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Minimal Spanning Tree Algorithms in Complex Bipolar Neutrosophic Environments: Prim's and Kruskal's

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ABSTRACT: In real-world circumstance where conclusion are ambiguity and inconsistent, fuzzy model graphs frequently have a no good time appearing bipolar details that is confusion and complex. The present study involving a new methodology for point-out minimal spanning trees (MSTs) in Bipolar Complex Neutrosophic Weighted Connected Graphs (BCNWCGs) to correct this defect. We better the conventional algorithms, specifically Kruskal's and Prim's , by attaching new function called score functions that are block out to deal with the different levels of values like falsehood membership, truth, uncertainty memberships, and these values are define in different intervals with positive and negative values these are involving in Bipolar Complex Neutrosophic Weighted Connected Graphs (BCNWCGs). This work is completely based on the algorithms and neutrosophic graph theory and its complex bipolar extension. The study decorate the two algorithms Prim's and Kruskal's produces compatible weights which are minimal within the bipolar neutrosophic complex framework, achieved through careful algorithmic mathematical formulation and comparing numerical values. For Wi-Fi apps to work well, the signal strength must be right and the signal loss over distance must be kept to a minimum.

Keywords: Complex bipolar neutrosophic set, Minimal Spanning Tree, Prim's Algorithm(PA), Kruskal's Algorithm(KA), score function, uncertainty.

Contents

1. Introduction

Classical graph-based optimization presumes deterministic edge weights, which is inappropriate for systems marked by ambiguity, contradiction, and incomplete information. Smarandache's neutrosophic set theory builds on fuzzy and intuitionistic fuzzy frameworks by adding three separate parts: truth, indeterminacy, and falsity. The bipolar extension makes things even more expressive by modeling both positive and negative information at the same time. Prim's and Kruskal's Minimal Spanning Tree (MST) algorithms are very important for designing networks, communication systems, and wireless routing. But they can't be used directly in situations where things aren't clear. This research addresses this deficiency by expanding MST computation to Complex Bipolar Neutrosophic Connected Weighted Graphs (CB-NCWGs). The integration of MST computation into Complex Bipolar Neutrosophic Connected Weighted

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Graphs. Enhancement of Prim's and Kruskal's algorithms through the application of neutrosophic score functions. Mathematical formulation of score-based edge comparison in the context of complex bipolar uncertainty. Demonstration via numerical example that both algorithms produce the same minimal weights. Application significance for Wi-Fi signal enhancement amid interference and ambiguity. Graph theory has become one of the most important mathematical tools for modeling, analyzing, and optimizing complicated systems in science, engineering, and technology. Graphs are a natural way to model networks that show communication systems, transportation networks, electrical grids, social interactions, and biological systems.

In graphs, vertices stand for entities and edges stand for relationships or interactions between them. The *Minimal Spanning Tree* (MST) problem is one of the most important graph-theoretic optimization problems because it is both theoretically beautiful and very useful in the real world. The purpose of an MST is to link all the vertices of a weighted graph with the least amount of total edge weight while making sure that no cycles are made. Many people use classical greedy algorithms like Prim's and Kruskal's because they are easy to use, fast, and always give the best answer in deterministic environments [6]. In conventional graph models, edge weights are regarded as exact numerical values that accurately represent metrics such as cost, distance, delay, or capacity. But this assumption is not always true in real life [16]. In these situations, deterministic MST models might give answers that aren't the best or aren't reliable. Zadeh introduced fuzzy set theory to deal with uncertainty in mathematical modeling [1].Fuzzy sets allow elements to be only partially members, which makes them a flexible way to show imprecise information. This idea was naturally expanded to fuzzy graphs, where membership degrees, not crisp values, are linked to vertices and edges. After that, researchers looked into the MST problem in fuzzy settings by changing classical algorithms through ranking and defuzzification or α-cut techniques [7,8,9,10,17]. These studies showed that fuzzy graph models could better show uncertainty than classical graphs. Even with these improvements, fuzzy graph models still have some problems. Fuzzy sets represent uncertainty solely through a singular membership degree, failing to explicitly incorporate hesitation or indeterminacy. Intuitionistic fuzzy sets were suggested to address this constraint by integrating both membership and non-membership degrees. But intuitionistic fuzzy sets limit the sum of membership and non-membership degrees, which makes it hard for them to model information that is very inconsistent or incomplete. Additionally, numerous real-world decision-making dilemmas encompass conflicting evidence from various sources, which cannot be sufficiently depicted through fuzzy or intuitionistic fuzzy frameworks [13,14].

Smarandache's introduction of neutrosophic set theory [2] marks a major step forward in how we model uncertainty. A neutrosophic set comprises three distinct components: truth-membership, indeterminacy-membership, and falsity-membership. These components are not limited by strict rules like fuzzy and intuitionistic fuzzy sets are. This means that neutrosophic sets can show incomplete, uncertain, and even contradictory information in a very flexible way.SVNs stands for Single-valued Neutrosophic sets were introduced to fine the membership values in on interval from 0 to 1 and these are very use full for the decision making problems in different fields. With these sets, for each membership truth valu , indeterminacy and falsity will come in the unit interval [3]. The significant attention for this theory will utilized in various domains,like engineering, technology, medical etc. [18,4].

The Bipolar modeling, involving two different intervals in which one in negative part and other one is positive part it indicates in degree and out degree in graph theory. the conjunction of bipolar and complex neutrosophic graphical theory, includes and edge with six numbers and vertex with six numbers in which three members are positive and remaining members are negative and more over the complex systems are studied by [5]. Complex numbers improve modeling even more by letting you show both phase and magnitude information. Complex-valued fuzzy and neutrosophic sets have significant applications in signal processing, electrical engineering, and communication systems, where phase is essential. Complex bipolar neutrosophic sets combine complexity, bipolarity, and neutrosophic uncertainty into one system. As a result, complex bipolar neutrosophic graphs are especially good for modeling advanced communication networks like Wi-Fi systems, where signal strength, interference, distance, and phase changes all happen at the same time [19,20]. Despite considerable advancements in neutrosophic graph theory, the examination of Minimal Spanning Tree problems within intricate bipolar neutrosophic contexts is still somewhat constrained. Several researchers have examined Minimum Spanning Trees (MSTs) in fuzzy, intuitionistic fuzzy, and neutrosophic graphs [7,8,15]. Nonetheless, extensive frameworks for Minimum

Spanning Tree (MST) computation in complex bipolar neutrosophic connected weighted graphs (CBNCWGs) remain limited. One of the biggest problems in this case is that there is no natural order for complex bipolar neutrosophic numbers. In classical MST algorithms, the minimum edge is chosen at each step by comparing the weights of the edges. However, this is not easy to do in neutrosophic environments.

To tackle this problem, several score and ranking functions have been suggested to convert neutrosophic data into real-valued metrics appropriate for comparison [11,12]. These score functions mix truth, uncertainty, and falsehood in different ways. Although these methods facilitate the adaptation of greedy algorithms to neutrosophic graphs, their effects on the consistency and optimality of Minimum Spanning Tree (MST) solutions—especially in intricate bipolar neutrosophic contexts—remain inadequately investigated. Motivated by these research gaps, the present study initiate a new mathematical framework for finding the minimal numerical values corresponding to minimal weighted spanning trees in complicated connected neutrosophic weighted bipolar graphs by using two different algorithms with the help of Score functions. A notable handout of this work is the explain two algorithms for giving minimal vales together with same minimal spanning trees. The suggested take aside of practical consequences is explained through its applications in Wi-Fi network enhancement.Due to distance materials thickness, the signal associated with Wi-Fi systems has to to work well when it keeps the signal is very strong. These things will happen only when the fundamental ambiguity in frequency variations has to be modified by time being contradictory. For modeling the type of real world problems we will use CBNCWGs and computing numerical vales by transforming the given graph in to MSTs under uncertainty in neutrosophic theory. Also by using bipolar graph theory is a good mathematical tool to establish a good relation between real life problem with graph theory [16].The existing literature on neutrosophic graph is with gaps to full fill these gaps we introduced a new technique called bipolar complex neutrosophic modeling with conventional Minimum Spanning Tree (MST) methods like Prim's and Kruskal's algorithms.

2. Title Material

2.1. Neutrosophic Set

A Neutrosophic Set (NS) is defined as

$$\aleph = \langle m, \Re_\aleph(m), \wp_\aleph(m), \Im_\aleph(m)\rangle : m \in \mathbb{M}, \tag{2.1}$$

where $\Re_\aleph, \wp_\aleph, \Im_\aleph \in [0,1]$ and $0 \leq \Re_\aleph + \wp_\aleph + \Im_\aleph \leq 3$.
where the symbols denotes $\Re_\aleph$ truth, $\wp_\aleph$ falsity and $\Im_\aleph$ indeterminacy memberships

2.2. Single-Valued Neutrosophic Set

A Single-Valued Neutrosophic Set (SVNS) restricts $\Re_\aleph, \wp_\aleph, \Im_\aleph$ to real numbers in $[0,1]$.
From here on wards these symbols $\Re_\aleph, \wp_\aleph, \Im_\aleph$ are replased by $\mathbb{T}, \mathbb{F}$ and $\mathbb{I}$

2.3. Complex Bipolar Neutrosophic Set

A *complex bipolar neutrosophic set* $\mathbb{A}$ on a universe $\mathbb{X}$ is defined as

$$A = \left\{\langle x, \mathbb{T}^+(x), \mathbb{I}^+(x), \mathbb{F}^+(x), \mathbb{T}^-(x), \mathbb{I}^-(x), \mathbb{F}^-(x)\rangle : x \in \mathbb{X}\right\},$$

where $\mathbb{T}^+, \mathbb{I}^+, \mathbb{F}^+ : \mathbb{X} \to [0,1]$ and $\mathbb{T}^-, \mathbb{I}^-, \mathbb{F}^- : \mathbb{X} \to [-1,0]$ are complex-valued membership functions.

2.4. Complex Bipolar Neutrosophic Graph

A CBNCWG is defined as $\mathbb{G} = (\mathbb{V}, \mathbb{E}, \mathbb{W})$ where each edge weight is expressed as a complex bipolar neutrosophic number

$$\mathbb{W}(e_{ij}) = (\mathbb{T}^+ + i\mathbb{T}^-, \mathbb{I}^+ + i\mathbb{I}^-, \mathbb{F}^+ + i\mathbb{F}^-)). \tag{2.2}$$

2.5. Complex Bipolar Neutrosophic Complete Weighted Graph

A *complex bipolar neutrosophic complete weighted graph* $\mathbb{G} = (\mathbb{V}, \mathbb{E})$ is a graph in which each edge $e \in \mathbb{E}$ is assigned a complex bipolar neutrosophic weight.

2.6. Score Function

A general score function for single valued Neutrosophic graph is defined as

$$S(e) = \frac{\mathbb{T} - \mathbb{I} - \mathbb{F}}{3}. \tag{2.3}$$

This function maps neutrosophic weights into real values for ranking edges. This is the score function for Neutrosophic graph. Like wise there is different score function has been defined. particularity we will apply the score function for complex bipolar NG also. the formula is given as

$$S(e) = e^{(T^+ - 2F^+ + I^+) + i(2I^- - T^- - F^-)} \frac{T^+ + 1 - F^+ - I^+ + T^- + 2 + F^- - I^- + 3}{6}$$

.

2.7. Example

Let the vertex set be

$$V = \{p, q, r, t\}$$

and the edge set be

$$E = \{(p, q), (p, r), (p, t), (q, t)\}.$$

The positive complex truth–membership(CTM), complex indeterminate–membership (CIM) and complex false–membership(CFM) values of the vertices are given as follows,

	p	q	r	t
ϱ_T^+	$0.6e^{i0.9}$	$0.8e^{i0.7}$	$0.4e^{i0.5}$	$0.9e^{i0.3}$
ϱ_I^+	$0.4e^{i\frac{2\pi}{3}}$	$0.3e^{i\frac{3\pi}{2}}$	$0.2e^{i2\pi}$	$0.4e^{i3\pi}$
ϱ_F^+	$0.2e^{i0.2}$	$0.3e^{i0.4}$	$0.9e^{i0.4}$	$0.5e^{i0.6}$

The negative CTM, CIM and CFM values of the vertices are defined as

	p	q	r	t
ϱ_T^-	$-0.7e^{-i0.7}$	$-1e^{-i\pi}$	$-0.3e^{-i0.4}$	$-0.7e^{-i0.3}$
ϱ_I^-	$-0.5e^{-i\pi}$	$-0.4e^{i0.1}$	$-0.3e^{-i0.2}$	$-0.7e^{-i0.1}$
ϱ_F^-	$-0.3e^{-i0.4}$	$-0.6e^{-i0.5}$	$-0.8e^{-i3\pi}$	$-0.6e^{-i2\pi}$

Similarly complex truth +Ve and -Ve, complex indeterminacy +Ve and -Ve, complex falsity +Ve and -Ve memberships of edges can calculated.

Using the defining functions

$$\nu, \xi, \varpi : [0, 1]^2 \to [0, 1], \qquad \tau, \omega, \chi : [-1, 0]^2 \to [-1, 0],$$

the corresponding bipolar complex neutrosophic edge–membership values are computed, and hence the triad

$$\zeta = (V, \varrho, \varpi)$$

forms a *bipolar complex neutrosophic graph (BCNG).*

3. Neutrosophic MST Algorithms

3.1. Prim's Algorithm

1. **Input:** A bipolar complex weighted neutrosophic complete graph $\zeta = (V, \varrho, \varpi)$.
2. Construct the adjacency matrix corresponding to ζ.
3. Transform the complex bipolar neutrosophic edge weights into real values using the proposed score function $S(e)$.
4. Choose an arbitrary vertex from V as the initial vertex.
5. While not all vertices of ζ are included in the spanning tree, select the edge with the minimum score value that does not form a cycle.
6. **Output:** The minimal spanning tree of ζ.

3.2. Kruskal's Algorithm

1. **Input:** A bipolar complex weighted neutrosophic complete graph $\zeta = (V, \varrho, \varpi)$.
2. Convert the complex bipolar neutrosophic edge weights into real values using the score function $S(e)$.
3. Arrange all edges of ζ in ascending order according to their score values.
4. Sequentially select edges from the ascending ordered list and connect the respective vertices with an edge in MST if and only if it does not create a cycle.
5. **Output:** The minimum spanning tree of ζ.

4. Figures and Tables

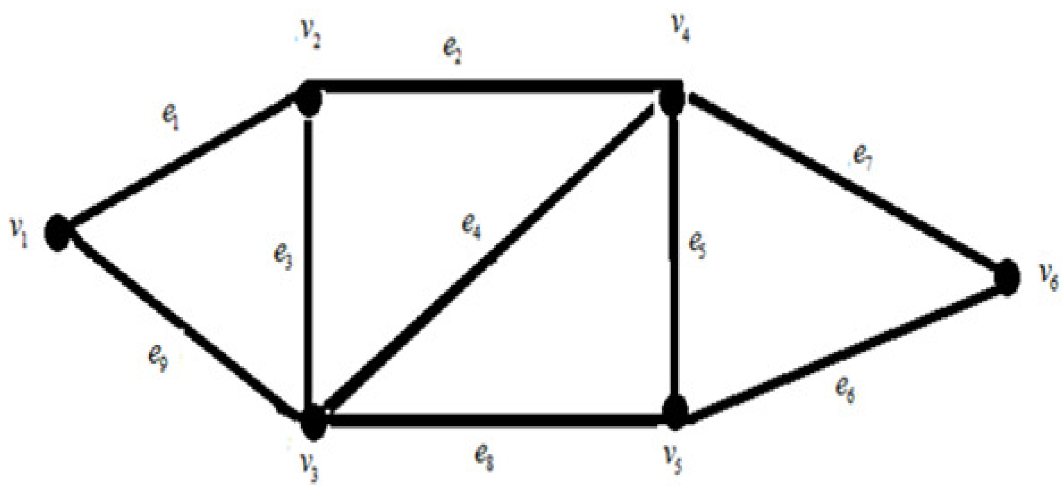

Figure 1: Complex Bipolar Neutrosophic Weighted Graph

Weighted Edges of BNCWG

The weighted edges of BNCWG are shown in Table 1.

Table 1: Weighted Edges of the BNCWG

Edge	End Vertices	Positive Weight (T^+, I^+, F^+)	Negative Weight (T^-, I^-, F^-)
e_1	v_1v_2	$(0.5, 0.4, 0.6)$	$(-0.2, -0.3, -0.1)$
e_2	v_2v_4	$(0.3, 0.5, 0.2)$	$(-0.3, -0.1, -0.5)$
e_3	v_2v_3	$(0.6, 0.4, 0.1)$	$(-0.4, -0.6, -0.8)$
e_4	v_3v_4	$(0.5, 0.1, 0.6)$	$(-0.7, -0.1, -0.8)$
e_5	v_4v_5	$(0.1, 0.3, 0.4)$	$(-0.4, -0.6, -0.1)$
e_6	v_5v_6	$(0.1, 0.4, 0.7)$	$(-0.1, -0.5, -0.6)$
e_7	v_4v_6	$(0.3, 0.5, 0.8)$	$(-0.1, -0.6, -0.4)$
e_8	v_3v_5	$(0.6, 0.1, 0.4)$	$(-0.1, -0.7, -0.6)$
e_9	v_1v_3	$(0.6, 0.4, 0.5)$	$(-0.7, -0.4, -0.5)$

Construction of the Score Matrix

In view of constructing Score Matrix we will use the Score function $S(e)$, by applying it on Table 3 and 4 we will get,

$$S = \begin{bmatrix} 0 & 0.517 & 0.617 & 0 & 0 & 0 \\ 0.517 & 0 & 0.633 & 0.500 & 0 & 0 \\ 0.617 & 0.633 & 0 & 0.450 & 0.567 & 0 \\ 0 & 0.500 & 0.450 & 0 & 0.550 & 0.533 \\ 0 & 0 & 0.567 & 0.550 & 0 & 0.317 \\ 0 & 0 & 0 & 0.533 & 0.317 & 0 \end{bmatrix}$$

Table 2: Complex Neutrosophic Membership Values of Vertices

Vertex	$\boldsymbol{T_V(v_i)}$	$\boldsymbol{I_V(v_i)}$	$\boldsymbol{F_V(v_i)}$
v_1	$0.5e^{i\phi_{T_1}}$	$0.3e^{i\phi_{I_1}}$	$0.2e^{i\phi_{F_1}}$
v_2	$0.8e^{i\phi_{T_2}}$	$0.5e^{i\phi_{I_2}}$	$0.4e^{i\phi_{F_2}}$
v_3	$0.6e^{i\phi_{T_3}}$	$0.4e^{i\phi_{I_3}}$	$0.3e^{i\phi_{F_3}}$
v_4	$0.4e^{i\phi_{T_4}}$	$0.6e^{i\phi_{I_4}}$	$0.2e^{i\phi_{F_4}}$
v_5	$0.6e^{i\phi_{T_5}}$	$0.3e^{i\phi_{I_5}}$	$0.5e^{i\phi_{F_5}}$
v_6	$0.1e^{i\phi_{T_6}}$	$0.5e^{i\phi_{I_6}}$	$0.7e^{i\phi_{F_6}}$

Table 3: Adjacency Matrix of Complex Neutrosophic Weighted Graph

	v_1	v_2	v_3
v_1	$(0, 0, 0)$	$\left(0.5e^{i\phi^T_{12}}, 0.3e^{i\phi^I_{12}}, 0.2e^{i\phi^F_{12}}\right)$	$(0, 0, 0)$
v_2	$\left(0.3e^{i\phi^T_{21}}, 0.4e^{i\phi^I_{21}}, 0.3e^{i\phi^F_{21}}\right)$	$(0, 0, 0)$	$\left(0.1e^{i\phi^T_{23}}, 0.2e^{i\phi^I_{23}}, 0.4e^{i\phi^F_{23}}\right)$
v_3	$(0, 0, 0)$	$\left(0.4e^{i\phi^T_{32}}, 0.3e^{i\phi^I_{32}}, 0.2e^{i\phi^F_{32}}\right)$	$(0, 0, 0)$

The resultant graphs are with same minimal weight(i.e.,2.317). So, This study specify the argumentation of complex neutrosophic bipolar graph theory with minimal spanning tree algorithms. Hear we learn how Minimal Spanning Tree will be obtained by using different algorithms by extending these algorithms neutrosophic graphs with bipolar environment. Especially by using score functions we calculated the weights of complex neutrosophic edges. we got same minimal values in both the algorithms which shows the identical nature of PA & KA .From this we can conclude that bipolar complex neutrosophic weighted graphs satisfies both algorithms

Table 4: Adjacency Matrix of Complex Neutrosophic Weighted Graph

	v_4	v_5	v_6
v_4	$(0,0,0)$	$\left(0.3e^{i\phi^T_{42}}, 0.4e^{i\phi^I_{42}}, 0.3e^{i\phi^F_{42}}\right)$	$(0,0,0)$
v_5	$\left(0.3e^{i\phi^T_{51}}, 0.5e^{i\theta^I_{51}}, 0.2e^{i\phi^F_{51}}\right)$	$(0,0,0)$	$\left(0.4e^{i\phi^T_{53}}, 0.3e^{i\phi^I_{53}}, 0.5e^{i\phi^F_{53}}\right)$
v_6	$(0,0,0)$	$\left(0.1e^{i\phi^T_{62}}, 0.3e^{i\phi^I_{62}}, 0.4e^{i\phi^F_{62}}\right)$	$(0,0,0)$

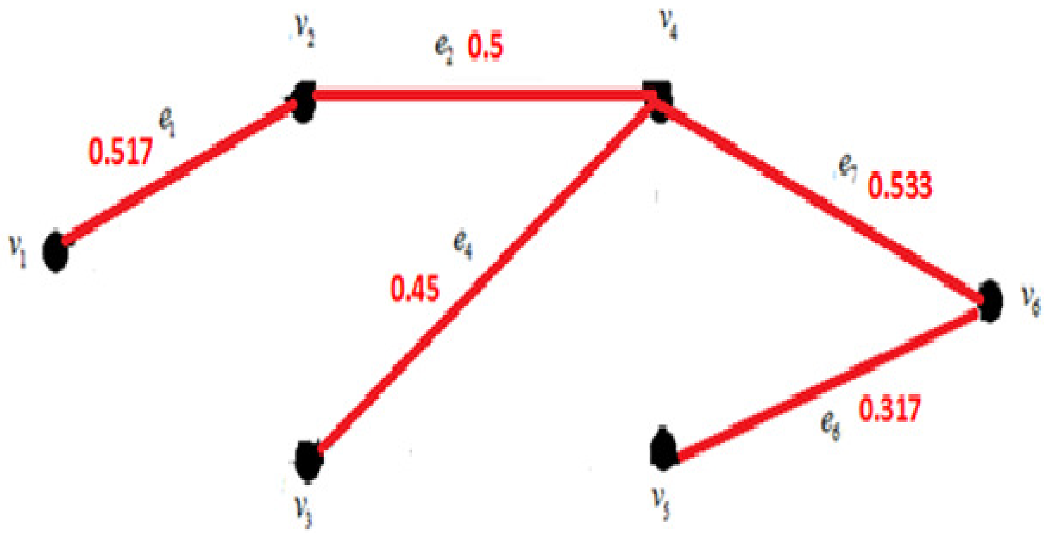

Figure 2: Final minimal spanning tree corresponding to given CBNWG using PA and KA

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