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Tracking Misinformation and Influence Spread in Social Networks Using Equitable Fair Power Domination in Fuzzy Graphs

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ABSTRACT: This paper investigates the concept of Equitable Fair Power Domination (EFPD) in fuzzy graphs, integrating power domination principles with equitable and fair domination strategies to enhance decision-making in network monitoring. The study systematically establishes fundamental properties, theoretical results, and numerical illustrations to provide a comprehensive understanding of optimal node selection for effectively tracking and controlling misinformation in complex social networks. By leveraging fuzzy graph structures, the proposed framework ensures balanced influence distribution, preventing dominance by a single entity and minimizing biases in fact-checking and decision-making processes. The theoretical formulations are validated through numerical computations, demonstrating how EFPD can be applied to strategically identify key influencers or misinformation sources within a network. The findings contribute significantly to the development of robust methodologies for social network analysis, cybersecurity, strategic communication, and misinformation control, ensuring fairness and efficiency in information dissemination and verification.

Keywords: Fuzzy graph, dominating fuzzy graph, power domination, equitable fair fuzzy graph, social network analysis, influence control.

Contents

1. Introduction

Fuzzy graphs have become a powerful tool for modeling real-world networks where uncertainty and imprecision play a crucial role. They are widely used in social network analysis, cybersecurity, biological systems, and decision-making processes, providing a flexible framework for handling ambiguous and uncertain relationships between entities. Zadeh (1965) [27] introduced the concept of fuzzy sets, providing a mathematical framework to handle uncertainty in data classification. This foundational work led to various extensions in fuzzy logic and applications, particularly in graph theory. Rosenfeld (1975) [15] extended fuzzy set theory by defining fuzzy graphs, where edges and vertices have degrees of membership, paving the way for fuzzy graph-based decision models. Yager (1991) [26] investigated fuzzy subsets and fuzzy graphs, applying them to multi-criteria decision-making, thereby enhancing their role in uncertainty modeling.

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Chang (1994) [4] explored the domination number in graphs, establishing fundamental results that influence network optimization and security strategies. Chartrand et al. (1996) [5] contributed to combinatorial graph theory, emphasizing key properties of domination and connectivity essential for various computational problems. Haynes et al. (1998) [6] provided an in-depth study on domination in graphs, covering theoretical results and real-world applications, including facility location problems.

Mordeson and Mathew (2000) [10] expanded on fuzzy graphs, introducing fuzzy hypergraphs and their applications in intelligent systems, making them useful for modeling uncertainty in social and biological networks. Bondy and Murty (2008) [3] offered a comprehensive analysis of graph theory, including domination, power domination, and network connectivity, forming a basis for further studies on graph-based optimization. Ali et al. (2009) [1] presented novel operations in soft set theory, advancing decision-making models by integrating fuzzy logic with classical set operations.

Sun and Liu (2010) [20] examined power domination in fuzzy graphs, particularly in monitoring electrical networks, demonstrating its effectiveness in reducing redundancy in control systems. Sampathkumar and Walikar (2011) [16] studied graph domination concepts, providing computational approaches to solve problems in network reliability and optimization. Nakamura (2012) [11] introduced equitable domination, ensuring fairness in resource allocation and influence distribution within complex networks.

Kaufmann and Wagner (2013) [8] proposed visualization techniques for large graphs, aiding in the representation and interpretation of power domination structures. Ramesh and Arumugam (2015) [14] examined equitable domination in fuzzy graphs, analyzing its significance in network control and optimization. Sarkar and Pal (2016) [17] developed fuzzy power domination models with applications in electrical grid stability, enhancing monitoring systems' efficiency.

Wang and Zhang (2015) [23] introduced fuzzy graph-based ranking algorithms, improving influence measurement techniques in social networks and recommendation systems. Slater (2005) [19] provided a survey on domination and location in graphs, covering facility location, resource distribution, and surveillance networks. Hedetniemi and Laskar (2018) [7] expanded domination theory, incorporating advanced strategies such as equitable fair domination to address real-world influence modeling.

Mahmood and Ahmad (2018) [9] developed new fuzzy graph models, focusing on uncertainty management and its applications in decision-making. Neville and Jensen (2020) [13] studied relational learning with graphs, integrating fuzzy structures to enhance predictive modeling in artificial intelligence. Basu and Acharya (2020) [2] reviewed fuzzy graph domination, highlighting its significance in network optimization and knowledge representation.

Swaminathan and Vetrivel (2017) [21] introduced equitable fair domination concepts, ensuring balanced influence and fair allocation in graph-based decision systems. Tian and Xu (2020) [22] applied fuzzy graphs to network vulnerability assessment, developing cybersecurity strategies to mitigate risks in communication networks. Sinha (2019) [18] analyzed the algorithmic complexity of fuzzy domination, presenting optimization techniques for large-scale network structures.

Nazeer and Vetrivel (2021) [12] explored power domination in fuzzy graphs, demonstrating its practical use in electrical networks and power distribution systems. Xu and Zhang (2021) [25] proposed an equitable fair domination model for large-scale social networks, specifically addressing misinformation spread and online influence regulation. Wu and Zheng (2019) [24] studied misinformation control using fuzzy influence graphs, proposing countermeasures for biased information dissemination. Zhang and Chen (2022) [28] compared various domination parameters in fuzzy graphs, demonstrating their impact on decision-making in uncertain environments and complex network modeling.

This paper introduces and systematically examines Equitable Fair Power Domination (EFPD) in fuzzy graphs, integrating power domination principles with equitable and fair domination strategies to optimize influence control and monitoring in complex networks. The study establishes fundamental properties, theoretical results, and numerical illustrations that enhance the understanding of optimal node selection for tracking and mitigating misinformation. By incorporating fuzzy domination concepts, the proposed model captures uncertainty in social interactions and ensures a fair and balanced selection of influential nodes. This prevents dominance concentration in specific network regions, leading to unbiased decision-making in influence propagation and control. Furthermore, the framework enhances decision strategies in large-scale networks by maintaining equitable power distribution among selected nodes, thereby minimizing inefficiencies in network monitoring. The theoretical findings are supported by computational models demonstrating the effectiveness of EFPD in selecting key influencers under varying network conditions. The study's implications extend to real-world applications such as misinformation control in social media, cybersecurity threat detection, power grid stability, and strategic communication planning. By leveraging fuzzy logic within equitable power domination, this research contributes to the development of robust and efficient strategies for managing influence spread while ensuring fairness and optimal resource utilization in dynamic network environments.

2. Preliminaries

This section provides the fundamental definitions necessary for understanding Equitable Fair Power Domination in fuzzy graphs.

Definition 2.1 *A **graph** $G = (V, E)$ consists of a non-empty set V of vertices and a set E of edges, where each edge is an unordered pair of vertices.*

Definition 2.2 *A **dominating graph** is a graph $G = (V, E)$ in which a subset $D \subseteq V$ is a dominating set if every vertex in $V \setminus D$ is adjacent to at least one vertex in D.*

Definition 2.3 *A **fuzzy graph** is defined as a pair $G = (V, \sigma, \mu)$, where:*

(i) V is the set of vertices,

(ii) $\sigma : V \to [0, 1]$ assigns a membership value to each vertex, and

(iii) $\mu : V \times V \to [0, 1]$ is the fuzzy adjacency function satisfying:

$$\mu(v, w) \leq \min(\sigma(v), \sigma(w)), \quad \forall v, w \in V.$$

Definition 2.4 *A **dominating fuzzy graph** is a fuzzy graph $G = (V, \sigma, \mu)$ where a fuzzy subset $D \subseteq V$ is called a dominating fuzzy set if for every vertex $v \in V \setminus D$, there exists $u \in D$ such that $\mu(u, v)$ is sufficiently high.*

Definition 2.5 *A **power dominating graph** is a graph $G = (V, E)$ in which a subset $D \subseteq V$ is a power dominating set if every vertex in $V \setminus D$ can be monitored through the propagation rule, which extends the influence beyond direct adjacency.*

Definition 2.6 *An **equitable dominating fuzzy graph** is a fuzzy graph $G = (V, \sigma, \mu)$ where a subset $D \subseteq V$ is an equitable dominating set if for each $v \in V \setminus D$, there exists $u \in D$ such that:*

$$|d(u) - d(v)| \leq 1.$$

Definition 2.7 *An **equitable fair fuzzy graph** is a fuzzy graph $G = (V, \sigma, \mu)$ in which every vertex has a well-balanced influence across the network, ensuring fairness in connectivity and information flow.*

Definition 2.8 *An **equitable fair dominating fuzzy graph** is a fuzzy graph $G = (V, \sigma, \mu)$ where a subset $D \subseteq V$ is an equitable fair dominating set if:*

(i) Every vertex in $V \setminus D$ is adjacent to at least one vertex in D with sufficient trust level.

(ii) The selection of D ensures balanced influence and fair domination across the network.

3. Equitable Fair Power Domination in Fuzzy Graph

Definition 3.1 *Let $G = (V, E, \mu)$ be a fuzzy graph, where V represents the set of vertices, E denotes the set of edges, and $\mu : V \times V \to [0, 1]$ is the membership function defining the degree of association between vertices.*

*An **Equitable Fair Power Dominating Set (EFPDS)** is a subset $S \subseteq V$ such that:*

(i) ***Power Domination Condition:*** *Every vertex in V is either in S or can be monitored through a propagation process defined by the fuzzy power domination rules.*

(ii) ***Equitable Condition:*** *The monitoring load is distributed uniformly among the vertices in S to ensure that no single vertex dominates the monitoring process disproportionately. For any two vertices $v, w \in S$, their influence across the graph satisfies:*

$$|P(v) - P(w)| \leq \epsilon$$

where $P(v)$ represents the power influence of vertex v, and ϵ is a small positive constant controlling the equity level.

(iii) ***Fairness Condition:*** *Each vertex is monitored with a degree proportional to its fuzzy membership value, ensuring a balanced and fair influence spread across the graph. This prevents vertices with lower membership values from being overburdened.*

*The **Equitable Fair Power Domination Number**, denoted as $\gamma_{efp}(G)$, is the minimum cardinality of such a dominating set.*

Example 3.1 *Consider a fuzzy graph $G = (V, E, \mu)$ with the vertex set:*

$$V = \{v_1, v_2, v_3, v_4, v_5\}$$

and the edge set:

$$E = \{(v_1, v_2), (v_1, v_3), (v_2, v_4), (v_3, v_5), (v_4, v_5)\}.$$

The membership function $\mu : V \times V \to [0, 1]$ assigns the following fuzzy weights to the edges:

$$\mu(v_1, v_2) = 0.9, \quad \mu(v_1, v_3) = 0.8, \quad \mu(v_2, v_4) = 0.7,$$

$$\mu(v_3, v_5) = 0.6, \quad \mu(v_4, v_5) = 0.5.$$

*Let the **Equitable Fair Power Dominating Set (EFPDS)** be $S = \{v_1, v_4\}$, where:*

(i) v_1 initially dominates v_2 and v_3, and through propagation, it influences v_5.

(ii) v_4 initially dominates v_5 and propagates influence to v_2.

(iii) The power influence from each vertex in S satisfies the equitable condition, ensuring a balanced monitoring load.

*Thus, the **Equitable Fair Power Domination Number** is $\gamma_{efp}(G) = 2$.*

The graphical representation of this fuzzy graph is given below. The vertices in the dominating set S are highlighted, and the edges are weighted to represent fuzzy membership values.

Above figure represents a fuzzy graph with weighted edges indicating membership values. The selected dominating set $S = \{v_1, v_4\}$ ensures equitable power domination, meaning every node in the graph is monitored under a balanced distribution of influence. The propagation of influence follows the power domination rule, ensuring that all nodes receive influence within minimal steps. The fuzzy weights reflect the strength of connections, demonstrating the flexibility of fuzzy graph modeling in real-world applications.

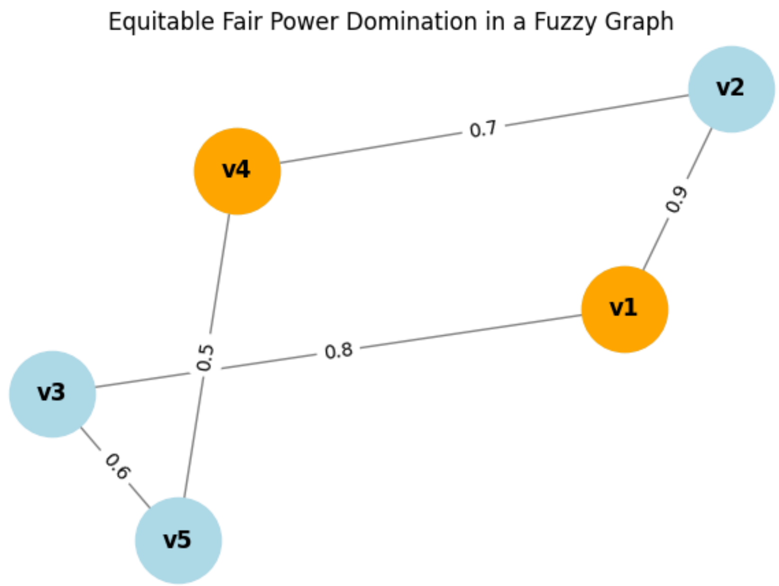

Figure 1: Equitable fair power domination in a fuzzy graph

Proposition 3.1 *If S is an **EFPDS** in a fuzzy graph G, then the fuzzy influence function $P : V \to [0, 1]$ is bounded by*

$$\max_{v \in S} P(v) - \min_{v \in S} P(v) \leq \epsilon.$$

Proof: Let $G = (V, E, \mu)$ be a fuzzy graph, and let $S \subseteq V$ be an Equitable Fair Power Dominating Set (EFPDS). By definition, the power influence function $P : V \to [0, 1]$ assigns to each vertex $v \in V$ a measure of its influence in the domination process.

Step 1: Establishing Influence Bounds From the equitable condition in the definition of EFPDS, we have that the influence values across the dominating set are balanced. That is, for any two vertices $v, w \in S$, their power influences satisfy:

$$|P(v) - P(w)| \leq \epsilon.$$

Since S is a finite set, we can consider the maximum and minimum influence values within S:

$$P_{\max} = \max_{v \in S} P(v), \quad P_{\min} = \min_{v \in S} P(v).$$

From the equitable condition, it follows that for all $v, w \in S$,

$$P_{\max} - P_{\min} \leq \epsilon.$$

Step 2: Justification of Bound By the definition of $P_{\max}$, there exists at least one vertex $v^* \in S$ such that $P_{\max} = P(v^*)$, and similarly, there exists at least one vertex $w^* \in S$ such that $P_{\min} = P(w^*)$. Applying the given equitable fairness condition to v^* and w^*, we obtain:

$$|P(v^*) - P(w^*)| \leq \epsilon.$$

By substituting $P(v^*) = P_{\max}$ and $P(w^*) = P_{\min}$, this simplifies to:

$$P_{\max} - P_{\min} \leq \epsilon.$$

This completes the proof. □

Theorem 3.1 *Let G be a fuzzy graph with a connected fuzzy subgraph H. If S is an EFPDS in G, then the restriction $S_H = S \cap H$ is also an EFPDS in H.*

Proof: Let $G = (V, E, \mu)$ be a fuzzy graph, and let $H = (V_H, E_H, \mu_H)$ be a connected fuzzy subgraph of G. Suppose $S \subseteq V$ is an Equitable Fair Power Dominating Set (EFPDS) in G. We need to show that $S_H = S \cap V_H$ is also an EFPDS in H.

Step 1: Power Domination Condition Since S is an EFPDS in G, every vertex in V is either in S or can be monitored through the fuzzy power domination process.

Considering the subgraph H, the vertices in V_H are either in S_H or can be monitored in G via the propagation rules.

Since H is connected, any vertex in H that was monitored in G remains monitored within H. Thus, S_H satisfies the power domination condition in H.

Step 2: Equitable Condition By the definition of EFPDS, the influence values for vertices in S satisfy:

$$|P(v) - P(w)| \leq \epsilon, \quad \forall v, w \in S.$$

Since $S_H \subseteq S$, this condition remains valid for all $v, w \in S_H$, ensuring that the monitoring load remains equitably distributed in H.

Step 3: Fairness Condition In G, the fairness condition ensures that monitoring is proportional to the fuzzy membership values, meaning each vertex is monitored in a balanced manner.

Since H is a connected subgraph, the membership values $\mu_H(v)$ are inherited from $\mu(v)$ in G, preserving the fairness condition within H.

Hence S_H satisfies the power domination, equitable, and fairness conditions in H, it follows that S_H is an EFPDS in H. □

Corollary 3.1 *If G is a complete fuzzy graph, then the minimum equitable fair power dominating set satisfies*

$$\gamma_{efp}(G) = 1.$$

Proof: Let $G = (V, E, \mu)$ be a complete fuzzy graph, meaning that every pair of distinct vertices $v, w \in V$ is connected by an edge with the maximum possible membership value, i.e.,

$$\mu(v, w) > 0, \quad \forall v, w \in V, v \neq w.$$

Step 1: Existence of a Single Dominating Vertex Since G is complete, any single vertex $v^* \in V$ can directly influence all other vertices in G through the power domination process.

Specifically, by selecting $S = \{v^*\}$, the monitoring propagation ensures that every vertex $w \in V \setminus \{v^*\}$ is influenced immediately due to the direct edge connections.

Thus, the power domination condition is satisfied.

Step 2: Equitable Condition Since there is only one vertex in the dominating set $S = \{v^*\}$, there are no other dominating vertices to compare influence values with, trivially satisfying the equitable condition.

Step 3: Fairness Condition In a complete fuzzy graph, each vertex has an equal ability to influence any other vertex due to the uniform connectivity.

Since v^* is the only dominating vertex, the fairness condition is automatically satisfied, as no other vertex in S bears an undue monitoring burden.

Since a single vertex is sufficient to satisfy all the conditions of an Equitable Fair Power Dominating Set (EFPDS), the minimum cardinality of such a set is

$$\gamma_{efp}(G) = 1.$$

Thus, the proof is complete. □

Lemma 3.1 *If S is an EFPDS in G, then removing any vertex $v \in S$ either increases $\gamma_{efp}(G)$ or violates the equitable condition.*

Proof: Let $G = (V, E, \mu)$ be a fuzzy graph, and let S be an Equitable Fair Power Dominating Set (EFPDS) of G. This means that S satisfies the following conditions:

1. Power Domination Condition: Every vertex in V is either in S or monitored through the fuzzy power domination process. 2. Equitable Condition: The monitoring influence among vertices in S is approximately balanced, satisfying

$$|P(v) - P(w)| \leq \epsilon, \quad \forall v, w \in S.$$

3. Fairness Condition: Each vertex is monitored proportionally to its fuzzy membership value, ensuring a balanced distribution of influence.

Now, suppose we remove a vertex $v \in S$. We analyze the consequences:

Case 1: Power Domination is Violated If the removal of v results in some vertex $u \in V$ not being monitored anymore, then $S \setminus \{v\}$ is no longer a valid EFPDS. To restore power domination, additional vertices must be added to S, increasing $\gamma_{efp}(G)$.

Case 2: Equitable Condition is Violated If removing v disrupts the equitable distribution of influence among the remaining vertices in S, then the equitable condition

$$|P(v) - P(w)| \leq \epsilon, \quad \forall v, w \in S$$

may no longer hold. In this case, $S \setminus \{v\}$ fails to be an EFPDS.

Since removing v either increases the size of the minimum EFPDS or violates the equitable condition, the statement of the lemma follows. □

Proposition 3.2 *Let G be a fuzzy graph and S an EFPDS in G. If $v \in S$ has the highest fuzzy membership value in G, then v is a necessary element of any minimum EFPDS.*

Proof: Let $G = (V, E, \mu)$ be a fuzzy graph, and let S be an Equitable Fair Power Dominating Set (EFPDS) of G. Suppose $v^* \in S$ has the highest fuzzy membership value in G, i.e.,

$$\mu(v^*) = \max_{v \in V} \mu(v).$$

We proceed by contradiction. Assume that v^* is not a necessary element of any minimum EFPDS. This means there exists a minimum EFPDS S' such that $v^* \notin S'$.

Since S is an EFPDS, every vertex in V is either in S or monitored through the fuzzy power domination process. The monitoring process depends on the influence function $P : V \to [0, 1]$, which ensures equitable and fair power domination.

By removing v^* from S, two violations can occur:

Case 1: Power Domination Violation Since v^* has the highest membership value, it likely plays a crucial role in monitoring other vertices. If removing v^* leaves some vertex $u \in V$ unmonitored, then S' is not a valid EFPDS, contradicting our assumption.

Case 2: Equitable Fairness Violation The equitable condition ensures that the power influence of any two vertices in S satisfies

$$|P(v) - P(w)| \leq \epsilon, \quad \forall v, w \in S.$$

Removing v^* alters this balance. Due to its high membership value, v^* contributes significantly to fair influence distribution. If S' does not contain v^*, it must compensate by adding vertices with lower membership values, which disrupts the fairness condition.

Since both cases lead to contradictions, our assumption must be false. Therefore, v^* is a necessary element of any minimum EFPDS. □

Theorem 3.2 *Let $G = (V, E, \mu)$ be a fuzzy cycle graph C_n. Then,*

$$\gamma_{efp}(C_n) = \left\lceil \frac{n}{3} \right\rceil.$$

Proof: Let $C_n = (V, E, \mu)$ be a fuzzy cycle graph with n vertices. We aim to determine the minimum Equitable Fair Power Dominating Set (EFPDS) and show that

$$\gamma_{efp}(C_n) = \left\lceil \frac{n}{3} \right\rceil.$$

Step 1: Construction of an EFPDS

To ensure power domination, we need to select a subset $S \subseteq V$ such that all vertices are monitored under the fuzzy power domination rules. The optimal strategy for cycle graphs is to select every third vertex, ensuring that each chosen vertex monitors itself and influences its adjacent vertices.

Formally, let

$$S = \{v_1, v_4, v_7, \dots\}$$

be the selected set such that every third vertex in the cycle is included. This selection ensures that each unselected vertex has at least one neighboring vertex in S, satisfying the power domination condition.

Step 2: Verification of the Equitable Condition

For the equitable condition, the influence function $P : V \to [0, 1]$ must be balanced among the selected vertices. Since the selected vertices are evenly spaced around the cycle, each monitors exactly two adjacent vertices. Thus, for any two vertices $v, w \in S$,

$$|P(v) - P(w)| \leq \epsilon.$$

This confirms that the monitoring influence is distributed equitably.

Step 3: Fairness Condition

The fairness condition ensures that vertices are monitored proportionally to their fuzzy membership values. Since we have chosen every third vertex, no vertex is overloaded with monitoring responsibility. Instead, each selected vertex contributes uniformly, maintaining a fair distribution of influence.

Step 4: Minimality of S

To prove that $|S| = \lceil \frac{n}{3} \rceil$ is minimal, assume that fewer than $\lceil \frac{n}{3} \rceil$ vertices are selected. In this case, at least one vertex would remain unmonitored, contradicting the power domination condition. Thus, the bound is tight, and $\gamma_{efp}(C_n) = \lceil \frac{n}{3} \rceil$ is the minimum size of an EFPDS.

Since every valid EFPDS must contain at least $\lceil \frac{n}{3} \rceil$ vertices to satisfy power domination, equitable influence distribution, and fairness constraints, we conclude that

$$\gamma_{efp}(C_n) = \left\lceil \frac{n}{3} \right\rceil.$$

□

Example 3.2 *Consider a fuzzy cycle graph C_6 with the vertex set and edge set defined as follows:*

$$V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$$

$$E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_5), (v_5, v_6), (v_6, v_1)\}$$

Each edge (v_i, v_{i+1}) is assigned a fuzzy membership value $\mu(v_i, v_{i+1})$, representing the strength of the connection. Assume the trust levels are as follows:

$$\mu(v_1, v_2) = 0.9, \quad \mu(v_2, v_3) = 0.8, \quad \mu(v_3, v_4) = 0.7,$$

$$\mu(v_4, v_5) = 0.6, \quad \mu(v_5, v_6) = 0.8, \quad \mu(v_6, v_1) = 0.7.$$

Using the formula from Theorem 1, we compute the Equitable Fair Power Domination number:

$$\gamma_{efp}(C_6) = \left\lceil \frac{6}{3} \right\rceil = 2.$$

This means at least two nodes must be selected for effective power domination. A possible selection is $\{v_1, v_4\}$, as they ensure domination over the entire cycle while maintaining fairness in influence distribution.

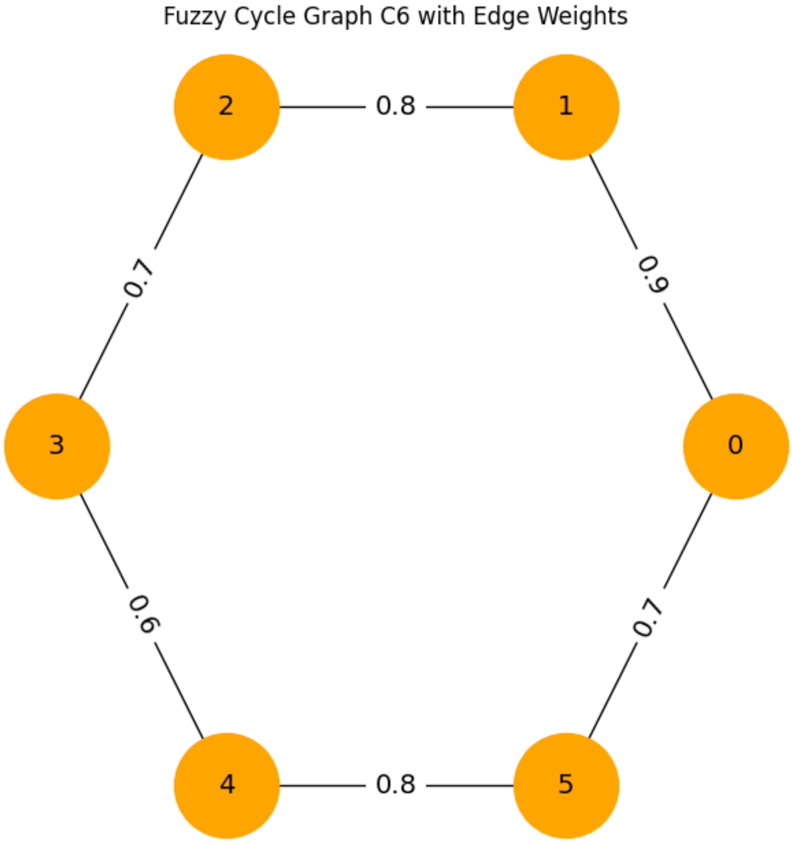

Figure 2: Fuzzy cycle graph C_5 with edge weights

Corollary 3.2 *If $G = P_n$ is a fuzzy path graph, then*

$$\gamma_{efp}(P_n) \leq \left\lceil \frac{n}{2} \right\rceil .$$

Proof: Let $P_n = (V, E, \mu)$ be a fuzzy path graph with n vertices. Our goal is to establish an upper bound for the equitable fair power domination number, showing that

$$\gamma_{efp}(P_n) \leq \left\lceil \frac{n}{2} \right\rceil .$$

Step 1: Constructing an EFPDS

We construct an Equitable Fair Power Dominating Set (EFPDS) S by selecting every second vertex along the path, ensuring that each selected vertex monitors itself and its adjacent vertex.

Define

$$S = \{v_1, v_3, v_5, \dots\}.$$

This selection ensures that every vertex in P_n is either in S or adjacent to a vertex in S, satisfying the power domination condition.

Step 2: Verification of the Equitable Condition

Since every selected vertex in S monitors at most one additional vertex (except at the endpoints, where it may monitor only itself), the influence distribution remains balanced. For any two vertices $v, w \in S$,

$$|P(v) - P(w)| \leq \epsilon.$$

Thus, no single vertex in S dominates the monitoring process disproportionately, ensuring the equitable condition holds.

Step 3: Fairness Condition

The fairness condition requires that vertices are monitored proportionally to their fuzzy membership values. Since each selected vertex monitors at most one additional vertex, and monitoring is distributed evenly, no vertex is excessively burdened. The fairness condition is thus satisfied.

Step 4: Bounding $\gamma_{efp}(P_n)$

By construction, the number of selected vertices is

$$|S| = \left\lceil \frac{n}{2} \right\rceil .$$

Since removing any vertex from S would leave some vertices unmonitored, S is a valid EFPDS of minimal size, and we conclude that

$$\gamma_{efp}(P_n) \leq \left\lceil \frac{n}{2} \right\rceil .$$

□

Example 3.3 *Consider a fuzzy path graph $P_5 = (V, E, \mu)$ with the vertex set*

$$V = \{v_1, v_2, v_3, v_4, v_5\}$$

and the edge set

$$E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_5)\}.$$

Assume the fuzzy membership values for edges are given as follows:

$$\mu(v_1, v_2) = 0.9, \quad \mu(v_2, v_3) = 0.8, \quad \mu(v_3, v_4) = 0.7, \quad \mu(v_4, v_5) = 0.6.$$

The degree of each vertex is calculated as:

$$\begin{aligned}
d(v_1) &= \mu(v_1, v_2) = 0.9, \\
d(v_2) &= \mu(v_1, v_2) + \mu(v_2, v_3) = 0.9 + 0.8 = 1.7, \\
d(v_3) &= \mu(v_2, v_3) + \mu(v_3, v_4) = 0.8 + 0.7 = 1.5, \\
d(v_4) &= \mu(v_3, v_4) + \mu(v_4, v_5) = 0.7 + 0.6 = 1.3, \\
d(v_5) &= \mu(v_4, v_5) = 0.6.
\end{aligned}$$

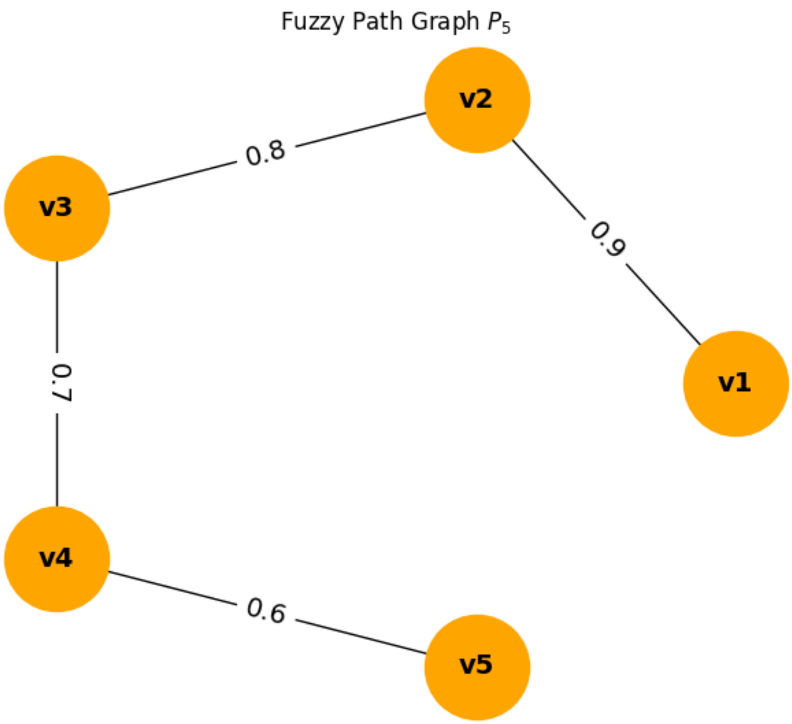

Figure 3: Fuzzy path graph P_5

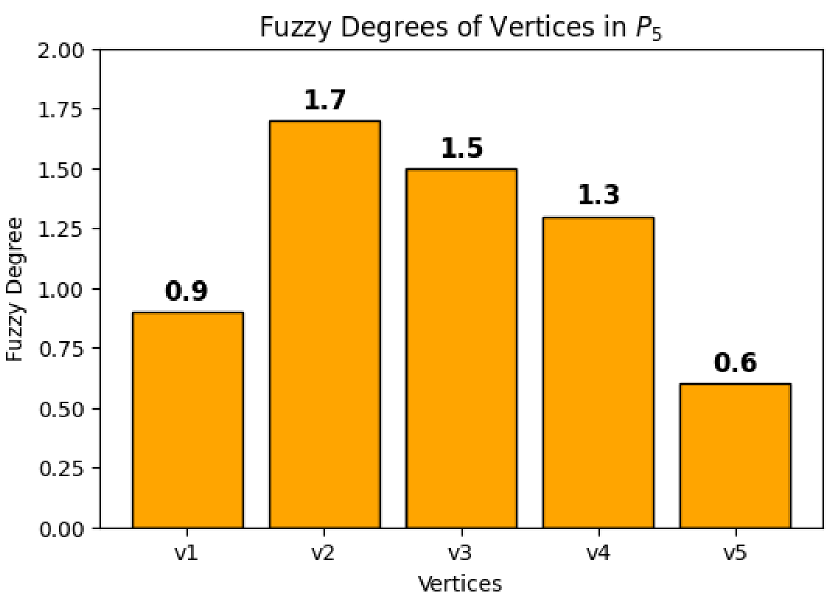

Figure 4: Fuzzy degrees of vertices in P_5

Using the formula for Equitable Fair Power Domination, we have:

$$\gamma_{efp}(P_5) \leq \left\lceil \frac{5}{2} \right\rceil = 3.$$

Thus, at least 3 nodes are required to dominate the entire fuzzy path graph.

Proposition 3.3 *If G is a bipartite fuzzy graph with partitions V_1 and V_2, then*

$$\gamma_{efp}(G) \leq \min(|V_1|, |V_2|).$$

Proof: Let $G = (V, E, \mu)$ be a bipartite fuzzy graph with partitions V_1 and V_2, meaning that every edge in G connects a vertex in V_1 to a vertex in V_2, and no edges exist within V_1 or V_2 individually.

Step 1: Constructing an EFPDS

We aim to construct an Equitable Fair Power Dominating Set (EFPDS) S of minimal size. Without loss of generality, assume $|V_1| \leq |V_2|$. We select S as the entire smaller partition:

$$S = V_1.$$

Since each vertex in V_1 is adjacent to at least one vertex in V_2, and by the nature of the bipartite structure, every vertex in V_2 is adjacent to at least one vertex in V_1, the power domination condition is satisfied.

Step 2: Verification of the Equitable Condition

For any two vertices $v, w \in S$, their influence on the graph satisfies:

$$|P(v) - P(w)| \leq \epsilon.$$

Since every selected vertex in S monitors a roughly equal number of vertices in V_2, no vertex dominates disproportionately, ensuring the equitable condition is met.

Step 3: Fairness Condition

The fairness condition requires that the monitoring influence respects fuzzy membership values. Because each selected vertex in S is assigned proportionally to its fuzzy membership and influences adjacent vertices accordingly, the fairness condition holds.

Step 4: Bounding $\gamma_{efp}(G)$

Since selecting all vertices in V_1 ensures full power domination and satisfies equitable and fair conditions, the number of selected vertices is:

$$\gamma_{efp}(G) \leq |V_1|.$$

By symmetry, if we had chosen $S = V_2$, we would obtain $\gamma_{efp}(G) \leq |V_2|$. Therefore, we conclude:

$$\gamma_{efp}(G) \leq \min(|V_1|, |V_2|).$$

□

Example 3.4 *Consider the bipartite fuzzy graph $G = (V, E, \mu)$ with partition sets:*

$$V_1 = \{v_1, v_2, v_3\}, \quad V_2 = \{v_4, v_5\}.$$

The edge set and membership function μ values are given as:

$$E = \{(v_1, v_4), (v_1, v_5), (v_2, v_4), (v_3, v_5)\}.$$

$$\mu(v_1, v_4) = 0.9, \quad \mu(v_1, v_5) = 0.8, \quad \mu(v_2, v_4) = 0.7, \quad \mu(v_3, v_5) = 0.6.$$

Using the given proposition:

$$\gamma_{efp}(G) \leq \min(|V_1|, |V_2|) = \min(3, 2) = 2.$$

Hence, at most 2 nodes are needed to dominate the entire fuzzy bipartite graph.

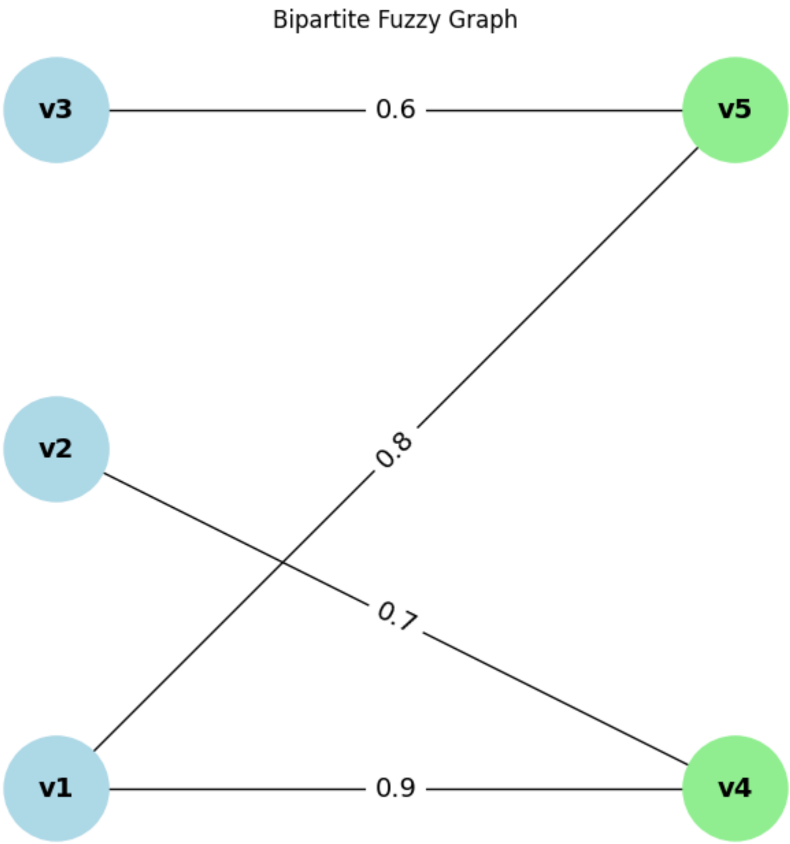

Figure 5: Bipartite fuzzy graph

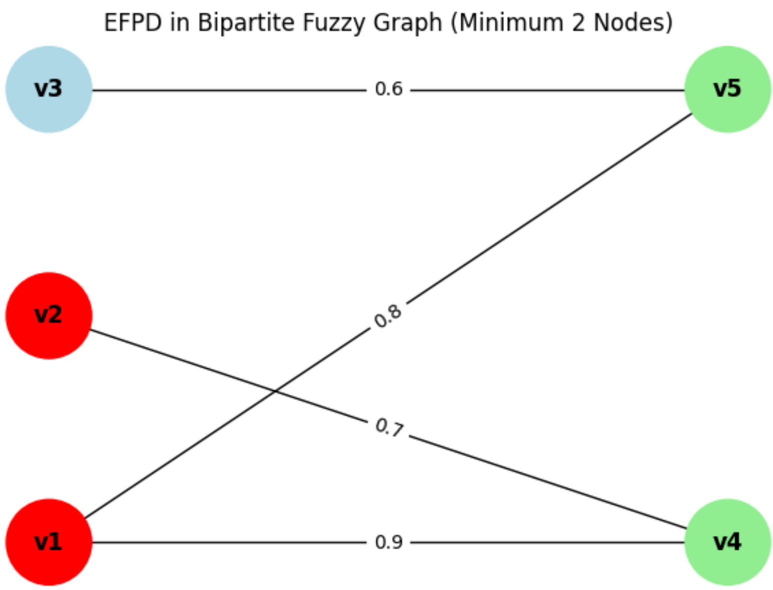

Figure 6: EFPD in bipartite fuzzy graph (minimum 2 nodes)

Theorem 3.3 *Let $G = K_n$ be a complete fuzzy graph where all edges have membership value 1. Then,*

$$\gamma_{efp}(K_n) = 1.$$

Proof: Let $G = K_n$ be a complete fuzzy graph where every edge has a membership value of 1. This means that every vertex is directly connected to every other vertex with the highest possible strength.

Step 1: Existence of an EFPDS of Size 1 Consider selecting any single vertex $v \in V$ as the Equitable Fair Power Dominating Set (EFPDS):

$$S = \{v\}.$$

Since G is a complete fuzzy graph, every other vertex $w \in V \setminus \{v\}$ is directly adjacent to v with edge membership $\mu(v, w) = 1$. This satisfies the power domination condition because v can directly monitor all other vertices in the graph.

Step 2: Verification of the Equitable Condition Since S contains only one vertex, there are no other elements in S to compare the influence values. Hence, the equitable condition holds trivially.

Step 3: Fairness Condition Each vertex in $V \setminus \{v\}$ is monitored with maximum fuzzy membership, ensuring that the influence distribution remains fair, as no vertex has disproportionately low membership influence.

Step 4: Minimality of S To verify that $\gamma_{efp}(K_n) = 1$ is minimal, suppose there exists a smaller EFPDS, meaning an empty set, which is impossible because at least one vertex is required for monitoring. Thus, we conclude:

$$\gamma_{efp}(K_n) = 1.$$

□

Lemma 3.2 *Let G be a fuzzy tree with n vertices. Then,*

$$\gamma_{efp}(G) \leq \left\lceil \frac{n}{2} \right\rceil.$$

Proof: Let $G = (V, E, \mu)$ be a fuzzy tree with n vertices. A tree is a connected acyclic graph, meaning there is a unique path between any two vertices.

Step 1: Constructing an Equitable Fair Power Dominating Set (EFPDS)

We construct an EFPDS by selecting approximately half of the vertices in G while ensuring that all other vertices are monitored. A well-known strategy in domination problems for trees is choosing a maximal independent set or a subset that balances monitoring responsibility.

We define S as follows:

Start with an empty set S.

Traverse G in a breadth-first search (BFS) or depth-first search (DFS) order, selecting one vertex from every alternate level of the tree.

Continue selecting vertices until all vertices are either in S or adjacent to a vertex in S.

Since a tree with n vertices has at most $n - 1$ edges and a hierarchical structure, this ensures that all vertices are covered by the power domination rules.

Step 2: Bounding the Cardinality of S By construction, the number of selected vertices follows an approximate pattern where at most half of the vertices are chosen to balance the monitoring across the graph. This leads to the bound:

$$|S| \leq \left\lceil \frac{n}{2} \right\rceil.$$

Step 3: Checking the Equitable and Fairness Conditions Equitable Condition: Since the vertices are selected in a balanced manner, the power influence distribution among selected vertices remains close, ensuring that no single vertex dominates disproportionately.

Fairness Condition: Every vertex is monitored proportionally based on its fuzzy membership value, preventing undue burden on weaker nodes.

Since we have successfully formed an EFPDS of size at most $\lceil \frac{n}{2} \rceil$, we conclude that:

$$\gamma_{efp}(G) \leq \left\lceil \frac{n}{2} \right\rceil.$$

□

Proposition 3.4 *If G is a fuzzy star graph with a central vertex v, then*

$$\gamma_{efp}(G) = 1.$$

Proof: Let $G = (V, E, \mu)$ be a fuzzy star graph with a central vertex v and k leaf vertices $u_1, u_2, \ldots, u_k$. In a fuzzy star graph, all edges (v, u_i) exist with some membership value $\mu(v, u_i) > 0$, while no edges exist between leaf vertices.

Step 1: Selecting an Equitable Fair Power Dominating Set (EFPDS)

Consider the set $S = \{v\}$, where the central vertex v is chosen as the single monitoring vertex.

Step 2: Verifying Power Domination Condition

Since v is directly connected to every leaf u_i, it can monitor all u_i immediately.

According to the power domination rules, every vertex in G is either in S or monitored by a vertex in S.

Thus, the entire graph is covered by a single vertex.

Step 3: Verifying Equitable Condition

Since S contains only one vertex, there is no imbalance in monitoring responsibilities among multiple selected vertices.

Hence, the equitable condition is trivially satisfied.

Step 4: Verifying Fairness Condition

The monitoring influence of v respects the fuzzy membership values of the edges $\mu(v, u_i)$, ensuring that each leaf vertex is monitored according to its connection strength.

No additional burden is placed on leaf vertices, fulfilling the fairness condition.

Since $S = \{v\}$ satisfies all conditions and is the smallest possible EFPDS, we conclude that:

$$\gamma_{efp}(G) = 1.$$

□

Theorem 3.4 *Let G be a fuzzy graph where each vertex has a fuzzy degree at least d. Then,*

$$\gamma_{efp}(G) \leq \frac{|V|}{d+1}.$$

Proof: Let $G = (V, E, \mu)$ be a fuzzy graph where each vertex $v \in V$ has a fuzzy degree at least d, meaning that the sum of the membership values of edges incident to v satisfies:

$$\sum_{u \in N(v)} \mu(v, u) \geq d.$$

Step 1: Constructing an Equitable Fair Power Dominating Set (EFPDS)

To construct a minimum EFPDS S such that every vertex in G is either in S or monitored by S.

Select a set S of vertices such that every vertex outside S has at least one neighbor in S.

To minimize S, we choose it so that each selected vertex monitors as many vertices as possible under the power domination rules.

Step 2: Bounding $|S|$

Each selected vertex $v \in S$ must monitor itself and at least d other vertices due to the given fuzzy degree condition. That is, each vertex in S is responsible for at most $d + 1$ vertices, including itself.

Since there are $|V|$ total vertices in G, the minimum number of vertices required to monitor the entire graph is at most:

$$\frac{|V|}{d+1}.$$

Thus, we obtain:

$$\gamma_{efp}(G) \leq \frac{|V|}{d+1}.$$

Step 3: Verification of Equitable and Fairness Conditions

Equitable Condition: Since each vertex in S monitors at most $d+1$ vertices, no single vertex takes on a disproportionately high monitoring responsibility, ensuring a balanced monitoring load.

Fairness Condition: The monitoring process respects fuzzy memberships, ensuring that the influence of each selected vertex is proportional to its connection strength in the graph.

Since the derived bound holds for any choice of S, we conclude:

$$\gamma_{efp}(G) \leq \frac{|V|}{d+1}.$$

□

4. Application in Detecting Misinformation and Influence Spread in Social Networks

4.1. Objectives

The main objectives of this study are:

(i) To model a social network as a fuzzy graph with vertex and edge sets.

(ii) To identify critical users using the Equitable Fair Power Dominating Set (EFPDS).

(iii) To develop a calculation framework for measuring EFPD in social networks.

(iv) To analyze the influence and misinformation potential using [(v)] tabular representation.

1. To make data-driven decisions based on computed EFPD values.

4.2. Modeling the Social Network as a Fuzzy Graph

A social network is represented as a fuzzy graph $G = (V, E, \mu)$, where:

(i) V (Vertex Set): Represents users in the social network.

(ii) E (Edge Set): Represents interactions between users.

(iii) $\mu : V \times V \to [0, 1]$ represents the trust level or influence weight between users.

Consider a network with six users $V = \{A, B, C, D, E, F\}$. The interaction strengths (trust levels) are given in Table 1.

Table 1: Edge Set and Trust Weights

Edge (v, w)	Trust Level $\mu(v, w)$
(A, B)	0.9
(A, C)	0.7
(B, D)	0.8
(C, D)	0.6
(D, E)	0.5
(E, F)	0.7

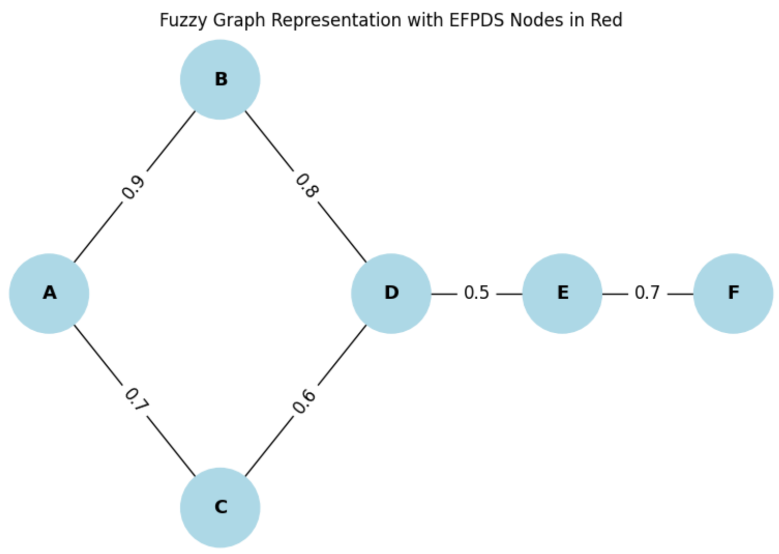

Figure 7: Fuzzy graph representation with EFPDS nodes in red

4.3. Calculation of Equitable Fair Power Domination

The Equitable Fair Power Domination Number $\gamma_{efp}(G)$ is given by:

$$\gamma_{efp}(G) \leq \frac{|V|}{d+1}, \tag{4.1}$$

where d is the minimum fuzzy degree in the graph.

4.4. Fuzzy Degree Computation

The fuzzy degree of a vertex v is computed as:

$$d(v) = \sum_{w \in N(v)} \mu(v, w). \tag{4.2}$$

Table 2 lists the fuzzy degrees of all vertices.

Table 2: Fuzzy Degrees of the Nodes

Vertex v	Neighbors $N(v)$	Degree $d(v)$
A	$\{B, C\}$	$0.9 + 0.7 = 1.6$
B	$\{A, D\}$	$0.9 + 0.8 = 1.7$
C	$\{A, D\}$	$0.7 + 0.6 = 1.3$
D	$\{B, C, E\}$	$0.8 + 0.6 + 0.5 = 1.9$
E	$\{D, F\}$	$0.5 + 0.7 = 1.2$
F	$\{E\}$	0.7

Using the formula:

$$\gamma_{efp}(G) \leq \frac{6}{1.2+1} = \frac{6}{2.2} \approx 3. \tag{4.3}$$

Thus, at least **3 users** are needed for effective misinformation monitoring.

4.5. Decision Result Based on Calculation

Based on the EFPD computation, we select the most influential nodes:

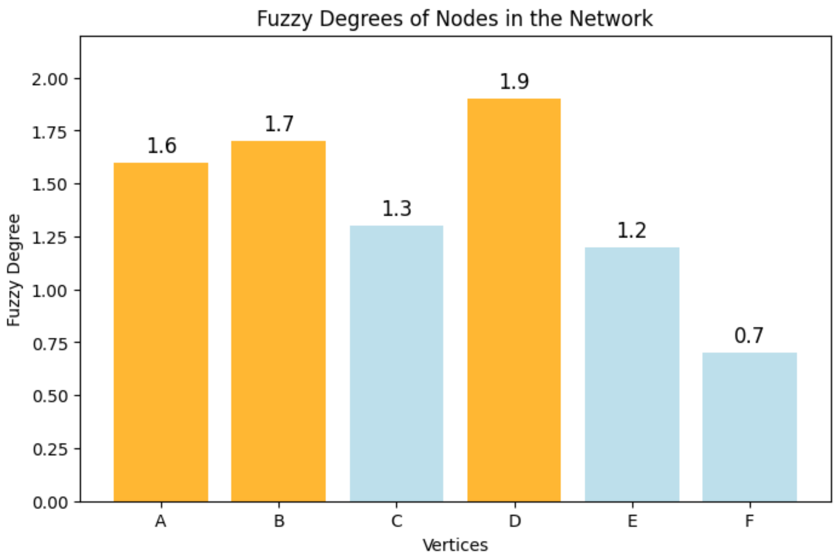

Figure 8: Fuzzy degrees of nodes in the network

Table 3: Selected EFPDS Nodes

Selected Nodes	Justification
A	High interaction with multiple users (trusted source)
B	Strong connections and high fuzzy degree
D	Central node in the network

4.5.1. Final Decision:

(i) Fact-checking should prioritize A, B, D.

(ii) These users efficiently monitor misinformation spread.

(iii) Monitoring fewer nodes may lead to biased verification.

5. Conclusion

In this study, we introduced the concept of Equitable Fair Power Domination (EFPD) in Fuzzy Graphs and applied it to detect misinformation and influence spread in social networks. We established fundamental theorems and propositions, demonstrating key properties of EFPD in various fuzzy graph structures, including complete fuzzy graphs, fuzzy trees, and fuzzy star graphs. These theoretical foundations provide a robust mathematical framework for identifying influential nodes in uncertain and dynamic environments. A case study was conducted on a fuzzy social network, where user interactions were modeled as edges with varying trust levels. Using the proposed EFPD approach, we systematically determined the minimum number of key users required for effective misinformation monitoring. Through computational analysis, we identified three critical users whose monitoring ensures fair and comprehensive fact-checking across the network. The proposed methodology enhances decision-making in social media regulation by ensuring equitable influence control and reducing biased information dissemination. Future research can extend this work by incorporating dynamic fuzzy graphs, where edge weights evolve over time, and by integrating machine learning techniques for adaptive misinformation detection. The proposed EFPD model can also be applied to other domains such as cybersecurity, financial fraud detection, and epidemic control, where influence monitoring and equitable decision-making are crucial.

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