



Dynamics and Stability in a Mathematical Model of Poverty and Alcohol Consumption

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ABSTRACT: This paper develops and analyzes a mathematical model describing the dynamic interaction between poverty and alcohol consumption within a population. Both deterministic and stochastic formulations are considered to capture variability in individual and collective behavior. The model incorporates assumptions about how individuals' economic conditions and alcohol dependency influence each other over time. Equilibrium points are derived and their stability properties are investigated analytically. Numerical simulations illustrate the system's dynamics under varying parameters and intervention strategies. The results provide insight into the mechanisms linking social behavior and economic status, contributing to the understanding of poverty persistence and recovery within a mathematical social science framework.

Keywords: Poverty and alcohol consumption, bifurcation analysis, sensitivity analysis, numerical simulation, stochastic model.

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1. Introduction

The mathematical model illustrates how poverty and alcoholism are directly related. These days, there are more and more practical approaches to endemic research that are largely focused on social issues. We have begun to create mathematical models based on the relationships between factors such as alcohol use and poverty. This latest toolkit provides researchers with support for:

- (i) understand the significance of gender, sex, and intersectionality in the research on infectious diseases and their implementation;
- (ii) design, develop, and report studies using an intersectional gender lens in accordance with a step-by-step guide;

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(iii) comprehend the obstacles to the successful application of health interventions to prevent and control infectious diseases; and

(iv) investigate ways to improve equality in access to healthcare so that no one is left behind. Other health research and activities are as significant, even if the toolkit focuses on studies that prioritize the prevention and management of infectious diseases linked to poverty [1].

African drylands, for example, would be impacted by increased rainfall, put further pressure on already destitute regions due to vector-borne water-related diseases [2].

Heart disease and respiratory conditions may be significantly exacerbated by tobacco usages. Because nicotine is highly addictive, there are more than 20 distinct types of cancer and a number of other serious health issues [6,4].

Tobacco usage may also kill nonsmokers. Every year, 1.2 million people die as a result of secondhand smoking, which has also been linked to negative health impacts. Approximately 50% of children are exposed to tobacco-polluted air, and diseases linked to secondhand smoke claim the lives of 65,000 children annually. If you smoke during pregnancy, you may cause long-term health issues for the unborn child. Alcohol is a highly addictive, pleasurable, and poisonous drug. Nowadays alcoholic drinks constitutes a typical component of social interactions in many cultures. This is particularly true for those with a positive reputation and cultural impact in social settings where alcohol is also used, both domestically and abroad [5,6].

Dangerous alcohol consumption accounts for 7.1% and 2.2% of the global disease burden for men and women, respectively. Alcohol is the leading cause of injury and early mortality among those aged 15 to 49, making about 10% of all deaths in this age range. The prevalence of alcohol-related diseases and mortality is higher among marginalized and poor populations [?].

The analytical formula describing the models' equilibrium between high and low alcohol levels, as well as its tipping points. Poverty is directly or indirectly connected to a number of factors, including illiteracy, an undesirable public atmosphere, an incapacity to handle pain and reality, a lack of psychological and social support, harm to families and society, family issues and forced relations, and failure to complete school. Alcohol addiction is common among those who are poor. We utilize a mathematical model to better understand the dynamic behavior of the system and how poverty and alcohol consumption might be reduced to the lowest possible levels. For people who are addicted, our approach has a part that permits a mix of government and private therapy [7].

This work is an extension of [7] by considering standard incidence interaction. Based on the results also varies phenomenally better than that of [7].

The paper is structured with first assumption of the model and analysis is explained in Section 2, which includes the subsections on basic reproduction number, equilibria, bifurcation analysis, and their stability analysis. In Section 3 we have to extend this work to sensitivity analysis. Section 4 presents the stochastic model, Section 5 shows the numerical findings. The deterministic and stochastic models of the numerical simulation are calculated in a Python program to display the figures. Section 6's findings, discussion, and conclusion have finally been presented.

2. The Model and Analysis

The non-linear model for poverty and alcohol use is proposed and examined, and it divides the population into four groups. Adults are the age group most affected by alcohol addiction. These allow us to determine the age range of persons living in poverty. Let $N(t)$, $A(t)$, $P(t)$, and $R(t)$ represent the proportions of the population that are not poor, those who suffer from alcoholism, and those who are in poverty and recovery, respectively, at time t . The overall populations are thought to be diverse and evenly distributed [7].

α_1 denote the rates of change from Non-impooverished people to the alcohol addicted people, that is depending on unemployed and underemployed peoples are directly linked into the alcohol. β denotes the value of individuals in Non-impooverished People to move into the poverty/impooverished people. Due to the natural disaster/the lose of business/health issues. In the γ value is those who are binge drinking in the alcohol they have to move in poverty population. Individuals α_2 and α_3 have recovered from alcohol addiction, and the poor have received counseling and rehabilitation at lower rates of alcohol addiction thanks to the assistance of some kind of non-governmental organization. The term $\alpha_1 N \frac{A}{T}$ represents the

transformation value from N class to A class, where the communication value is α_1 . Similarly, the term $\alpha_2 R \frac{A}{T}$ represent the transformation value from R class to A class, where the transmission value is α_2 . μ is natural death rate.

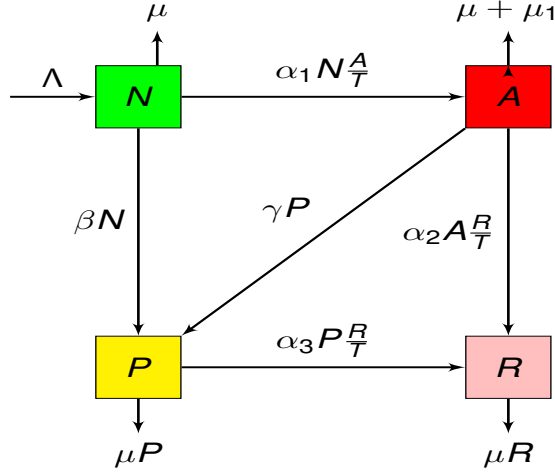


Figure 1: Model Flow Diagram

$$\begin{aligned}
 \frac{dN}{dt} &= \Lambda - \mu N - \alpha_1 N \frac{A}{T} - \beta N, \\
 \frac{dA}{dt} &= \alpha_1 N \frac{A}{T} - \gamma P - (\mu + \mu_1) A - \alpha_2 R \frac{A}{T}, \\
 \frac{dP}{dt} &= \beta N + \gamma P - \alpha_3 R \frac{P}{T} - \mu P, \\
 \frac{dR}{dt} &= \alpha_2 R \frac{A}{T} + \alpha_3 R \frac{P}{T} - \mu R.
 \end{aligned} \tag{2.1}$$

where $T = N + A + P + R$

The model (2.1) can be simplified as following:

$$\begin{aligned}
 T' &= \Lambda - \mu T - \mu_1 A, \\
 A' &= \alpha_1 (T - A - P - R) \frac{A}{T} - \gamma P - (\mu + \mu_1) A - \alpha_2 R \frac{A}{T}, \\
 P' &= \beta (T - A - P - R) + \gamma P - \alpha_3 R \frac{P}{T} - \mu P, \\
 R' &= \alpha_2 R \frac{A}{T} + \alpha_3 R \frac{P}{T} - \mu R.
 \end{aligned} \tag{2.2}$$

2.1. The Basic Reproduction Number R_0

The basic reproduction number, or R_0 , is the number of secondary alcohol addictions that the average person in poverty causes in a population that is otherwise disease-free. The reproduction number (R_0) for our model is determined following the procedure described in [8], and the matrices \mathcal{F} and \mathcal{V} are

followed by the same notation as in [8].

$$\mathcal{F} = \begin{pmatrix} \alpha_1 A \\ 0 \end{pmatrix}, \quad \mathcal{V} = \begin{pmatrix} \alpha_1(A + P + R)\frac{A}{T} + \gamma P + (\mu + \mu_1)A + \alpha_2 R\frac{A}{T} \\ \beta(A + P + R) + \alpha_3 R\frac{P}{T} + \mu P - (\beta T + \gamma P) \end{pmatrix}.$$

Now, the matrix F and V evaluates to both alcohol and poverty free equilibrium point is given by

$$F = \begin{pmatrix} \alpha_1 & 0 \\ 0 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} \mu + \mu_1 & \gamma \\ \beta & \beta + \mu - \gamma \end{pmatrix}.$$

The model 2.2 is given by the basix reproduction number to the largest eigenvalue of FV^{-1} is called the R_0 and attains the following:

$$R_0 = \frac{\alpha_1(\beta + \mu - \gamma)}{(\mu + \mu_1)(\beta - \gamma) - \beta\gamma}$$

In this case, the R_0 indicates the number of secondary alcohol addictions that are caused by the single impoverished person's lifetime alcohol use. when the whole population is seen as being impoverished.

2.2. Existence of Equilibria

The equilibria of the model is determined by the right hand side of the system (2.2) is equal to zero [9,10]. The following equilibria are ,

- (i) Both alcohol and poverty free equilibrium $E_0(T^0, 0, 0, 0) = (\frac{\Lambda}{\mu}, 0, 0, 0)$
- (ii) Alcohol Free Equilibrium (AFE) $E_1(T^1, 0, P^1, R^1) = (\frac{\Lambda}{\mu}, 0, \frac{\Lambda}{\alpha_3}, \frac{\beta T + P(\gamma - \mu - \beta)}{\beta + \mu})$,
- (iii) Poverty free Equilibrium (PFE) $E_2(T^2, A^2, 0, R^2) = (\frac{\alpha_2 A}{\mu}, \frac{\Lambda}{\alpha_2 + \mu_1}, 0, \frac{\Lambda}{\alpha_2 + \mu_1}(\frac{\alpha_2}{\mu} - 1))$ and
- (iv) Endemic Equilibrium (EE) $E_3(T^*, A^*, P^*, R^*)$, where

$$\begin{aligned} T^* &= \frac{\Lambda - \mu_1 A}{\mu}, \\ P^* &= \frac{\Lambda - \mu_1 A - \alpha_2 A}{\alpha_3}, \\ R^* &= \frac{(\Lambda - \mu_1 A)(u)}{(\mu\alpha_3\beta\Lambda - \mu\alpha_3\beta\mu_1 A + \mu^2\alpha_3\Lambda - \mu^2\alpha_3\mu_1 A - \mu^2\alpha_3\alpha_2 A)} \end{aligned}$$

Here, $u = \alpha_3\beta(\Lambda - \mu_1 A - \mu A) + \mu\beta(-\Lambda + \mu_1 A + \alpha_2 A) + \gamma\mu(\Lambda - \mu_1 A - \alpha_2 A) - 2\mu^2\Lambda + \mu^2 A(-\mu + \alpha_2)$
and

$$A^* = l_1 A^3 + l_2 A^2 + l_3 A - l_4 \tag{2.3}$$

Where,

$$l_1 = l_{11} + l_{12}, \quad l_2 = l_{21} + l_{22}, \quad l_3 = l_{31} + l_{32}, \quad l_4 = \mu\Lambda^3\gamma(\beta + \mu)$$

$$\begin{aligned} l_{11} &= (b_2 + b_3 + b_4 + b_5 + b_6 + b_8), & l_{12} &= -(b_1 + b_7 + b_9 + b_{10} + b_{11}). \\ l_{21} &= (c_2 + c_3 + c_6 + c_8 + c_9), & l_{22} &= -(c_1 + c_4 + c_5 + c_7 + c_{11} + c_{10} + c_{12} + c_{13}). \\ l_{31} &= (d_1 + d_2 + d_4 + d_6 + d_8), & l_{32} &= -(d_3 + d_5 + d_7). \\ b_1 &= \alpha_1\mu_1^2\mu^2(\gamma + \beta + \mu), & b_7 &= \alpha_2^2\mu^3\alpha_1, \\ b_2 &= \alpha_1\mu_1\mu^2(\mu^2 + \mu_1), & b_8 &= \alpha_1\alpha_3\mu^2\mu_1(\mu_1 + \mu), \\ b_3 &= \alpha_2\mu\mu_1(\mu_1\beta\gamma + \mu\mu_1\gamma + \mu\mu_1\beta + \mu^3), & b_9 &= \alpha_1\alpha_2\mu^2\mu_1(\mu + \gamma), \\ b_4 &= \alpha_1\alpha_2\alpha_3\mu^2(\mu + \mu_1), & b_{10} &= \alpha_2\mu\mu_1\beta(\alpha_2\mu + \alpha_1\mu_1 + \mu\mu_1), \\ b_5 &= \mu_1^3\mu\gamma(\beta + \mu), & b_{11} &= \alpha_3\mu^3\mu_1(\mu_1 + \alpha_2), \\ b_6 &= \alpha_2^2\mu^2\mu_1(\beta + \mu), & & \end{aligned}$$

Table 2: Number of Positive roots in equation (3)

S. No	l_1	l_2	l_3	l_4	Sign	No. of +Ve Roots
1	$l_{11} > l_{12}$	$l_{21} > l_{22}$	$l_{31} > l_{32}$	-	+ + + -	1 [Fig.2a]
2	$l_{11} > l_{12}$	$l_{21} > l_{22}$	$l_{31} < l_{32}$	-	+ + - -	1 [Fig.2b]
3	$l_{11} > l_{12}$	$l_{21} < l_{22}$	$l_{31} > l_{32}$	-	+ - + -	3 [Fig.4]
4	$l_{11} < l_{12}$	$l_{21} > l_{22}$	$l_{31} > l_{32}$	-	- + + -	2 [Fig.3a]
5	$l_{11} < l_{12}$	$l_{21} < l_{22}$	$l_{31} > l_{32}$	-	- - + -	2 [(Fig.3b]
6	$l_{11} < l_{12}$	$l_{21} > l_{22}$	$l_{31} < l_{32}$	-	- + - -	2 [Fig.3c]
7	$l_{11} > l_{12}$	$l_{21} < l_{22}$	$l_{31} < l_{32}$	-	+ - - -	1 [Fig.2c]
8	$l_{11} < l_{12}$	$l_{21} < l_{22}$	$l_{31} < l_{32}$	-	- - - -	0

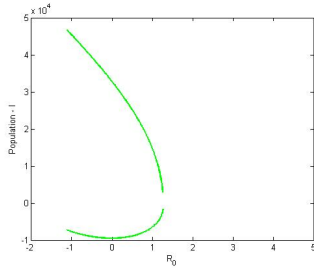
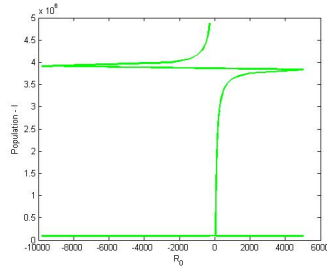
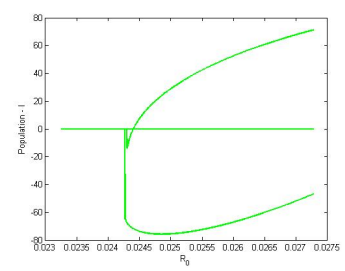

 (a) $\Lambda = 100, \alpha_1 = 0.207, \alpha_2 = 0.01, \alpha_3 = 0.01, \mu = 0.014, \mu_1 = 0.0167, \gamma = 0.051$.

 (b) $\Lambda = 100, \alpha_1 = 0.7, \alpha_2 = 0.5, \alpha_3 = 0.2, \mu = 0.014, \mu_1 = 0.0167, \gamma = 0.7$.

 (c) $\Lambda = 100, \alpha_1 = 0.07, \alpha_2 = 0.7, \alpha_3 = 0.7, \mu = 0.4, \mu_1 = 0.7, \gamma = 0.7$.

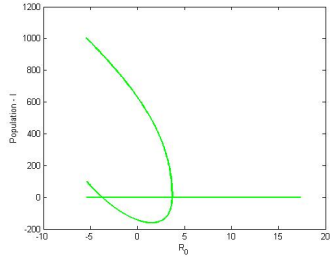
Figure 2: We have to see the above results are shown in one positive root results in different values of the parameter set.

$$\begin{aligned}
 c_1 &= 2\mu\mu_1\Lambda\beta(\mu_1\gamma + \alpha_2\gamma + \mu\alpha_2), & c_8 &= \alpha_2\mu^2\Lambda(\alpha_3(\mu + \beta) + \alpha_1\gamma), \\
 c_2 &= 2\mu\mu_1\Lambda\beta\alpha_3(\mu + \alpha_2), & c_9 &= \mu\Lambda(\mu_1^2\alpha_3\beta + \mu^2\Lambda\alpha_1), \\
 c_3 &= 2\mu^3\Lambda(\mu_1\alpha_3 + \alpha_1\alpha_2), & c_{10} &= \mu\mu_1^2\alpha_3(\mu\alpha_2 + \beta\mu_1 + \mu\mu_1), \\
 c_4 &= \alpha_1\alpha_3\mu^2\Lambda(\mu + \mu_1), & c_{11} &= \mu\mu_1\gamma(\beta\mu_1 + \mu\mu_1 + \mu^2\alpha_2), \\
 c_5 &= \mu^2\mu_1\Lambda(\alpha_1\alpha_2 + \mu_1\gamma + 2\alpha\gamma + \alpha_2\mu), & c_{12} &= \alpha_1\alpha_2\mu^2\Lambda(\alpha_3 + \mu), \\
 c_6 &= \mu^2\mu_1\Lambda(\gamma(\mu_1 + 2\alpha_1 + \alpha_2) + \alpha_3(\mu_1 + \alpha_2) + \alpha_1\mu), & c_{13} &= \mu^4\Lambda(\alpha_1 + \alpha_2), \\
 c_7 &= \alpha_2^2\mu^2\Lambda(\beta + \mu), & & \\
 d_1 &= 2\mu\mu_1\Lambda^2\gamma(\beta + \mu), & d_5 &= \mu^3\Lambda^2(\alpha_1 + \alpha_3), \\
 d_2 &= \mu^2\mu_1\Lambda(\gamma + \mu_1\alpha_3), & d_6 &= \mu\Lambda^2\gamma(\beta\alpha_2 + \alpha_1\mu), \\
 d_3 &= \mu^2\mu_1\Lambda^2\alpha_3, & d_7 &= \mu\beta\alpha_3\Lambda^2(\mu + \mu_1 + \alpha_2), \\
 d_4 &= \mu^2\Lambda^2(\alpha_1\alpha_3 + \alpha_2(\mu + \beta + \gamma)), & d_8 &= \beta\mu\mu_1\Lambda(\gamma + \mu_1\alpha_3).
 \end{aligned}$$

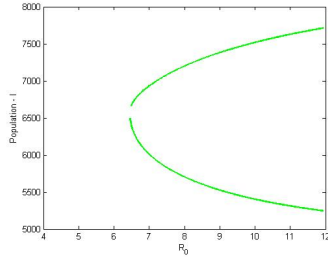
by using Descart rule to finding the number of positive roots in analytically to see the results in Table 2. we have to identify more than one positive roots are come to A^* values in E_3 points getting the numerical results by using MATLAB.

2.3. Bifurcation analysis

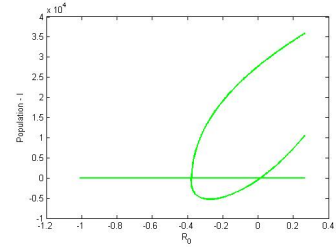
Let $T = x_1, A = x_2, P = x_3$ and $R = x_4$, Further using the vector notation $X = (x_1, x_2, x_3, x_4)^T$, the model (2.2) can be written with the following forms $\frac{dX}{dt} = (f_1, f_2, f_3, f_4)^T$



(a) $\Lambda = 100, \mu = 0.014, \mu_1 = 0.0167, \gamma = 0.048, \alpha_1 = 0.75, \alpha_2 = 0.65, \alpha_3 = 0.81.$



(b) $\Lambda = 100, \alpha_1 = 0.378, \alpha_2 = 0.905, \alpha_3 = 0.0681, \mu = 0.0143, \mu_1 = 0.0167, \gamma = 0.68.$



(c) $\Lambda = 100, \alpha_1 = 0.017, \alpha_2 = 0.17, \alpha_3 = 0.07, \mu = 0.014, \mu_1 = 0.0167, \gamma = 0.027.$

Figure 3: We have to see the above results are shown in two positive roots results in different values of the parameter set.

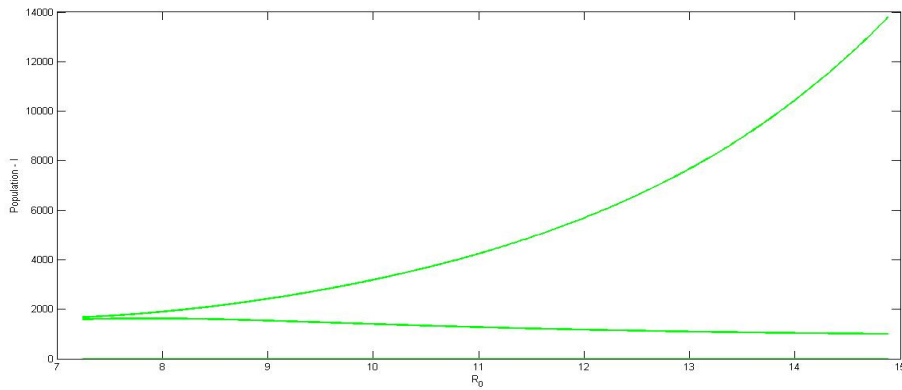


Figure 4: We have to see the above results are shown in three positive roots in the following values of the parameters like $\Lambda = 100, \alpha_1 = 0.81, \alpha_2 = 0.9, \alpha_3 = 0.3, \mu = 0.004, \mu_1 = 0.05, \gamma = 0.5.$

$$\begin{aligned}
 x'_1 &= \Lambda - \mu x_1 - \mu_1 x_2, \\
 x'_2 &= \alpha_1(x_1 - x_2 - x_3 - x_4) \frac{x_2}{x_1} - \gamma x_3 - (\mu + \mu_1)x_2 - \alpha_2 x_4 \frac{x_2}{x_1}, \\
 x'_3 &= \beta(x_1 - x_2 - x_3 - x_4) + \gamma x_3 - \alpha_3 x_4 \frac{x_3}{x_1} - \mu x_3, \\
 x'_4 &= \alpha_2 x_4 \frac{x_2}{x_1} + \alpha_3 x_4 \frac{x_3}{x_1} - \mu x_4.
 \end{aligned} \tag{2.4}$$

The Jacobian of the above system at Both alcohol and poverty free equilibrium point is given by

$$J_1 = \begin{pmatrix} -\mu & -\mu_1 & 0 & 0 \\ 0 & \alpha_1 - (\mu + \mu_1) & -\gamma & 0 \\ \beta & -\beta & \gamma - \beta - \mu & -\beta \\ 0 & 0 & 0 & -\mu \end{pmatrix}$$

Consider the case when $R_0 = 1$. Lets take $\beta_1 = \beta_1^*$ is a bifurcation parameter. Solving from $R_0 = 1$ gives

$$\beta = \beta^* = \frac{(\mu - \gamma)\alpha_1 + (\mu + \mu_1)\gamma}{(\mu + \mu_1 - \alpha_1 - \gamma)}$$

Use the following theorem which is reproduced from the reference [11] we will be able to find whether or not the system (2.4) exhibits backward bifurcation at $R_0 = 1$.

Theorem 2.1 Consider the following system of ordinary differential equations with the parameter ϕ ,

$$\frac{dx}{dt} = f(x, \phi),$$

$$f : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R},$$

and

$$f \in \mathbb{C}^2(\mathbb{R}^n \times \mathbb{R})$$

where 0 is an equilibrium point of the system (i.e. $f(0, \phi) \equiv 0$ for all ϕ) and

1. $A = D_x f(0, 0) = (\frac{\partial f_i}{\partial x_j}(0, 0))$ is the linearization matrix of the system around the equilibrium 0 with f evaluated at 0 ;
2. Zero is a simple eigenvalue of A and other eigenvalues of A have negative real parts;
3. Matrix A has a right eigenvector w and a left eigenvector v corresponding to the zero eigenvalue.

Let f_k be the k^{th} component of f and

$$a_1 = \sum_{i,j,k=1}^n v_k w_i w_j \frac{\partial^2 f_k}{\partial x_i \partial x_j}(0, 0)$$

$$b_1 = \sum_{i,j,k=1}^n v_k w_i \frac{\partial^2 f_k}{\partial x_i \partial \phi}(0, 0)$$

then the local dynamics of the system around the equilibrium point 0 is totally determined by the signs of a_1 and b_1 . Particularly, if $a_1 > 0$ and $b_1 > 0$, then a backward bifurcation occurs at $\phi = 0$.

2.3.1. *Eigenvalues of J_{β^*} .* It can be seen the Jacobian with $\beta = \beta^*$ of the linearized system has to simple 0 eigenvalue and all the remaining eigenvalues are negative real parts. Hence, the manifold theory can be analyzed the dynamics of the system (4) near $\beta = \beta^*$. In this case when $R_0 = 1$, use the technique in Castillo-Chavez and Song [11], Athithan. S and Mini Ghosh [12], Akhil Kumar Srivastav and Mini Ghosh [13], it can be displayed the matrix J_{β^*} has a right eigenvector (in the 0 eigenvalue), is given by $w = [w_1 \ w_2 \ w_3 \ w_4]^T$, where

$$w_1 = \frac{-\mu_1 w_2}{\mu}, w_2 = w_2 > 0, w_3 = \frac{\alpha_1 - (\mu + \mu_1)w_2}{\gamma}, w_4 = 0.$$

Similarly, the matrix J_{β^*} has a left eigenvector (in the 0 eigenvalue), denoted by $v = [v_1 \ v_2 \ v_3 \ v_4]$, where

$$v_1 = \frac{\beta^* v_3}{\mu_1}, v_2 = \frac{(\gamma - \beta^* - \mu)v_3}{\gamma}, v_3 = v_3 > 0, v_4 = \frac{-\beta^* v_3}{\mu}.$$

2.3.2. *Computation of a_1 .* To the system (2.4), is associated non-zero partial derivatives are given by

$$\begin{aligned} \frac{\partial^2 f_2}{\partial x_2^2} &= \frac{-2\alpha_1}{x_1}, \\ \frac{\partial^2 f_2}{\partial x_2 x_3} &= \frac{-\alpha_1}{x_1}, \\ \frac{\partial^2 f_2}{\partial x_4 x_2} &= \frac{-(\alpha_1 + \alpha_2)}{x_1}, \\ \frac{\partial^2 f_3}{\partial x_4 x_3} &= \frac{-\alpha_3}{x_1}. \end{aligned}$$

It follows from the above expressions that

$$a_1 = v_2 \left[w_2^2 \left(\frac{-2\alpha_1 \mu}{\Lambda} \right) + w_2 \left(\frac{\alpha_1 - (\mu + \mu_1)w_2}{\gamma} \right) \left(\frac{-\alpha_1 \mu}{\Lambda} \right) \right]$$

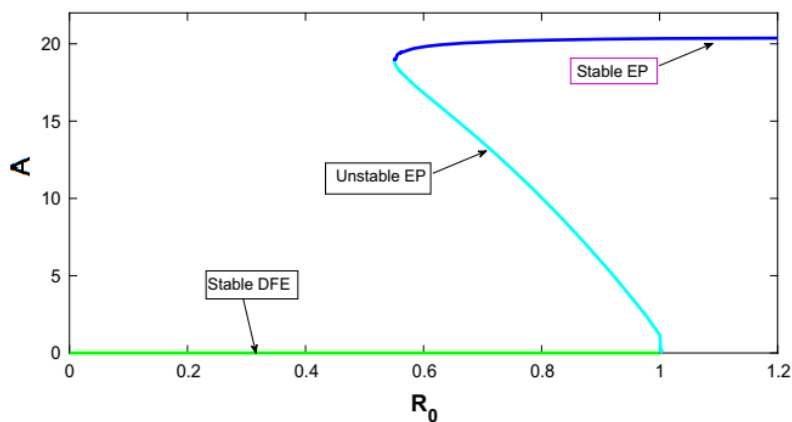


Figure 5: Plot of A with R_0 showing backward bifurcation by considering β as bifurcation parameter and other parameters are $\mu = 0.0000421$, $\mu_1 = 0.0328$, $\gamma = 0.02$, $\alpha_1 = 0.6$, $\alpha_2 = 0.02$.

2.3.3. *Computation of b_1 .* To the computation of b_1 is find out the associated non-zero partial derivatives are

$$\frac{\partial^2 f_3}{\partial x_3 \partial \beta} = 1$$

It follows from the above expression

$$\begin{aligned} b_1 &= v_3 w_3 \beta \\ &= \beta v_3 \left(\frac{\alpha_1 - (\mu + \mu_1) w_2}{\gamma} \right) > 0 \end{aligned}$$

Thus, the following results are established.

Theorem 2.2 *The model expose backward bifurcation at $R_0 = 1$ whenever a_1 is positive. This fact is showed in Fig. (5).*

2.4. Stability Analysis

2.4.1. *Stability analysis of alcohol free equilibrium.*

Theorem 2.3 *The alcohol free equilibrium E_1 is locally and asymptotically stable provided $R_0 < 1$ [20] - [30].*

Proof: The study of stability to the alcohol free equilibria of the matrix M_1 corresponding to the system E_1 is obtained as

$$M_1 = \begin{pmatrix} b_{11} & b_{12} & 0 & 0 \\ 0 & b_{22} & b_{23} & 0 \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{pmatrix}$$

where,

$$\begin{aligned} b_{11} &= -\mu, & b_{33} &= \gamma - \beta - \mu - \frac{\alpha_3 R}{T}, \\ b_{12} &= -\mu_1, & b_{34} &= -\beta - \frac{\alpha_3 P}{T}, \\ b_{22} &= \alpha_1 - (\mu + \mu_1) + \frac{-\alpha_1(P + R) - (\alpha_2 R)}{T}, & b_{41} &= \frac{-R\alpha_3 P}{T^2}, \\ b_{23} &= -\gamma, & b_{42} &= \frac{\alpha_2 R}{T}, \\ b_{31} &= \beta + \frac{\alpha_3 P R}{T^2}, & b_{43} &= \frac{\alpha_3 R}{T}, \\ b_{32} &= -\beta, & b_{44} &= \frac{\alpha_3 P}{T} - \mu. \end{aligned}$$

The eigenvalues of the variational matrix are given by the roots of the following characteristic equation in λ .

$$\lambda^4 + y_1 \lambda^3 + y_2 \lambda^2 + y_3 \lambda + y_4 = 0$$

where,

$$\begin{aligned} y_1 &= -(b_{11} + b_{12} + b_{13} + b_{14}), \\ y_2 &= b_{11}(b_{22} + b_{33} + b_{44}) + b_{22}(b_{33} + b_{44}) + b_{33}b_{44} - b_{43}b_{34} - b_{23}b_{32}, \\ y_3 &= -b_{11}b_{22}(b_{33} + b_{44}) + (b_{11} + b_{22})(b_{43}b_{34} - b_{33}b_{44}) + b_{23}b_{32}(b_{11} + b_{44}) - b_{23}(b_{42}b_{34} - b_{12}b_{31}), \\ y_4 &= b_{11}b_{22}(b_{33}b_{44} - b_{43}b_{34}) + b_{11}b_{23}(b_{42}b_{34} - b_{32}b_{44}) + b_{12}b_{23}(b_{31}b_{44} - b_{34}b_{41}). \end{aligned}$$

by Routh Hurwitz criteria, roots of the bi-quadratic equation will have negative real parts is positive. Now $b_1 > 0, b_3 > 0, b_4 > 0$ and $b_1 b_2 b_3 > b_3^2 + b_1^2 b_4$. Hence the equilibrium point E_1 is locally asymptotically stable provided $R_0 < 1$ [20] - [30]. \square

2.4.2. Stability analysis of Poverty free equilibrium.

Theorem 2.4 *The Poverty Free Equilibrium E_2 is locally asymptotically stable provided $R_0 < 1$.*

Proof: The study the stability of disease free equilibrium the matrix M_2 corresponding to the system E_2 is obtained as

$$M_2 = \begin{pmatrix} c_{11} & c_{12} & 0 & 0 \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{pmatrix}$$

where,

$$\begin{aligned} c_{11} &= -\mu, & c_{32} &= -\beta, \\ c_{12} &= -\mu_1, & c_{33} &= \gamma - \beta - \mu - \frac{\alpha_3 R}{T}, \\ c_{21} &= \frac{\alpha_1 A(A+R)}{T^2} + \frac{\alpha_2 AR}{T^2}, & c_{34} &= -\beta, \\ c_{22} &= \alpha_1 + \frac{-\alpha_1(2A+R) - (\alpha_2 R)}{T} - (\mu + \mu_1), & c_{41} &= \frac{-R(\alpha_2 A)}{T^2}, \\ c_{23} &= -\frac{\alpha_1 A}{T} - \gamma, & c_{42} &= \frac{\alpha_2 R}{T}, \\ c_{24} &= -\frac{A(\alpha_1 + \alpha_2)}{T}, & c_{43} &= \frac{\alpha_3 R}{T}, \\ c_{31} &= \beta, & c_{44} &= \frac{\alpha_2 A}{T} - \mu. \end{aligned}$$

The eigenvalues of this variational matrix are given by the roots of the following characteristic equation in λ .

$$\lambda^4 + z_1 \lambda^3 + z_2 \lambda^2 + z_3 \lambda + z_4 = 0$$

where,

$$\begin{aligned} z_1 &= -(c_{11} + c_{12} + c_{13} + c_{14}), \\ z_2 &= c_{11}(c_{22} + c_{33} + c_{44}) + c_{22}(c_{33} + c_{44}) + c_{33}c_{44} - c_{43}c_{34} - c_{23}c_{32} - c_{42}c_{24} - c_{12}c_{21}, \\ z_3 &= -c_{11}c_{22}(c_{33} + c_{44}) + (-c_{33}c_{44} + c_{43}c_{34})(c_{11} + c_{22}) + c_{23}c_{32}(c_{11} + c_{44}) \\ &\quad + c_{12}c_{21}(c_{33} + c_{44}) + c_{24}c_{42}(c_{11} + c_{33}) - c_{23}(c_{42}c_{34} + c_{12}c_{31}) - c_{24}(c_{32}c_{43} + c_{12}c_{41}), \\ z_4 &= c_{11}c_{22}(c_{33}c_{44} - c_{43}c_{34}) + c_{11}c_{23}(c_{42}c_{34} - c_{32}c_{44}) + c_{11}c_{24}(c_{32}c_{43} - c_{42}c_{33}) \\ &\quad + c_{12}c_{21}(c_{34}c_{43} - c_{33}c_{44}) + c_{12}c_{31}(c_{23}c_{44} - c_{24}c_{43}) + c_{12}c_{41}(c_{24}c_{33} - c_{23}c_{34}). \end{aligned}$$

by Routh Hurwitz criteria, roots of the bi-udratic equation will have negative real parts is positive. Now $c_1 > 0, c_3 > 0, c_4 > 0$ and $c_1 c_2 c_3 > c_3^2 + c_1^2 c_4$ Hence the equilibrium point E_1 is locally asymptotically stable provided $R_0 < 1$. \square

2.4.3. Stability analysis of endemic equilibrium.

Theorem 2.5 *The Endemic Equilibrium E_3 is locally asymptotically stable when $R_0 > 1$ [14].*

Proof: E_3 corresponding to the variational matrix M_3 is given by

$$M_3 = \begin{pmatrix} d_{11} & d_{12} & 0 & 0 \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{pmatrix}$$

where,

$$\begin{aligned}
 d_{11} &= -\mu, & d_{32} &= -\beta, \\
 d_{12} &= -\mu_1, & d_{33} &= \gamma - \beta - \mu - \frac{\alpha_3 R}{T}, \\
 d_{21} &= \frac{\alpha_1 A(A + P + R)}{T^2} + \frac{\alpha_2 AR}{T^2}, & d_{34} &= -\beta - \frac{\alpha_3 P}{T}, \\
 d_{22} &= \alpha_1 + \frac{-\alpha_1(2A + P + R) - (\alpha_2 R)}{T} - (\mu + \mu_1), & d_{41} &= \frac{-R(\alpha_2 A + \alpha_3 P)}{T^2}, \\
 d_{23} &= -\frac{\alpha_1 A}{T} - \gamma, & d_{42} &= \frac{\alpha_2 R}{T}, \\
 d_{24} &= -\frac{A(\alpha_1 + \alpha_2)}{T}, & d_{43} &= \frac{\alpha_3 R}{T}, \\
 d_{31} &= \beta + \frac{\alpha_3 PR}{T^2}, & d_{44} &= \frac{\alpha_2 A + \alpha_3 P}{T} - \mu.
 \end{aligned}$$

The eigenvalues of the M_3 is given by the roots of the following equation in λ .

$$\lambda^4 + d_1 \lambda^3 + d_2 \lambda^2 + d_3 \lambda + d_4 = 0$$

Where,

$$\begin{aligned}
 d_1 &= -(d_{11} + d_{12} + d_{13} + d_{14}), \\
 d_2 &= d_{11}(d_{22} + d_{33} + d_{44}) + d_{22}(d_{33} + d_{44}) + d_{33}d_{44} - d_{43}d_{34} - d_{23}d_{32} - d_{42}d_{24} - d_{12}d_{21}, \\
 d_3 &= -d_{11}d_{22}(d_{33} + d_{44}) + (-d_{33}d_{44} + d_{43}d_{34})(d_{11} + d_{22}) + d_{23}d_{32}(d_{11} + d_{44}) \\
 &\quad + d_{12}d_{21}(d_{33} + d_{44}) + d_{24}d_{42}(d_{11} + d_{33}) - d_{23}(d_{42}d_{34} + d_{12}d_{31}) - d_{24}(d_{32}d_{43} + d_{12}d_{41}), \\
 d_4 &= d_{11}d_{22}(d_{33}d_{44} - d_{43}d_{34}) + d_{11}d_{23}(d_{42}d_{34} - d_{32}d_{44}) + d_{11}d_{24}(d_{32}d_{43} - d_{42}d_{33}) \\
 &\quad + d_{12}d_{21}(d_{34}d_{43} - d_{33}d_{44}) + d_{12}d_{31}(d_{23}d_{44} - d_{24}d_{43}) + d_{12}d_{41}(d_{24}d_{33} - d_{23}d_{34}).
 \end{aligned}$$

by Routh Hurwitz criteria, roots of the bi-quadratic equation have negative real part is positive. Now, $d_1 > 0, d_3 > 0, d_4 > 0$ and $d_1 d_2 d_3 > d_3^2 + d_1^2 d_4$

Hence, the equilibrium point E_3 is locally asymptotically stable when $R_0 > 1$. \square

3. Sensitivity analysis

We do sensitivity analysis on the reproduction number (R_0) parameter. Sensitivity indices may be used to evaluate the relative change in a variable when a parameter is changed. It is used to identify factors that have a significant impact on R_0 and should be addressed via intervention strategies. We do this by examining the forward sensitivity index of a variable with respect to a certain parameter, which is described by the magnitude relationship between the relative amendment in the variable and the relative amendment in the parameter. The sensitivity index is described using partial derivatives if the variable is differentiable with respect to the parameter [15] - [30]. R_0 defines the forward sensitivity index of, which is differentiable depending on a specified parameter β .

$$\frac{\partial R_0}{\partial \alpha_1} \frac{\alpha_1}{R_0} = 1 > 0$$

$$\frac{\partial R_0}{\partial \beta} \frac{\beta}{R_0} = \frac{\beta(\mu(\gamma - \mu - \mu_1) - \gamma^2)}{(\beta + \mu - \gamma)(\beta(\mu + \mu_1 - \gamma) - \gamma(\mu + \mu_1))} > 0$$

$$\frac{\partial R_0}{\partial \gamma} \frac{\gamma}{R_0} = \frac{\gamma(\gamma(\mu + \mu_1) - \beta(\mu + \mu_1 - \gamma) + (\beta + \mu - \gamma)(\mu + \mu_1 + \beta))}{(\beta + \mu - \gamma)(\beta(\mu + \mu_1 - \gamma) - \gamma(\mu + \mu_1))} < 0$$

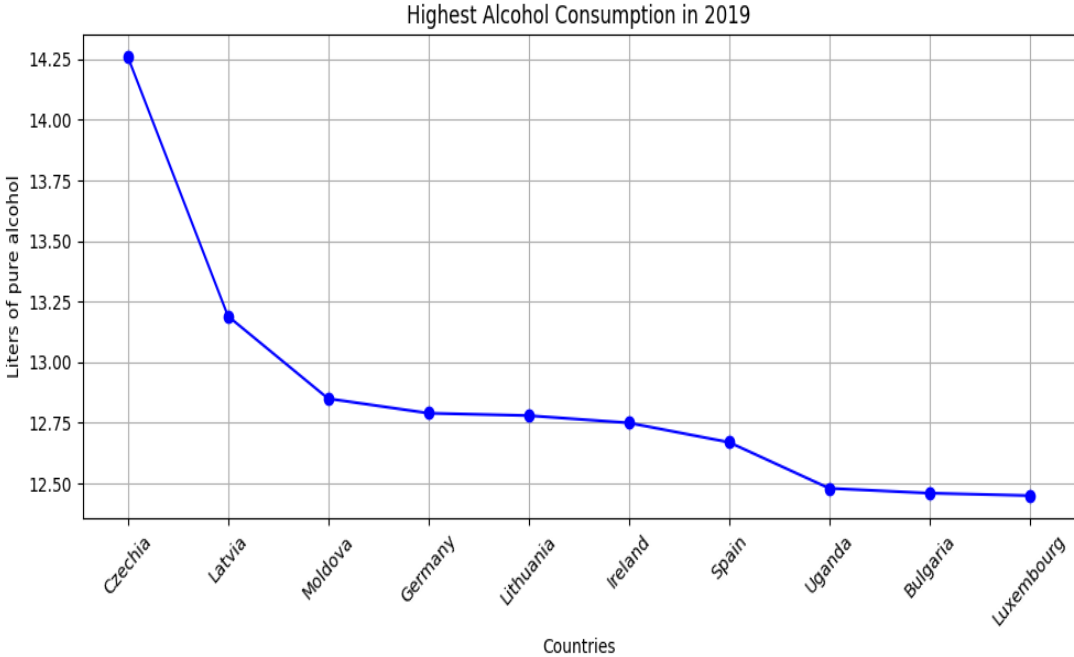


Figure 6: In 2019, the 10 most popular nations with the highest alcohol consumption are as follows: (in litres of pure alcohol per capita)

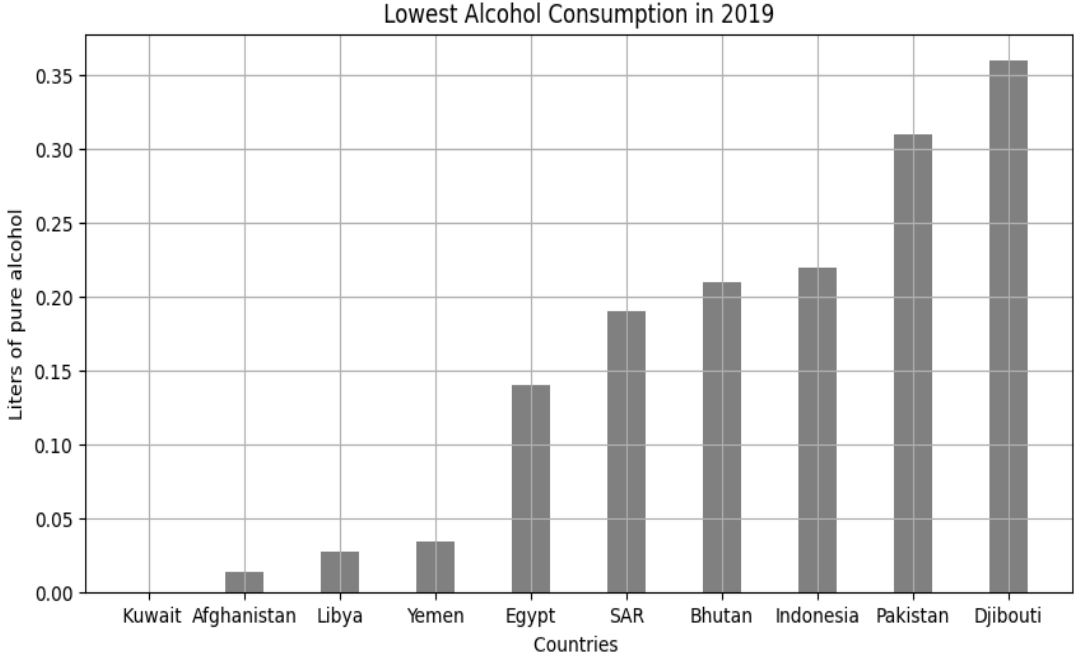


Figure 7: In 2019, the 10 most popular Nations with the Minimum Alcohol Consumption (in litres of pure alcohol per capita)

Using the method above, one can easily get an analytical equation for the sensitivity of R_0 to each of its parameters. The R_0 in Fig. 8 also shows the influence of the factors. We may infer from the figures that β , γ , μ , μ_1 , and α_1 have the same response on the R_0 . This indicates that if we have to increase the value of these parameters, the population’s rate of poverty and alcohol use will increase. The prevalence of poverty must be reduced because of the binge drinking habits of both the impoverished and the non-poor. Additionally, it is noted that the parameters α_1 and β are quite sensitive, and that adjusting them may significantly lower the value of R_0 .

The amount of alcohol consumed in each nation varies greatly and is impacted by its laws, customs, and other elements. The WHO recorded the amount of alcohol consumed in each nation in liters of pure alcohol for all types of beer, wine, spirits, and other alcoholic beverages.

According to the WHO, Czech citizens consumed the most alcohol in 2019—14.26 liters of pure alcohol per person—as seen in Fig. (6). Second and third place go to Latvia and Moldova, with 13.19 and 12.85 points, respectively. People in 38 countries drink more than 10 liters of pure alcohol annually. Europe is home to the great bulk of these countries.

<https://worldpopulationreview.com/country-rankings/alcohol-consumption-by-country>

The Middle East’s primary faiths forbid drinking alcohol, as shown by the region’s very low consumption rates, as seen in Fig. (7).

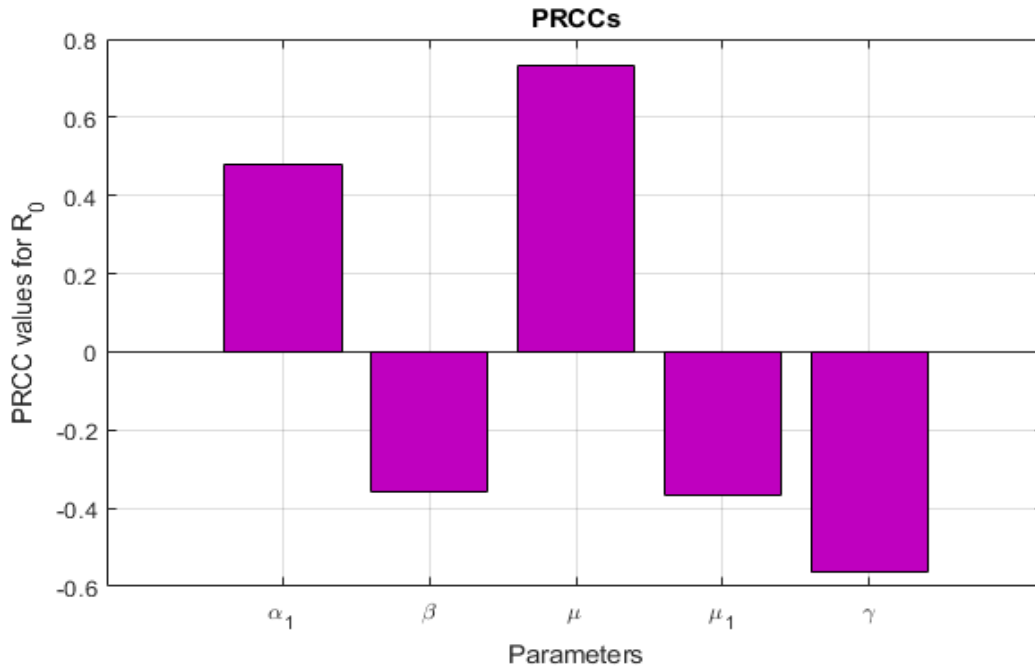


Figure 8: PRCC findings with R_0 displaying model parameter sensitivity indices

4. Stochastic Model

Here we are extending our ODE model to stochastic model, due to the fact stochastic fashions are extra able to detecting natural changes withinside the hassle of decreasing the poverty in the global population. Derivation of an SDE model is based on these approaches [16]–[19]. Let $X(t) = (X_1(t), X_2(t), X_3(t), X_4(t))^T$ be a continuous random variable for $(T(t), A(t), P(t), R(t))^T$ and T denotes the transpose of a matrix.

Let $\Delta X = X(t + \Delta t) - X(t) = (\Delta X_1, \Delta X_2, \Delta X_3, \Delta X_4)^T$ denotes the random vector for the change in random variables during time interval Δt . Here, we create a transition map that defines all the changes that may occur between the states of the SDE model. Based on our ODE model system (2.2), here we see that there exist 18 possible changes between states in a small time interval Δt . State changes and their probabilities are discussed in Table 3. Let us consider the case the recruitment of Non-imphoverised people becomes Alcohol addiction / poverty people. In this case, the state change ΔX is denoted by $\Delta X = (1, 0, 0, 0)$ its probability of the change is given by $Prob(\Delta X_1, \Delta X_2, \Delta X_3, \Delta X_4) = (1, 0, 0, 0) | (X_1, X_2, X_3), X_4 = P_1 = \Lambda \Delta t + o(\Delta t)$

One will quickly work out the expectation change $E(\Delta X)$ and its covariance matrix $V(\Delta X)$ associated with ΔX by neglecting the terms higher than $o(\Delta t)$. The expectation of ΔX is given by

$$\begin{aligned} E(\Delta X) &= \sum_{i=1}^{18} P_i(\Delta X)_i \Delta t \\ &= \left(\begin{array}{c} \Lambda - \mu X_1 - \mu_1 X_2 \\ \alpha_1 X_1 \frac{X_2}{X_1} - \alpha_1 X_2 \frac{X_2}{X_1} - \alpha_1 X_3 \frac{X_2}{X_1} - \alpha_1 X_4 \frac{X_2}{X_1} - \gamma X_3 - (\mu + \mu_1) X_2 - \alpha_2 X_4 \frac{X_2}{X_1} \\ \beta X_1 - \beta X_2 - \beta X_3 - \beta X_4 + \gamma X_3 - \alpha_3 X_4 \frac{X_3}{X_1} - \mu X_3 \\ \alpha_2 X_4 \frac{X_2}{X_1} + \alpha_3 X_4 \frac{X_3}{X_1} - \mu X_4 \end{array} \right) \Delta t \\ &= f(X_1, X_2, X_3, X_4) \Delta t \end{aligned}$$

Here, it can be noted that the expectation vector and the function f are in the same form as those in ODE system (2.2). Since, $V(\Delta X) = E((\Delta X)(\Delta X)^T) - E(\Delta X)(E(\Delta X))^T$ and $E((\Delta X)(\Delta X)^T) = f(X)(f(X))^T \Delta t$, it can be approximately diffusion matrix V times Δt by neglecting the term of $(\Delta t)^2$ such that $V(\Delta X) \approx E((\Delta X)(\Delta X)^T)$. Hence,

$$E((\Delta X)(\Delta X)^T) = \sum_{i=1}^{17} P_i((\Delta X)_i(\Delta X)_i^T) \Delta t = \begin{pmatrix} V_{11} & V_{12} & 0 & 0 \\ V_{21} & V_{22} & V_{23} & 0 \\ 0 & V_{32} & V_{33} & V_{34} \\ 0 & 0 & V_{43} & V_{44} \end{pmatrix} \Delta t = \Omega \Delta t$$

where the above diffusion matrix is symmetric, positive-definite and each component of this 4 x 4 diffusion matrix can be obtained as follows:

$$\begin{aligned} V_{11} &= P_1 + P_2 + P_3, & V_{12} &= V_{21} = -P_3, \\ V_{22} &= P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9 + P_{10}, & V_{23} &= V_{32} = -P_8, \\ V_{33} &= P_8 + P_{11} + P_{12} + P_{13} + P_{14} + P_{15} + P_{16}, & V_{34} &= V_{43} = P_{15}, \\ V_{44} &= P_{10} + P_{15} + P_{17}, \end{aligned}$$

We are going to following the approach discussed [16] and constructed with a matrix M such that $V = MM^T$, where M is a 3 x 8 matrix.

$$M = \begin{pmatrix} m_{11} & m_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{22} & m_{23} & m_{24} & m_{25} & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{34} & 0 & m_{36} & m_{37} & 0 \\ 0 & 0 & 0 & 0 & m_{45} & 0 & m_{47} & m_{48} \end{pmatrix}$$

Table 3: The Probability of Possible states and changes.

Possible Change of States		Probability of Change of state
$(\Delta X)_1 = (1, 0, 0, 0)^T$	Change when the recruitment growth	$P_1 = \Lambda \Delta t + o(\Delta t)$
$(\Delta X)_2 = (-1, 0, 0, 0)^T$	Change when the natural death	$P_2 = \mu X_1 \Delta t + o(\Delta t)$
$(\Delta X)_3 = (-1, -1, 0, 0)^T$	Change when the alcohol addiction death rate	$P_3 = \mu_1 X_2 \Delta t + o(\Delta t)$
$(\Delta X)_4 = (0, 1, 0, 0)^T$	Change when the Alcohol population growth	$P_4 = \alpha_1 X_1 X_2 / X_1 \Delta t + o(\Delta t)$
$(\Delta X)_5 = (0, -1, 0, 0)^T$	Change when the alcohol consumed populations are interaction to total population	$P_5 = \alpha_1 X_2^2 / X_1 \Delta t + o(\Delta t)$
$(\Delta X)_6 = (0, -1, 0, 0)^T$	Change when the alcohol and poverty are interaction to total population	$P_6 = (\alpha_1 X_3 X_2) / X_1 \Delta t + o(\Delta t)$
$(\Delta X)_7 = (0, -1, 0, 0)^T$	Change when the alcohol and Recovered population are interaction to total population	$P_7 = (\alpha_1 X_4 X_2) / X_1 \Delta t + o(\Delta t)$
$(\Delta X)_8 = (0, -1, 1, 0)^T$	Change when the Alcohol consumed population goes to poverty population	$P_8 = \gamma X_3 \Delta t + o(\Delta t)$
$(\Delta X)_9 = (0, -1, 0, 0)^T$	Change when the alcohol consumed death	$P_9 = \mu X_2 \Delta t + o(\Delta t)$
$(\Delta X)_{10} = (0, -1, 0, 1)^T$	Changed into Alcohol Consumed to recovery growth	$P_{10} = (\alpha_2 X_4 X_2) / X_1 \Delta t + o(\Delta t)$
$(\Delta X)_{11} = (0, 0, 1, 0)^T$	Change into the Poverty growth	$P_{11} = \beta X_1 \Delta t + o(\Delta t)$
$(\Delta X)_{12} = (0, 0, -1, 0)^T$	Change into the poverty death rate	$P_{12} = \beta X_2 \Delta t + o(\Delta t)$
$(\Delta X)_{13} = (0, 0, -1, 0)^T$	Change into the poverty reduction rate due to alcohol control	$P_{13} = \beta X_3 \Delta t + o(\Delta t)$
$(\Delta X)_{14} = (0, 0, -1, 0)^T$	Change when the alcohol addiction into poverty	$P_{14} = \beta X_4 \Delta t + o(\Delta t)$
$(\Delta X)_{15} = (0, 0, -1, 1)^T$	Change due to the Poverty Reduced then Recovered	$P_{15} = \alpha_3 X_3 X_4 / X_1 \Delta t + o(\Delta t)$
$(\Delta X)_{16} = (0, 0, -1, 0)^T$	Change due to death of Poverty	$P_{16} = \mu X_3 \Delta t + o(\Delta t)$
$(\Delta X)_{17} = (0, 0, 0, -1)^T$	Change into natural death after recover	$P_{17} = \mu X_4 \Delta t + o(\Delta t)$
$(\Delta X)_{18} = (0, 0, 0, 0)^T$	No Change	$P_{18} = 1 - \sum_{i=1}^{17} P_i + o(\Delta t)$

where,

$$\begin{aligned}
m_{11} &= \sqrt{\Lambda - \mu X_1}, & m_{34} &= \sqrt{\gamma X_3}, \\
m_{12} &= -\sqrt{\mu_1 X_2}, & m_{36} &= \sqrt{\beta(X_1 - X_2 - X_3 - X_4) - \mu X_3}, \\
m_{22} &= -\sqrt{\mu_1 X_2}, & m_{37} &= -\sqrt{\alpha_3 X_4 \frac{X_3}{X_1}}, \\
m_{23} &= \sqrt{\alpha_1(X_1 - X_2 - X_3 - X_4) \frac{X_2}{X_1}} - \sqrt{\mu X_2}, & m_{45} &= \sqrt{\alpha_2 \frac{X_2}{X_4}}, \\
m_{24} &= -\sqrt{\gamma X_3}, & m_{47} &= \sqrt{\alpha_3 X_4 \frac{X_3}{X_1}}, \\
m_{25} &= -\sqrt{\alpha_2 X_4 \frac{X_2}{X_1}}, & m_{48} &= -\sqrt{\mu X_4}.
\end{aligned}$$

Then, the Ito-Stochastic differential model has the following form:

$$d(\mathbf{X}(t)) = f(X_1, X_2, X_3, X_4)dt + \mathbf{M}.d\mathbf{W}(t)$$

with initial condition $X(0) = (X_1(0), X_2(0), X_3(0), X_4(0))^T$ and a Wiener process,

$\mathbf{W}(t) = (W_1(t), W_2(t), W_3(t), W_4(t), W_5(t), W_6(t), W_7(t), W_8(t))^T$. we are looking the facts , we will get the SDE model as followings:

$$\begin{aligned}
dT &= (\Lambda - \mu X_1 - \mu_1 X_2)dt + \sqrt{\Lambda - \mu X_1}dW_1 - \sqrt{\mu X_2}dW_2 \\
dA &= \left(\alpha_1(X_1 - X_2 - X_3 - X_4) \frac{X_2}{X_1} - \gamma X_3 - (\mu_1 + \mu)X_2 - \alpha_2 X_4 \frac{X_2}{X_1} \right) dt - \sqrt{\mu_1 X_2}dW_2 \\
&\quad + \left(\sqrt{\alpha_1(X_1 - X_2 - X_3 - X_4) \frac{X_2}{X_1}} - \sqrt{\mu X_2} \right) dW_3 - \sqrt{\gamma X_3}dW_4 - \sqrt{\alpha_2 X_4 \frac{X_2}{X_1}}dW_5 \\
dP &= \left(\beta(X_1 - X_2 - X_3 - X_4) + \gamma X_3 - \alpha_3 X_4 \frac{X_3}{X_1} - \mu X_3 \right) dt + \sqrt{\gamma X_3}dW_4 \\
&\quad + \sqrt{\beta(X_1 - X_2 - X_3 - X_4) - \mu X_3}dW_6 - \sqrt{\alpha_3 X_4 \frac{X_3}{X_1}}dW_7 \\
dR &= \left(\alpha_2 X_4 \frac{X_2}{X_1} + \alpha_3 X_4 \frac{X_3}{X_1} - \mu X_4 \right) + \sqrt{\alpha_2 X_4 \frac{X_2}{X_1}}dW_5 + \sqrt{\alpha_3 X_4 \frac{X_3}{X_1}}dW_7 - \sqrt{\mu X_4}dW_8.
\end{aligned}$$

5. Numerical simulation

We need to apply numerical simulation in this part to verify our analytical conclusions. We used numerical simulation to confirm the analytical findings. Results that have been verified via experimentation are given and justified. In this part, the numerical simulations will assist us in discussing and evaluating the efficacy of our analytical method.

The system (2.2) is simulated for a set of parameters that are locally asymptotically stable and satisfy numerous values (see Fig. 12) the EE condition E_3 . We considered the parameters set $S_1 = \{\Lambda = 5, \mu = 0.014, \mu_1 = 0.02, \beta = 0.05, \gamma = 0.001, \alpha_1 = 0.21, \alpha_2 = 0.05, \alpha_3 = 0.02\}$, and some of the parameters are assum with feasible values.

The set of parameters are E_2 the system (2.2) has only the PFE is locally asymptotically stable [7] (see Fig. 11). The parameter values as follows $S_2 = \{\Lambda = 5, \mu = 0.014, \mu_1 = 0.02, \beta = 0.002, \gamma = 0.03, \alpha_1 = 0.4, \alpha_2 = 0.06, \alpha_3 = 0.64\}$

The set parameters are E_1 the system (2.2) has only the AFE is locally asymptotically stable (see Fig. 10). The parameters values are $S_3 = \{\Lambda = 5, \mu = 0.01, \mu_1 = 0.02, \beta = 0.01, \gamma = 0.001, \alpha_1 = 0.01, \alpha_2 = 0.31, \alpha_3 = 0.05\}$

There are three feasible equilibrium states in the system (2.2). EE, AFE, and PFE are locally asymptotically stable [9] (refer to Fig. 12, 10, and 11, respectively).

The system's parameters (2.2) $(\alpha_1, \alpha_2, \alpha_3, \beta, \gamma)$ must be adjusted to ensure the stability of the equilibrium point at various positions in the parameter. This is demonstrated by the variation of pointwise effects, as illustrated in Figures 13, 14, 15, 16, and 17.

Now we change the values of α_1 increasing the alcohol consumed people level step by step it is increasing . The parameter α_2 increasing the alcohol consumed people are interact the recovered people that automatically they have to reduce the alcohol usages in his/her life. So they are move to recovered class .

In this way parametr effect of α_3 value is increasing the poverty people go to recover class. In scenarios where individuals interact, social workers can contribute to the advancement of society by addressing issues such as human behavior, development, the economy, society, and cultural institutions and relationships.

Now the β values is increased its shown the Non-imphoverished people how go to the poverty people like Binge Drinking (alcohol) Chronic maladies and other significant issues, including high blood pressure, heart disease, stroke, liver disease, and digestive issues, can result from use over time. Breast, mouth, pharynx, trachea, vocal box, liver, colon, and rectum malignancies are common.

There was a lot of drinking going on at the time, as well as unprotected sex or intercourse with several people. Unintended pregnancy or sexually transmitted illnesses, such as HIV, can occur as a result of these practises.

The parameter γ value is increse on that time the poverty population level also rise. Because they maintain the poverty environment on same level and didnt try develop his/her social improvement in economically or personally in these aspect. Alcohol use does not appear to have a beneficial impact on earnings and wages [20].For prime-age guys, the connection between alcohol intake and wages. A polychotomous choice model, which accounts for the endogenous link between drinking and wages, is used to estimate income inequalities for nondrinkers, moderate drinkers, and heavy drinkers. In comparison to abstinence, the authors find that moderate alcohol intake leads to higher profits. Heavy drinking, on the other hand, results in lower earnings than moderate drinking. Binge drinkers have poorer returns on higher education and have flatter age-earnings profiles than teetotallers and moderate drinkers [?].

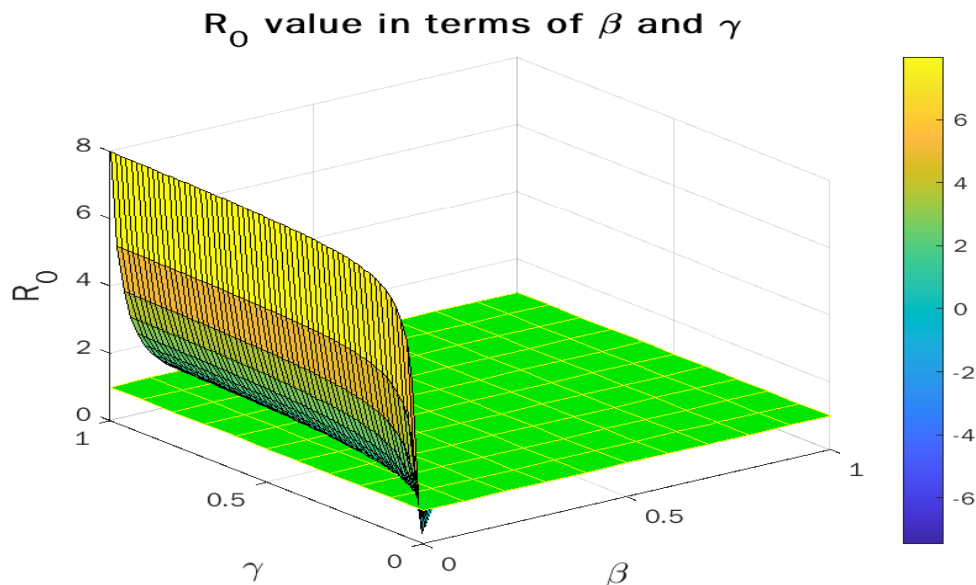


Figure 9: In terms of the parameters β and γ are showing the effect of R_0

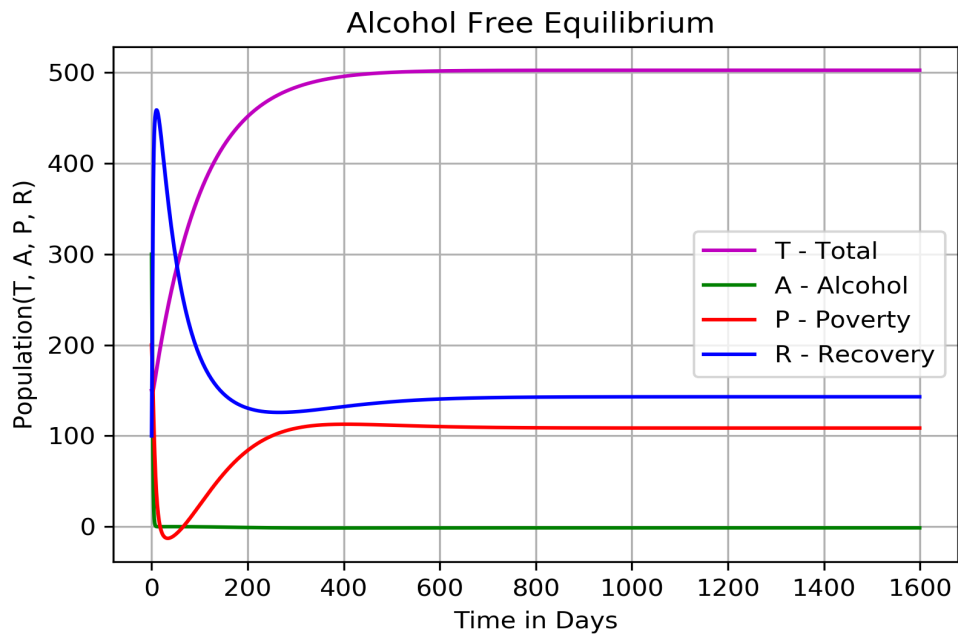


Figure 10: Variation of T , A , P and R with time showing the stability of alcohol free equilibrium E_1

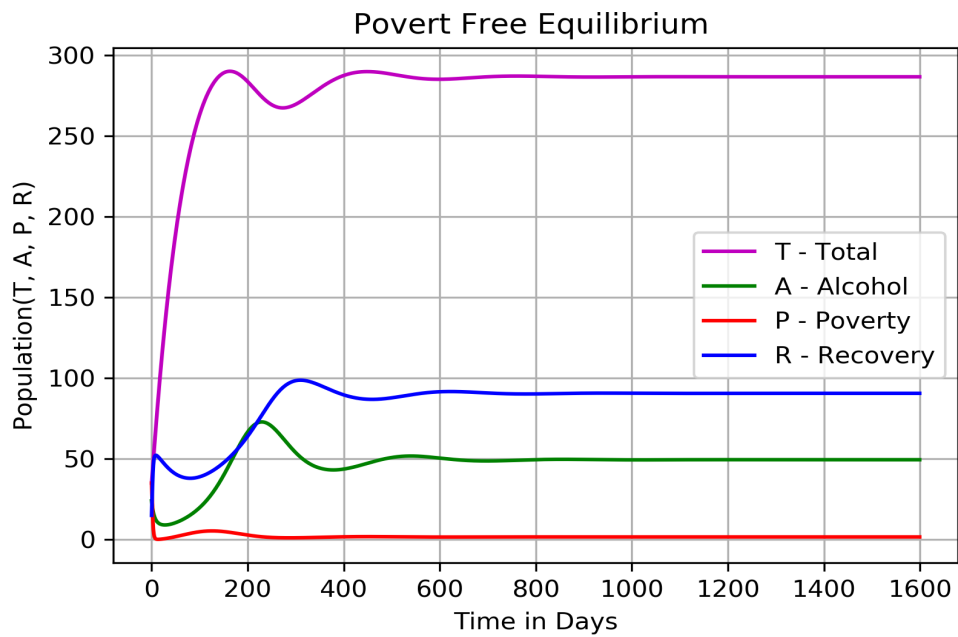


Figure 11: Variation of T , A , P and R with time showing the stability of poverty free equilibrium E_2

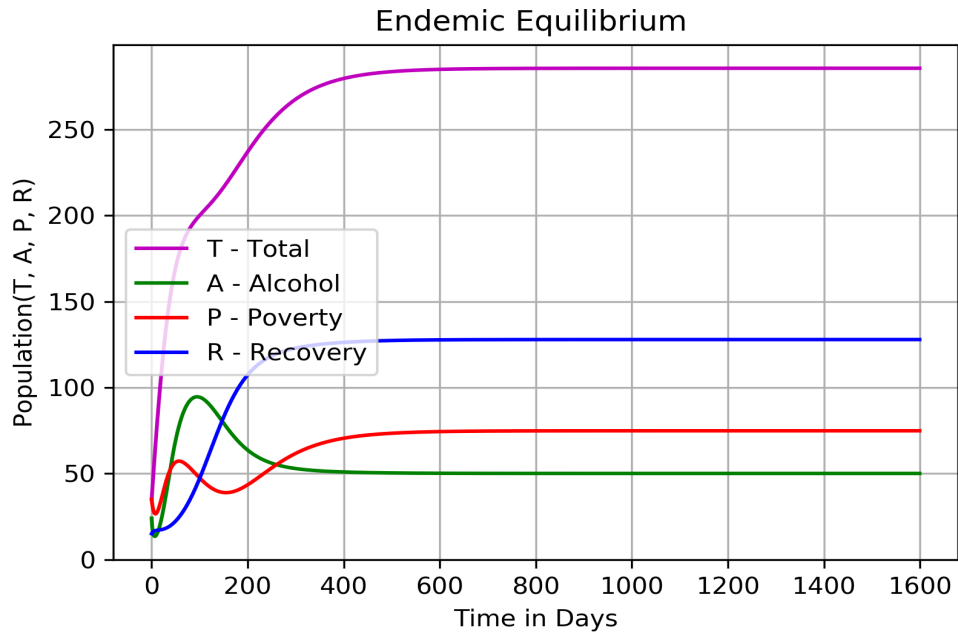


Figure 12: Time evolution depicting the dynamical behavior of the systems stable at which point in E_3

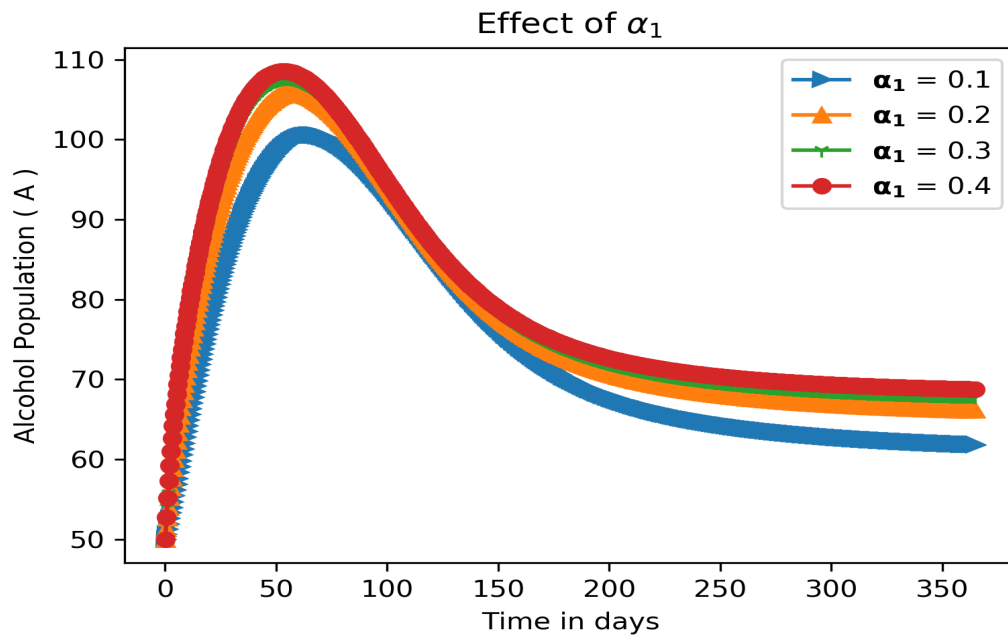
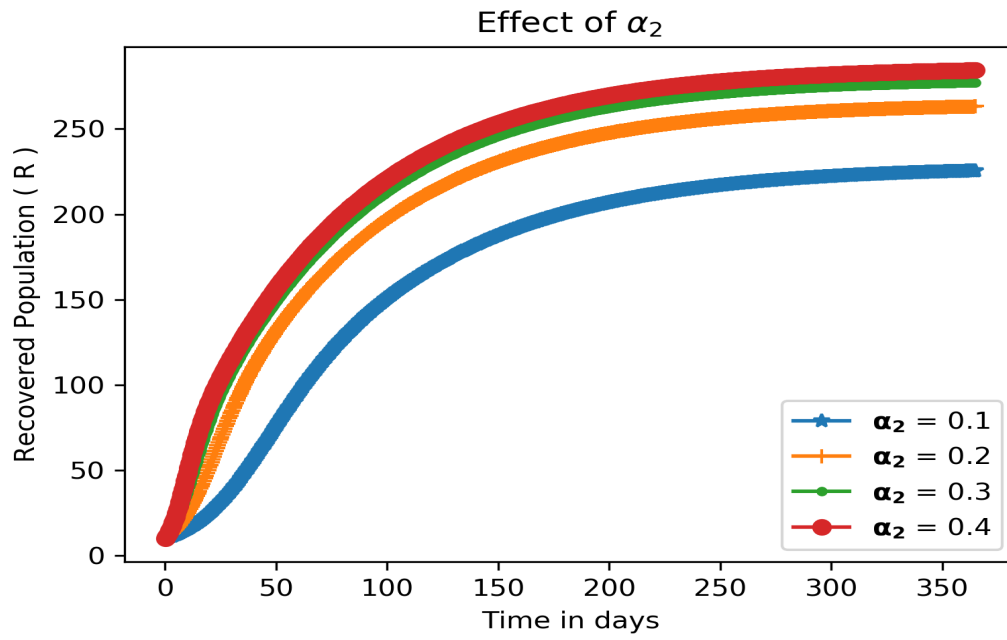
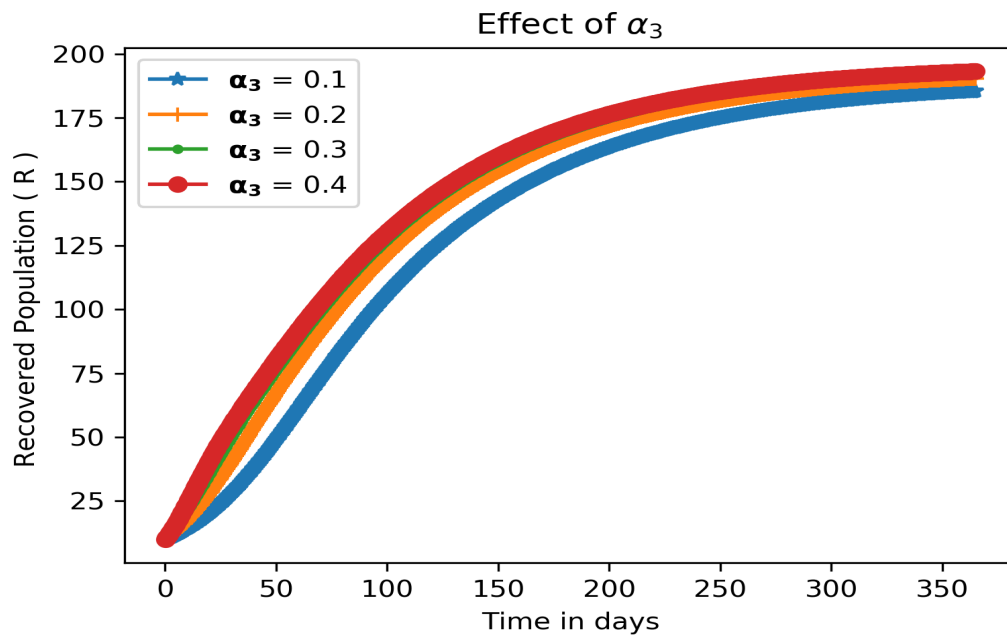


Figure 13: Variation of A with time for different values of the parameter effects at each point α_1 .

Figure 14: Variation of R with time for different values of the parameter effects at each point α_2 Figure 15: Variation of R with time for different values of the parameter effects at each point α_3

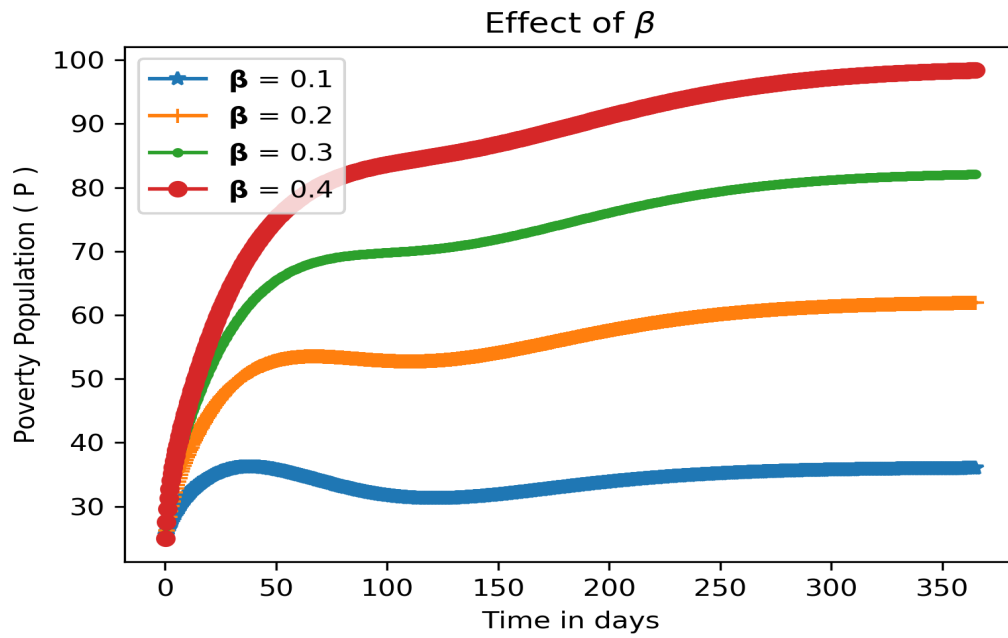


Figure 16: Variation of P with time for different values of the parameter effects at each point of β

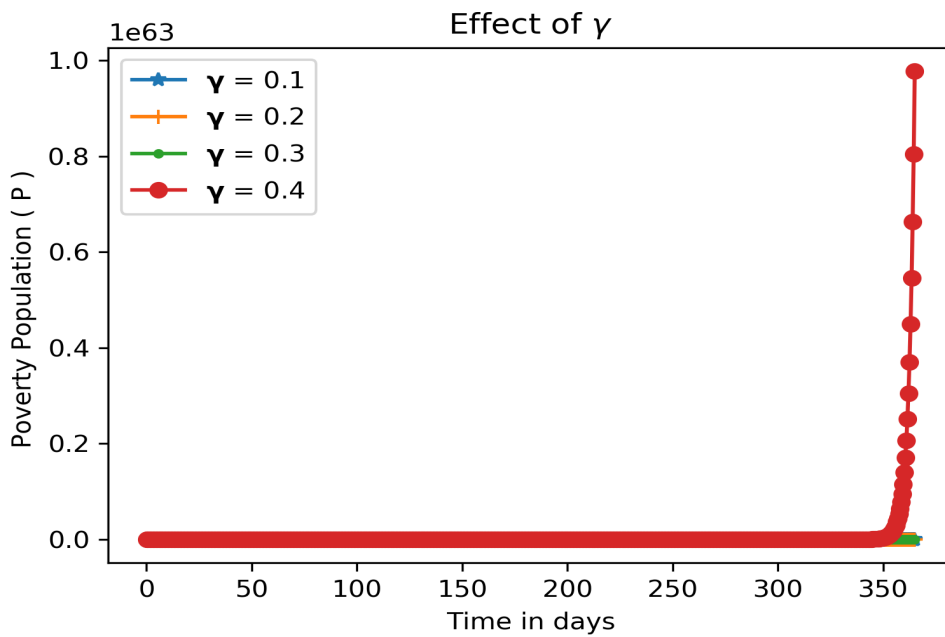


Figure 17: Variation of P with time for different values of the parameter effects at each point of γ

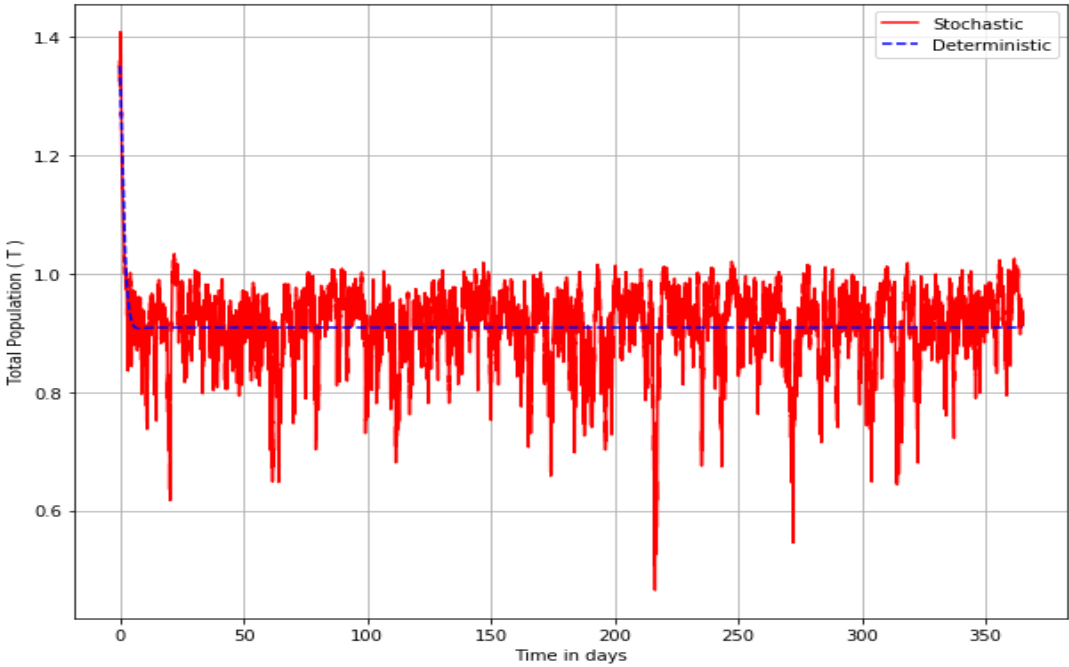


Figure 18: Comparison of Deterministic and Stochastic model effect on Total Population

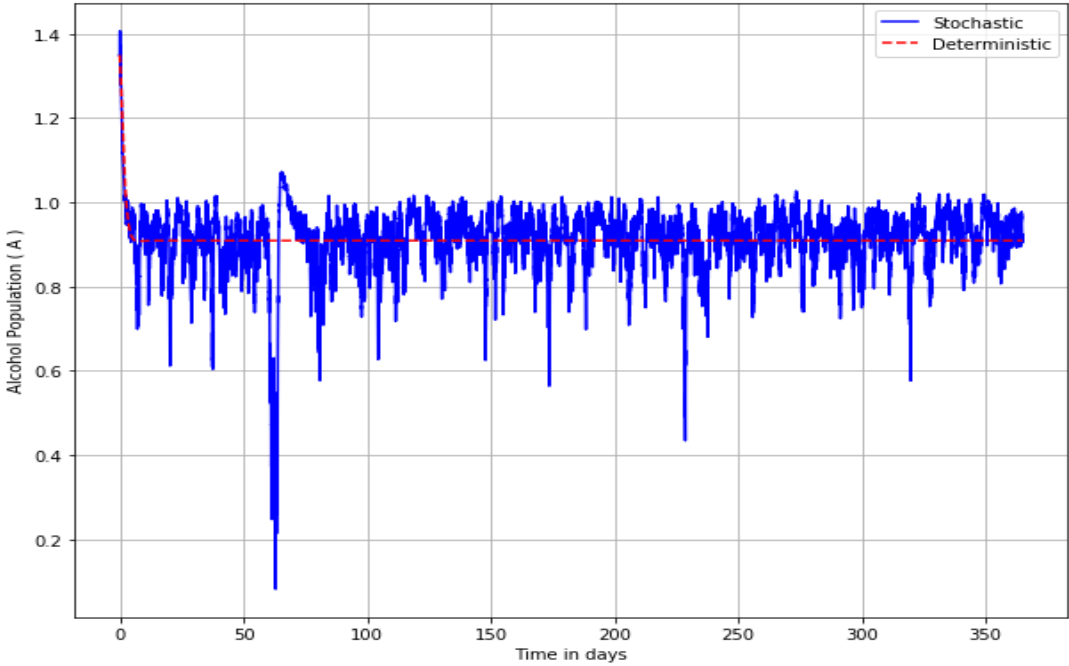


Figure 19: Comparison of Deterministic and Stochastic model effect on Alcohol Population

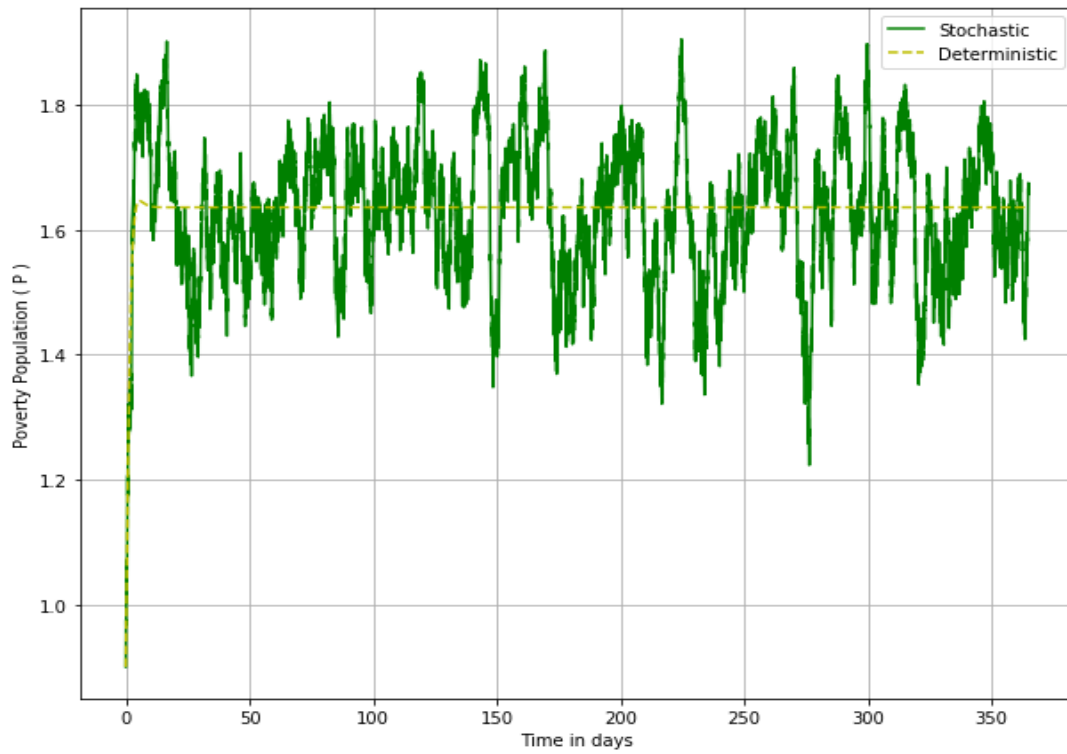


Figure 20: Comparison of Deterministic and Stochastic model effect on Poverty Population

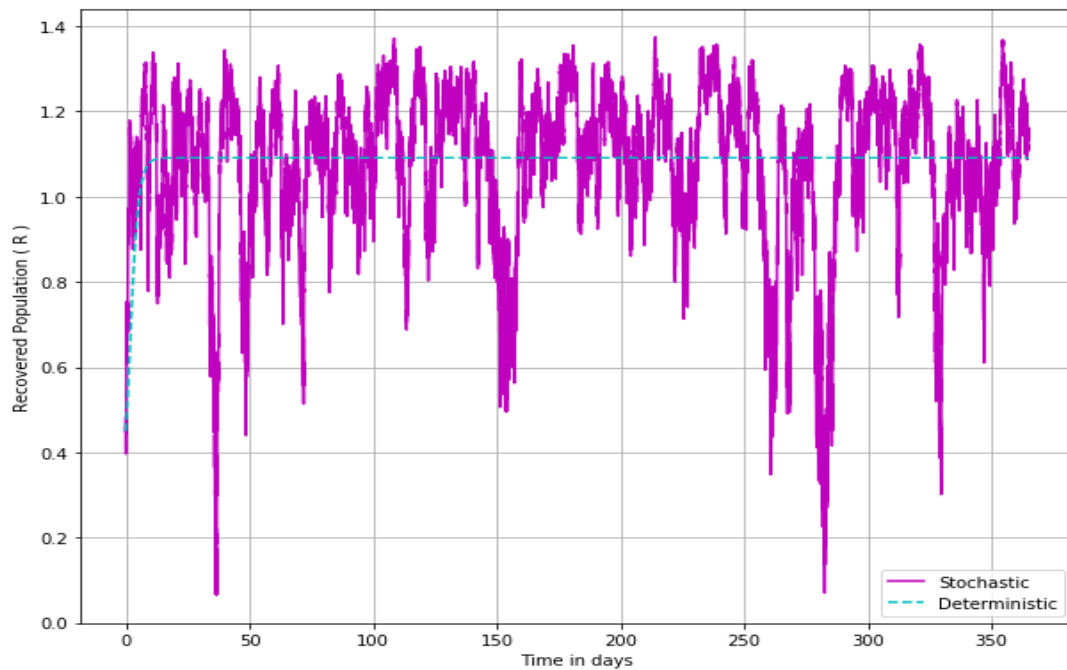


Figure 21: Comparison of Deterministic and Stochastic model effect on Recovered Population

6. Result Discussion and Conclusion

In this section, we examine how the compartments and parameters of our model contribute to reducing poverty and addressing alcohol intervention. It is widely acknowledged that tackling poverty reduction is a challenging issue both socially and economically. We explore various strategies aimed at curbing poverty growth and alcoholism, highlighting both generalized and specific approaches. Additionally, we discuss the widespread effects of poverty and its impact on the population.

Consequently, identification/detection is necessary to reduce destitution and its underlying causes. All of these are likely to occur if we mobilize ourselves to eliminate distress in order to create a prosperous world. The social responsibility of each of us is to reduce the persistence of poverty reduction in society. Through the implementation of mathematical models, we are now taking steps to predict poverty reduction utilizing various strategies. In order to mitigate destitution, this paradigm is proposed and assessed. We determined the potential for the population to recover from addiction to destitution through numerical - simulation using this model.

As β raises the poverty level is the rate of progression, to the α_2 and α_3 raise would decrease the poverty and alcohol population.

The population's variation at various points during a specific period is illustrated through simulations of the systems. The objective is to determine the maximal intervention rates that will reduce poverty and alcohol addiction. The utmost number of members in both poverty and alcohol consumption will be reduced as a result of the findings of this paper.

Declarations

- **Ethical Approval:** Not applicable.
- **Conflict of Interests:** The authors declare there is no conflict of interest.
- **Authors' Contributions:** All authors have equally contributed to the paper.
- **Funding:** No funding was received for this work.
- **Availability of Data and Materials:** As no datasets were created or examined for this study, data sharing is not relevant to this article.

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