Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2026 (44) 17** : 1–10.
ISSN-0037-8712
doi:10.5269/bspm.82363

Investigation of Nanofluid-Induced Heat Transfer Enhancement in a Vertical Rotating System Using CFD

[1] G. Mahadevi, [2] Bala Siddulu Malga, [3] B. R. Sreedhar, [4] G. Narsimlu

ABSTRACT: The use of nano-fluids to improve heat transmission has recently attracted a lot of interest. As an alternative to traditional coolants, nano-fluids show promise in a wide range of industries, including electronics, transportation, HVAC, electricity production, and nuclear power. The analytical examination of water-based heat-transfer performance is presented in this work. The system is vertically spinning and contains nanofluids of copper oxide (CuO), aluminum oxide (Al_2O_3), and zinc oxide (ZnO). We take a close look at how the thermal and hydrodynamic behavior changes depending on the nanoparticle composition, volume concentration, and rotating speed. We simplify the governing partial differential equations to a system of nonlinear ordinary differential equations by utilizing similarity transformations and boundary-layer theory. Then, we solve these equations numerically. The findings show that compared to the basic fluid, the heat-transfer rate is significantly improved when nanoparticles are added. When it comes to thermal conductivity, CuO-water, Al_2O_3-water, and ZnO-water are the nanofluids that stand out. Rotational motion also has a major impact on the thermal boundary layer thickness and velocity distribution. Taken together, the results show that nanofluids have a lot of promise for making spinning systems more efficient heat exchangers. Rotating equipment and associated industrial applications may benefit from the knowledge gained from this study in terms of heat management and design.

Keywords: Vertical rotating system, nano fluid, heat transfer.

Contents

1. Introduction

The increased heat conductivity and micro-convection produced by the nanoparticles' mobility make nanofluids a vast improvement over conventional fluids for heat transfer in vertically rotating systems. The enhancement is the result of a complex interplay between many aspects of the system's dynamics and the nanofluid. Extensive progress has been consistently shown by numerical and experimental studies. The heat transfer coefficient may be up to 74.5 percent higher in agitated vessels with propellers and water nanofluids containing Al_2O_3, according to the study. Researchers studying nanofluid applications in vertically rotating systems often use advanced simulation methods including computational fluid dynamics (CFD) and numerical methodologies to characterize the complex flow behavior and predict performance. Various systems for cooling turbine blades and electronic cooling systems are examples of industrial heat exchangers that use rotational forces. The current focus of research is on finding ways to improve heat transfer while reducing stability problems and pressure loss.

Investing in heat exchangers could pay out handsomely for a variety of manufacturing operations. In this context, "augmentation" means that, under the same conditions of operation, there is an improvement in heat transfer compared to the reference surface. If heat exchangers function better, we may design

---

2020 *Mathematics Subject Classification:* 80A20, 76D10.
Submitted March 03, 2026. Published June 19, 2026.

them to be more efficient, which in turn reduces the amount of energy, materials, and money used in heat exchange operations. A nanofluid is a solution of metal nanoparticles in water. The effective thermal conductivity of a solid/fluid combination is expected to be higher than that of the base fluid due to the fact that metallic solids often possess a thermal conductivity that is many orders of magnitude larger than that of fluids.

Hence, nanoparticles enhance nanofluid heat transfer rates by changing the transport properties of the base fluid, leading to an increase in the fluid's effective thermal conductivity and heat capacity. Since inserts can be easily manufactured and integrated into existing heat exchangers, passive methods—which include inserts inserted in the flow channel to enhance the heat transfer rate—are preferable to active alternatives. Consequently, this research enhanced the heat exchanger's efficiency by increasing the exchange rate by the use of twisted tape, a passive approach. Passive methods include things like treated or structured surfaces, rough or stretched surfaces, devices that increase swirl flow or encourage displacement, compounds for gases and liquids, and so on.

Convective heat transfer is significantly impacted by the complex flow patterns produced by a vertically rotating system due to the combined effects of centrifugal forces, Coriolis forces, and imposed temperature gradients. Conventional fluids often fail to provide adequate thermal performance in these systems due to uneven temperature fields and limited thermal conductivity. A novel class of enhanced heat-transfer media called nanofluids holds the key to resolving these issues. Nanofluids enhance convective heat transfer by dispersing nanoparticles in a base liquid, which alters flow patterns and significantly improves effective thermal conductivity. A number of thermophysical properties, such as specific heat and density, as well as the kind of nanoparticle and the volume concentration of the particles, determine the magnitude of the improvement.

The flow of nanofluids between vertical plates has been the subject of much study for the last 20 years, shedding light on their thermophysical behavior and practical significance. Choi first suggested the idea of nanofluids as synthetic suspensions of uniformly dispersed solid nanoparticles in a base fluid. In comparison to the base fluid in its most basic state, these particles significantly improve the thermal conductivity and convective heat transfer coefficient. Using analytical and semi-empirical models that include phenomena such as thermophoresis and Brownian motion, further theoretical research have evaluated the thermal conductivity of nanofluids.

In most cases, nanofluids boost heat-transfer capability, which in turn increases the efficiency of thermal systems. Base fluids may include water, oils, hydrogen-based liquids, or any number of other industrial working fluids; nanoparticle materials typically include diamonds, graphite, carbon nanotubes, carbon, metal oxides, and carbon-based structures. Radiators for cars, cooling electronics, and other high-performance heat-exchange systems have made heavy use of nanofluids due to their ability to reduce thermal resistance. Because of their superior thermal qualities, next-generation coolants are very attractive for use in complicated industrial processes and electrical equipment.

Nanoparticles change the flow behavior and heat transfer processes by influencing the base fluid's thermophysical properties, such as density, specific heat, viscosity, and thermal conductivity. Much study has focused on the theoretical and practical aspects of producing nanofluids, the variations in thermal conductivity and viscosity, and the benefits to performance that these factors provide. Metal oxide nanofluids have outstanding physical characteristics and are hence ideal for application in industrial systems operating at high temperatures. You can regulate properties like optical transcalency by adjusting the concentration of nanoparticles, which increases their flexibility.

Nanofluid flows have the potential to improve thermal management in engineering, according to new study. This is because, when flowing over surfaces that are rotating or have vertical plates, they transport more mass and heat.

2. Literature Survey

According to a literature review, nanofluids significantly improve heat transfer in rotating systems compared to traditional base fluids due to their increased thermal conductivity, induction of micro-convection and chaotic mixing, and other properties. However, in most cases, the viscosity and pressure drop both increase simultaneously.

Awais et al. describe nanofluids, a state-of-the-art advancement in thermal engineering, as colloidal suspensions of nanoparticles equally dispersed in a base fluid [1]. Their ability to significantly enhance the thermal properties of the base fluid has piqued the curiosity of many in these nanocomposite fluids. This enhancement is due, in part, to nanoparticles' ability to alter the thermal and transport characteristics of fluids, as well as their tiny size and extraordinary surface-to-volume ratio, as reported by Awua et al. [2]. A nanofluid combination including nanoparticles has a far greater effective thermal conductivity than the individual components used alone, according to Aybar et al. [3]. This includes water, ethylene glycol, refrigerant, paraffin oil, and phase change materials (PCMs). Nanofluids have several applications in heat and mass transfer systems due to their enhanced thermal conductivity. Complex thermal systems in the aerospace and biomedical areas, as well as HVAC systems, cooling modules for electronics, and engine cooling systems are all examples of such systems, as discussed by by Babar et al. [5], Bacha et al. [6], Bhattad et al. [8].

Following post-treatment, phase transfer, one-step, or two-step techniques are often used ways for producing nanofluids. The one-step method, which is characterized by its stability and comprises producing and dispersing particles concurrently, has been extensively used in several research projects. The one-step process may be divided into physical vapor condensation techniques and chemical ones, as described by Aziz and Kraus [4]. While the two-step process is less stable but more versatile, enabling the dispersion of various nanoparticle types in various base fluids, the one-step method is recognized for producing uniformly distributed and stable nanofluids, as reviewed by Bacha et al. [6]. Babar et al. provide a visual representation of the approaches that were used for preparation [5]. A key current challenge in nanofluid preparation, based on the author's experience, is the difficulty of obtaining scalable mass production. Although it works, the one-step procedure can't be scaled up because of how complicated it is; each batch can only produce around 100 liters. Inadequate capacity of existing equipment for large-scale production might hinder the two-step process of effective ultrasonication. Nanofluid synthesis has made extensive use of the two-step procedure owing to its dependability and ease of usage, although there is a known issue with it: stability, as reported by Bacha et al. [6].

In their computational investigation of natural convection flows in an eccentric annulus filled with Cu-water nanofluid and a constant heat flux wall, Hu et al. found that the average Nusselt number is strongly influenced by the eccentricity and radial ratio [16]. The direction of the field significantly affects heat transfer and fluid flow, as shown by Mahmoudi et al. in their study on the effect of a magnetic field on nanofluid flow in a cavity [17]. In a square cavity exposed to MHD natural convection flow, Tayebi and Chamkha discovered that an angled magnetic field impacted the flow and heat transfer rates [10]. The impacts of the volume concentration of nanoparticles were examined in detail by Alsabery et al. [11]. One approach to finding the effective conductivity of nanofluids was to simulate their behavior when Brownian motion was present. Results from the simulations for Cu-ethylene glycol and Al_2O_3-ethylene glycol were within 3% and 5% of the experimental data, respectively. Xuan and Yao developed a Lattice Boltzmann model to study the distribution and flow pattern of nanoparticles [29]. Researchers discovered that improving energy transfer in nanofluids may be as simple as increasing the fluid's temperature and primary flow. This would increase the dispersion of nanoparticles. The fluids in question may include acids or bases, which would render the electrostatic repulsion approach ineffective, as pointed out by Choi et al. [30]. The finding remained, nonetheless, that shear rate was uncorrelated with viscosity. The intrinsic convective heat transfer coefficient was evidently reduced concentration-dependently by the mediation of nanofluids. Khanafer et al. used a computational model to predict how nanofluids behave during convective heat transfer in a two-dimensional horizontal container; nevertheless, the findings do not match their predictions [31]. This experiment confirms the findings of Putra et al. that the natural convective heat transfer coefficient for aqueous CuO and Al_2O_3 nanofluids within a horizontal cylinder is reduced [32]. Researchers blamed particle/fluid slippage and nanoparticle sedimentation for the drop. Scientists speculate that changes in concentration, differences in dispersion properties, or interactions between particles and surfaces might be causing convection, which in turn reduces heat transfer.

Ahmed et al. carried out a numerical study of unstable free convection of MHD nanofluid flow using a first-order chemical reaction on a semi-infinite vertically rotating plate submerged in a porous medium [33]. In the mathematical model of the fluid's flow, it is assumed that the viscosity of the fluid changes with temperature. Magyari and Rees presented a mathematical model of the axisymmetric discharge of an electrically conducting fluid over a surface during its radial expansion, taking into account the effect of the Soret number [18]. Analysis of the flow dynamics is carried out when the surface experiences exponential radial expansion. Toki investigated the effect of the Dufour number on flow patterns and heat transmission by considering an infinite vertical plate in a porous medium subjected to a heat source [19]. Bala Siddhulu Malga and colleagues conducted a numerical study on the MHD viscous nanofluid stretched by convective forces, providing improved insight into mass and heat transport through chemical processes and steady-state dissipations of viscous fluids [15].

3. Methodology

The experimental or numerical setup, nanofluid preparation, system configuration, parameters measured, and analysis approach should all be thoroughly covered in the methodology for investigating heat transfer enhancement in a vertically rotating system with the addition of various nanofluids (CuO, Al_2O_3, ZnO).

3.1 Mathematical Tasks:

At a ratio of 60, the shape of the cylinders appears. Assigning a constant velocity at the entry yields the Reynolds number. It is thought that the flow is completely developed at the exit if all derivatives are zero. A constant supply of heat is sent into the inner shaft, which is in a non-slip condition, while the outer tube and walls of the inner shaft spin. Due of the chaotic nature of the setting, gravity was ignored in this simulation.

The cylinders' geometry when L/DH = 60. The Reynolds number is determined by assigning a uniform velocity at the entrance. Assuming all derivatives are zero, it is believed that the flow is fully developed at the outlet. The inner shaft is heated by a steady stream of heat while the outer tube and walls of the inner shaft rotate in a non-slip state. Gravity was disregarded in this simulation because it took place in a chaotic environment.

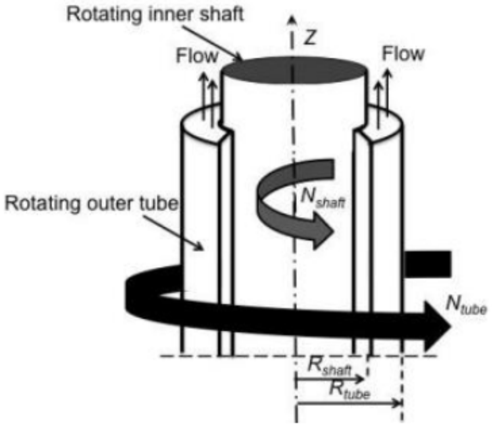

Figure 1: Geometry of the study

Characteristics of Flow

Examine the thermal and physical properties of nanofluids with 2% volume concentrations of copper (Cu), aluminum oxide (Al_2O_3), and zinc oxide (ZnO) nanoparticles in relation to water, the base fluid. Commonly used in heat transfer applications, they are typically evaluated at ambient temperature (∼25°C).

Table 1: Nanofluids with 2% Volume Concentration

Property	**CuO**	**Al_2O_3**	**ZnO**
Density (kg/m^3)	∼1025–1035	∼1015–1025	1010–1020
Specific Heat Capacity (J/kg·K)	3850	∼4000	4100
Thermal Conductivity (W/m·K)	0.75–0.85	∼0.68–0.75	0.65–0.73
Dynamic Viscosity (Pa·s)	1.1×10^{-3}	1.05×10^{-3}	1.02×10^{-3}

3.2 Principles of Control:

We have assumed stable and incompressible flow conditions to simplify the numerical investigation. The two-equation "k–ε" turbulence model was used for turbulence modeling since the mass flow rates in this investigation suggest turbulent flow conditions. Due to its adaptability and robustness, the "k–ε" turbulence model has been used to handle several kinds of turbulent flows. The model is stable and there are less convergence concerns. Below are the equations that control the flow.

Modeling of Turbulence: If the Reynolds number predicts that the flow regime will be turbulent, employ an appropriate turbulence model (such k–ε, k–ω, or Reynolds Stress Model) to account for the turbulent variations in velocity and temperature. When selecting a turbulence model, it is critical to think about the computing resources that are at your disposal as well as the flow characteristics.

Conservation of Mass:

$$\nabla \cdot (\rho \vec{V}) = 0 \tag{3.1}$$

Momentum Equation:

X-momentum:

$$\nabla \cdot (\rho u \vec{V}) = \frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \tag{3.2}$$

Y-momentum:

$$\nabla \cdot (\rho v \vec{V}) = \frac{\partial p}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + \rho g \tag{3.3}$$

Z-momentum:

$$\nabla \cdot (\rho w \vec{V}) = \frac{\partial p}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} \tag{3.4}$$

Energy equation:

$$\nabla \cdot (\rho e \vec{V}) = -p \nabla \cdot \vec{V} + \nabla \cdot (k \nabla T) + q + \Phi \tag{3.5}$$

4. The Findings and Analysis

To get a more in-depth physical insight, we tested three different kinds of water-based nanofluids with known values for the equation's parameters to determine the non-dimensional velocities, temperatures, species concentrations, skin-friction, and Nusselt numbers.

Using the following parameters: $S = 1.0$, $\varepsilon = 0.02$, $R = 0.02$, $n = 10.0$, $t = 0.1$, $M = 0.5$, $F = 1.0$, $Q = 10.0$, $\phi = 0.15$, and $K = 0.05$.

Data from Table 2 showing values of ϕ and skin friction (M) under different scenarios.

Different values of the parameter $F = 0.3, 0.5, 0.7$, and 0.9 are shown in the figure, which compares two nanofluids: CuO-Water (solid lines) and ZnO-Water (dashed lines) under the identical physical circumstances, showing the velocity profile (χ vs z). The fact that CuO-Water (solid lines) and ZnO-Water (dashed lines) exhibit different velocities for the same F indicates that CuO-Water has better momentum transfer properties in this particular configuration.

Table 2: Skin Friction values under different scenarios

M	ϕ	**Skin Friction**		
		$\varepsilon = 0.01;\ n = 10.0;\ t = \pi/2;\ F = 1.0;$ $Pr = 6.2;\ Q = 10.0;\ K = 0.05;\ R = 0.02;\ S = 1.0$		
		CuO	**ZnO**	**Al_2O_3**
0.0	0.1	0.602330	0.466185	0.382986
	0.2	0.395930	0.315610	0.270562
	0.3	0.175042	0.158834	0.135920
	0.4	0.028650	0.025187	0.024582
0.5	0.1	0.602547	0.473703	0.383277
	0.2	0.396003	0.319424	0.270715
	0.3	0.175100	0.162048	0.135992
	0.4	0.028734	0.025256	0.024630
1.0	0.1	0.602758	0.481499	0.383573
	0.2	0.396079	0.323406	0.270871
	0.3	0.175162	0.166150	0.136063
	0.4	0.028820	0.025313	0.024682

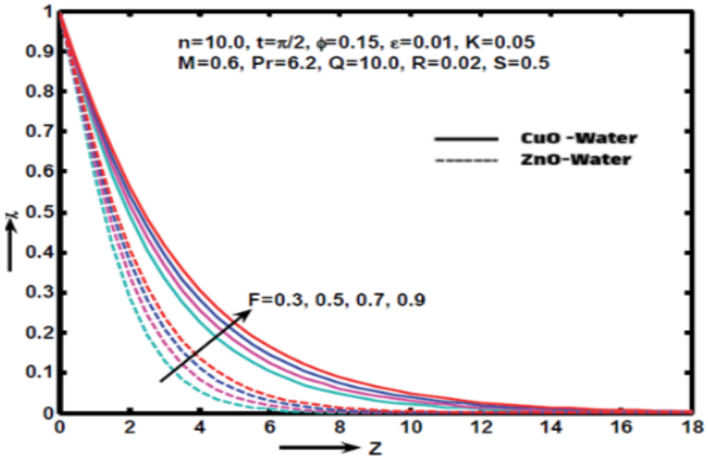

Figure 2: **Profile of velocity for different levels of F**

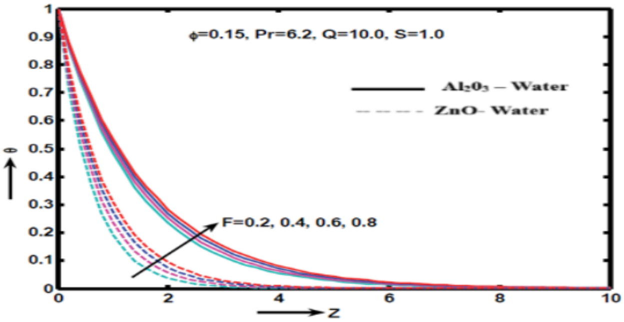

Figure 3: **Temperature profile for various values of F**

The thickening of the thermal boundary layer with increasing temperature is the reason it manifests in Figure 3. Because of this, we may deduce that less radiation is needed to accelerate cooling. Compared to ZnO-Water and Al_2O_3-Water combinations, nanofluid velocities and temperatures in ZnO-Water are lower. The physical world provides a more consistent basis for our understanding of this occurrence.

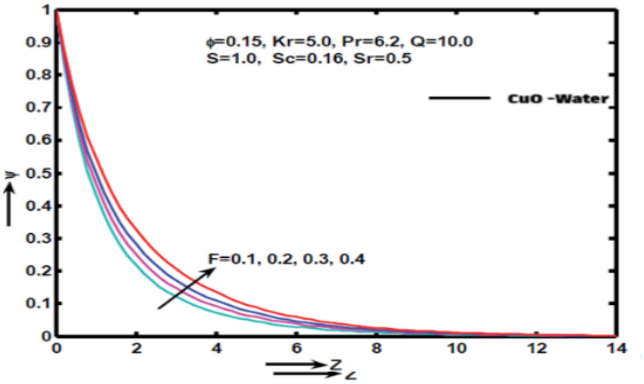

Figure 4: **Concentration profile for different values of F**

Under varying values of F, the CuO-Water nanofluid's concentration profile as a function of z for $F = 0.1, 0.2, 0.3$, and 0.4, under the given physical circumstances, is shown in Figure 4. It seems that the mass transfer parameter F improves things; increasing F causes the diffusion effects to be larger or maybe the mass flow from the border to be higher.

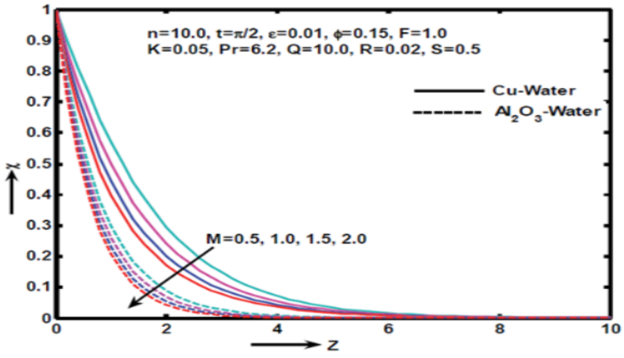

Figure 5: **Velocity profile for different kinds of M**

With $M = 0.5, 1.0, 1.5$, and 2.0, Figure 5 shows the velocity profile χ vs z for Cu-Water (solid lines) and Al_2O_3-Water (dashed lines) nanofluids. The lower resistance to magnetic suppression in Cu-Water nanofluid results in higher velocity profiles than in Al_2O_3-Water, most likely because of differences in thermal conductivity and viscosity.

5. Conclusions

Results demonstrate that all three nanofluids (CuO, Al_2O_3, and ZnO) considerably enhance heat transfer in comparison to the base fluid. The increased effective thermal conductivity, improved Brownian motion, and energy transfer channels boosted by dispersed nanoparticles are the main reasons for this improvement. The heat-transfer performance of CuO was consistently higher than that of the other nanofluids examined across all operating conditions. The improved intrinsic thermal conductivity of the spinning system substantially improves its conductive and convective heat-transfer processes.

While the findings demonstrate that increasing the volume % of nanoparticles improves heat- transfer rates, doing so also increases the viscosity, which in turn raises the resistance to flow and the power required to pump the solution. The optimal thermal enhancement and viscosity increase balance was achieved by CuO nanofluid at low concentrations (about 0.5–1%). Hence, in applications where heat transfer efficiency is critical, such cooling systems for heat exchangers in motion or machinery with a rotating base, CuO nanofluid is the ideal working fluid to use.

References

1. M. Awais, N. Ullah, J. Ahmad, F. Sikandar, M. M. Ehsan, S. Salehin, and A. A. Bhuiyan, "Heat transfer and pressure drop performance of nanofluid: A state-of-the-art review," *Int. J. Thermofluids*, vol. 9, p. 100065, 2021.

2. J. T. Awua, J. S. Ibrahim, S. A. Adio, M. Mehrabi, M. Sharifpur, and J. P. Meyer, "Experimental investigations into viscosity, pH and electrical conductivity of nanofluid prepared from palm kernel fibre and a mixture of water and ethylene glycol," *Bull. Mater. Sci.*, vol. 41, no. 6, pp. 1–7, 2018.

3. H. Aybar, M. Sharifpur, M. R. Azizian, M. Mehrabi, and J. P. Meyer, "A review of thermal conductivity models for nanofluids," *Heat Transfer Eng.*, vol. 36, no. 13, pp. 1085–1110, 2015.

4. A. Aziz and A. D. Kraus, "Transient heat transfer in extended surfaces," *Appl. Mech. Rev.*, vol. 48, no. 7, pp. 317–350, 1995.

5. H. Babar, H. Wu, W. Zhang, T. R. Shah, D. McCluskey, and C. Zhou, "The promise of nanofluids: A bibliometric journey through advanced heat transfer fluids in heat exchanger tubes," *Adv. Colloid Interface Sci.*, vol. 325, p. 103112, 2024.

6. H. B. Bacha, N. Ullah, A. Hamid, and N. A. Shah, "A comprehensive review on nanofluids: Synthesis, cutting-edge applications, and future prospects," *Int. J. Thermofluids*, vol. 22, p. 100595, 2024.

7. G. Bhakta, R. K. Sharma, N. Gupta, S. Cool, V. Nurcombe, and A. Maitra, "Multifunctional silica nanoparticles with potentials of imaging and gene delivery," *Nanomedicine*, vol. 7, no. 4, pp. 472–479, 2011.

8. A. Bhattad, J. Sarkar, and P. Ghosh, "Improving the performance of refrigeration systems by using nanofluids: A comprehensive review," *Renew. Sustain. Energy Rev.*, vol. 82, pp. 3656–3669, 2018.

9. H. R. Ashorynejad and A. Zarghami, “Magnetohydrodynamics flow and heat transfer of Cu-water nanofluid through a partially porous wavy channel,” *Int. J. Heat Mass Transf.*, vol. 119, pp. 247–258, 2018.

10. T. Tayebi and A. J. Chamkha, “Entropy generation analysis due to MHD natural convection flow in a cavity occupied with hybrid nanofluid and equipped with a conducting hollow cylinder,” *J. Therm. Anal. Calorim.*, vol. 139, pp. 2165–2179, 2020.

11. A. I. Alsabery, M. A. Ismael, A. J. Chamkha, and I. Hashim, “Mixed convection of Al_2O_3-water nanofluid in a double lid-driven square cavity with a solid inner insert using Buongiorno’s two-phase model,” *Int. J. Heat Mass Transf.*, vol. 119, pp. 939–961, 2018.

12. A. S. Dogonchi, T. Tayebi, A. J. Chamkha, and D. D. Ganji, “Natural convection analysis in a square enclosure with a wavy circular heater under magnetic field and nanoparticles,” *J. Therm. Anal. Calorim.*, vol. 139, pp. 661–671, 2020.

13. A. I. Alsabery, M. A. Ismael, A. J. Chamkha, I. Hashim, and H. Abulkhair, “Unsteady flow and entropy analysis of nanofluids inside cubic porous container holding inserted body and wavy bottom wall,” *Int. J. Mech. Sci.*, vol. 193, p. 106161, 2021.

14. A. Radwan and M. Ahmed, “Thermal management of concentrator photovoltaic systems using microchannel heat sink with nanofluids,” *Sol. Energy*, vol. 171, pp. 229–246, 2018.

15. B. Shankar Goud, P. Pramod Kumar, and B. Siddulu Malga, “Effect of heat source on an unsteady MHD free convection flow of Casson fluid past a vertical oscillating plate in porous medium using finite element analysis,” *Partial Differential Equations in Applied Mathematics*, vol. 2, p. 100015, 2020.

16. Y. Hu, D. Li, S. Shu, and X. Niu, “Natural convection in a nanofluid-filled eccentric annulus with constant heat flux wall,” *Int. Commun. Heat Mass Transf.*, vol. 86, pp. 262–273, 2017.

17. A. Mahmoudi, I. Mejri, M. A. Abbassi, and A. Omri, “Lattice Boltzmann simulation of MHD natural convection in a nanofluid-filled cavity,” *Powder Technol.*, vol. 256, pp. 257–271, 2014.

18. E. Magyari and D. A. S. Rees, “Effect of viscous dissipation on the Darcy free convection boundary layer flow over a vertical plate with exponential temperature distribution in a porous medium,” *Fluid Dyn. Res.*, vol. 38, pp. 405–429, 2006.

19. C. J. Toki, “Free convection and mass transfer flow near a moving vertical porous plate,” *J. Appl. Math.*, vol. 75, pp. 1–8, 2008.

20. C. J. Toki, “Unsteady free convection flow on a vertical oscillating porous plate with constant heating,” *J. Appl. Math.*, vol. 76, pp. 1–4, 2009.

21. O. D. Makinde and P. O. Olanrewaju, “Buoyancy effects on thermal boundary layer over a vertical plate with a convection surface boundary condition,” *J. Fluid Eng.*, vol. 132, pp. 1–4, 2010.

22. W. A. Khan, O. D. Makinde, and Z. H. Khan, “MHD boundary layer flow of a nanofluid containing gyrotactic microorganisms past with Navier slip,” *Int. J. Heat Mass Transf.*, vol. 74, pp. 285–291, 2014.

23. M. Mustafa *et al.*, “Nonlinear radiation heat transfer effects in the natural convection boundary layer flow of nanofluid past a vertical plate,” *PLoS ONE*, vol. 9, no. 9, pp. 1–10, 2014.

24. D. Srinivasacharya and O. Surender, “Mixed convection boundary layer flow of a nanofluid past a vertical plate in porous medium,” *Appl. Nanosci.*, vol. 5, no. 1, pp. 1–10, 2014.

25. M. Narahari, “Unsteady free convection flow past semi-infinite vertical plate with constant heat flux in water based nanofluids,” *IOP Conf. Ser.: Mater. Sci. Eng.*, vol. 342, pp. 1–2, 2018.

26. A. Rauf and Y. Mahsud, “Unsteady two dimensional flows of a nanofluid over a vertical plate with time exponential temperature,” *Arab J. Basic Appl. Sci.*, vol. 25, pp. 150–157, 2018.

27. H. E. Patel, T. Sundararajan, and T. Pradeep, “A micro-convection model for thermal conductivity of nanofluids,” *Pramana – J. Phys.*, vol. 65, pp. 863–869, 2005.

28. P. Bhattacharya, S. K. Saha, A. Yadav, P. E. Phelan, and R. S. Prasher, “Brownian dynamics simulation to determine the effective thermal conductivity of nanofluids,” *J. Appl. Phys.*, vol. 95, no. 11, pp. 6492–6494, 2004.

29. Y. Xuan and Z. Yao, “Lattice Boltzmann model for nanofluids,” *Heat Mass Transf.*, vol. 41, no. 3, pp. 199–205, 2005.

30. S. U. S. Choi, Z. G. Zhang, and P. Keblinski, “Nanofluids,” in *Encyclopedia of Nanoscience and Nanotechnology*, vol. 6, pp. 757–773, 2004.

31. K. Khanafer, K. Vafai, and M. Lightstone, “Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids,” *Int. J. Heat Mass Transf.*, vol. 46, pp. 3639–3653, 2003.

32. N. Putra, W. Roetzel, and S. K. Das, “Natural convection of nanofluids,” *Heat Mass Transf.*, vol. 39, no. 8–9, pp. 775–784, 2003.

33. W. Ahmed, E. Onyango, D. Theuri, and F. Awad, “MHD nanofluid flow past a vertical plate embedded in a rotating porous medium with chemical reaction,” *J. Appl. Math. Phys.*, vol. 12, pp. 4242–4273, 2024.

[1] *Research Scholar, Gitam School of Science and Assistant Professor,*
Department of Mathematics, CMR Technical Campus, Medchal, Hyderabad 501401, India.
[2] *Assistant Professor, Department of Mathematics, Gitam School of Science,*
GITAM University, Hyderabad, India.
[3] *Assistant Professor, Department of Mathematics, Chaitanya Bharathi Institute of Technology,*
Gandipet, Hyderabad, Telangana, India.
[4] *Assistant Professor, Department of Mathematics, Chaitanya Bharathi Institute of Technology,*
Gandipet, Hyderabad, Telangana, India.

E-mail address: `mgunnall@gitam.in`
E-mail address: `drbsmalga@gmail.com`
E-mail address: `brsreedhar_maths@cbit.ac.in`
E-mail address: `gnarsimlu_maths@cbit.ac.in`