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Fully Linear Diophantine Fuzzy Linear Programming Problems Based on Triangular LD-Fuzzy Numbers

Naveed Yaqoob* and Salma Iqbal

ABSTRACT: Operations research, control theory, management sciences, and other domains have all used fuzzy set theory. Triangular linear Diophantine fuzzy numbers is a new generalization of the triangular fuzzy numbers. In linear Diophantine fuzzy sets, we use the reference parameters, which allow us to choose the grades without any limitation, this helps us in obtaining better results. In this paper, a technique for solving a fully linear Diophantine fuzzy linear programming problem has been proposed. We split the fully LDF-linear programming problem (LDFLPP) to the classical linear programming problems and then solve all the classical linear programming problems by simplex method. At the end we combine all of our results to get the solution of our LDFLP problem.

Keywords: Linear programming problems, linear Diophantine fuzzy sets, linear Diophantine fuzzy numbers.

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1. Introduction

Linear programming is a crucial approach in operational research. Numerous authentic problems can indeed be modeled using linear programming. As a result, this model is significant for present arena applications including power, transport, and production. In actual applications, data consistency, reliability, and correctness are frequently imagined. Engineers also use linear programming to the project management offices of construction companies and also offices of projects management. For example, two major constituents of building construction costs are materials cost and labor cost. Linear programming (LP) is a quantitative technique by means of a mathematical modeling technique designs to solve allocation problem (resource allocation) to achieve the maximum profit or lowest cost. Examples of engineering applications of linear programming include optimizing the energy use cost in a building system analysis, material analysis of a truss, and space optimization in city planning, office design and grocery store shelves. Since this optimal solution of an LP is solely determined by a small range of constraints, the majority of the evidence gathered has a minute impression on the solution. It is logical to think of expert knowledge regarding attributes as fuzzy data. Zadeh [28] came up with the concept of fuzzy set. Afterwards, Bellman and Zadeh [4] presented fuzzy optimization problems in which they investigated how a fuzzy decision might be perceived as the convergence of fuzzy goals and problem restrictions. Zimmermann [29] established the first concept of the fuzzy linear programming problem (FLPP). Many scholars, including Campos and Verdegay [7], Tanaka [25], Cadenas and Verdegay [6], and Rommelfanger et al. [24] later investigated this topic when coping with the notion of interacting

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* Corresponding author.
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with fuzzy optimization problems. Recent years have seen a surge of interest in fuzzy optimization. Buckley and Feuring [5] presented a general type of fuzzy linear programming issues known as wholly fuzzified linear programming problems, in which all optimal solutions and parameters are fuzzy numbers. Lodwick and Bachman [17] presented the large scale fuzzy and possible optimization difficulties. In fuzzy queuing theory, Abdalla and Buckley [1] developed Monte Carlo approaches. In [2], [11], [16], [22] and [23] the authors provided new approaches to solve fuzzy linear programming problems. Some research proposes fuzzy linear programming, in which merely some parts of the problem for instance, just the variables or just the right side and objective function coefficients were intended to be fuzzy. Fully fuzzy linear programming (FFLP) problems are defined as those in which all of the parameters and variables are represented by fuzzy numbers. Many scholars investigated the FFLP with inequality constraints. The problem of the present methods' solutions is that they do not entirely fulfill the requirements. This asserts that it is not possible to acquire the fuzzy number of the right-hand side of the constraint by swapping the solution on the left-hand side of the constraint.

Dehghan et al. [8] developed methods for solving the fully fuzzy linear system (FFLS) that are identical to well-known methods. Lotfi et al. [18] created a novel approach for addressing FFLP by converting an FFLP into two associated LPs using the idea of the symmetric triangular fuzzy number. Kumar et al. [15] highlighted the inadequacies of the preceding techniques. They suggested a new approach for determining the fuzzy optimal solution to FFLP issues with linear constraints to address these drawbacks. Najafi and Edalatpanah [20] highlighted out that the strategy requires some tweaks to make the model work properly in general. Kaur and Kumar [13] presented a new approach for solving FFLP issues with equality requirements that use non-negative fuzzy coefficients and unlimited fuzzy variables.

Ganesan and Veeramani [9] investigated a novel type of fuzzy arithmetic for symmetric trapezoidal fuzzy numbers and offered a method for addressing fuzzy linear programming problems. Mahapatra & Mahapatra [19] presented trapezoidal intuitionistic fuzzy number analysis for intuitionistic fuzzy fault tree analysis. Angelov [3] developed optimization in an intuitionistic fuzzy setting. Gani and Abbas [10] offered a solution for tackling the intuitionistic fuzzy transportation problem. Ye [27] investigated the anticipated value approach for intuitive trapezoidal fuzzy multicriteria decision-making issues. Wan and Dong [26] proposed the possibility degree approach for decision-making with interval-valued intuitionistic fuzzy numbers. Kabiraj et al. [12] presented a strategy for tackling intuitionistic fuzzy linear programming problems (IFLPP) based on Zimmermann's prior technique for solving fuzzy linear programming issues.

In this paper, we first illustrated a triangular linear Diophantine fuzzy number TLDFN. Our primary goal is to investigate a meaningful strategy to dealing with linear programming problems (LPPs) with data represented as linear Diophantine fuzzy numbers.

2. Preliminaries and Basic Definitions

In this section we defined some basic definitions of linear Diophantine fuzzy numbers.

Definition 2.1 *[21] Let X be the universe. A linear Diophantine fuzzy set (LDFS) $_R$ on X is defined as follows:*

$$_R = \{(\theta, \langle \xi^{\tau}_{R}(\theta), \Im^{\upsilon}_{R}(\theta)\rangle, \langle \alpha(\theta), \beta(\theta)\rangle) : \theta \in X\},$$

where $\xi^{\tau}_{R}(\theta), \Im^{\upsilon}_{R}(\theta), \alpha(\theta), \beta(\theta) \in [0,1]$ such that

$$\begin{aligned} 0 &\leq \alpha(\theta)\xi^{\tau}_{R}(\theta) + \beta(\theta)\Im^{\upsilon}_{R}(\theta) \leq 1, \forall \theta \in X, \\ 0 &\leq \alpha(\theta) + \beta(\theta) \leq 1. \end{aligned}$$

The hesitation part can be written as

$$\varrho_{\pi_R} = 1 - (\alpha(\theta)\xi^{\tau}_{R}(\theta) + \beta(\theta)\Im^{\upsilon}_{R}(\theta)),$$

where ϱ is the reference parameter.

Definition 2.2 *[14] Let $\pounds_{\mathfrak{R}}$ be a LDFS on $\mathbb{R}$ with the following membership functions ($\xi^{\tau}_{\mathfrak{R}}$ and α) and non-membership functions ($\Im^{\nu}_{\mathfrak{R}}$ and β)*

$$\xi^{\tau}_{\mathfrak{R}}(x) = \begin{cases} \frac{x-\vartheta_1}{\vartheta_3-\vartheta_1} & \vartheta_1 \le x \le \vartheta_3 \\ \frac{\vartheta_5-x}{\vartheta_5-\vartheta_3} & \vartheta_3 \le x \le \vartheta_5 \\ 0 & otherwise \end{cases}, \quad \Im^{\nu}_{\mathfrak{R}}(x) = \begin{cases} \frac{\vartheta_3-x}{\vartheta_3-\vartheta_2} & \vartheta_2 \le x \le \vartheta_3 \\ \frac{x-\vartheta_3}{\vartheta_4-\vartheta_3} & \vartheta_3 \le x \le \vartheta_4 \\ 0 & otherwise, \end{cases}$$

and

$$\alpha(x) = \begin{cases} \frac{x-\vartheta_2'}{\vartheta_3'-\vartheta_2'} & \vartheta_2' \le x \le \vartheta_3' \\ \frac{\vartheta_4'-x}{\vartheta_4'-\vartheta_3'} & \vartheta_3' \le x \le \vartheta_4' \\ 0 & otherwise \end{cases}, \quad \beta(x) = \begin{cases} \frac{\vartheta_3'-x}{\vartheta_3'-\vartheta_1'} & \vartheta_1' \le x \le \vartheta_3' \\ \frac{x-\vartheta_3'}{\vartheta_5'-\vartheta_3'} & \vartheta_3' \le x \le \vartheta_5' \\ 0 & otherwise, \end{cases}$$

where $\vartheta_1' \le \vartheta_2' \le \vartheta_3' \le \vartheta_4' \le \vartheta_5'$ for all $x \in \mathbb{R}$. Then $\pounds_{\mathfrak{R}}$ is called a triangular LDFN if $\vartheta_3 = \vartheta_3'$ and $\vartheta_1 \le \vartheta_2 \le \vartheta_3 \le \vartheta_4 \le \vartheta_5$.

Definition 2.3 *[14] Let $\pounds_{\mathfrak{R}} = (\langle \xi^{\tau}_{\mathfrak{R}}, \Im^{\nu}_{\mathfrak{R}} \rangle, \langle \alpha, \beta \rangle)$ and $\pounds_{\mathfrak{P}} = \big(\langle \xi^{\tau}_{\mathfrak{P}}, \Im^{\nu}_{\mathfrak{P}} \rangle, \langle \gamma, \delta \rangle\big)$ be two TLDFN on $\mathbb{R}$. Then*

$$(i)\ \pounds_{\mathfrak{R}} + \pounds_{\mathfrak{P}} = \begin{cases} \sup\limits_{t=x+y} \{\min\{\xi^{\tau}_{\mathfrak{R}}(x), \xi^{\tau}_{\mathfrak{P}}(y)\}\} & \inf\limits_{t=x+y} \{\max\{\Im^{\nu}_{\mathfrak{R}}(x), \Im^{\nu}_{\mathfrak{P}}(y)\}\} \\ \sup\limits_{t=x+y} \{\min\{\alpha(x), \gamma(y)\}\} & \inf\limits_{t=x+y} \{\max\{\beta(x), \delta(y)\}\}, \end{cases}$$

$$(ii)\ \pounds_{\mathfrak{R}} - \pounds_{\mathfrak{P}} = \begin{cases} \sup\limits_{t=x-y} \{\min\{\xi^{\tau}_{\mathfrak{R}}(x), \xi^{\tau}_{\mathfrak{P}}(y)\}\} & \inf\limits_{t=x-y} \{\max\{\Im^{\nu}_{\mathfrak{R}}(x), \Im^{\nu}_{\mathfrak{P}}(y)\}\} \\ \sup\limits_{t=x-y} \{\min\{\alpha(x), \gamma(y)\}\} & \inf\limits_{t=x-y} \{\max\{\beta(x), \delta(y)\}\}, \end{cases}$$

$$(iii)\ \pounds_{\mathfrak{R}} \times \pounds_{\mathfrak{P}} = \begin{cases} \sup\limits_{t=x\times y} \{\min\{\xi^{\tau}_{\mathfrak{R}}(x), \xi^{\tau}_{\mathfrak{P}}(y)\}\} & \inf\limits_{t=x\times y} \{\max\{\Im^{\nu}_{\mathfrak{R}}(x), \Im^{\nu}_{\mathfrak{P}}(y)\}\} \\ \sup\limits_{t=x\times y} \{\min\{\alpha(x), \gamma(y)\}\} & \inf\limits_{t=x\times y} \{\max\{\beta(x), \delta(y)\}\}, \end{cases}$$

$$(iv)\ \pounds_{\mathfrak{R}} \div \pounds_{\mathfrak{P}} = \begin{cases} \sup\limits_{t=x\div y} \{\min\{\xi^{\tau}_{\mathfrak{R}}(x), \xi^{\tau}_{\mathfrak{P}}(y)\}\} & \inf\limits_{t=x\div y} \{\max\{\Im^{\nu}_{\mathfrak{R}}(x), \Im^{\nu}_{\mathfrak{P}}(y)\}\} \\ \sup\limits_{t=x\div y} \{\min\{\alpha(x), \gamma(y)\}\} & \inf\limits_{t=x\div y} \{\max\{\beta(x), \delta(y)\}\}. \end{cases}$$

Definition 2.4 *[14] A TLDFN $\pounds_{\mathfrak{R}_{TLDFN}} = \begin{cases} (\vartheta_1,\vartheta_2,\vartheta_3,\vartheta_4,\vartheta_5) \\ (\vartheta_1',\vartheta_2',\vartheta_3,\vartheta_4',\vartheta_5') \end{cases}$ is said to be positive if and only if $\vartheta_1 \ge 0$ and $\vartheta_1' \ge 0$.*

Definition 2.5 *[14] Two TLDFNs $\pounds_{\mathfrak{R}_{TLDFN}} = \begin{cases} (\vartheta_1,\vartheta_2,\vartheta_3,\vartheta_4,\vartheta_5) \\ (\vartheta_1',\vartheta_2',\vartheta_3,\vartheta_4',\vartheta_5') \end{cases}$ and $\mathfrak{N}_{\mathfrak{R}_{TLDFN}} = \begin{cases} (\delta_1,\delta_2,\delta_3,\delta_4,\delta_5) \\ (\delta_1',\delta_2',\delta_3,\delta_4',\delta_5') \end{cases}$ are said to be equal if and only if $\vartheta_1 = \delta_1$, $\vartheta_2 = \delta_2$, $\vartheta_3 = \delta_3$, $\vartheta_4 = \delta_4$, $\vartheta_5 = \delta_5$, $\vartheta_1' = \delta_1'$, $\vartheta_2' = \delta_2'$, $\vartheta_4' = \delta_4'$ and $\vartheta_5' = \delta_5'$.*

We now define the arithmetic operations on TLDFNs using the concept of interval arithmetic.

Definition 2.6 *[14] Consider two positive TLDFNs $\pounds_{\mathfrak{R}_{TLDFN}} = \begin{cases} (\vartheta_1,\vartheta_2,\vartheta_3,\vartheta_4,\vartheta_5) \\ (\vartheta_1',\vartheta_2',\vartheta_3,\vartheta_4',\vartheta_5') \end{cases}$ and $\mathfrak{N}_{\mathfrak{R}_{TLDFN}} = \begin{cases} (\delta_1,\delta_2,\delta_3,\delta_4,\delta_5) \\ (\delta_1',\delta_2',\delta_3,\delta_4',\delta_5') \end{cases}$, then*

$$(i)\ \pounds_{\mathfrak{R}_{TLDFN}} + \mathfrak{N}_{\mathfrak{R}_{TLDFN}} = \begin{cases} (\vartheta_1+\delta_1,\vartheta_2+\delta_2,\vartheta_3+\delta_3,\vartheta_4+\delta_4,\vartheta_5+\delta_5) \\ (\vartheta_1'+\delta_1',\vartheta_2'+\delta_2',\vartheta_3+\delta_3,\vartheta_4'+\delta_4',\vartheta_5'+\delta_5') \end{cases};$$

$$(ii)\ \pounds_{\mathfrak{R}_{TLDFN}} - \mathfrak{N}_{\mathfrak{R}_{TLDFN}} = \begin{cases} (\vartheta_1-\delta_5,\vartheta_2-\delta_4,\vartheta_3-\delta_3,\vartheta_4-\delta_2,\vartheta_5-\delta_1) \\ (\vartheta_1'-\delta_5',\vartheta_2'-\delta_4',\vartheta_3-\delta_3,\vartheta_4'-\delta_2',\vartheta_5'-\delta_1') \end{cases};$$

$$(iii)\ \pounds_{\mathfrak{R}_{TLDFN}} \times \mathfrak{N}_{\mathfrak{R}_{TLDFN}} = \begin{cases} (\vartheta_1\delta_1,\vartheta_2\delta_2,\vartheta_3\delta_3,\vartheta_4\delta_4,\vartheta_5\delta_5) \\ (\vartheta_1'\delta_1',\vartheta_2'\delta_2',\vartheta_3\delta_3,\vartheta_4'\delta_4',\vartheta_5'\delta_5') \end{cases};$$

$$(iv)\ \pounds_{\mathfrak{R}_{TLDFN}} \div \mathfrak{N}_{\mathfrak{R}_{TLDFN}} = \begin{cases} \left(\frac{\vartheta_1}{\delta_5},\frac{\vartheta_2}{\delta_4},\frac{\vartheta_3}{\delta_3},\frac{\vartheta_4}{\delta_2},\frac{\vartheta_5}{\delta_1}\right) \\ \left(\frac{\vartheta_1'}{\delta_5'},\frac{\vartheta_2'}{\delta_4'},\frac{\vartheta_3}{\delta_3},\frac{\vartheta_4'}{\delta_2'},\frac{\vartheta_5'}{\delta_1'}\right) \end{cases};$$

$$(v)\ k \times \pounds_{\mathfrak{R}_{TLDFN}} = \begin{cases} \begin{cases} (k\vartheta_1,k\vartheta_2,k\vartheta_3,k\vartheta_4,k\vartheta_5) \\ (k\vartheta_1',k\vartheta_2',k\vartheta_3,k\vartheta_4',k\vartheta_5') \end{cases} & if\ k > 0 \\ \begin{cases} (k\vartheta_5,k\vartheta_4,k\vartheta_3,k\vartheta_2,k\vartheta_1) \\ (k\vartheta_5',k\vartheta_4',k\vartheta_3,k\vartheta_2',k\vartheta_1') \end{cases} & if\ k < 0. \end{cases}$$

Definition 2.7 *[14] Consider a TLDFN*

$$\pounds_{\mathfrak{R}_{TLDFN}}=\begin{cases}(\vartheta_1,\vartheta_2,\vartheta_3,\vartheta_4,\vartheta_5)\\(\vartheta_1',\vartheta_2',\vartheta_3,\vartheta_4',\vartheta_5')\end{cases}.$$

Then, the ranking of $\pounds_{\mathfrak{R}_{TLDFN}}$ *is*

$$R(\pounds_{\mathfrak{R}_{TLDFN}})=\frac{\vartheta_3}{2}+\frac{\vartheta_1+\vartheta_2+\vartheta_4+\vartheta_5+\vartheta_1'+\vartheta_2'+\vartheta_4'+\vartheta_5'}{16}.$$

3. Fully LDF-linear Programming Problems

One of the most often used operations research techniques is linear programming. The value of the components of linear programming models must be explicitly defined and accurate in the usual approach. However, in the real world, this is not a plausible assumption. There may be ambiguity about the parameters in real-world problems. In this case, the parameters of linear programming problems can be represented as fuzzy numbers. FLDFLP problems with m fuzzy equality constraints and n fuzzy variables can be expressed in the following way:

$$\text{Maximize (or Minimize)}\quad \widetilde{Z}=\sum_{k=1}^{n}\widetilde{c_k}\widetilde{x_k}$$

$$\text{subject to}\ \sum_{k=1}^{n}\widetilde{a_{jk}}\widetilde{x_k}\leq\widetilde{b_{j,}}\quad 1\leq j\leq m;\ x_k\geq 0.$$

In this paper, we study the case in which all the coefficients, variables and right hand side constants are modeled as TLDFN and then redevelop as a LPP with linear Diophantine fuzzy inequalities and objective function.

$$\text{Min (or Max)}\quad \begin{cases}(\vartheta_1,\vartheta_2,\vartheta_3,\vartheta_4,\vartheta_5)\\(\vartheta_1',\vartheta_2',\vartheta_3,\vartheta_4',\vartheta_5')\end{cases}=\sum_{j=1}^{n}\left(\begin{cases}(c_j^1,c_j^2,c_j^3,c_j^4,c_j^5)\\(c_j^{1\prime},c_j^{2\prime},c_j^3,c_j^{4\prime},c_j^{5\prime})\end{cases}\otimes\begin{cases}(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5)\\(x_j^{1\prime},x_j^{2\prime},x_j^3,x_j^{4\prime},x_j^{5\prime})\end{cases}\right)$$

$$\text{subject to}\quad \sum_{j=1}^{n}\left(\begin{cases}(a_{ij}^1,a_{ij}^2,a_{ij}^3,a_{ij}^4,a_{ij}^5)\\(a_{ij}^{1\prime},a_{ij}^{2\prime},a_{ij}^3,a_{ij}^{4\prime},a_{ij}^{5\prime})\end{cases}\otimes\begin{cases}(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5)\\(x_j^{1\prime},x_j^{2\prime},x_j^3,x_j^{4\prime},x_j^{5\prime})\end{cases}\right)\leq\begin{cases}(b_i^1,b_i^2,b_i^3,b_i^4,b_i^5)\\(b_i^{1\prime},b_i^{2\prime},b_i^{3\prime},b_i^{4\prime},b_i^{5\prime})\end{cases}$$

$$\begin{cases}(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5)\\(x_j^{1\prime},x_j^{2\prime},x_j^3,x_j^{4\prime},x_j^{5\prime})\end{cases}\geq 0,\quad\forall\ i=1,2,...,m\text{ and }j=1,2,...,n.$$

To find the solution of this problem, we further divide this problem into nine parts which will be the classical linear programming problems (CLP-problems) and then we solve each part by simplex method. The parts are:

1. Min (or Max) $\theta_1=\sum_{j=1}^{n}c_j^1x_j^1$
   subject to $\sum_{j=1}^{n}a_{ij}^1x_j^1\leq b_i^1$
   $x_j^1\geq 0,\ \forall\ i\in\mathbb{N}_m$
2. Min (or Max) $\theta_2=\sum_{j=1}^{n}c_j^2x_j^2$
   subject to $\sum_{j=1}^{n}a_{ij}^2x_j^2\leq b_i^2$
   $x_j^2\geq 0,\ \forall\ i\in\mathbb{N}_m$
3. Min (or Max) $\theta_3=\sum_{j=1}^{n}c_j^3x_j^3$
   subject to $\sum_{j=1}^{n}a_{ij}^3x_j^3\leq b_i^3$
   $x_j^3\geq 0,\ \forall\ i\in\mathbb{N}_m$
4. Min (or Max) $\theta_4=\sum_{j=1}^{n}c_j^4x_j^4$
   subject to $\sum_{j=1}^{n}a_{ij}^4x_j^4\leq b_i^4$
   $x_j^4\geq 0,\ \forall\ i\in\mathbb{N}_m$
5. Min (or Max) $\theta_5=\sum_{j=1}^{n}c_j^5x_j^5$
   subject to $\sum_{j=1}^{n}a_{ij}^5x_j^5\leq b_i^5$
   $x_j^5\geq 0,\ \forall\ i\in\mathbb{N}_m$
6. Min (or Max) $\theta_1'=\sum_{j=1}^{n}c_j^{1\prime}x_j^{1\prime}$
   subject to $\sum_{j=1}^{n}a_{ij}^{1\prime}x_j^{1\prime}\leq b_i^{1\prime}$
   $x_j^{1\prime}\geq 0,\ \forall\ i\in\mathbb{N}_m$

7. $\text{Min (or Max) } \theta'_2 = \sum_{j=1}^{n} c_j^{2\prime} x_j^{2\prime}$

$\text{subject to } \sum_{j=1}^{n} a_{ij}^{2\prime} x_j^{2\prime} \leq b_i^{2\prime}$

$x_j^{2\prime} \geq 0, \ \forall \ i \in \mathbb{N}_m$

8. $\text{Min (or Max) } \theta'_4 = \sum_{j=1}^{n} c_j^{4\prime} x_j^{4\prime}$

$\text{subject to } \sum_{j=1}^{n} a_{ij}^{4\prime} x_j^{4\prime} \leq b_i^{4\prime}$

$x_j^{4\prime} \geq 0, \ \forall \ i \in \mathbb{N}_m$

9. $\text{Min (or Max) } \theta'_5 = \sum_{j=1}^{n} c_j^{5\prime} x_j^{5\prime}$

$\text{subject to } \sum_{j=1}^{n} a_{ij}^{5\prime} x_j^{5\prime} \leq b_i^{5\prime}$

$x_j^{5\prime} \geq 0, \ \forall \ i \in \mathbb{N}_m.$

After combining all the values obtained from these nine parts, we get the values of $\left\{\begin{array}{l}(\vartheta_1,\vartheta_2,\vartheta_3,\vartheta_4,\vartheta_5)\\(\vartheta'_1,\vartheta'_2,\vartheta_3,\vartheta'_4,\vartheta'_5)\end{array}\right.$ and $\left\{\begin{array}{l}(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5)\\(x_j^{1\prime},x_j^{2\prime},x_j^3,x_j^{4\prime},x_j^{5\prime})\end{array}\right.$ for $j = 1, 2, ..., n$.

4. Numerical Example

In this section, we provide some numerical examples.

Example 4.1 *Consider the following triangular fully linear Diophantine fuzzy linear programming (TFLDFLP) problem and solve it by the proposed method;*

$$\begin{aligned}
&\mathit{Max}\ \widetilde{\theta} = \left\{\begin{array}{l}(3,5,7,8,9)\\(2,4,7,9,9)\end{array}\right. \widetilde{X} + \left\{\begin{array}{l}(2,4,5,7,9)\\(1,3,5,8,10)\end{array}\right. \widetilde{Y}\\
&\mathit{subject\ to}\ \left\{\begin{array}{l}(1,5,8,10,13)\\(0,2,8,12,14)\end{array}\right. \widetilde{X} + \left\{\begin{array}{l}(3,6,8,10,14)\\(2,5,8,12,17)\end{array}\right. \widetilde{Y} \leq \left\{\begin{array}{l}(2,13,31,51,97)\\(1,7,31,75,111)\end{array}\right.\\
&\left\{\begin{array}{l}(5,8,10,13,16)\\(4,6,10,15,19)\end{array}\right. \widetilde{X} + \left\{\begin{array}{l}(2,5,6,8,11)\\(1,4,6,9,13)\end{array}\right. \widetilde{Y} \leq \left\{\begin{array}{l}(3,17,35,59,100)\\(2,12,35,79,119)\end{array}\right.\\
&\widetilde{X}, \widetilde{Y} \geq 0.
\end{aligned}$$

Let $\widetilde{\theta} = \left\{\begin{array}{l}(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5)\\(\theta'_1,\theta'_2,\theta_3,\theta'_4,\theta'_5)\end{array}\right.$ $\widetilde{X} = \left\{\begin{array}{l}(x_1,x_2,x_3,x_4,x_5)\\(x'_1,x'_2,x_3,x'_4,x'_5)\end{array}\right.$ *and* $\widetilde{Y} = \left\{\begin{array}{l}(y_1,y_2,y_3,y_4,y_5)\\(y'_1,y'_2,y_3,y'_4,y'_5)\end{array}\right.$ *. From the above given method we can divide the problem in the following nine steps;*

$$\begin{aligned}
&\mathit{max}\ \theta_1 = 3x_1 + 2y_1\\
&\mathit{subject\ to}\ 1x_1 + 3y_1 \leq 2\\
&\qquad 5x_1 + 2y_1 \leq 3\\
&\qquad x_1, y_1 \geq 0
\end{aligned}$$

$$\begin{aligned}
&\mathit{max}\ \theta_2 = 5x_2 + 4y_2\\
&\mathit{subject\ to}\ 5x_2 + 6y_2 \leq 13\\
&\qquad 8x_2 + 5y_2 \leq 17\\
&\qquad x_2, y_2 \geq 0
\end{aligned}$$

$$\begin{aligned}
&\mathit{max}\ \theta_3 = 7x_3 + 5y_3\\
&\mathit{subject\ to}\ 8x_3 + 8y_3 \leq 31\\
&\qquad 10x_3 + 6y_3 \leq 35\\
&\qquad x_3, y_3 \geq 0
\end{aligned}$$

$$\begin{aligned}
&\mathit{max}\ \theta_4 = 8x_4 + 7y_4\\
&\mathit{subject\ to}\ 10x_4 + 10y_4 \leq 51\\
&\qquad 13x_4 + 8y_4 \leq 59\\
&\qquad x_4, y_4 \geq 0
\end{aligned}$$

$$\begin{aligned}
&\mathit{max}\ \theta_5 = 9x_5 + 9y_5\\
&\mathit{subject\ to}\ 13x_5 + 14y_5 \leq 97\\
&\qquad 16x_5 + 11y_5 \leq 100\\
&\qquad x_5 \geq 0
\end{aligned}$$

$$\begin{aligned}
&\mathit{max}\ \theta'_1 = 2x'_1 + 1y'_1\\
&\mathit{subject\ to}\ 0x'_1 + 2y'_1 \leq 1\\
&\qquad 4x'_1 + 1y'_1 \leq 2\\
&\qquad x'_1 \geq 0
\end{aligned}$$

$$\begin{aligned}
&\mathit{max}\ \theta'_2 = 4x'_2 + 3y'_2\\
&\mathit{subject\ to}\ 2x'_2 + 5y'_2 \leq 7\\
&\qquad 6x'_2 + 4y'_2 \leq 12\\
&\qquad x'_2, y'_2 \geq 0
\end{aligned}$$

$$\begin{aligned}
&\mathit{max}\ \theta'_4 = 9x'_4 + 8y'_4\\
&\mathit{subject\ to}\ 12x'_4 + 12y'_4 \leq 75\\
&\qquad 15x'_4 + 9y'_4 \leq 79\\
&\qquad x'_4, y'_4 \geq 0
\end{aligned}$$

$$\begin{aligned} \textit{max } \theta_5' = &\ 9x_5' + 10y_5' \\ \textit{subject to } &\ 14x_5' + 17y_5' \leq 111 \\ &\ 19x_5' + 13y_5' \leq 119 \\ &\ x_5', y_5' \geq 0. \end{aligned}$$

Now we can represent these nine classical linear programming problems in Table 1.

θ_1'	θ_1	θ_2'	θ_2	θ_3	θ_4	θ_4'	θ_5	θ_5'
2	3	4	5	7	8	9	9	9
1	2	3	4	5	7	8	9	10
0	1	2	5	8	10	12	13	14
2	3	5	6	8	10	12	14	17
1	2	7	13	31	51	75	97	111
4	5	6	8	10	13	15	16	19
1	2	4	5	6	8	9	11	13
2	3	12	17	35	59	79	100	119

Table 1. Tabular form of nine CLP-problems

After solving these classical linear programming problems with simplex method, we get the solution given in Table 2.

$\widetilde{X} =$	$\frac{3}{8}$	$\frac{5}{13}$	$\frac{16}{11}$	$\frac{37}{23}$	$\frac{47}{16}$	$\frac{91}{25}$	$\frac{91}{24}$	$\frac{37}{9}$	$\frac{580}{141}$
$\widetilde{Y} =$	$\frac{1}{2}$	$\frac{7}{13}$	$\frac{9}{11}$	$\frac{19}{23}$	$\frac{15}{16}$	$\frac{73}{50}$	$\frac{59}{24}$	$\frac{28}{9}$	$\frac{443}{141}$
$\widetilde{\theta} =$	$\frac{5}{4}$	$\frac{29}{13}$	$\frac{91}{11}$	$\frac{261}{23}$	$\frac{101}{4}$	$\frac{1967}{50}$	$\frac{1291}{24}$	$\frac{130}{2}$	$\frac{136}{2}$

Table 2. Solution table

Hence the optimal solution of the given TFLDFLP problem is

$$\begin{gathered} \left\{ \begin{array}{l} (x_1, x_2, x_3, x_4, x_5) \\ (x_1', x_2', x_3, x_4', x_5') \end{array} \right. = \left\{ \begin{array}{l} (\frac{5}{13}, \frac{37}{23}, \frac{47}{16}, \frac{91}{25}, \frac{37}{9}) \\ (\frac{3}{8}, \frac{16}{11}, \frac{47}{16}, \frac{91}{24}, \frac{580}{141}) \end{array} \right. \\ \left\{ \begin{array}{l} (y_1, y_2, y_3, y_4, y_5) \\ (y_1', y_2', y_3, y_4', y_5') \end{array} \right. = \left\{ \begin{array}{l} (\frac{7}{13}, \frac{19}{23}, \frac{15}{16}, \frac{73}{50}, \frac{28}{9}) \\ (\frac{1}{2}, \frac{9}{11}, \frac{15}{16}, \frac{59}{24}, \frac{443}{141}) \end{array} \right. \\ \left\{ \begin{array}{l} (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) \\ (\theta_1', \theta_2', \theta_3, \theta_4', \theta_5') \end{array} \right. = \left\{ \begin{array}{l} (\frac{29}{13}, \frac{261}{23}, \frac{101}{4}, \frac{1967}{50}, \frac{130}{2}) \\ (\frac{5}{4}, \frac{91}{11}, \frac{101}{4}, \frac{1291}{24}, \frac{136}{2}) \end{array} \right. . \end{gathered}$$

Example 4.2 *Let us consider the following TFLDFLP problem and solve by proposed method.*

$$\begin{aligned} \textit{Min } \widetilde{\theta} &= \left\{ \begin{array}{l} (2,4,5,7,9) \\ (1,3,5,8,10) \end{array} \right. \widetilde{X} + \left\{ \begin{array}{l} (3,5,6,8,10) \\ (2,4,6,9,11) \end{array} \right. \widetilde{Y} \\ \textit{subject to } & \left\{ \begin{array}{l} (1,4,5,6,8) \\ (0,1,5,7,9) \end{array} \right. \widetilde{X} + \left\{ \begin{array}{l} (4,7,8,9,11) \\ (1,5,8,10,12) \end{array} \right. \widetilde{Y} \geq \left\{ \begin{array}{l} (8,19,27,39,61) \\ (1,11,27,49,75) \end{array} \right. \\ & \left\{ \begin{array}{l} (4,7,8,9,11) \\ (1,5,8,10,12) \end{array} \right. \widetilde{X} + \left\{ \begin{array}{l} (5,8,9,10,12) \\ (4,6,9,11,13) \end{array} \right. \widetilde{Y} \geq \left\{ \begin{array}{l} (11,24,34,48,74) \\ (6,16,34,60,89) \end{array} \right. \\ & \widetilde{X}, \widetilde{Y} \geq 0. \end{aligned}$$

Let $\widetilde{\theta} = \left\{ \begin{array}{l} (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) \\ (\theta_1', \theta_2', \theta_3, \theta_4', \theta_5') \end{array} \right.$, $\widetilde{X} = \left\{ \begin{array}{l} (x_1, x_2, x_3, x_4, x_5) \\ (x_1', x_2', x_3, x_4', x_5') \end{array} \right.$ *and* $\widetilde{Y} = \left\{ \begin{array}{l} (y_1, y_2, y_3, y_4, y_5) \\ (y_1', y_2', y_3, y_4', y_5') \end{array} \right.$. *From the above given method we can divide the problem in the following nine steps:*

$$\begin{aligned} \textit{min } \theta_1 = &\ 2x_1 + 3y_1 \\ \textit{subject to } &\ 1x_1 + 4y_1 \geq 8 \\ &\ 4x_1 + 5y_1 \geq 11 \\ &\ x_1, y_1 \geq 0 \end{aligned} \qquad \begin{aligned} \textit{min } \theta_2 = &\ 4x_2 + 5y_2 \\ \textit{subject to } &\ 4x_2 + 7y_2 \geq 19 \\ &\ 7x_2 + 8y_2 \geq 24 \\ &\ x_2, y_2 \geq 0 \end{aligned}$$

$$\begin{array}{rl} \textit{min } \theta_3 = & 5x_3 + 6y_3 \\ \textit{subject to} & \begin{array}{l} 5x_3 + 8y_3 \geq 27 \\ 8x_3 + 9y_3 \geq 34 \end{array} \\ & x_3, y_3 \geq 0 \end{array} \qquad \begin{array}{rl} \textit{min } \theta_4 = & 7x_4 + 8y_4 \\ \textit{subject to} & \begin{array}{l} 6x_4 + 9y_4 \geq 39 \\ 9x_4 + 10y_4 \geq 48 \end{array} \\ & x_4, y_4 \geq 0 \end{array}$$

$$\begin{array}{rl} \textit{min } \theta_5 = & 9x_5 + 10y_5 \\ \textit{subject to} & \begin{array}{l} 8x_5 + 11y_5 \geq 61 \\ 11x_5 + 12y_5 \geq 74 \end{array} \\ & x_5, y_5 \geq 0 \end{array} \qquad \begin{array}{rl} \textit{min } \theta'_1 = & 1x'_1 + 2y'_1 \\ \textit{subject to} & \begin{array}{l} 0x'_1 + 1y'_1 \geq 1 \\ 1x'_1 + 4y'_1 \geq 6 \end{array} \\ & x'_1, y'_1 \geq 0 \end{array}$$

$$\begin{array}{rl} \textit{min } \theta'_2 = & 3x'_2 + 4y'_2 \\ \textit{subject to} & \begin{array}{l} 1x'_2 + 5y'_2 \geq 11 \\ 5x'_2 + 6y'_2 \geq 16 \end{array} \\ & x'_2, y'_2 \geq 0 \end{array} \qquad \begin{array}{rl} \textit{min } \theta'_4 = & 8x'_4 + 9y'_4 \\ \textit{subject to} & \begin{array}{l} 7x'_4 + 10y'_4 \geq 49 \\ 10x'_4 + 11y'_4 \geq 60 \end{array} \\ & x'_4, y'_4 \geq 0 \end{array}$$

$$\begin{array}{rl} \textit{min } \theta'_5 = & 10x'_5 + 11y'_5 \\ \textit{subject to} & \begin{array}{l} 9x'_5 + 12y'_5 \geq 75 \\ 12x'_5 + 13y'_5 \geq 89 \end{array} \\ & x'_5, y'_5 \geq 0. \end{array}$$

Now we can represent these nine classical linear programming problems in Table 3.

θ'_1	θ_1	θ'_2	θ_2	θ_3	θ_4	θ'_4	θ_5	θ'_5
1	2	3	4	5	7	8	9	10
2	3	4	5	6	8	9	10	11
0	1	1	4	5	6	7	8	9
1	4	5	7	8	9	10	11	12
1	8	11	19	27	39	49	61	75
1	4	5	7	8	9	10	11	12
4	5	6	8	9	10	11	12	13
6	11	16	24	34	48	60	74	89

Table 3. Tabular form of nine CLP-problems

After solving these classical linear programming problems with simplex method, we get the solution given in Table 4.

$\widetilde{X} =$	0	$\frac{4}{11}$	$\frac{14}{19}$	$\frac{16}{17}$	$\frac{29}{19}$	2	$\frac{61}{23}$	$\frac{82}{25}$	$\frac{31}{9}$
$\widetilde{Y} =$	$\frac{3}{2}$	$\frac{21}{11}$	$\frac{39}{19}$	$\frac{37}{17}$	$\frac{46}{19}$	3	$\frac{70}{23}$	$\frac{79}{25}$	$\frac{11}{3}$
$\widetilde{\theta} =$	3	$\frac{71}{11}$	$\frac{198}{19}$	$\frac{249}{17}$	$\frac{421}{19}$	38	$\frac{1118}{23}$	$\frac{1528}{25}$	$\frac{673}{9}$

Table 4. Solution table

The optimal solution of the above TFLDFLP problem is

$$\left\{ \begin{array}{l} (x_1, x_2, x_3, x_4, x_5) \\ (x'_1, x'_2, x_3, x'_4, x'_5) \end{array} \right. = \left\{ \begin{array}{l} (\frac{4}{11}, \frac{16}{17}, \frac{29}{19}, 2, \frac{82}{25}) \\ (0, \frac{14}{19}, \frac{29}{19}, \frac{61}{23}, \frac{31}{9}) \end{array} \right.$$

$$\left\{ \begin{array}{l} (y_1, y_2, y_3, y_4, y_5) \\ (y'_1, y'_2, y_3, y'_4, y'_5) \end{array} \right. = \left\{ \begin{array}{l} (\frac{21}{11}, \frac{37}{17}, \frac{46}{19}, 3, \frac{79}{25}) \\ (\frac{3}{2}, \frac{39}{19}, \frac{46}{19}, \frac{70}{23}, \frac{11}{3}) \end{array} \right.$$

$$\left\{ \begin{array}{l} (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) \\ (\theta'_1, \theta'_2, \theta_3, \theta'_4, \theta'_5) \end{array} \right. = \left\{ \begin{array}{l} (\frac{71}{11}, \frac{249}{17}, \frac{421}{19}, 38, \frac{1528}{25}) \\ (3, \frac{198}{19}, \frac{421}{19}, \frac{1118}{23}, \frac{673}{9}) \end{array} \right. .$$

Now consider the following fully linear Diophantine fuzzy linear programming (FLDFLP) with m constraints and n variables:

$$\text{Maximize (or Minimize) } \widetilde{Z} = \sum_{k=1}^{n} \widetilde{c_k} \widetilde{x_k}$$

$$\text{subject to } \sum_{k=1}^{n} \widetilde{a_{jk}} \widetilde{x_k} \leq \widetilde{b_j}, \ \ 1 \leq j \leq m;$$

where $\widetilde{x_k}$ is unrestricted for some k.

Example 4.3 *Consider the following TFLDFLP problem*

$$\text{Max}\begin{cases}(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5)\\(\theta'_1,\theta'_2,\theta_3,\theta'_4,\theta'_5)\end{cases}=\begin{cases}(3,7,9,10,13)\\(2,5,9,12,15)\end{cases}\widetilde{X}+\begin{cases}(4,8,10,11,14)\\(3,6,10,13,16)\end{cases}\widetilde{Y}$$

$$\text{subject to}\quad\begin{cases}(6,8,9,10,12)\\(5,7,9,11,13)\end{cases}\widetilde{X}+\begin{cases}(5,7,8,9,11)\\(4,6,8,10,12)\end{cases}\widetilde{Y}\leq\begin{cases}(13,37,47,57,77)\\(7,23,47,67,87)\end{cases}$$

$$\begin{cases}(4,6,7,8,10)\\(3,5,7,9,11)\end{cases}\widetilde{X}+\begin{cases}(9,11,12,13,15)\\(8,10,12,14,16)\end{cases}\widetilde{Y}\leq\begin{cases}(21,45,55,65,85)\\(13,35,55,75,95)\end{cases}$$

$$\widetilde{X}\geq 0,\quad \widetilde{Y}\ \text{is unrestricted.}$$

Let $\widetilde{\theta}=\begin{cases}(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5)\\(\theta'_1,\theta'_2,\theta_3,\theta'_4,\theta'_5)\end{cases}$, $\widetilde{X}=\begin{cases}(x_1,x_2,x_3,x_4,x_5)\\(x'_1,x'_2,x_3,x'_4,x'_5)\end{cases}$ *and* $\widetilde{Y}=\begin{cases}(y_1,y_2,y_3,y_4,y_5)\\(y'_1,y'_2,y_3,y'_4,y'_5)\end{cases}$. *The above TFLDFLP problem may be written as:*

$$\begin{array}{ll}\text{max } \theta_1 = & 3x_1+4y_1\\ \text{subject to} & 6x_1+5y_1\leq 13\\ & 4x_1+9y_1\leq 21\\ & x_1\geq 0,\ y_1 \text{ is unrestricted}\end{array}\qquad\begin{array}{ll}\text{max } \theta_2 = & 7x_2+8y_2\\ \text{subject to} & 8x_2+7y_2\leq 37\\ & 6x_2+11y_2\leq 45\\ & x_2\geq 0,\ y_2 \text{ is unrestricted}\end{array}$$

$$\begin{array}{ll}\text{max } \theta_3 = & 9x_3+10y_3\\ \text{subject to} & 9x_3+8y_3\leq 47\\ & 7x_3+12y_3\leq 55\\ & x_3\geq 0,\ y_3 \text{ is unrestricted}\end{array}\qquad\begin{array}{ll}\text{max } \theta_4 = & 10x_4+11y_4\\ \text{subject to} & 10x_4+9y_4\leq 57\\ & 8x_4+13y_4\leq 65\\ & x_4\geq 0,\ y_4 \text{ is unrestricted}\end{array}$$

$$\begin{array}{ll}\text{max } \theta_5 = & 13x_5+14y_5\\ \text{subject to} & 12x_5+11y_5\leq 77\\ & 10x_5+15y_5\leq 85\\ & x_5\geq 0,\ y_5 \text{ is unrestricted}\end{array}\qquad\begin{array}{ll}\text{max } \theta'_1 = & 2x'_1+3y'_1\\ \text{subject to} & 5x'_1+4y'_1\leq 7\\ & 3x'_1+8y'_1\leq 13\\ & x'_1\geq 0,\ y'_1 \text{ is unrestricted}\end{array}$$

$$\begin{array}{ll}\text{max } \theta'_2 = & 5x'_2+6y'_2\\ \text{subject to} & 7x'_2+6y'_2\leq 23\\ & 5x'_2+10y'_2\leq 35\\ & x'_2\geq 0,\ y'_2 \text{ is unrestricted}\end{array}\qquad\begin{array}{ll}\text{max } \theta'_4 = & 12x'_4+13y'_4\\ \text{subject to} & 11x'_4+10y'_4\leq 67\\ & 9x'_4+14y'_4\leq 75\\ & x'_4\geq 0,\ y'_4 \text{ is unrestricted}\end{array}$$

$$\begin{array}{ll}\text{max } \theta'_5 = & 15x'_5+16y'_5\\ \text{subject to} & 13x'_5+12y'_5\leq 87\\ & 11x'_5+16y'_5\leq 95\\ & x'_5\geq 0,\ y'_5 \text{ is unrestricted.}\end{array}$$

Now we can represent these nine classical linear programming problems in Table 5.

θ'_1	θ_1	θ'_2	θ_2	θ_3	θ_4	θ'_4	θ_5	θ'_5
2	3	5	7	9	10	12	13	15
3	4	6	8	10	11	13	14	16
5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12
7	13	23	37	47	57	67	77	87
3	4	5	6	7	8	9	10	11
8	9	10	11	12	13	14	15	16
13	21	35	45	55	65	75	85	95

Table 5. Tabular form of nine CLP-problems

After solving these classical linear programming problems with simplex method, we get the solution given in Table 6.

$\widetilde{X} =$	$\frac{1}{7}$	$\frac{6}{17}$	$\frac{1}{2}$	2	$\frac{31}{13}$	$\frac{78}{29}$	$\frac{47}{16}$	$\frac{22}{7}$	$\frac{63}{19}$
$\widetilde{Y} =$	$\frac{11}{7}$	$\frac{37}{17}$	$\frac{13}{4}$	3	$\frac{83}{26}$	$\frac{97}{29}$	$\frac{111}{32}$	$\frac{25}{7}$	$\frac{139}{38}$
$\widetilde{\theta} =$	5	$\frac{166}{17}$	22	38	$\frac{694}{13}$	$\frac{1847}{29}$	$\frac{2571}{32}$	$\frac{636}{7}$	$\frac{2057}{19}$

Table 6. Solution table

The optimal solution of the above problem is

$$\left\{\begin{array}{l}(x_1,x_2,x_3,x_4,x_5)\\(x_1',x_2',x_3,x_4',x_5')\end{array}\right. = \left\{\begin{array}{l}(\frac{6}{17},2,\frac{31}{13},\frac{78}{29},\frac{22}{7})\\(\frac{1}{7},\frac{1}{2},\frac{31}{13},\frac{47}{16},\frac{63}{19})\end{array}\right.$$

$$\left\{\begin{array}{l}(y_1,y_2,y_3,y_4,y_5)\\(y_1',y_2',y_3,y_4',y_5')\end{array}\right. = \left\{\begin{array}{l}(\frac{37}{17},3,\frac{83}{26},\frac{97}{29},\frac{25}{7})\\(\frac{11}{7},\frac{13}{4},\frac{83}{26},\frac{111}{32},\frac{139}{38})\end{array}\right.$$

$$\left\{\begin{array}{l}(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5)\\(\theta_1',\theta_2',\theta_3,\theta_4',\theta_5')\end{array}\right. = \left\{\begin{array}{l}(\frac{166}{17},38,\frac{694}{13},\frac{1847}{29},\frac{636}{7})\\(5,22,\frac{694}{13},\frac{2571}{32},\frac{2057}{19})\end{array}\right. .$$

5. Application

An oil industry produces two types of oil. Two types of oils are olive and canola oil are considered in this empirical analysis. Table 7 shows the data of consumptions, productions and availability of

Category	Olive	Canola	Availability
Value	6	7	
Recorded production (ton)	5	7	
Consumption (ton)	7	6	
Live Stock (ton)			35
Output of Livestock (ton)			39

Table 7: Oil categories.

The above information from Table 7 is converted into the linear programming problem.

Let us suppose the variable X and Y ; where X = olive and Y = canola. The LPP can be written in the following form,

$$\begin{array}{ll}\text{Maximize } \theta = & c_1X + c_2Y\\ \text{subject to} & \begin{array}{l} a_{11}X + a_{12}Y \leq b_1\\ a_{21}X + a_{22}Y \leq b_2\end{array}\\ & X, Y \geq 0\end{array}$$

The constraints are defined as follows,

$$\begin{aligned} a_{11}, a_{12} &= \text{Recorded production}\\ a_{21}, a_{22} &= \text{Consumption}\\ b_1, b_2 &= \text{Availability.}\end{aligned}$$

Now we convert the problem in fully linear Diophantine fuzzy linear programming problem. From the Table 7, the problem can be written as,

$$\begin{array}{ll}\text{Maximize } \widetilde{\theta} = & 6\widetilde{X} + 7\widetilde{Y}\\ \text{subject to} & \begin{array}{l} 5\widetilde{X} + 7\widetilde{Y} \leq 35\\ 7\widetilde{X} + 6\widetilde{Y} \leq 39\end{array}\\ & \widetilde{X}, \widetilde{Y} \geq 0\end{array} \qquad \begin{array}{ll}\text{Maximize } \widetilde{\theta} = & 6\widetilde{X}' + 7\widetilde{Y}'\\ \text{subject to} & \begin{array}{l} 5\widetilde{X}' + 7\widetilde{Y}' \leq 35\\ 7\widetilde{X}' + 6\widetilde{Y}' \leq 39\end{array}\\ & \widetilde{X}', \widetilde{Y}' \geq 0\end{array}$$

Now, from the crisp amount in Table 7, the part $(\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \vartheta_5)$ of LDF numbers for each coefficient are presented in Table 8,

Coefficient	Extremely left side	Left side	Crisp number	Right side	Extremely right side
$\widetilde{c}_1$	3	5	6	7	9
$\widetilde{c}_2$	4	6	7	8	10
$\widetilde{a}_{11}$	2	4	5	6	8
$\widetilde{a}_{12}$	4	6	7	8	10
$\widetilde{a}_{21}$	4	6	7	8	10
$\widetilde{a}_{22}$	3	5	6	7	9
$\widetilde{b}_1$	13	29	35	41	57
$\widetilde{b}_{23}$	17	33	39	45	61

Table 8: The part $(\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \vartheta_5)$ of LDF numbers.

The part $(\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \vartheta_5)$ of FLDFLPP can be written as follows,

$$\begin{array}{ll} \text{Maximize } \widetilde{\theta} = & (3,5,6,7,9)\widetilde{X} + (4,6,7,8,10)\widetilde{Y} \\ \text{subject to} & \begin{array}{l} (2,4,5,6,8)\widetilde{X} + (4,6,7,8,10)\widetilde{Y} \leq (13,29,35,41,57) \\ (4,6,7,8,10)\widetilde{X} + (3,5,6,7,9)\widetilde{Y} \leq (17,33,39,45,61) \end{array} \\ & \widetilde{X} = (x_1,x_2,x_3,x_4,x_5),\ \widetilde{Y} = (y_1,y_2,y_3,y_4,y_5) \geq 0. \end{array}$$

Now, the problem is divided into five subproblems.

$$\begin{array}{ll} \max \theta_1 = & 3x_1 + 4y_1 \\ \text{subject to} & \begin{array}{l} 2x_1 + 4y_1 \leq 13 \\ 4x_1 + 3y_1 \leq 17 \end{array} \\ & x_1, y_1 \geq 0 \end{array} \qquad \begin{array}{ll} \max \theta_2 = & 5x_2 + 6y_2 \\ \text{subject to} & \begin{array}{l} 4x_2 + 6y_2 \leq 29 \\ 6x_2 + 5y_2 \leq 33 \end{array} \\ & x_2, y_2 \geq 0 \end{array}$$

$$\begin{array}{ll} \max \theta_3 = & 6x_3 + 7y_3 \\ \text{subject to} & \begin{array}{l} 5x_3 + 7y_3 \leq 35 \\ 7x_3 + 6y_3 \leq 39 \end{array} \\ & x_3, y_3 \geq 0 \end{array} \qquad \begin{array}{ll} \max \theta_4 = & 7x_4 + 8y_4 \\ \text{subject to} & \begin{array}{l} 6x_4 + 8y_4 \leq 41 \\ 8x_4 + 7y_4 \leq 45 \end{array} \\ & x_4, y_4 \geq 0 \end{array}$$

$$\begin{array}{ll} \max \theta_5 = & 9x_5 + 10y_5 \\ \text{subject to} & \begin{array}{l} 8x_5 + 10y_5 \leq 57 \\ 10x_5 + 9y_5 \leq 61 \end{array} \\ & x_5, y_5 \geq 0. \end{array}$$

After solving these sub problems with simplex method, we get the optimal solution as follows:

$$\begin{aligned} (x_1, y_1) &= (2.9, 1.8),\ \theta_1 = 15.9 \\ (x_2, y_2) &= (3.312, 2.62),\ \theta_2 = 32.31 \\ (x_3, y_3) &= (3.315, 2.63),\ \ \theta_3 = 38.31 \\ (x_4, y_4) &= (3.318, 2.636),\ \theta_4 = 44.31 \\ (x_5, y_5) &= (3.46, 2.92),\ \ \theta_5 = 60.46. \end{aligned}$$

Similarly, from the crisp amount in Table 7, the part $(\vartheta_1', \vartheta_2', \vartheta_3, \vartheta_4', \vartheta_5')$ of LDF numbers for each coefficient are presented in Table 9,

Coefficient	Extremely left side	Left side	Crisp number	Right side	Extremely right side
$\widetilde{c}_1'$	2	4	6	8	10
$\widetilde{c}_2'$	3	5	7	9	11
$\widetilde{a}_{11}'$	1	3	5	7	9
$\widetilde{a}_{12}'$	3	5	7	9	11
$\widetilde{a}_{21}'$	3	5	7	9	11
$\widetilde{a}_{22}'$	2	4	6	8	10
$\widetilde{b}_1'$	7	19	35	47	67
$\widetilde{b}_{23}'$	11	23	39	51	71

Table 9: The part $(\vartheta_1^{'}, \vartheta_2^{'}, \vartheta_3, \vartheta_4^{'}, \vartheta_5^{'})$ of LDF numbers.

The part $(\vartheta_1', \vartheta_2', \vartheta_3, \vartheta_4', \vartheta_5')$ of FLDFLPP can be written as follows,

$$\begin{aligned} \text{Maximize } \widetilde{\theta} = \;& (2,4,6,8,10)\widetilde{X}' + (3,5,7,9,11)\widetilde{Y}' \\ \text{subject to } \;& (1,3,5,7,9)\widetilde{X}' + (3,5,7,9,11)\widetilde{Y}' \leq (7,19,35,47,67) \\ & (3,5,7,9,11)\widetilde{X}' + (2,4,6,8,10)\widetilde{Y}' \leq (11,23,39,51,71) \\ & \widetilde{X}' = (x_1', x_2', x_3, x_4', x_5'),\ \widetilde{Y} = (y_1', y_2', y_3, y_4', y_5') \geq 0. \end{aligned}$$

Now, the problem is divided into five subproblems.

$$\begin{aligned} \max\ \theta_1' = \;& 2x_1' + 3y_1' \\ \text{subject to } \;& 1x_1' + 3y_1' \leq 7 \\ & 3x_1' + 2y_1' \leq 11 \\ & x_1', y_1' \geq 0 \end{aligned} \qquad \begin{aligned} \max\ \theta_2' = \;& 4x_2' + 5y_2' \\ \text{subject to } \;& 3x_2' + 5y_2' \leq 19 \\ & 5x_2' + 4y_2' \leq 23 \\ & x_2', y_2' \geq 0 \end{aligned}$$

$$\begin{aligned} \max\ \theta_3 = \;& 6x_3 + 7y_3 \\ \text{subject to } \;& 5x_3 + 7y_3 \leq 35 \\ & 7x_3 + 6y_3 \leq 39 \\ & x_3, y_3 \geq 0 \end{aligned} \qquad \begin{aligned} \max\ \theta_4' = \;& 8x_4' + 9y_4' \\ \text{subject to } \;& 7x_4' + 9y_4' \leq 47 \\ & 9x_4' + 8y_4' \leq 51 \\ & x_4', y_4' \geq 0 \end{aligned}$$

$$\begin{aligned} \max\ \theta_5' = \;& 10x_5' + 11y_5' \\ \text{subject to } \;& 9x_5' + 11y_5' \leq 67 \\ & 11x_5' + 10y_5' \leq 71 \\ & x_5', y_5' \geq 0. \end{aligned}$$

After solving these sub problems with simplex method, we get the optimal solution as follows:

$$\begin{aligned} (x_1', y_1') &= (2.71, 1.42),\ \theta_1' = 9.714 \\ (x_2', y_2') &= (3, 2),\ \theta_2' = 22 \\ (x_3, y_3) &= (3.315, 2.63),\quad \theta_3 = 38.31 \\ (x_4', y_4') &= (3.32, 2.64),\ \theta_4' = 50.32 \\ (x_5', y_5') &= (3.58, 3.16),\ \theta_5' = 70.58. \end{aligned}$$

The optimal solution of the FLDFLP problem is

$$\begin{aligned} \left\{ \begin{array}{l} (x_1, x_2, x_3, x_4, x_5) \\ (x_1', x_2', x_3, x_4', x_5') \end{array} \right. &= \left\{ \begin{array}{l} (2.9, 3.312, 3.315, 3.318, 3.46) \\ (2.71, 3, 3.315, 3.32, 3.58) \end{array} \right. \\ \left\{ \begin{array}{l} (y_1, y_2, y_3, y_4, y_5) \\ (y_1', y_2', y_3, y_4', y_5') \end{array} \right. &= \left\{ \begin{array}{l} (1.8, 2.62, 2.63, 2.636, 2.92) \\ (1.42, 2, 2.63, 2.64, 3.16) \end{array} \right. \\ \left\{ \begin{array}{l} (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) \\ (\theta_1', \theta_2', \theta_3, \theta_4', \theta_5') \end{array} \right. &= \left\{ \begin{array}{l} (15.9, 32.31, 38.31, 44.31, 60.46) \\ (9.71, 22, 38.31, 50.32, 70.58) \end{array} \right. . \end{aligned}$$

Now,

$$\begin{aligned} X &= R\left(\left\{\begin{array}{l}(2.9, 3.312, 3.315, 3.318, 3.46)\\(2.71, 3, 3.315, 3.32, 3.58)\end{array}\right)\right. = 3.2572\\ Y &= R\left(\left\{\begin{array}{l}(1.8, 2.62, 2.63, 2.636, 2.92)\\(1.42, 2, 2.63, 2.64, 3.16)\end{array}\right)\right. = 2.5147\\ Z &= R\left(\left\{\begin{array}{l}(15.9, 32.31, 38.31, 44.31, 60.46)\\(9.71, 22, 38.31, 50.32, 70.58)\end{array}\right)\right. = 38.25 \end{aligned}$$

Thus the profit can be maximized with $X = 3.2572$ and $Y = 2.5147$.

6. Conclusion

This manuscript is devoted to studying the concept of FLDFLP problems. This new concept of FLDFLP problem removes the limitations of FLP problem and enhances the space of membership and non-membership grades by adding the reference or control parameters. In this paper, we have proposed a new method for solving FLDFLP problems and provided some illustrative examples. By using the proposed method we successfully solve the following types of FLDFLP problems :

(i) FLDFLP having nonnegative fuzzy coefficients and non negative fuzzy variables,

(ii) FLDFLP having nonnegative fuzzy coefficients and unrestricted fuzzy variable.

Compliance with ethical standards:

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Naveed Yaqoob,
Department of Mathematics and Statistics,
Riphah International University,
I-14 Campus, Islamabad,
Pakistan.
E-mail address: `nayaqoob@ymail.com`

and

Salma Iqbal,
Department of Mathematics,
Pakistan International School,
Buraydah Al-Qassim,
Saudi Arabia.
E-mail address: `salmaiqbalawan64@gmail.com`