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Completeness of Fuzzy Soft m-Normed Linear Space

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ABSTRACT: The aim of this research is originate the hypothesis of fuzzy soft subspaces with its properties and fuzzy soft m-normed linear space which is accordance with the theory of fuzzy n-normed linear space and soft n-normed linear space. We presented formally apprehension of fuzzy soft m-normed linear space and provided some results with example of convergence sequence and Cauchy sequence in the fuzzy soft m-normed linear space. **Methods:** In this research paper we instituted the definition of fuzzy soft 2-normed linear space, fuzzy soft m-normed linear space and its properties. The fuzzy soft m-normed linear space can be analyzed by using the fuzzy m-normed linear space and soft m-normed linear space. **Findings:** In this research paper we construct the m-norm function that satisfies the properties of fuzzy soft m-normed linear space, and provided the suitable example with proof which is a convergence sequence and cauchy sequence in fuzzy soft m-normed function if and only it is convergence sequence and cauchy sequence fuzzy soft m-normed linear space. Also proved a theorem for completeness of a sequence in fuzzy soft m-normed linear space. **Novelty:** we have well-defined definitions and theorems with proofs on fuzzy norm, fuzzy n-normed linear space, soft norms and soft n-normed linear space. We established the abstraction of fuzzy soft m-normed linear space and given a results to theorems on properties of fuzzy soft m-normed linear space also prepared a necessary axioms to completeness of a sequence in the fuzzy soft m-normed linear space.

Keywords: Fuzzy softest, fuzzy soft vector space, fuzzy soft m-normed linear space.

Contents

1. Introduction

The fuzzy soft set theory was presented formally first time by Maji, Biswas and Roy in 2003 [1]-[4] accordance with the fuzzy set theory and soft set theory also they instituted the fuzzy soft set theory and it is a combination of concepts of A L Zadhe [5]fuzzy set theory which is dealing with which the problems contains the imprecise and vagueness set theory, soft set theory. According to D Moldtsov [6]-[7]soft set theory is new mathematical conceptualization to deal the effectiveness in various situations with incomplete information, simplification of complex data sets and ability to handle uncertainty and imprecations. Also he explained the soft set theory is free from the parameterization insufficiency condition of fuzzy set theory, rough set theory, probability theory and game theory. [8]-[10]. Fuzzy set theory is applied to many fields which are medical diagnosis, data mining, machine learning and data analysis. Conceptions of the theory of 2-norm and n-norm were introduced by Gahler [11] and initially concepts of fuzzy norm on a linear space is formulated by A K Katsaras [12]. The definition of fuzzy normed linear space was introduced by Sadaati [13]. Subsequently Bag and Samantha [14] originated the view of fuzzy norm on a linear space accordance with fuzzy norm [15], Samantha developed the definition of convergence sequence and Cauchy sequence in fuzzy normed linear space [16]-[17]. AL.Narayanan and S.Vijayabalaji were introduced the Fuzzy n-normed linear space [18]-[20]

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M.Mursaleem and S.A.M.Razvi were originated the abstraction of soft norm, soft n-norm, soft n-normed linear space which is generalization of classical n-normed linear space with a combination of soft sets , n-norm and linear space. A soft norm is a assignment of magnitude or size to a vector in a more nuanced and flexible way allowing for uncertainty and imprecision in the norm values. Later on Thangaraj defined the fuzzy soft normed linear space as a fuzzy set defined on the soft normed linear space [21]- [22] also provided theorems with its proofs of cauchy sequence, completeness and bounded sequences in fuzzy soft norm.

Motivated by the previously mentioned theory, we put forwarded the hypothesis of fuzzy soft m-normed linear spaces and study of its characteristic properties. We suggested theorems with proofs of convergence sequence and Cauchy sequence in fuzzy soft m-normed linear spaces.

2. Figures and Tables

Let the universal set $U = \{I = India, F = Finland, D = Denmork, M = Moldova, R = Russia, L = Lebanon, A = Afghanistan\}$be the set countries and varies world indexes as a parameter set $M = \{I_1 = World\ Health\ Index(WHI), I_2 = Press\ Freedom\ Index(PFI), I_3 = Global\ Peace\ Index(GPI), I_4 = Human\ Development\ Index(HDI), I_5 = Gender\ Inequality\ Index(GII)\}$ then

Fuzzy Set

	I_1	I_2	I_3	I_4	I_5
I	0.32	0.43	0.52	0.52	0.65
F	0.99	0.90	0.87	0.87	0.98
D	0.95	0.95	0.90	0.90	0.97
M	0.92	0.7	0.50	0.55	0.69
R	0.50	0.005	0.10	0.60	0.70
L	0.46	0.20	0.30	0.40	0.30
A	0.002	0.20	0.009	0.05	0.09

Soft Set

	I_1	I_2	I_3	I_4	I_5
I	0	0	1	0	1
F	1	1	0	1	1
D	1	1	1	1	1
M	1	1	0	0	1
R	0	0	0	1	1
L	0	0	0	0	0
A	0	0	0	0	0

Fuzzy Soft Set

	I_1	I_2	I_3	I_4	I_5
I	0.39	0.36	0.58	0.36	0.72
F	1	0.97	0.95	0.96	0.99
D	0.98	0.96	0.97	0.96	1
M	0.86	0.68	0.45	0.58	0.73
R	0.50	0	0	0.69	0.76
L	0.46	0.20	0.22	0.31	0.20
A	0	0	0.007	0	0.003

3. Mathematics

Definition 3.1 *Fuzzy sets theory is an extension of classical set theory and elements have varying degree of membership a logic based on two truth values. If U is the universe of discourse, a set F is said to be fuzzy set in F if there exists a function $\mu : F \to [0,1]$ and it is denoted by a set of ordered pairs as $F = (u, \mu F(u))/u \in U$.*

Definition 3.2 *Suppose U is a discrete and finite universe of discourse the fuzzy set F is written as $F = \mu_F(u_1)/u_1 + \mu_F(u_1)/u_1 + \mu_F(u_1)/u_1 + \ldots = \Sigma U \mu_F(u_1) = \{(u_l, \mu_F(u_1))/u_l \in U\}$*

Definition 3.3 *Suppose U is a continuous and finite universe of discourse the fuzzy set F is written as $F = \displaystyle\int_U \mu_F(u_l)$,here the summation and integration signs indicate the collection of all elements u in the universe of discourse U along with their associated membership values $\mu_F(u_l)$.*

Definition 3.4 *If U is the universe of discourse, a set F_1 and F_2 fuzzy sets with characteristic functions $\mu_{F1}(u)$ and $\mu_{F2}(u)$ respectively the fuzzy set operations are defined as given bellow*

1. ***Union:*** *the union of two fuzzy sets F_1 and F_2 is defined as*

$$\mu_{(F_1 \cup F_2)}(u) = \mu_{F_1}(u) \vee \mu_{F_2}(u) = \max\{\mu_{F_1}(u), \mu_{F_2}(u)\}$$

$$\textit{for all } u \in U$$

2. ***Intersection:*** *the intersection of two fuzzy sets F_1 and F_2 is defined as for all $u \in U$,*

$$\mu_{(F_1 \cap F_2)}(u) = \mu_{F_1}(u) \wedge \mu_{F_2}(u) = \min\{\mu_{F_1}(u), \mu_{F_2}(u)\}$$

3. ***Complement:*** *The compliment of F is denoted by F^C and it is defined as*

$$\mu_{F^C}(u) = 1 - \mu_F(u)$$

4. ***Algebraic Sum:*** *the sum of the two fuzzy sets $\mu_{(F_1)}, \mu_{(F_2)}$ is $\mu_{(F_1)} + \mu_{(F_2)}$it is defined as*

$$\mu_{(F_1+F_2)}(u) = \mu_{F_1}(u) + \mu_{F_2}(u) - \mu_{F_1}(u) \cdot \mu_{F_2}(u)$$

5. ***Algebraic Product:*** *The product of the two fuzzy sets is $\mu_{(F_1)} \cdot \mu_{(F_2)}$ it is defined as*

$$\mu_{(F_1 \cdot F_2)}(u) = \mu_{F_1}(u) \cdot \mu_{F_2}(u)$$

6. ***Bounded Sum:*** *the bounded sum of the two fuzzy sets $\mu_{(F_1)}, \mu_{(F_2)}$ is $\mu_{(F_1)} \oplus \mu_{(F_2)}$ it is defined as*

$$\mu_{(F_1 \oplus F_2)}(u) = \min\{1, \mu_{F_1}(u) + \mu_{F_2}(u)\}$$

7. ***Bounded Difference:*** *the bounded difference $\ominus$ of the two fuzzy sets $\mu_{(F_1)}, \mu_{(F_2)}$ is $\mu_{(F_1)} \ominus \mu_{(F_2)}$ it is defined as*

$$\mu_{(F_1 \ominus F_2)}(u) = \max\{0, \mu_{F_1}(u) - \mu_{F_2}(u)\}$$

Definition 3.5 *Let U be an universe and M be a set of parameters. Let $P(U)$ denote the power set of U and S be a non-empty subset of U. A pair (X, S) is called a soft set over U, where X is a mapping given by $X : S \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\delta \in S, X(\delta)$ may be considered as the set of δ – approximate elements of the $P(U)$ soft set (X, S).*

Example 3.1 *A soft set (X, S) describes the attractiveness of the houses which Mr. Alice is going to buy. U = the set of houses under consideration. M= the set of parameters. Each parameter is a word or a sentence. S=expensive; beautiful; wooden; cheap: in the green surroundings; modern; in good repair; in bad repair. In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. It is worth noting that the sets $X(\delta)$ may be empty for some $\delta \in S$.*

Definition 3.6 *Let X be a non-empty set and M be a non-empty parameter set. Then a function $\delta : M \to X$ is said to be a soft element of X. A soft element δ of X is said to belongs to a soft set S of X, which is denoted by $\delta \in S(m)$, if $\delta(m) \in S(m), \forall m \in M$. Thus for a soft set S of X with respect to the index set M, we have $S\ (m) = \delta(m), \delta \in M, m \in M$ it is to be noted that every singleton soft set (a soft set (X, M) for which $X(M)$ is a singleton set, $\forall m \in M$ can be identified with a soft element by simply identifying the singleton set with the element that it contains $\forall e \in M$.*

Definition 3.7 *If for every $m \in M, X(m) = \Phi$, then a soft set (X, M) over U is a null soft set, represented by Φ.*

Definition 3.8 *If $X(M) = U$ for every $m \in M$, then a soft set (X, M) over U is considered an absolute soft set, represented by U.*

Definition 3.9 *For two soft sets (X_1, S_1) and (X_2, S_2) over a common universe U, we say that (X_1, S_1) is a soft subset of (X_2, S_2) if $S_1 \subseteq S_2$ and for all $\delta \in, X_2(\delta) \subseteq X_2(\delta)$. We write $(X_1, S_1) \subset \delta(X_2, S_2)$.*

Definition 3.10 *Let (X_2, S_2) is said to be a soft superset of (X_1, S_1) if (X_2, S_2) is a soft subset of (X_1, S_1) We denote it by $(X_1, S_1) \supset (X_2, S_2)$*

Definition 3.11 *If (X_1, S_1) is a soft subset of (X_2, S_2) and (X_2, S_2) is a soft subset of (X_1, S_1), then two soft sets (X_1, S_1) and (X_2, S_2) over a common universe U are equivalent.*

Example 3.2 *Assuming S to be a set of parameters, let $B(\mathbb{R})$ be the collection of all non-empty bounded subsets of $\mathbb{R}$ and $\mathbb{R}$ be the set of real numbers. A mapping $X : S \to B(\mathbb{R})$ is hence referred to as a soft real set. (X, S) is used to represent it. (X, S) will be referred to as a soft real number once it has been determined that it is a singleton soft set by matching it with the appropriate soft element.*

Definition 3.12 *A soft set (X, S) has a complement represented by (X^C, S), Where $X^C : S \to P(U)$ is a mapping such that $X^C(s) = U - X(s)$, for all $s \in S$.*

Definition 3.13 *The union of two soft sets (X_1, S_1) and (X_2, S_2) with the common universe U is the soft set (X, S), where $S = S_1 \cup S_2$ For every $s \in S$,* $X(s) = \begin{cases} X_1(s), \ if \ s \in S_1 - S_2 \\ X_2(s), \ if \ s \in S_2 - S_1 \\ X_1(s) \cup X_2(s), \ if \ s \in S_1 \cap S_2 \end{cases}$
We write it as $(X_1, S_1) \cup (X_2, S_2) = (X, S)$.

Definition 3.14 *The union of three soft sets $(X_1, S_1), (X_2, S_2)$ and (X_3, S_3) over the common universe U is the soft set (X, S), where $S = S_1 \cup S_2 \cup S_3$ and for all $s \in S$,* $X(s) = \begin{cases} X_1(s), \ if \ s \in S_1 - (S_2 \cup S_3) \\ X_2(s), \ if \ s \in S_2 - (S_1 \cup S_3) \\ X_3(s), \ if \ s \in S_1 - (S_1 \cup S_2) \\ X_1(s) \cup X_2(s), \ if \ s \in S_1 \cap S_2 \\ X_2(s) \cup X_3(s), \ if \ s \in S_2 \cap S_3 \\ X_1(s) \cup X_3(s), \ if \ s \in S_1 \cap S_3 \\ X_1(s) \cup X_2(s) \cup X_3(S), \\ \ if \ s \in S_1 \cap S_2 \cap S_3 \end{cases}$

We express it as $(X_1, S_1) \cup (X_2, S_2) \cup (X_3, S_3) = (X, S)$.

Definition 3.15 *Let U is the common universe of two soft sets (X_1, S_1) and (X_2, S_2) and intersection of these two set is the soft set $(X, S), if (i) S = S_1 \cap S_2$, (ii) $X(s) = X_1(s) \cap X_2(s)$, for all $s \in S_1 \cap S_2$.This relation is denoted by $(X_1, S_1) \cap (X_2, S_2) = (X, S)$*

Definition 3.16 *Let U is the common universe of two soft sets (X_1, S_1) and (X_2, S_2) and intersection of these two set is the soft set $(X, S), if (i) S = S_1 \cap S_2$, (ii) $X(s) = X_1(s) \cap X_2(s)$, for all $s \in S_1 \cap S_2$.This relation is denoted by $(X_1, S_1) \cap (X_2, S_2) = (X, S)$*

Remark 3.1 *The fact that $(X_1, S_1) \cap (X_2, S_2) = (X, S)$ does not exist in many circumstances was highlighted as evidence that the concept of intersection of soft sets is not well-defined. For this, the example 3.3 is given.*

Example 3.3 *Let (S_1, M_1) and (S_2, M_2) be two soft sets ,and let (S, M) be the soft set which is intersection of two sets $(S_1, M_1), (S_2, M_2)$ where U is a set of cars; $U = \{C^1, C^2, C^3, C^4, C^5, C^6\}$ and $M1 =${Luxurious, Attractive color, Driving comfort }, and $M_2 =$ {Fuel efficiency, Driving comfort, More safety } two parameter sets. Noticing the ϵ-approximate elements may differ from person to person, we assume that S_1 (Luxurious) $= \{C^2, C^3, C^5\}, S_1$ (Driving Comfort) $= \{C^1, C^2, C^4, C^6\}, S_1$ (Attractive color) $= \{C^1, C^2, C^6\}, S_2$ (Fuel efficiency) $= \{C^1, C^3, C^5\} S_2$ (Driving comfort) $= \{C^2, C^3, C^5, C^6\}, S_2$ (more safety) $= \{C^2, C^4, C^6\}$, we have "Driving comfort" $M_1 \cap M_2$ then S_1 (Driving Comfort) $= \{C^1, C^2, C^4, C^6\}, S$ (Driving Comfort) and S_2 (Driving comfort) $= \{C^2, C^3, C^5, C^6\} = S_2$ (Driving comfort) which is a contradiction to the fact that $(S_1, M_1) \cap (S_2, M_2)$ does not exist in many cases makes it is impossible to check the validity of some of the assertions.*

Definition 3.17 *Assume that M is any nonempty set with an arbitrary binary relation of σ between a member of M and an element of G. As a result, σ is a subset of (M, G) and a set valued function $S_G : M \to P(G)$, which is defined as follows $S_G(m) = \{n \in G/n \propto m \Rightarrow (n, m) \in \sigma \text{ for all } m \in M\}$. Consequently, if the pair (S_G, M) is a soft set over G that is generated from the relation σ.*

Definition 3.18 *Suppose F_s is a soft set over U and let a function $f_s : M \to P(U)$ such that $f_s(s) = \phi$ if $s \notin S$ where f_s is called a fuzzy approximation of the fuzzy soft set F_s it is denoted by the pair $(s, f_s(s))$ that is $F_s = \{(s, f_s(s))\}/s \in f_s(s) \in P(U)$ where the value $f_s(s)$ is a set called s-element of the fuzzy soft set for all $s \in U$*

Definition 3.19 *Let $\mathbb{R}$ be a real vector space over with dimension m over a field F a real valued function $\|\cdots, \cdots, .\| :\to [0, 1]$and it satisfies the following properties, for any $w_1, w_2, w_3, \cdots w_{m-1}, w_m \in \mathbb{R}$*
$(R1)\|w_1, w_2, w_3, \cdots w_{m-1}, w_m\|$ if and only if $w_1, w_2, w_3, \cdots w_{m-1}, w_m$ are linearly independent over F.
$(R2)\|w_1, w_2, w_3, \cdots w_{m-1}, w_m\|$ is invariant under any permutation $w_1, w_2, w_3, \cdots w_{m-1}, w_m$.
$(R3)\|w_1, w_2, w_3, \cdots w_{m-1}, \sigma w_m\| = |\sigma|\|w_1, w_2, w_3, \cdots w_{m-1}, w_m\|$
$(R4)\|w_1, w_2, w_3, \cdots w_{m-1}, w_m + w'_m\| = \|w_1, w_2, w_3, \cdots w_{m-1}, w_m\| + \|w_1, w_2, w_3, \cdots w_{m-1}, w'_m\|$ is called the m-norm on $\mathbb{R}$ and the pair $(\mathbb{R}, \|.., .., .\|)$ is called the m-normed linear space.

Definition 3.20 *Let V is a fuzzy vector space over a field F a real valued norm function $\|\cdots, \cdots, .\| : V \times V \times V \times V \cdots \times V(mtimes) \to [0, 1]$the pair $(V, \|\cdots, \cdots, .\|)$ is called as the fuzzy m-normed linear space, and is called the fuzzy m-norm on V if it has the following properties for all $w_1, w_2, w_3, \cdots w_{m-1}, w_m \in V$*
$(V1)\|w_1, w_2, w_3, \cdots w_{m-1}, w_m\|$ if and only if $w_1, w_2, w_3, \cdots w_{m-1}, w_m$ are linearly independent over F.
$(V2)\|w_1, w_2, w_3, \cdots w_{m-1}, w_m\|$ is invariant under any permutation $w_1, w_2, w_3, \cdots w_{m-1}, w_m$.
$(V3)\|w_1, w_2, w_3, \cdots w_{m-1}, \sigma w_m\| = |\sigma|\|w_1, w_2, w_3, \cdots w_{m-1}, w_m\|$
$(V4)\|w_1, w_2, w_3, \cdots w_{m-1}, w_m + w'_m\| \leqslant \|w_1, w_2, w_3, \cdots w_{m-1}, w_m\| + \|w_1, w_2, w_3, \cdots w_{m-1}, w'_m\|$

Definition 3.21 *A in a Let $(V, \|, \cdots, .\|)$ be a fuzzy m-normed linear space[FMNLS] and the sequence $\{w_r\}_{r=1}^{\infty}$ said to convergence to $w \in V$ if $\lim\limits_{r\to\infty} \|w_1, w_2, w_3, \cdots w_{r-1}, w_r - w\| = 0$ A sequence$\{w_r\}_{r=1}^{\infty}$ in FMNLS $(V, \|, \cdots, .\|)$ is a Cauchy sequence if*
$\lim_{r,q\to\infty} \|w_1, w_2, w_3, \cdots, w_r - w_q\| = 0$

Remark 3.2 *Every convergent sequence is Cauchy and norm bounded in the fuzzy m-normed linear space $(V, \|\cdot, \cdots, \cdot\|)$*

Definition 3.22 *A fuzzy m-normed linear space is said to be complete If every Cauchy sequence in a $(V, \|\cdot, \cdots, \cdot\|)$ is convergent.*

Definition 3.23 *Let S_V is a linear space over a soft field F a real valued function $\|\cdot, \cdots, \cdot\| : s_V \times s_V \times s_V \cdots \times s_V(mtimes) \to \{0, 1\}$ and it has the following properties*
$(S_N^m - 1)\|s_{v1}, s_{v2}, s_{v3}, \cdots s_{vm-1}, s_{vm}\| = 0$ if and only if $s_{v1}, s_{v2}, s_{v3}, \cdots s_{vm-1}, s_{vm}$ are linearly independent over F.
$(S_N^m - 2)\|s_{v1}, s_{v2}, s_{v3}, \cdots s_{vm-1}, s_{vm}\| = 0$ is invariant under any permutation
$s_{v1}, s_{v2}, s_{v3}, \cdots s_{vm-1}, s_{vm}$.
$(S_N^m - 3)\|s_{v1}, s_{v2}, s_{v3}, \cdots s_{vm-1}, ks_{vm}\| = k\|s_{v1}, s_{v2}, s_{v3}, \cdots s_{vm-1}, s_{vm}\|$ where $k \in F$

$(S_N^m-4)\|s_{v1},s_{v2},s_{v3},\cdots s_{vm-1},s_{vm}+s'_{vm}\| \leqslant \|s_{v1},s_{v2},s_{v3},\cdots s_{vm-1},s_{vm}\|+$ $\|s_{v1},s_{v2},s_{v3},\cdots s_{vm-1},s'_{vm}\|$ the pair $(s_V,\|\cdot,\cdots,\cdot\|)$ is referred to as the soft m-normed linear space, and is called the soft m-norm on S_V.

Definition 3.24 *A sequence $\{s_{vk}\}_{k=1}^{\infty}$ in a SMNLS $(S_V,\|\cdot,\cdots,\cdot\|)$ is said to convergence to $s_v \in S_v$ if* $\lim\limits_{r\to\infty}\|s_{v1},s_{v2},s_{v3},\cdots s_{vk-1},s_{vk}-s_v\|=0$
A Cauchy sequence is defined as $\{s_{vk}\}_{k=1}^{\infty}$ in a SMNLS $(S_V,\|\cdot,\cdots,\cdot\|)$ if $\lim\limits_{r,p\to\infty}\|s_{v1},s_{v2},s_{v3},\cdots s_{vk-1},s_{vk}-s_{vp}\|=0$

Definition 3.25 *If every Cauchy sequence in a SMNLS $(S_V,\|\cdot,\cdots,\cdot\|)$ is convergent, then the space is a complete.*

4. Main Results and Discussions

Definition 4.1 *Let F_s is a fuzzy soft vector space over a soft field F a real valued norm function $\|,\cdots,\|: F_s\times F_s\times F_s\cdots F_s(mtimes)\to I$ where $I\subseteq[0,1]$ the pair $(F_s,\|\cdot,\cdots,\cdot\|)$ is called as the fuzzy soft m-normed linear space(FSMNLS), and is called the fuzzy soft m-norm on F_s if it has the following properties for all $w_1,w_2,w_3,\cdots w_{m-1},w_m\in F_s$*

1. *$\|w_1,w_2,w_3,\cdots w_{m-1},w_m\|$ if and only if $w_1,w_2,w_3,\cdots w_{m-1},w_m$ are linearly independent over F.*
2. *$\|w_1,w_2,w_3,\cdots w_{m-1},w_m\|$ is invariant under any permutation $w_1,w_2,w_3,\cdots w_{m-1},w_m$.*
3. *$\|w_1,w_2,w_3,\cdots w_{m-1},\rho w_m\|=|\rho|\|w_1,w_2,w_3,\cdots w_{m-1},w_m\|$*
4. *$\|w_1,w_2,w_3,\cdots w_{m-1},w_m+w'_m\|\leq\|w_1,w_2,w_3,\cdots w_{m-1},w_m\|+\|w_1,w_2,w_3,\cdots w_{m-1},w'_m\|$*

Example 4.1 *Let $V=\mathbb{R}^2$ be the 2-dimensional real vector space over $\mathbb{R}$ and define a fuzzy soft 2-norm $\|\cdot,\cdots,\cdot\|: V^2\to[0,1]$ by $\|w_1,w_2\|=\frac{|det(w_1,w_2)|}{1+|det(w_1,w_2)|}$, where $w_1=(x_1,y_1),w_2=(x_2,y_2),det(w_1,w_2)=x_1y_2-x_2y_1$ This is a fuzzy soft version of the area of the parallelogram spanned by normalized to map into $[0,1]$, if $\|w_1,w_2\|=0$ if and only if w_1,w_2 are linearly dependent (since the determinant is zero). The value lies in [0,1) approaching 1 as the determinant increases when the vectors are more strongly linearly independent. It is symmetric: that is $\|w_1,w_2\|=\|w_2,w_1\|$. thus $\|\cdot,\cdots,\cdot\|$ is a fuzzy soft 2-norm and $(\mathbb{R}^2,\|\cdot,\cdots,\cdot\|)$ is a fuzzy soft 2-normed linear space.*

Definition 4.2 *A in a Let $(F_s,\|\cdot,\cdots,\cdot\|)$ be a fuzzy soft m-normed linear space and the sequence $\{w_r\}_{r=1}^{\infty}$ said to convergence to $w\in F_s$, if* $\lim\limits_{r,q\to\infty}\|w_1,w_2,w_3,\cdots,w_r-w_q\|=0$

Definition 4.3 *In the fuzzy soft m-normed linear space $(F_s,\|\cdot,\cdots,\cdot\|)$ a sequence $\{w_r\}_{r=1}^{\infty}$ is called bounded or norm bounded if there is a constant η such that $\|w_1,w_2,w_3,\cdots,w_{r-1},w_r\|\leq\eta$ for all r.*

Remark 4.1 *Every convergent sequence is Cauchy and norm bounded in the fuzzy m-normed linear space $(F_s,\|\cdot,\cdots,\cdot\|)$.*

Definition 4.4 *A fuzzy m-normed linear space is said to be complete If every Cauchy sequence in a $(F_s,\|\cdot,\cdots,\cdot\|)$ is convergent.*

Definition 4.5 *Let $(F_s,\|\cdot,\cdots,\cdot\|)$ be a fuzzy soft m-normed linear space and let $\{w_r\}_{r=1}^{\infty}$ and $\{w'_r\}_{r=1}^{\infty}$ be two Cauchy sequences in $(F_s,\|\cdot,\cdots,\cdot\|)$ such that $\|w_1,w_2,w_3,\cdots,w_{r-1},w_r\|\to w$ and $\|w'_1,w'_2,w'_3,\cdots,w'_{r-1},w'_r\|\to w'$ as $r\to\infty$*
Let $\{k_r\}_{r=1}^{\infty}$ be a sequence in K where K is being the field of soft scalars be such that $k_r\to k_0$ as $r\to\infty$ then the following axioms holds:

i. $\|w_1 \pm w'_1, w_2 \pm w'_2, \cdots, w_r, \pm w'_r\| \to w \pm w', asr \to \infty$

ii. $\|w_1, w_2, w_3, \cdots, w_{r-1}, k_r w_r\| \to k_0 w$ *as* $r \to \infty$

iii. *Let* $\{w_r\}_{r=1}^{\infty}$ *and* $\{w'_r\}_{r=1}^{\infty}$ *be two Cauchy sequences in* $(F_s, \|\cdot, \cdots, \cdot\|)$ *and* $\{k_r\}_{r=1}^{\infty}$ *be a Cauchy sequence in then* $\{w_r + w'_r\}_{r=1}^{\infty}$ *and* $\{k_r w_r\}_{r=1}^{\infty}$ *are also Cauchy sequence in* $(F_s, \|\cdot, \cdots, \cdot\|)$

Example 4.2 *Let* $\{w_r\}_{r=1}^{\infty}$ *and* $\{w'_r\}_{r=1}^{\infty}$*be two Cauchy sequences in* $(V, \|\cdot, \cdots, \cdot)$ *and* $\{k_r\}_{r=1}^{\infty}$ *be a Cauchy sequence in K then* $\{w_r + w'_r\}_{r=1}^{\infty}$ *and* $\{k_r w_r\}_{r=1}^{\infty}$ *are also Cauchy sequence in* $(V, \|\cdot, \cdots, \cdot)$.
Let $V = \mathbb{R}^2$ *be the linear space over* $K = \mathbb{R}$ *and define*
a fuzzy soft 2-norm μ *as* $\mu(w_1, w_2) = \begin{cases} \frac{z}{1+|det(w_1,w_2)|}, & z \geq 0 \\ 0, & z < 0 \end{cases}$

Define the sequences $w_1^r = (1 - \frac{1}{r}, 0), w_2^r = (0, 1 + \frac{1}{r})$ *and which are Cauchy sequences in* $\mathbb{R}^2$ *as and both convergent to (1,0) and (0,1) respectively, let the scalar sequence* $k_{vr} = 1 + \frac{1}{r}$ *a Cauchy sequence in* $\mathbb{R}$ *that converges to 1. This shows that under the fuzzy soft m-norm* μ *the sequences* $\{w_1^r + w_2^r\}_{r=1}^{\infty}$ *and* $\{k_{vr} w_1^r\}_{r=1}^{\infty}$ *remain Cauchy sequences.*

Definition 4.6 *A binary operation* $\oplus : [0,1] \times [0,1] \to [0,1]$ *is continuous Triangular-norm or t-norm if it satisfies the following conditions for every* $t_1, t_2, t_3, \& t_4 in I$

i. $\oplus$ *is associative and commutative,*

ii. $\oplus$ *is continuous*

iii. $t_1 \oplus 1 = t_1$

iv. $t_1 \oplus t_2 \leq t_3 \oplus t_4$ *whenever* $t_1 \leq t_3$ *and* $t_2 \leq t_4$

Definition 4.7 *let* F_s *be a linear space over a soft field F and let* $\widetilde{F_s}$ *is a fuzzy soft sub set of* $F_s \times F_s \times \cdots \times F_s(m - times) \times (0, \infty)$ *is called as fuzzy norm on* F_s *iff it satisfies the following axioms for every* $w_1, w_2, w_3, \cdots, w_{m-1}, w_m \in F_s$ *and* $\xi \in F$
$[FS-1].\widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w_m, z) = 0$ *if for all* $Z \in (-\infty, 0)$
$[FS-2].\widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w_m, z) = 0 \Leftrightarrow w_1, w_2, w_3, \cdots, w_{m-1}, w_m$ *are linearly dependent if for all* $z \in (0, \infty)$
$[FS-3].\widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w_m, z)$ *is invariant under any permutation of*
$w_1, w_2, w_3, \cdots, w_{m-1}, w_m$

$[FS-4].\widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w_m, \rho z) = \widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w_m, \frac{z}{\rho})$ *if for all* $z \in (0, \infty)$

$[FS-5].\widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w_{m1} + w_{m2}, z_1, z_2) \in (0, \infty)$
$[FS-6].\widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w_m, z)$ *is continuous non decreasing function of*
$\lim_{m \to \infty} \widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w_m, z) = 1$ *if for all* $z \in (0, \infty)$
Then the pair $(F_s, \widetilde{F_s})$ *is called as a fuzzy soft m-norm on* F_s.

Definition 4.8 *Let the fuzzy soft m-normed linear space* $(F_s, \|\cdot, \cdots, \cdot\|)$, *let* $\{w_r\}_{r=1}^{\infty}$ *be a sequence is convergent to* $\widetilde{v}$ *where* $w \in F_s$ *in* $(F_s, \|\cdot, \cdots, \cdot\|)$ *if for every* $\delta > 0$ *there exists a* $M \in \mathbb{Z}^+$ *such that* $\lim_{n \to \infty} \|w_1, w_2, w_3, \cdots, w_{n-1}, w_n - w\| < \delta$ *it is expressed as*

$$\lim_{n \to \infty} \|w_1, w_2, w_3, \cdots, w_{n-1}, w_n\| = w$$

Definition 4.9 *Let* $(F_s, \|\cdot, \cdots, \cdot\|)$ *be the fuzzy n-normed linear space,* $\{w_r\}_{r=1}^{\infty}$ *be a sequence is said to be Cauchy sequence in* $(F_s, \|\cdot, \cdots, \cdot\|)$ *if for every* $\delta > 0$ *there exists a* $M \in \mathbb{Z}^+$
such that $\lim_{n,p \to \infty} \|w_1, w_2, w_3, \cdots, w_{n-1}, w_n - w^p\| < \delta$ *whenever* $n, p \geq M$ *and it is represented as*
$\lim_{n,p \to \infty} \|w_1, w_2, w_3, \cdots, w_{n-1}, w_n - w^p\| = 0$

Definition 4.10 *If every Cauchy sequence is convergent in the fuzzy soft n-normed linear space* $(F_s, \|\cdot, \cdots, \cdot\|)$ *then it is a complete space.*

Definition 4.11 *Let* $(F_s, \|\cdot, \cdots, \cdot\|)$ *is a fuzzy m-normed linear space and let U be subset of* $(F_s, \|\cdot, \cdots, \cdot\|)$ *is said to be bounded if there exists* $\kappa \in \mathbb{R}^+$ *such that* $\|\widetilde{u}^1, \widetilde{u}^2, \widetilde{u}^3 \cdots \widetilde{u}^m\| \leq \kappa, \forall \widetilde{u}^1, \widetilde{u}^2, \widetilde{u}^3 \cdots \widetilde{u}^m \in U$

Definition 4.12 *Let* $(F_s, \|\cdot, \cdots, \cdot\|)$ *be a fuzzy soft m-normed linear space, an open ball with center* $\widetilde{v}^0$ *and the radius r, is denoted by* $B(w^0, r)$ *and is defined as for any*
$w_1, w_2, w_3, \cdots, w_{m-1}, w_n \in F_s^m$ *such that* $\|w_1, w_2, w_3, \cdots, w_{m-1}, w_m - w^0\| < r$
that is $B(w^0, r) = \{\|w_1, w_2, w_3, \cdots, w_{m-1}, w_m - w^0\| < r$ *for any* $(w_1, w_2, w_3, \cdots, w_{m-1}, w_m) \in F_s^m\}$

Example 4.3 *Let* $(F_s, \|\cdot, \cdots, \cdot\|)$ *be a fuzzy soft m-normed linear space, We define continuous t-norm* $\widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w_m, z) = \dfrac{z + \|w_1, w_2, w_3, \cdots, w_{m-1}, w_m\|}{z - \|w_1, w_2, w_3, \cdots, w_{m-1}, w_m\|}$ *Then the set* $(F_s, \widetilde{F_s})$ *where* $(w_1, w_2, w_3, \cdots, w_{m-1}, w_m) \in F_s^m$ *is a fuzzy m-normed linear space.*

$[FS-1].\widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w_m, z) = 0,$ *if for all* $z \in (-\infty, 0)$

$[FS-2].\widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w_m, z) = 1 \Leftrightarrow \dfrac{z + \|w_1, w_2, w_3, \cdots, w_{m-1}, w_m\|}{z - \|w_1, w_2, w_3, \cdots, w_{m-1}, w_m\|} = 1$
$\Leftrightarrow \|w_1, w_2, w_3, \cdots, w_{m-1}, w_m\| = 0 \Leftrightarrow w_1, w_2, w_3, \cdots, w_{m-1}, w_m$ *are linearly dependent if for all* $z \in (0, \infty)$

$[FS-3].\widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w_m, z) = \dfrac{z + \|w_1, w_2, w_3, \cdots, w_{m-1}, w_m\|}{z - \|w_1, w_2, w_3, \cdots, w_{m-1}, w_m\|}$
$= \dfrac{z + \|w_1, w_2, w_3, \cdots, w_{m-1}\|}{z - \|w_1, w_2, w_3, \cdots, w_{m-1}\|} \Leftrightarrow \widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w_m, z) = \ldots\ldots$ *so on*
Hence $\widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w_m, z)$ *is invariant under any permutation of* $w_1, w_2, w_3, \cdots, w_{m-1}, w_m.$

$[FS-4].We\ have \widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, \rho w_m, z) = \dfrac{z + \|w_1, w_2, w_3, \cdots, w_{m-1}, \rho w_m\|}{z - \|w_1, w_2, w_3, \cdots, w_{m-1}, \rho w_m\|}$
$= \dfrac{z + \rho\|w_1, w_2, w_3, \cdots, w_{m-1}, w_m\|}{z - \rho\|w_1, w_2, w_3, \cdots, w_{m-1}, w_m\|}$
$= \dfrac{\frac{z}{|\rho|} + \|w_1, w_2, w_3, \cdots, w_{m-1}, w_m\|}{\frac{z}{|\rho|} - \|w_1, w_2, w_3, \cdots, w_{m-1}, w_m\|} = \widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, \rho w_m, \dfrac{z}{|\rho|})$ *where* $\rho \neq 0 \in F$

$[FS-5].\widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w_m, z_1) \leq \widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w'_m, z_2)$
$\Leftrightarrow \dfrac{z_1 + \|w_1, w_2, w_3, \cdots, w_{m-1}, w_m\|}{z_1 - \|w_1, w_2, w_3, \cdots, w_{m-1}, w_m\|} \leq \dfrac{z_2 + \|w_1, w_2, w_3, \cdots, w_{m-1}, w'_m\|}{z_2 - \|w_1, w_2, w_3, \cdots, w_{m-1}, w'_m\|}$
$\Leftrightarrow z_2\|w_1, w_2, w_3, \cdots, w_{m-1}, w_m\| \leq z_1\|w_1, w_2, w_3, \cdots, w_{m-1}, w'_m\|$
$\Leftrightarrow [z_2\|w_1, w_2, w_3, \cdots, w_{m-1}, w_m\| - z_1\|w_1, w_2, w_3, \cdots, w_{m-1}, w'_m\|] \geq 0 \cdots\cdots\cdots\cdots (1)$
again we consider that
$\widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w_m, z_1 + z_2) - \widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w'_m, z_1)$
$\Leftrightarrow \dfrac{z_1 + z_2 + \|w_1, w_2, w_3, \cdots, w_{m-1}, w'_m\|}{z_1 + z_2 - \|w_1, w_2, w_3, \cdots, w_{m-1}, w'_m\|} - \dfrac{z_1 + \|w_1, w_2, w_3, \cdots, w_{m-1}, w_m\|}{z_1 - \|w_1, w_2, w_3, \cdots, w_{m-1}, w_m\|}$
$\Leftrightarrow z_1\|w_1, w_2, w_3, \cdots, w_{m-1}, w'_m\| - z_2\|w_1, w_2, w_3, \cdots, w_{m-1}, w_m\| \cdots\cdots\cdots\cdots (2)$
from (1) and (2) we obtain
$\widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w'_m, z_1 + z_2) - \widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w_m, z_1) \geq 0$
$\Leftrightarrow \widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w'_m, z_1 + z_2) \geq \widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w'_m, z_1)$
$\Leftrightarrow \widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w'_m, z_1 + z_2) \geq min\{\widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w_m, z_1),$
$\widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w_m, z_1)\}$

$[FS-6].clearly \widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w_m, z)$ *is left continuous and is a non decreasing function of z such tha* $\lim\limits_{z \to \infty} \widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{m-1}, w_m, z)$
$= \lim\limits_{z \to \infty} \dfrac{z + \|w_1, w_2, w_3, \cdots, w_{m-1}, w_m\|}{z - \|w_1, w_2, w_3, \cdots, w_{m-1}, w_m\|} = 1$

Remark 4.2 *shows that there may exist cauchy sequence in the fuzzy soft m-normed linear space* $(F_s, \widetilde{F_s})$ *which is not convergent.*

Remark 4.3 *For the Example 3.2 and Example 3.3 consider a fuzzy soft m-normed linear space* $(F_s, \widetilde{F_s})$ *as in previous example Example 3.1.*

Example 4.4 *Let* $(F_s, \widetilde{F_s})$ *be the fuzzy soft m-normed linear space, a sequence* $\{w_r\}_{r=1}^{\infty}$ *in* $(F_s, \widetilde{F_s})$ *then a sequence* $\{w_r\}_{r=1}^{\infty}$ *a convergence in* $(F_s, \widetilde{F_s})$ *a sequence* $\{w_r\}_{r=1}^{\infty}$ *a convergence in* $(F_s, \|\cdot, \cdots, \cdot\|)$
Proof:*we have a sequence* $\{w_r\}_{r=1}^{\infty}$ *is also a convergence* $(F_s, \widetilde{F_s})$
$\Leftrightarrow \lim_{r\to\infty} \widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{r-1}, w_r - w, z) = 0 \Leftrightarrow \lim_{r\to\infty} \dfrac{z + \|w_1, w_2, w_3, \cdots, w_{r-1}, w_r\|}{z - \|w_1, w_2, w_3, \cdots, w_{r-1}, w_r\|} = 1$
Hence a sequence
$\Leftrightarrow \lim_{r\to\infty} \|w_1, w_2, w_3, \cdots, w_{r-1}, w_r - w, z\| = 0$
$\{w_r\}_{r=1}^{\infty}$ *is a convergence in* $(F_s, \|\cdot, \cdots, \cdot\|)$
Conversely we have a sequence $\{w_r\}_{r=1}^{\infty}$ *is a convergence in* $(F_s, \|\cdot, \cdots, \cdot\|)$

$$\Leftrightarrow \lim_{r\to\infty} \|w_1, w_2, w_3, \cdots, w_{r-1}, w_r - w, z\| = 0 \Leftrightarrow \frac{z + \|w_1, w_2, w_3, \cdots, w_{r-1}, w_r\|}{z - \|w_1, w_2, w_3, \cdots, w_{r-1}, w_r\|} = 1$$

$\Leftrightarrow \lim_{r\to\infty} \widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{r-1}, w_r - w, z) = 1$
Hence a sequence $\{w_r\}_{r=1}^{\infty}$ *is also a convergence in* $(F_s, \widetilde{F_s})$

Example 4.5 *Let* $(F_s, \widetilde{F_s})$ *be the fuzzy soft m-normed linear space, a sequence* $\{w_r\}_{r=1}^{\infty}$ *in* $(F_s, \widetilde{F_s})$ *then a sequence* $\{w_r\}_{r=1}^{\infty}$ *is a Cauchy sequence in* $(F_s, \widetilde{F_s})$ *a sequence* $\{w_r\}_{r=1}^{\infty}$ *is a Cauchy sequence in* $(F_s, \|\cdot, \cdots, \cdot\|)$
Proof: *Suppose a sequence* $\{w_r\}_{r=1}^{\infty}$ *is a cauchy sequence* $(F_s, \widetilde{F_s})$
$\Leftrightarrow \lim_{r\to\infty} \widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{r-1}, w_r - w_l, z) = 1$
$\Leftrightarrow \lim_{r\to\infty} \dfrac{z + \|w_1, w_2, w_3, \cdots, w_{r-1}, w_r - w_l\|}{z - \|w_1, w_2, w_3, \cdots, w_{r-1}, w_r - w_l\|} = 1$
$\Leftrightarrow \lim_{r\to\infty} \|w_1, w_2, w_3, \cdots, w_{r-1}, w_r - w_l, z\| = 0$
Hence a sequence $\{w_r\}_{r=1}^{\infty}$ *is a Cauchy sequence in* $(F_s, \|\cdot, \cdots, \cdot\|)$

Conversely we prove that a sequence $\{w_r\}_{r=1}^{\infty}$ *is a Cauchy sequence in* $(F_s, \widetilde{F_s})$ *when it is a Cauchy sequence in* $(F_s, \|\cdot, \cdots, \cdot\|)$ *if for every* $\varepsilon > 0$ *there exists appositive number M such that*
$\|w_1, w_2, w_3, \cdots, w_{r-1}, w_r - w_l, z\| < \varepsilon$ *whenever* $r, l \geq M$

$$|\frac{z + \|w_1, w_2, w_3, \cdots, w_{r-1}, w_r - w_l\|}{z - \|w_1, w_2, w_3, \cdots, w_{r-1}, w_r - w_l\|}| < \varepsilon \text{ whenever } r, l \geq M$$

$$|\widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{r-1}, w_r - w_l, z)| < \varepsilon \text{ whenever } r, l \geq M$$

Hence a sequence $\{w_r\}_{r=1}^{\infty}$ *is a Cauchy sequence in* $(F_s, \widetilde{F_s})$.

Theorem 4.1 *In a fuzzy soft m-normed linear space* $(F_s, \widetilde{F_s})$ *every Cauchy sequence has a convergent sub-sequence is complete.*
Proof: *Let* $(F_s, \widetilde{F_s})$ *be the fuzzy soft -m-normed linear space and let a sequence* $\{w_r\}_{r=1}^{\infty}$ *is a Cauchy sequence in* $(F_s, \widetilde{F_s})$
Let $\{w_{r,l}\}_{l=1}^{\infty}$ *be a subsequence of* $\{w_r\}_{r=1}^{\infty}$ *and it is convergence to w now we need to prove that the sequence* $\{w_r\}_{r=1}^{\infty}$ *is convergence to w , for this let* ε *in* $(0,1)$ *and* $z \in (0, \infty)$, *choose* δ *in (0,1)such that* $\delta \oplus \delta < \varepsilon$

Given that a sequence $\{w_r\}_{r=1}^{\infty}$ *is a Cauchy sequence in* $(F_s, \widetilde{F_s})$ *that is if for every* ε *in* $(0,1)$ *and* $z \in (0, \infty)$ *there exists appositive number M such that*
$|\widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{r-1}, w_r - w_l, z)|\delta$ *for all* $r, l \geq M$

We have the sub-sequence $\{w_{r,l}\}_{l=1}^{\infty}$ *is also convergent to* w *there exists a* $r, l > M$ *such that*
$\widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{r-1}, w_r - w_l, \dfrac{z}{2}) < \delta$ *for all* $r, l \geq M$
Now $\widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{r-1}, w_r - w_l, z) \leq \{\widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{r-1}, w_r - w_l, \frac{z}{2})$
$\oplus \widetilde{F_s}(w_1, w_2, w_3, \cdots, w_{r-1}, w_r - w_l, \frac{z}{2})\} < \delta \oplus \delta < \varepsilon$
Therefore a sequence $\{w_r\}_{r=1}^{\infty}$ *is convergence to* w *in* $(F_s, \widetilde{F_s})$

Example 4.6 *Let* $F_V = \mathbb{R}$ *is vector space over* $\mathbb{R}$ *and define a fuzzy soft 2-norm by*
$\|x, y\|(t) = \frac{t}{1+\|x \bigwedge y\|}$ *for all* $t \geq 0$ *where* $x \bigwedge y$ *denotes the minimum of corresponding coordinates of* x *and* y*, and* $\| \cdots \|$ *is the usual Euclidean norm. Define the sequences:*
$f_{vr} = (1 - \frac{1}{r}, 0) \rightarrow f_v = (1, 0), f + vr' = (\frac{1}{r}, 0) \rightarrow f_v = (0, 0)$ *as* $r \rightarrow \infty$ *and scalar sequence* $k_{vr} = 1 + \frac{1}{r} \rightarrow 1 = k_v$ *as* $r \rightarrow \infty$,
then $f_{vr} + f_{vr'} = (1 - \frac{1}{r}, \frac{1}{r}) \rightarrow (1, 0), k_{vr} f_{vr} = ((1 + \frac{1}{r})(1 - \frac{1}{r}), 0) = (1, 0)$ *as* $r \rightarrow \infty$ *The sequence* $f_{vr} = (1 - \frac{1}{r}, 0)$ *is a Cauchy sequence in this fuzzy 2-normed space.*
Its limit is $f_v = (1, 0)$ *and thus the space is complete if such limits exist for all Cauchy sequences. Then we observe that fuzzy soft m-norm values of these sequences also converge accordingly. In any normed linear space (crisp or fuzzy soft), the sum of two Cauchy sequences is Cauchy, and scalar multiplication of a Cauchy sequence by a Cauchy scalar sequence remains Cauchy. Since the fuzzy soft m-norm is continuous with respect to the function (which itself is continuous), the fuzzy soft norm inherits this Cauchy-preserving property. This confirms that the completeness and closure properties hold even in higher dimensions. This theorem confirms that the class of Cauchy sequences is closed under standard vector space operations (addition and scalar multiplication) in a fuzzy soft-m-normed linear space. It underlines the necessity of completeness when analysing convergence behaviour of sequences, ensuring that limits behave consistently under fuzzy soft operations. That is, if sequences converge (or are Cauchy) in vector components, they do so in the fuzzy norm due to its continuity.*

5. Conclusions

In this research paper, we introduced the notion of fuzzy soft m-normed linear space by using the combination of fuzzy m-normed linear space and soft m-normed linear space. We obtained some results about the Cauchy sequence and convergence sequence in fuzzy soft m-normed linear space.Also provided theorems of completeness sequence and Cauchy sequence with suitable example in fuzzy soft m-normed linear space. This work can be extended to fuzzy soft m-normed linear space by introducing the concepts of fuzzy soft Gamma m-normed linear space.

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