{M^*}$ or $\lim\limits_{n \to \infty} \eta_n = x_s^{M^*}$ and $x_s^{M^*}$ is called soft limit of $\{\eta_n\}_{n \in \mathbb{N}}$.*

Definition 2.23 *[5] Let $(G_M, \widetilde{\tau})$ be a soft topological space . A sequence $\{\eta_n\}_{n\in\mathbb{N}}$ of soft points in G_M is said to be statistically soft convergent to a soft point $x_s^{M^*} \widetilde{\in} G_M$, if for any soft neighbourhood $H_{M'}$ of $x_s^{M^*}$, $d(\{n \in \mathbb{N} : \eta_n \widetilde{\notin} H_{M'}\}) = 0$. In this case, we write st-$\lim\limits_{n\to\infty} \eta_n = x_s^{M^*}$ or $\eta_n \xrightarrow{st} x_s^{M^*}$. $x_s^{M^*}$ is called a statistical soft limit of $\{\eta_n\}_{n\in\mathbb{N}}$.*

Definition 2.24 *[31] Let $(G_M, \widetilde{\tau})$ be a soft topological space. A soft point $x_s^{M^*}$ in G_M is said to be a soft limit point of a soft subset $K_{M'}$ of G_M, if for each soft neighbourhood $H_{M''}$ of $x_s^{M^*}$, $(H_{M''} \widetilde{-} \{x_s^{M^*}\}) \widetilde{\cap} K_{M'} \neq \widetilde{\emptyset}$.*

Definition 2.25 *[31] Let $(G_M, \widetilde{\tau})$ be a soft topological space. A soft point $x_s^{M^*}$ in G_M is said to be a soft limit point of a sequence $w = \{\eta_n\}_{n\in\mathbb{N}}$ of soft points in G_M, if for each soft neighbourhood $H_{M'}$ of $x_s^{M^*}$, the set $\{n \in \mathbb{N} : \eta_n \widetilde{\in} H_{M'}\}$ is infinite.*

The soft set of all soft limit points of $w = \{\eta_n\}_{n\in\mathbb{N}}$ is denoted by L_w.

3. $\mathcal{I}$-soft convergence in soft topological spaces

In this section following Kostyrko et al. [21] and Lahiri et al. [23], we introduce the notion of $\mathcal{I}$-soft convergence of sequences of soft points in a soft topological space.

Definition 3.1 *Let $(G_M, \widetilde{\tau})$ be a soft topological space. A sequence $\{\eta_n\}_{n\in\mathbb{N}}$ of soft points in G_M is said to be $\mathcal{I}$-soft convergent to a soft point $x_s^{M^*}$ in G_M, if for any soft neighbourhood $U_{M'}$ of $x_s^{M^*}$, $\{n \in \mathbb{N} : \eta_n \widetilde{\notin} U_{M'}\} \in \mathcal{I}$. In this case, $x_s^{M^*}$ is called $\mathcal{I}$-soft limit of $\{\eta_n\}_{n\in\mathbb{N}}$. In this case, we write $\mathcal{I} - \lim\limits_{n\to\infty} \eta_n = x_s^{M^*}$.*

Remark 3.1 *(i) If we take $\mathcal{I} = \mathcal{I}_f = \{A \subset \mathbb{N} : A$ is a finite subset of $\mathbb{N}\}$, then $\mathcal{I}_f$-soft convergent coincides with usual soft convergence [6] [10].*

(ii) If we take $\mathcal{I} = \mathcal{I}_d = \{A \subset \mathbb{N} : d(A) = 0\}$, then $\mathcal{I}_d$-soft convergence coincides with statistical soft convergence [5].

(iii) For an admissible ideal $\mathcal{I}$ of $\mathbb{N}$, the collection $\mathcal{S}(\mathcal{A}^{\mathcal{I}}) = \{D \subset \mathbb{N} : \delta_{\mathcal{A}^{\mathcal{I}}}(D) = 0\}$, where $\mathcal{A} = (a_{nk})$ is an $\mathbb{N} \times \mathbb{N}$ non-negetive regular summability matrix and $\delta_{\mathcal{A}^{\mathcal{I}}}(D) = \mathcal{I} - \lim\limits_{n\to\infty} \sum\limits_{m\in D} a_{nm}$, the $\mathcal{A}^{\mathcal{I}}$-density of D (see [13,16,20,25,28] for more details). Then the notion of $\mathcal{S}(\mathcal{A}^{\mathcal{I}})$-soft convergence of sequence of soft points gives the notion of $\mathcal{A}^{\mathcal{I}}$-statistical soft convergence. Further, if $\mathcal{I} = \mathcal{I}_f = \{A \subset \mathbb{N} : A$ is a finite subset of $\mathbb{N}\}$, then the notion of $\mathcal{S}(\mathcal{A}^{\mathcal{I}_f})$-soft convergence of sequence of soft points in a soft topological space gives the notion $\mathcal{A}$-statistical soft convergence. For the Cesaro matrix C_1, the notion of $\mathcal{S}({C_1}^{\mathcal{I}})$-soft convergence of sequence of soft points in a soft topological space gives the notion of $\mathcal{I}$-statistical soft convergence.

Note 1 *It is clear that for an admissible ideal $\mathcal{I}$ any soft convergent sequence of soft points in a soft topological space is $\mathcal{I}$-soft convergent with same limit. But the converse is not true. To show this we consider the following example.*

Example 3.1 *Let $X = \{x_1, x_2, x_3, x_4\}$ be an initial universe set, $S = \{s_1, s_2, s_3\}$ be a set of parameters, $M = \{s_1, s_2\} \subsetneq S$ and $G_M = \{(s_1, \{x_1, x_2\}), (s_2, \{x_2, x_3\})\}$ be a soft set over X. Let $\widetilde{\tau} = \{\widetilde{\emptyset}, G_M, P_{M'}\}$, where $P_{M'} = \{(s_2, \{x_2, x_3\})\}$. Let $\mathcal{I} = \mathcal{P}(2\mathbb{N}) \cup \mathcal{I}_f \cup \{A \cup B : A \in \mathcal{P}(2\mathbb{N}), B \in \mathcal{I}_f\}$, where $\mathcal{P}(2\mathbb{N})$ denotes power set of $2\mathbb{N}$, $\mathcal{I}_f$ denotes collection of all finite subset of $\mathbb{N}$. Then, $\mathcal{I}$ is an admissible ideal of $\mathbb{N}$ and $\{\eta_n\}_{n\in\mathbb{N}}$ be a sequence of soft points in G_M defined by*

$$\eta_n = \begin{cases} x_{2_{s_1}}^{M_1}, & \text{if } n \text{ is an even number} \\ x_{3_{s_2}}^{M_2}, & \text{if } n \text{ is an odd number.} \end{cases}$$

Now, G_M and $P_{M'}$ are only soft open sets in $(G_M, \widetilde{\tau})$ containing $x_{3_{s_2}}^{M_2}$.

Since $\{n \in \mathbb{N} : \eta_n \widetilde{\notin} G_M\} = \emptyset \in \mathcal{I}$ and $\{n \in \mathbb{N} : \eta_n \widetilde{\notin} P_{M'}\} = \{n \in \mathbb{N} : n$ is even$\} \in \mathcal{I}$, so for any soft neighbourhood $R_{M''}$ of $x_{3_{s_2}}^{M_2}$, $\{n \in \mathbb{N} : \eta_n \widetilde{\notin} R_{M''}\} \in \mathcal{I}$. Therefore, $\mathcal{I} - \lim \eta_n = x_{3_{s_2}}^{M_2}$. But the sequence is neither statistically soft convergent to $x_{3_{s_2}}^{M_2}$ nor soft convergent to $x_{3_{s_2}}^{M_2}$.

Note 2 *Note that $\mathcal{I}$-soft limit of a sequence of soft points in a soft topological space may not be unique. To show this we consider the following example.*

Example 3.2 *Let $X = \{x_1, x_2, x_3, x_4\}$ be an initial universe set, $S = \{s_1, s_2, s_3, s_4\}$ be a set of parameters, $M = \{s_1, s_2, s_3\} \subsetneq S$ and $G_M = \{(s_1, \{x_1, x_2\}), (s_2, \{x_3, x_4\}), (s_3, \{x_3, x_4\})\}$ be a soft set over X. Let $\widetilde{\tau} = \{\widetilde{\emptyset}, G_M, P_{M'}, Q_{M''}\}$, where $P_{M'} = \{(s_2, \{x_4\}), (s_3, \{x_4\})\}$, $Q_{M''} = \{(s_3, \{x_3, x_4\})\}$. Then $(G_M, \widetilde{\tau})$ is a soft topological space.*

Let $\{\eta_n\}_{n\in\mathbb{N}}$ be a sequence of soft points in G_M, defined by

$$\eta_n = \begin{cases} x_{2_{s_1}}^{M^1}, & \text{if } n \in T \\ x_{4_{s_2}}^{M^2}, & \text{if } n \in Q \\ x_{4_{s_3}}^{M^3}, & \text{otherwise,} \end{cases}$$

where, $T = \{n \in \mathbb{N} : n \text{ is prime}\}$ and $Q = \{2n : n \in (\mathbb{N} - \{1\})\}$.

Let $R = \{2n-1 : n \in \mathbb{N}\} - T$ and $\mathcal{I} = \mathcal{I}_f \cup \mathcal{P}(T) \cup \mathcal{P}(Q) \cup \{A \cup B \cup C : A \in \mathcal{I}_f, B \in \mathcal{P}(T), C \in \mathcal{P}(Q)\}$, where $\mathcal{P}(Y)$ denotes power set of Y. Then $\mathcal{I}$ is an admissible ideal of $\mathbb{N}$. Here, $P_{M'}, Q_{M''}$ and G_M are only soft open sets in $(G_M, \widetilde{\tau})$ containing $x_{4_{s_3}}^{M^3}$.

Since $\{n \in \mathbb{N} : \eta_n \widetilde{\notin} G_M\} = \emptyset \in \mathcal{I}$, $\{n \in \mathbb{N} : \eta_n \widetilde{\notin} Q_{M''}\} = T \cup Q \in \mathcal{I}$, $\{n \in \mathbb{N} : \eta_n \widetilde{\notin} P_{M'}\} = T \in \mathcal{I}$, so for any soft neighbourhood $H_{M''}$ of $x_{4_{s_3}}^{M^3}$, $\{n \in \mathbb{N} : \eta_n \widetilde{\notin} H_{M''}\} \in \mathcal{I}$. Therefore, $\mathcal{I} - \lim_{n\to\infty} \eta_n = x_{4_{s_3}}^{M^3}$.

Again, G_M and $P_{M'}$ are only soft open sets in $(G_M, \widetilde{\tau})$ containing $x_{4_{s_2}}^{M^2}$.

Since $\{n \in \mathbb{N} : \eta_n \widetilde{\notin} G_M\} = \emptyset \in \mathcal{I}$, $\{n \in \mathbb{N} : \eta_n \widetilde{\notin} P_{M'}\} = T \in \mathcal{I}$, so $\mathcal{I} - \lim_{n\to\infty} \eta_n = x_{4_{s_2}}^{M^2}$. Therefore $x_{4_{s_3}}^{M^3}$ and $x_{4_{s_2}}^{M^2}$ both are $\mathcal{I}$-soft limit of the sequence $\{\eta_n\}_{n\in\mathbb{N}}$.

If the soft topological space is soft Hausdorff then $\mathcal{I}$-soft limit of an $\mathcal{I}$-soft convergent sequence is unique, as proved in the following theorem.

Theorem 3.1 *$\mathcal{I}$-soft limit of an $\mathcal{I}$-soft convergent sequence in a soft Hausdorff space is unique.*

Proof: Let $(G_M, \widetilde{\tau})$ be a soft Hausdorff space and $\{\eta_n\}_{n\in\mathbb{N}}$ be a sequence of soft points in G_M, which is $\mathcal{I}$-soft convergent. If possible let $\mathcal{I} - \lim_{n\to\infty} = x_s^{M^*}$ as well as $\mathcal{I} - \lim_{n\to\infty} = y_{s'}^{M^{**}}$ in G_M. Then, there exists soft open sets $U_{M'}$ and $V_{M''}$ such that $x_s^{M^*} \widetilde{\in} U_{M'}$, $y_{s'}^{M^{**}} \widetilde{\in} V_{M''}$ and $U_{M'} \widetilde{\cap} V_{M''} = \widetilde{\emptyset}$. Then, $\{n \in \mathbb{N} : \eta_n \widetilde{\notin} U_{M'}\} \in \mathcal{I}$ and $\{n \in \mathbb{N} : \eta_n \widetilde{\notin} V_{M''}\} \in \mathcal{I}$. So, $\{n \in \mathbb{N} : \eta_n \widetilde{\in} U_{M'}\} \in \mathcal{F}(\mathcal{I})$ and $\{n \in \mathbb{N} : \eta_n \widetilde{\in} V_{M''}\} \in \mathcal{F}(\mathcal{I})$. Then $\{n \in \mathbb{N} : \eta_n \widetilde{\in} U_{M'}\} \cap \{n \in \mathbb{N} : \eta_n \widetilde{\in} V_{M''}\} \in \mathcal{F}(\mathcal{I})$. This implies, $U_{M'} \widetilde{\cap} V_{M''} \neq \widetilde{\emptyset}$, which is a contradiction. Thus, $\{\eta_n\}_{n\in\mathbb{N}}$ has unique $\mathcal{I}$-soft limit. □

Note 3 *Every subsequence of an $\mathcal{I}$-soft convergent sequence need not to be $\mathcal{I}$-soft convergent to the same $\mathcal{I}$-soft limit. To show this we consider the following example.*

Example 3.3 *Let $X = \mathbb{R}$ be an initial universe set, $S = \{s_1, s_2, s_3, s_4\}$ be a set of parameters, $M = \{s_1, s_2, s_3\} \subsetneq S$ and $G_M = \{(s_1, (-\infty, 1)), (s_2, \{2\}), (s_3, \{5\})\}$. Then G_M is a soft set over X.*

Let $\widetilde{\tau} = \{\widetilde{\emptyset}, G_M, P_{M'}, Q_{M''}\}$, where $P_{M'} = \{(s_2, \{2\}), (s_3, \{5\})\}$ and $Q_{M''} = \{(s_1, (-\infty, 1))\}$. Then $(G_M, \widetilde{\tau})$ is a soft topological space. Let $\{\eta_n\}_{n\in\mathbb{N}}$ be a sequence of soft points in G_M defined by,

$$\eta_n = \begin{cases} -n_{s_1}^{M_1}, & \text{if } n \in W \\ 2_{s_2}^{M_2}, & \text{if } n \in T \\ 5_{s_3}^{M_3}, & \text{otherwise} \end{cases}$$

where, $W = \{k \in \mathbb{N} : k \text{ is even}\}$, $T = \{k \in \mathbb{N} : k \text{ is prime and odd}\}$.

Let $\mathcal{I} = \mathcal{I}_f \cup \mathcal{P}(W) \cup \{A \cup B : A \in \mathcal{I}_f,\ B \in \mathcal{P}(W)\}$. Then $\mathcal{I}$ is an admissible ideal of $\mathbb{N}$.

Now, G_M and $P_{M'}$ are only soft open sets containing $5_{s_3}^{M_3}$ and $\{n \in \mathbb{N} : \eta_n \widetilde{\notin} G_M\} = \emptyset \in \mathcal{I}$ and $\{n \in \mathbb{N} : \eta_n \widetilde{\notin} P_{M'}\} = W \in \mathcal{I}$, so for any soft neighbourhood $R_{M'''}$ of $5_{s_3}^{M_3}$, $\{n \in \mathbb{N} : \eta_n \widetilde{\notin} R_{M'''}\} \in \mathcal{I}$. Therefore, $\mathcal{I} - \lim_{n\to\infty} \eta_n = 5_{s_3}^{M_3}$.

But the subsequence $\{\eta_{2k}\}_{k\in\mathbb{N}}$ of $\{\eta_n\}_{n\in\mathbb{N}}$ is not $\mathcal{I}$-soft convergent to $5_{s_3}^{M_3}$.

Theorem 3.2 *Let $(G_M, \widetilde{\tau})$ be a soft topological space and $\{\eta_n\}_{n\in\mathbb{N}}$ be a sequence of soft points in G_M. If every subsequence of $\{\eta_n\}_{n\in\mathbb{N}}$ has a subsequence which is $\mathcal{I}$-soft convergent to a soft point $x_s^{M^*} \widetilde{\in} G_M$, then $\{\eta_n\}_{n\in\mathbb{N}}$ is $\mathcal{I}$-soft convergent to $x_s^{M^*}$.*

Proof: Suppose $\{\eta_n\}_{n\in\mathbb{N}}$ is not $\mathcal{I}$-soft convergent to $x_s^{M^*}$. Then there exists a soft neighbourhood $U_{M'}$ of $x_s^{M^*}$ such that $\{n \in \mathbb{N} : \eta_n \widetilde{\notin} U_{M'}\} \notin \mathcal{I}$. Let $B = \{n \in \mathbb{N} : \eta_n \widetilde{\notin} U_{M'}\}$. Then B is an infinite set say, $B = \{n_1 < n_2 < \cdots < n_k < \cdots\}$. Then $\{\eta_{n_k}\}_{k\in\mathbb{N}}$ is a subsequence of $\{\eta_n\}_{n\in\mathbb{N}}$ which has no subsequence $\mathcal{I}$-soft convergent to $x_s^{M^*}$, a contradiction. Then, $\{\eta_n\}_{n\in\mathbb{N}}$ is $\mathcal{I}$-soft convergent to $x_s^{M^*}$. □

4. $\mathcal{I}^*$-soft convergence in soft topological spaces

In this section following Kostyrko et al. [21] and Lahiri et al. [23], we introduce the notion of $\mathcal{I}^*$-soft convergence of sequences of soft points in a soft topological space.

Definition 4.1 *Let $(G_M, \widetilde{\tau})$ be a soft topological space. Then, a sequence $\{\eta_n\}_{n\in\mathbb{N}}$ of soft points in G_M is said to be $\mathcal{I}^*$-soft convergent to a soft point $x_s^{M^*}$ in G_M if there exists $P = \{p_1 < p_2 < \cdots < p_k < \cdots\} \in \mathcal{F}(\mathcal{I})$ such that $\lim_{k\to\infty} \eta_{p_k} = x_s^{M^*}$. In this case, we write $\mathcal{I}^* - \lim_{n\to\infty} \eta_n = x_s^{M^*}$ and $x_s^{M^*}$ is called $\mathcal{I}^*$-soft limit of $\{\eta_n\}_{n\in\mathbb{N}}$.*

Theorem 4.1 *Let $(G_M, \widetilde{\tau})$ be a soft topological space and $\{\eta_n\}_{n\in\mathbb{N}}$ be a sequence of soft points in G_M. If $\mathcal{I}^* - \lim_{n\to\infty} \eta_n = x_s^{M^*}$, then $\mathcal{I} - \lim_{n\to\infty} \eta_n = x_s^{M^*}$ and if $(G_M, \widetilde{\tau})$ is soft Hausdorff then $\mathcal{I}^*$-soft limit of $\{\eta_n\}_{n\in\mathbb{N}}$ is unique.*

Proof: Since $\mathcal{I}^* - \lim_{n\to\infty} \eta_n = x_s^{M^*}$, so there exists $P = \{p_1 < p_2 < \cdots < p_k < \cdots\} \in \mathcal{F}(\mathcal{I})$ such that $\lim_{k\to\infty} \eta_{p_k} = x_s^{M^*}$. Let $U_{M'}$ be a soft neighbourhood of $x_s^{M^*}$. Then, there exists $k_0 \in \mathbb{N}$, such that $\eta_{p_k} \widetilde{\in} U_{M'}$, $\forall\ k > k_0$. Now, $\{n \in \mathbb{N} : \eta_n \widetilde{\notin} U_{M'}\} \subset (\mathbb{N} - P) \cup \{p_1, p_2, \cdots, p_{k_0}\}$. Since, $(\mathbb{N} - P) \cup \{p_1, p_2, \cdots, p_{k_0}\} \in \mathcal{I}$ so, $\{n \in \mathbb{N} : \eta_n \widetilde{\notin} U_{M'}\} \in \mathcal{I}$. Therefore, $\mathcal{I} - \lim_{n\to\infty} \eta_n = x_s^{M^*}$. Now if $(G_M, \widetilde{\tau})$ is soft Hausdorff, then 'Theorem 3.1', $\mathcal{I}^*$-soft limit of $\{\eta_n\}_{n\in\mathbb{N}}$ is unique. □

Theorem 4.2 *Let $(G_M, \widetilde{\tau})$ be a soft topological space, having no soft limit point. Then $\mathcal{I}$ and $\mathcal{I}^*$ soft convergence coincide.*

Proof: In the view of 'Theorem 4.1', we have only to prove $\mathcal{I}$-soft convergence implies $\mathcal{I}^*$-soft convergence.

Let $\{\eta_n\}_{n\in\mathbb{N}}$ be a sequence of soft points in G_M such that $\mathcal{I} - \lim_{n\to\infty} \eta_n = x_s^{M^*}$. Since, $(G_M, \widetilde{\tau})$ has no soft limit point, so $U_{M'} = x_s^{M^*}$ is soft open. So, $\{n \in \mathbb{N} : \eta_n \widetilde{\notin} U_{M'}\} \in \mathcal{I}$. Then, $\{n \in \mathbb{N} : \eta_n \widetilde{\in} U_{M'}\} \in \mathcal{F}(\mathcal{I})$, i.e, $\{n \in \mathbb{N} : \eta_n = x_s^{M^*}\} \in \mathcal{F}(\mathcal{I})$. let $\{n \in \mathbb{N} : \eta_n = x_s^{M^*}\} = P = \{p_1 < p_2 < \cdots < p_k < \cdots\}$. Then $P \in \mathcal{F}(\mathcal{I})$ and $\lim_{k\to\infty} \eta_{p_k} = x_s^{M^*}$. So, $\mathcal{I}^* - \lim_{n\to\infty} \eta_n = x_s^{M^*}$. □

Remark 4.1 *In an arbitrary soft topological space, equivalency between $\mathcal{I}$ and $\mathcal{I}^*$-soft convergence does not hold. To show this we consider the following result.*

Theorem 4.3 *Let $(G_M, \widetilde{\tau})$ be a soft first countable, soft Hausdorff space having at least one soft limit point $x_s^{M^*}$. Then there exists an admissible ideal $\mathcal{I}$ of $\mathbb{N}$ and a sequence $\{\gamma_n\}_{n\in\mathbb{N}}$ of soft points in G_M such that $\mathcal{I} - \lim_{n\to\infty} \gamma_n = x_s^{M^*}$ but $\mathcal{I}^* - \lim_{n\to\infty} \gamma_n$ does not exist.*

Proof: Since $(G_M, \widetilde{\tau})$ is soft first countable, soft Hausdorff and $x_s^{M^*}$ is a soft limit point of G_M, so there exists a soft local base $\{H_n(x_s^{M^*})\}_{n\in\mathbb{N}}$ at $x_s^{M^*}$ such that $H_{n+1}(x_s^{M^*})\widetilde{\subset}H_n(x_s^{M^*})$, $\forall\, n \in \mathbb{N}$ and a sequence $\{\eta_n\}_{n\in\mathbb{N}}$ of distinct soft points in G_M such that $\lim\limits_{n\to\infty} \eta_n = x_s^{M^*}$ and $\eta_n\widetilde{\in}H_n(x_s^{M^*})\widetilde{-}H_{n+1}(x_s^{M^*})$, $\forall n \in \mathbb{N}$.

Let $\mathbb{N} = \overset{\infty}{\underset{j=1}{\cup}} M_j$ be a decomposition of $\mathbb{N}$ such that M_j is infinite, $\forall\, j \in \mathbb{N}$ and $M_i \cap M_j = \emptyset$, for $i \neq j$, $i, j \in \mathbb{N}$. Let $\mathcal{I} = \{A \subset \mathbb{N} : A \text{ intersects at finitely many } M_j\text{'s}\}$. Then $\mathcal{I}$ is an admissible ideal of $\mathbb{N}$.

Let $\{\gamma_n\}_{n\in\mathbb{N}}$ be sequence of soft points in G_M such that $\gamma_n = \eta_j$, if $n \in M_j$. Let $V_{M'}$ be a soft neighbourhood of $x_s^{M^*}$. Then there exists $k' \in \mathbb{N}$ such that $H_n(x_s^{M^*})\widetilde{\subset}V_{M'}$, $\forall\, n > k'$. Then $\{n \in \mathbb{N} : \gamma_n\widetilde{\notin}V_{M'}\} \subset M_1 \cup M_2 \cup \cdots \cup M_{k'}$. Since, $M_1, M_2, \cdots, M_{k'} \in \mathcal{I}$, so $\{n \in \mathbb{N} : \gamma_n\widetilde{\notin}V_{M'}\} \in \mathcal{I}$. So, $\mathcal{I} - \lim\limits_{n\to\infty} \gamma_n = x_s^{M^*}$.

Now, suppose $\mathcal{I}^* - \lim\limits_{n\to\infty} \gamma_n = x_s^{M^*}$. Then there exists $Q = \{q_1 < q_2 < \cdots\} \in \mathcal{F}(\mathcal{I})$ such that $\lim\limits_{k\to\infty} \gamma_{q_k} = x_s^{M^*}$.

Let $P = \mathbb{N}-Q$. Since $P \in \mathcal{I}$, there exists $l \in \mathbb{N}$ such that $P \subset M_1\cup M_2\cup\cdots M_l$ and $M_j \subset \mathbb{N}-P$, $\forall\, j \geq l + 1$. So, for each each $j \geq l + 1$, there exists infinitely many k's such that $\gamma_{q_k} = \eta_j$. So for $i, j \geq l + 1$ with $i \neq j$ we get two subsequence for $\{\gamma_{q_k}\}_{k\in\mathbb{N}}$ converging to η_i and η_j respectively. Since $(G_M, \widetilde{\tau})$ is soft Hausdorff space and $\eta_i \neq \eta_j \neq x_s^{M^*}$, so we get a contradiction to the fact that $\lim\limits_{k\to\infty} \gamma_{q_k} = x_s^{M^*}$. Therefore $\mathcal{I}^* - \lim\limits_{n\to\infty} \gamma_n$ does not exist. □

We now consider a condition namely AP condition under which $\mathcal{I}$ and $\mathcal{I}^*$-soft convergence coincide.This condition (AP) was introduced by Kostyrko et al. [21] which is similar to the (APO) condition used in [8] and [12].

Definition 4.2 *[21] An admissible ideal $\mathcal{I}$ of $\mathbb{N}$ is said to satisfy the condition AP, if for every countable family of mutually disjoint sets $\{H_1, H_2, \cdots\ \cdots\}$ belonging to $\mathcal{I}$, there exists a countable family of sets $\{K_1, K_2, \cdots\ \cdots\}$ such that $H_i \Delta K_i$ is finite, for all $i \in \mathbb{N}$ and $K = \overset{\infty}{\underset{i=1}{\cup}} K_i \in \mathcal{I}$.*

Theorem 4.4 *(i) If $(G_M, \widetilde{\tau})$ is a soft first countable soft topological space and $\mathcal{I}$ satisfies condition (AP), then for any sequence $\{\eta_n\}_{n\in\mathbb{N}}$ of soft points in G_M, $\mathcal{I} - \lim\limits_{n\to\infty} \eta_n = x_s^{M^*} \implies \mathcal{I}^* - \lim\limits_{n\to\infty} \eta_n = x_s^{M^*}$.*

(ii) If $(G_M, \widetilde{\tau})$ is a soft first countable, soft T_1 space having at least one soft limit point and if for every sequence $\{\eta_n\}_{n\in\mathbb{N}}$ of soft points in G_M, $\mathcal{I}$-soft convergence of $\{\eta_n\}$ implies $\mathcal{I}^*$-soft convergence of $\{\eta_n\}$, then $\mathcal{I}$ has the property (AP).

Proof: (i) Let $\{\eta_n\}_{n\in\mathbb{N}}$ be a sequence of soft points in G_M such that $\mathcal{I}- \lim\limits_{n\to\infty} \eta_n = x_s^{M^*}$, where $x_s^{M^*}\widetilde{\in}G_M$. Since $(G_M, \widetilde{\tau})$ is soft first countable so by 'Theorem-2.2', there exists a countable collection of soft local base $\{G_n(x_s^{M^*}) : n \in \mathbb{N}\}$ at $x_s^{M^*}$, such that $G_{n+1}(x_s^{M^*})\widetilde{\subset}G_n(x_s^{M^*})$, $\forall\, n \in \mathbb{N}$.

Let $H_1 = \{n \in \mathbb{N} : \eta_n\widetilde{\notin}G_1(x_s^{M^*})\}$ and $H_m = \{n \in \mathbb{N} : \eta_n\widetilde{\notin}G_m(x_s^{M^*})\widetilde{-}G_{m-1}(x_s^{M^*})\}$, $\forall m \geq 2$. Then $H_m \in \mathcal{I}$, $\forall m \in \mathbb{N}$ and $H_i\cap H_j = \emptyset, \forall\, i, j \in \mathbb{N}$. Since $\mathcal{I}$ satisfies (AP) condition, so there exists a countable collection of sets, say $\{K_m : m \in \mathbb{N}\}$ such that $H_m\Delta K_m$ is finite, $\forall m \in \mathbb{N}$ and $K = \underset{m\in\mathbb{N}}{\cup} K_m \in \mathcal{I}$. Then $R = \mathbb{N} - K = \{r_1 < r_2 < \cdots < r_p < \cdots\ \cdots\} \in \mathcal{F}(\mathcal{I})$. Let $V_{M'}$ be a soft neighbourhood of $x_s^{M^*}$. Since $\{\eta_n\}_{n\in\mathbb{N}}$ is $\mathcal{I}$-soft convergent to $x_s^{M^*}$ so $\{n \in \mathbb{N} : \eta_n\widetilde{\notin}V_{M'}\} \in \mathcal{I}$. Then there exists $q \in \mathbb{N}$ such that $G_n(x_s^{M^*})\widetilde{\subset}V_{M'}, \forall\, n \geq q$. Then, $\{n \in \mathbb{N} : \eta_n\widetilde{\notin}V_{M'}\} \subset \overset{q}{\underset{i=1}{\cup}} H_i$. Since, $H_m\Delta K_m$ is finite, $\forall m \in \mathbb{N}$, so $\overset{q}{\underset{i=1}{\cup}} (H_i\Delta K_i)$ is also finite. So, there exists $n_0 \in \mathbb{N}$ such that $\forall\, n \in \mathbb{N}$ with $n > n_0$, $n \in \overset{q}{\underset{i=1}{\cup}} H_i$ if and only if $n \in \overset{q}{\underset{i=1}{\cup}} K_i$. We Choose, $t \in \mathbb{N}$ such that $r_t > n_0$. Since $r_j \notin K \supset \overset{q}{\underset{i=1}{\cup}} K_i$ for any $j \in \mathbb{N}$, so if $p > t$, then $r_p > r_t > n_0$ and so $r_p \notin \{n \in \mathbb{N} : \eta_n\widetilde{\notin}V_{M'}\}$, $\forall p > t \implies r_p \in \{n \in \mathbb{N} : \eta_n\widetilde{\in}V_{M'}\}$, $\forall p > t$. So, $\lim\limits_{p\to\infty} \eta_{r_p} = x_s^{M^*}$. Hence, $\mathcal{I}^* - \lim\limits_{n\to\infty} \eta_n = x_s^{M^*}$.

(ii) Let $x_s^{M^*}$ be a soft limit point of G_M. Since, $(G_M, \widetilde{\tau})$ is soft first countable, soft T_1 space, so there exist a soft local base $\{G_n(x_s^{M^*})\}_{n\in\mathbb{N}}$ at $x_s^{M^*}$ with $G_{n+1}(x_s^{M^*})\widetilde{\subset}G_n(x_s^{M^*})$, $\forall\ n \in \mathbb{N}$ and a sequence $\{\eta_n\}_{n\in\mathbb{N}}$ of distinct soft points in G_M such that $\lim_{n\to\infty}\eta_n = x_s^{M^*}$, $\eta_n\widetilde{\in}G_n(x_s^{M^*})$, $\forall n \in \mathbb{N}$ and $\eta_n \neq x_s^{M^*}$ for any $n \in \mathbb{N}$. Let $\{H_n\}_{n\in\mathbb{N}}$ be a sequence of mutually disjoint non-empty sets from $\mathcal{I}$. Let us define a sequence $\{\gamma_n\}_{n\in\mathbb{N}}$ of soft points in G_M by

$$\gamma_n = \begin{cases} \eta_j, & \text{if } n \in H_j \\ x_s^{M^*}, & \text{if } n \in \mathbb{N} - (\underset{j\in\mathbb{N}}{\cup} H_j). \end{cases}$$

Let $V_{M'}$ be a soft neighbourhood of $x_s^{M^*}$. Then, there exists $q \in \mathbb{N}$, such that $G_n(x_s^{M^*})\widetilde{\subset}V_{M'}$, $\forall\ n \geq q$. So, $\{n \in \mathbb{N} : \gamma_n\widetilde{\notin}V_{M'}\} \subset H_1 \cup H_2 \cup \cdots \cup H_{q-1}$. Since, $H_1, H_2, ..., H_{q-1} \in \mathcal{I}$, so $H_1 \cup H_2 \cup \cdots \cup H_{q-1} \in \mathcal{I}$ and so $\{n \in \mathbb{N} : \gamma_n\widetilde{\notin}V_{M'}\} \in \mathcal{I}$. So, $\mathcal{I}$-$\lim_{n\to\infty}\gamma_n = x_s^{M^*}$. Then, by the given condition, $\mathcal{I}^*$-$\lim_{n\to\infty}\gamma_n = x_s^{M^*}$. Then, there exists $R = \{r_1 < r_2 < \cdots < r_k < \cdots\} \in \mathcal{F}(\mathcal{I})$ such that $\lim_{k\to\infty}\gamma_{r_k} = x_s^{M^*}$. Let $H = \mathbb{N} - R$. Then $H \in \mathcal{I}$. Let $K_j = H_j \cap H$, $\forall j \in \mathbb{N}$. Since, $H \in \mathcal{I}$ and $K_j \subset H$, $\forall j \in \mathbb{N}$, so $K_j \in \mathcal{I}$, $\forall j \in \mathbb{N}$ and since $\underset{j\in\mathbb{N}}{\cup}K_j \subset H$, $\underset{j\in\mathbb{N}}{\cup}K_j \in \mathcal{I}$.

Let $j \in \mathbb{N}$. If $H_j \cap R$ is infinite, then $\gamma_{r_k} = \eta_j$ for infinitely many r_k's. But $\eta_j \neq x_s^{M^*}$ and so $G_M\widetilde{-}\{\eta_j\}$ is a soft neighbourhood of $x_s^{M^*}$ and $\gamma_{r_k}\widetilde{\notin}G_M\widetilde{-}\{\eta_j\}$ for infinitely many r_k's which contradicts that $\lim_{k\to\infty}\gamma_{r_k} = x_s^{M^*}$. So $H_j \cap R$ is a finite set.

So, there exists $l_0 \in \mathbb{N}$ such that $H_j \subset (H_j \cap K_j) \cup \{r_1, r_2, \cdots, r_{l_0}\}$. Then, $H_j \Delta K_j = H_j - K_j \subset \{r_1, r_2, \cdots, r_{l_0}\}$. So, $H_j \Delta K_j$ is finite.

Thus, we get a countable family of sets $\{K_j : j \in \mathbb{N}\}$ such that $H_j \Delta K_j$ is finite, $\forall j \in \mathbb{N}$ and $\underset{j\in\mathbb{N}}{\cup}K_j \in \mathcal{I}$. So, '$\mathcal{I}$' satisfies (AP) condition. □

5. $\mathcal{I}$-soft limit point and $\mathcal{I}$-soft cluster point

In this section, following Kostyrko et al. [21] and Lahiri et al. [23], we introduce the notion of $\mathcal{I}$-soft limit point and $\mathcal{I}$-soft cluster point of a sequence of soft points in a soft topological space.

Definition 5.1 *Let $(G_M, \widetilde{\tau})$ be a soft topological space and $w = \{\eta_n\}_{n\in\mathbb{N}}$ be a sequence of soft points in G_M. A soft point $x_s^{M^*}$ is said to be an $\mathcal{I}$-soft limit point of $w = \{\eta_n\}_{n\in\mathbb{N}}$, if there exists a subset $P = \{p_1 < p_2 < \cdots < p_k < \cdots\}$ of $\mathbb{N}$ such that $P \notin \mathcal{I}$ and $\lim_{k\to\infty}\eta_{p_k} = x_s^{M^*}$.*

The soft set of all $\mathcal{I}$-soft limit points of $w = \{\eta_n\}_{n\in\mathbb{N}}$ is denoted by $\mathcal{I}(\widetilde{\Lambda}_w)$.

Definition 5.2 *Let $(G_M, \widetilde{\tau})$ be a soft topological space and $w = \{\eta_n\}_{n\in\mathbb{N}}$ be a sequence of soft points in G_M. A soft point $x_s^{M^*}$ is said to be an $\mathcal{I}$-soft cluster point of $w = \{\eta_n\}_{n\in\mathbb{N}}$, if for any soft neighbourhood $U_{M''}$ of $x_s^{M^*}$, $\{n \in \mathbb{N} : \eta_n\widetilde{\in}U_{M''}\} \notin \mathcal{I}$.*

The soft set of all $\mathcal{I}$-soft cluster points of $w = \{\eta_n\}_{n\in\mathbb{N}}$ is denoted by $\mathcal{I}(\widetilde{\Gamma}_w)$.

Theorem 5.1 *Let $(G_M, \widetilde{\tau})$ be a soft topological space. Then, for any sequence $w = \{\eta_n\}_{n\in\mathbb{N}}$ of soft points in G_M, we have $\mathcal{I}(\widetilde{\Lambda}_w)\widetilde{\subset}\mathcal{I}(\widetilde{\Gamma}_w)$.*

Proof: Let $x_s^{M^*}\widetilde{\in}\mathcal{I}(\widetilde{\Lambda}_w)$. Then, there exists a set $P = \{p_1 < p_2 < \cdots < p_k < \cdots\} \notin \mathcal{I}$ such that $\lim_{k\to\infty}\eta_{p_k} = x_s^{M^*}$. Let $U_{M'}$ be a soft neighbourhood of $x_s^{M^*}$. Then, there exists $k_0 \in \mathbb{N}$ such that $\eta_{p_k}\widetilde{\in}U_{M'}$, $\forall\ k > k_0$. Then, $\{n \in \mathbb{N} : \eta_n\widetilde{\in}U_{M'}\} \supset P - \{p_1, p_2, \cdots, p_{k_0}\}$. If $\{n \in \mathbb{N} : \eta_n\widetilde{\in}U_{M'}\} \in \mathcal{I}$, then, $P - \{p_1, p_2, \cdots, p_{k_0}\} \in \mathcal{I}$ and so $P = (P - \{p_1, p_2, \cdots, p_{k_0}\}) \cup \{p_1, p_2, \cdots, p_{k_0}\} \in \mathcal{I}$, a contradiction. Therefore, $\{n \in \mathbb{N} : \eta_n\widetilde{\in}U_{M'}\} \notin \mathcal{I}$. So, $x_s^{M^*}$ is an $\mathcal{I}$-soft cluster point of $\{\eta_n\}_{n\in\mathbb{N}}$. Hence $\mathcal{I}(\widetilde{\Lambda}_w)\widetilde{\subset}\mathcal{I}(\widetilde{\Gamma}_w)$. □

Theorem 5.2 *Let $(G_M, \widetilde{\tau})$ be a soft topological space. For any sequence $w = \{\eta_n\}_{n\in\mathbb{N}}$ of soft points in G_M, $\mathcal{I}(\widetilde{\Gamma}_w)$ is soft closed in $(G_M, \widetilde{\tau})$.*

Proof: Let $y_s^{M^{**}} \widetilde{\in} \mathrm{cl}(\mathcal{I}(\widetilde{\Gamma}_w))$ and let $U_{M''}$ be a soft neighbourhood of $y_s^{M^{**}}$. Then, $U_{M''} \widetilde{\cap} \mathcal{I}(\widetilde{\Gamma}_w) \neq \widetilde{\emptyset}$. Let $w_{s'}^{M^{***}} \widetilde{\in} \mathcal{I}(\widetilde{\Gamma}_w) \widetilde{\cap} U_{M''}$. Since, $w_{s'}^{M^{***}}$ is an $\mathcal{I}$-soft cluster point of $\{\eta_n\}_{n\in\mathbb{N}}$ and $U_{M''}$ is a soft neighbourhood of $w_{s'}^{M^{***}}$, so $\{n \in \mathbb{N} : \eta_n \widetilde{\in} U_{M''}\} \notin \mathcal{I}$. So, $y_s^{M^{**}} \widetilde{\in} \mathcal{I}(\widetilde{\Gamma}_w)$. $\mathcal{I}(\widetilde{\Gamma}_w)$ is soft closed in $(G_M, \widetilde{\tau})$. □

Theorem 5.3 *Let $(G_M, \widetilde{\tau})$ be soft second countable and there exists a disjoint sequence $\{R_n\}_{n\in\mathbb{N}}$ of natural numbers such that $R_n \notin \mathcal{I}$, $\forall n \in \mathbb{N}$. Then for every non-empty soft closed set $H_{M'} \widetilde{\subset} G_M$, there exists a sequence $w = \{\gamma_n\}_{n\in\mathbb{N}}$ of soft points in G_M such that $H_{M'} = \mathcal{I}(\widetilde{\Gamma}_w)$.*

Proof: Let $H_{M'}$ be a non-empty soft closed set in G_M. Since $(G_M, \widetilde{\tau})$ is soft second countable, so the soft subspace $(H_{M'}, \widetilde{\tau}_{H_{M'}})$ is soft separable. So there exists a countable collection of soft points $x = \{\eta_n\}_{n\in\mathbb{N}}$ in $H_{M'}$ such that $cl_{H_{M'}}(\underset{n\in\mathbb{N}}{\widetilde{\cup}} \{\eta_n\}) = H_{M'}$.

Let $w' = \{\beta_n\}_{n\in\mathbb{N}}$ be a sequence of soft points in G_M such that $\beta_n = \eta_i$, when $n \in R_i$, $i \in \mathbb{N}$. Let $P = \overset{\infty}{\underset{i=1}{\cup}} R_i$. Then P is an infinite set. Let $P = \{n_1 < n_2 < \cdots < n_k < \cdots\}$. Then we can get a subsequence $w = \{\beta_{n_k}\}_{k\in\mathbb{N}}$ of w'.

Now we show that $\mathcal{I}(\widetilde{\Gamma}_w) \widetilde{\subset} H_{M'}$. Let $y_s^{M^{**}} \widetilde{\in} \mathcal{I}(\widetilde{\Gamma}_w)$ and $V_{M''}$ be a soft neighbourhood of $y_s^{M^{**}}$ in $(G_M, \widetilde{\tau})$. Then $\{k \in \mathbb{N} : \beta_{n_k} \widetilde{\in} V_{M''}\} \notin \mathcal{I}$ and so $\{k \in \mathbb{N} : \gamma_{n_k} \widetilde{\in} V_{M''}\} \neq \emptyset$. Then there exists $i_0 \in \mathbb{N}$ such that $\eta_{i_0} \widetilde{\in} V_{M''}$. So $V_{M''} \widetilde{\cap} H_{M'} \neq \widetilde{\emptyset}$ and so $y_s^{M^{**}} \widetilde{\in} cl(H_{M'}) = H_{M'}$ and so $\mathcal{I}(\widetilde{\Gamma}_w) \widetilde{\subset} H_{M'}$.

Conversely, let $y_s^{M^{**}} \widetilde{\in} H_{M'}$ and $W_{M'''}$ be a soft neighbourhood of $y_s^{M^{**}}$ in $(G_M, \widetilde{\tau})$. Then, $W_{M'''} \widetilde{\cap} H_{M'}$ is a soft neighbourhood of $y_s^{M^{**}}$ in $(H_{M'}, \widetilde{\tau}_{H_{M'}})$. Then there exists $\eta_{j_0} \in \mathbb{N}$ such that $\eta_{j_0} \widetilde{\in} W_{M'''} \widetilde{\cap} H_{M'}$ and $R_{j_0} \subset \{k \in \mathbb{N} : \beta_{n_k} \widetilde{\in} W_{M'''}\}$. Since $R_{j_0} \notin \mathcal{I}$, so $\{k \in \mathbb{N} : \beta_{n_k} \widetilde{\in} W_{M'''}\} \notin \mathcal{I}$. So, $y_s^{M^{**}} \widetilde{\in} \mathcal{I}(\widetilde{\Gamma}_w)$ and so $H_{M'} \widetilde{\subset} \mathcal{I}(\widetilde{\Gamma}_w)$. Hence, $H_{M'} = \mathcal{I}(\widetilde{\Gamma}_w)$.

□

Theorem 5.4 *Let $(G_M, \widetilde{\tau})$ be a soft topological space and $w = \{\eta_n\}_{n\in\mathbb{N}}$ and $v = \{\gamma_n\}_{n\in\mathbb{N}}$ are sequences of soft points in G_M such that $\{n \in \mathbb{N} : \eta_n \neq \gamma_n\} \in \mathcal{I}$. Then*

(i) $\mathcal{I}(\widetilde{\Lambda}_w) = \mathcal{I}(\widetilde{\Lambda}_v)$

(ii) $\mathcal{I}(\widetilde{\Gamma}_w) = \mathcal{I}(\widetilde{\Gamma}_v)$.

Proof: (i) Let $x_s^{M^*} \widetilde{\in} \mathcal{I}(\widetilde{\Lambda}_w)$. Then there exists a set $W = \{w_1 < w_2 < \cdots < w_k < \cdots\} \subset \mathbb{N}$ such that $W \notin \mathcal{I}$ and $\lim_{k\to\infty} \eta_{w_k} = x_s^{M^*}$.

Let $Y = \{n \in \mathbb{N} : \eta_n \neq \gamma_n\}$. We claim that $W - Y \notin \mathcal{I}$. In contrary, suppose $W - Y \in \mathcal{I}$. Since $Y \in \mathcal{I}$, so $W = (W - Y) \cup Y \in \mathcal{I}$, a contradiction. So, $W - Y \notin \mathcal{I}$. Then $W - Y$ is an infinite set. Let $W - Y = \{t_1 < t_2 < \cdots < t_l < \cdots\}$ and $V_{M'}$ be a soft neighbourhood of $x_s^{M^*}$. Since $\lim_{k\to\infty} \eta_{w_k} = x_s^{M^*}$, so there exists $k_0 \in \mathbb{N}$, such that $\eta_{w_k} \widetilde{\in} V_{M'}$, $\forall\, k > k_0$. Then, their exists $k_0' \in \mathbb{N}$ such that $\gamma_{t_l} \widetilde{\in} V_{M'}$, $\forall\, l > k_0'$. Therefore, $\lim_{l\to\infty} \gamma_{t_l} = x_s^{M^*}$ and hence $x_s^{M^*} \widetilde{\in} \mathcal{I}(\widetilde{\Lambda}_v)$. So, $\mathcal{I}(\widetilde{\Lambda}_w) \widetilde{\subset} \mathcal{I}(\widetilde{\Lambda}_v)$. Similarly, we have $\mathcal{I}(\widetilde{\Lambda}_v) \widetilde{\subset} \mathcal{I}(\widetilde{\Lambda}_w)$. Thus, $\mathcal{I}(\widetilde{\Lambda}_w) = \mathcal{I}(\widetilde{\Lambda}_v)$.

(ii) Let $x_s^{M^*} \widetilde{\in} \mathcal{I}(\widetilde{\Gamma}_w)$ and $R_{M''}$ be a soft neighbourhood of $x_s^{M^*}$. Then $\{n \in \mathbb{N} : \eta_n \widetilde{\in} R_{M''}\} \notin \mathcal{I}$. If possible, let $\{n \in \mathbb{N} : \gamma_n \widetilde{\in} R_{M''}\} \in \mathcal{I}$.

Since $\{n \in \mathbb{N} : \eta_n \widetilde{\in} R_{M''}\} \subset \{n \in \mathbb{N} : \eta_n \neq \gamma_n\} \cup \{n \in \mathbb{N} : \gamma_n \widetilde{\in} R_{M''}\}$ and $\{n \in \mathbb{N} : \eta_n \neq \gamma_n\} \in \mathcal{I}$, so $\{n \in \mathbb{N} : \eta_n \neq \gamma_n\} \cup \{n \in \mathbb{N} : \gamma_n \widetilde{\in} R_{M''}\} \in \mathcal{I}$ and so $\{n \in \mathbb{N} : \eta_n \widetilde{\in} R_{M''}\} \in \mathcal{I}$, a contradiction. Therefore, $\{n \in \mathbb{N} : \gamma_n \widetilde{\in} R_{M''}\} \notin \mathcal{I}$. This implies, $x_s^{M^*} \widetilde{\in} \mathcal{I}(\widetilde{\Gamma}_v)$. So, $\mathcal{I}(\widetilde{\Gamma}_w) \widetilde{\subset} \mathcal{I}(\widetilde{\Gamma}_v)$. Similarly we have $\mathcal{I}(\widetilde{\Gamma}_v) \widetilde{\subset} \mathcal{I}(\widetilde{\Gamma}_w)$. Hence, $\mathcal{I}(\widetilde{\Gamma}_w) = \mathcal{I}(\widetilde{\Gamma}_v)$.

□

Theorem 5.5 *Let $(G_M, \widetilde{\tau})$ be a soft topological space. Let $\{\eta_n\}_{n\in\mathbb{N}}$ be a sequence of soft points in G_M which is $\mathcal{I}$-soft convergent to $x_s^{M^*} \widetilde{\in} G_M$ and $\{\gamma_n\}_{n\in\mathbb{N}}$ be a sequence of soft points in G_M such that $\{n \in \mathbb{N} : \eta_n \neq \gamma_n\} \in \mathcal{I}$. Then $\{\gamma_n\}_{n\in\mathbb{N}}$ is $\mathcal{I}$-soft convergent to $x_s^{M^*}$.*

Proof: Let $R_{M''}$ be a soft neighbourhood of $x_s^{M^*}$. Since $\{\eta_n\}_{n\in\mathbb{N}}$ is $\mathcal{I}$-soft convergent to $x_s^{M^*}$, so $\{n \in \mathbb{N} : \eta_n \widetilde{\notin} R_{M''}\} \in \mathcal{I}$.

Since, $\{n \in \mathbb{N} : \gamma_n \widetilde{\notin} R_{M''}\} \subset \{n \in \mathbb{N} : \eta_n \neq \gamma_n\} \cup \{n \in \mathbb{N} : \eta_n \widetilde{\notin} R_{M''}\}$ and $\{n \in \mathbb{N} : \eta_n \neq \gamma_n\} \in \mathcal{I}$, so $\{n \in \mathbb{N} : \eta_n \neq \gamma_n\} \cup \{n \in \mathbb{N} : \eta_n \widetilde{\notin} R_{M''}\} \in \mathcal{I}$ and so, $\{n \in \mathbb{N} : \gamma_n \widetilde{\notin} R_{M''}\} \in \mathcal{I}$. Hence, $\mathcal{I} - \lim_{n\to\infty} \gamma_n = x_s^{M^*}$. □

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