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# Cost Optimization of System in Single-Valued Neutrosophic Fuzzy Queueing Systems Using Genetic Algorithm

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ABSTRACT: Queueing theory constitutes a fundamental analytical framework for evaluating the performance, stability, and operational behavior of manufacturing and service systems subjected to stochastic arrival and service processes. The Neutrosophic paradigm offers a powerful extension for modeling uncertainty, imprecision, indeterminacy, and inconsistency inherent in real-world system parameters—capabilities that surpass those of classical probabilistic and fuzzy approaches. The study includes Single-Valued Neutrosophic Queueing Systems (SVNQSs) as a generalized formulation of traditional and fuzzy queueing structures, providing an enriched mathematical apparatus for system analysis, control, and optimization. Within the SVNQS framework, both arrival and service rates are represented by single-valued neutrosophic numbers, enabling a more comprehensive characterization of ambiguous and fluctuating system conditions. Consequently, the associated probability measures, performance indices, and operational metrics are likewise expressed in neutrosophic form. The proposed model further examines the influence of neutrosophic parameters on queueing dynamics and investigates the optimization of total system cost through the application of a Genetic Algorithm (GA). This integrative approach yields deeper insights into decision-making under uncertainty and facilitates enhanced optimization strategies applicable to diverse manufacturing and operational environments. Numerical case studies are provided and successfully solved to demonstrate the applicability of the framework, and due to the computational intricacies involved, dedicated MATLAB routines have been developed to streamline and automate the required computations.

Keywords: Markovian Queueing Systems, single-valued neutrosophic numbers, neutrosophic arithmetic operations, Single-Valued Neutrosophic Queueing Systems (SVNQSs), Genetic Algorithm (GA).

## Contents



## 1. Introduction

The concept of a fuzzy set (FS) was originally formulated by L. A. Zadeh in 1965 [1], wherein each element of a universe is characterized by a degree of membership $\mu \in [0,1]$. Subsequently, in 1986, the intuitionistic fuzzy set (IFS) was introduced by K. Atanassov [2] as a generalization of FS, as shown in Figure 1 and Figure 2. In an intuitionistic fuzzy set (IFS), each element is associated with both a membership degree $\mu(x)$ and a non-membership degree $\nu(x)$, subject to the condition:

$$0 \leq \mu(x) + \nu(x) \leq 1.$$

This formulation accounts for an additional component, often interpreted as hesitation or uncertainty. Building further on these generalizations, F. Smarandache introduced the Neutrosophic set (NS), incorporating an additional and independent degree of indeterminacy alongside the membership and

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non-membership degrees. This framework enables a more comprehensive representation of uncertainty, vagueness, and inconsistency.

Traditional models require precise probability distributions and deterministic parameters. In contrast, fuzzy-based approaches incorporate imprecise and uncertain event times directly into the mathematical framework. This allows for more realistic modeling of communication networks, healthcare systems, and service environments with ambiguous data. In complex systems where arrival patterns fluctuate unpredictably and service times vary due to immeasurable factors, fuzzy queueing systems provide the mathematical sophistication needed to make reliable predictions and optimize operations despite incomplete information [3]. Common queue types include linear queues, virtual waiting systems, and priority-based service models, which are widely used across industries ranging from retail to telecommunications. Neutrosophic numbers extend classical mathematics by embracing uncertainty, indeterminacy,


**Fig.1** Timeline

and contradiction-elements that traditional systems often ignore. Unlike binary logic that deals only with true or false, neutrosophy introduces a revolutionary third dimension that captures the ambiguity inherent in real-world phenomena.

A neutrosophic number is expressed as

$$N = (x, \mu_t, \mu_i, \mu_f),$$

where $x$ represents a crisp value, and $\mu_t$, $\mu_i$, and $\mu_f$ are membership values and are independent components, allowing for a more nuanced representation than classical binary systems.

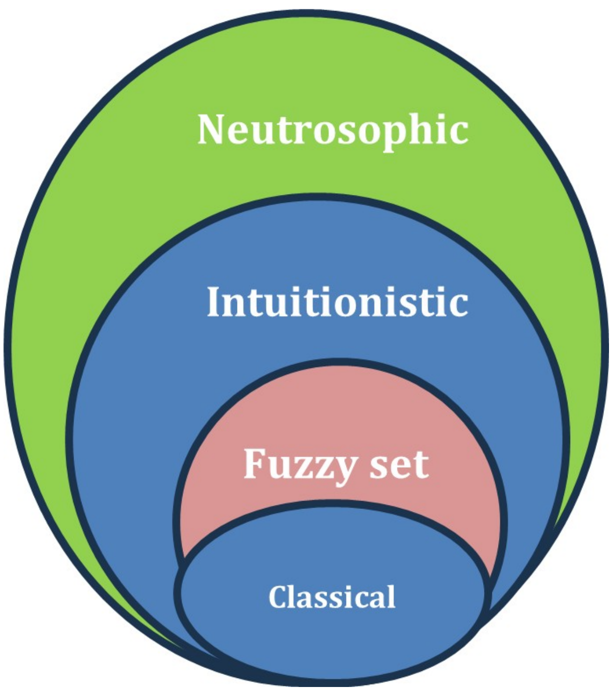


**Fig.2** Concept of Neutrosophic Set

*Truth* ($\mu_t$) represents the degree of truthfulness or membership in a set. Values range from 0 to 1, where 1 indicates complete truth and 0 indicates no truth.

*Indeterminacy* ($\mu_i$) captures uncertainty, unknown information, or neutrality. This unique component distinguishes neutrosophic logic from fuzzy logic, representing the degree of hesitation or lack of knowledge.

*Falsity* ($\mu_f$) measures the degree of falsehood or non-membership. Like truth, it ranges from 0 to 1, where 1 indicates complete falsity.

This research has clearly examined the integration of neutrosophic numbers into the M/M/1 queuing model and showcased the potential for enhancing the accuracy of queuing models through such integration. In study [4], the effects of neutrosophic parameters on key performance indicators are explored by considering various distributions such as Exponential, Erlang, and Deterministic.

Study [5] developed a novel approach for ranking single-valued trapezoidal neutrosophic numbers (SVTN-numbers) using defuzzification and applied this approach to multi-criteria decision-making (MCDM) problems. In [6], a new method for solving intuitionistic fuzzy queuing systems using non-linear programming (NLP) has been proposed and analyzed, and the approach is clearly illustrated with an example.

The modeling of uncertainty by proposing NM/NM/1 in terms of single-valued trapezoidal neutrosophic numbers (SVTNN) has been further enriched in [7-8]. In [9], the performance measures of a bulk queuing system in a neutrosophic environment have been examined and improved. Additionally, the computation of NM[b]/NM/1 performance measures has been described for systems with two and three parameters.

Neutrosophic queuing systems and Neutrosophic Little's Formulas (NLF), along with neutrosophic performance measures, are defined in [10], with wide applications in networking and communication sectors. The efficacy of neutrosophic fuzzy sets in solving transportation problems has been described and evaluated in [11], along with comparisons to traditional methods. This study also reflects on the impact of neutrosophic pentagonal and octagonal numbers.

In [12], the primary challenges and conditions related to neutrosophic traffic flow congestion have been summarized, providing insights for future research in optimization and management. The use of linguistic neutrosophic sets and Hausdorff distance to analyze Markovian queuing systems has been introduced in [13].

Cost optimization of the overall system plays a crucial role in improving the profitability of organizations operating large queuing systems. The application of genetic algorithms provides a novel and efficient approach to this problem, as discussed in [14-15].

### 1.1. Neutrosophic Arithmetic Operations

Let $A_1 = (x_1, \mu_{t1}, \mu_{i1}, \mu_{f1})$ and $A_2 = (x_2, \mu_{t2}, \mu_{i2}, \mu_{f2})$ then the arithmetic operations on these are defined in equations (1.1)-(1.5) as follows:

- **Neutrosophic Summation:**

$$A_1 \oplus A_2 = \left(x_1 + x_2,\ \mu_{t1} \wedge \mu_{t2},\ \mu_{i1} \vee \mu_{i2},\ \mu_{f1} \vee \mu_{f2}\right), \tag{1.1}$$

- **Neutrosophic Multiplication:**

$$A_1 \otimes A_2 = \left(x_1 x_2,\ \mu_{t1} \wedge \mu_{t2},\ \mu_{i1} \vee \mu_{i2},\ \mu_{f1} \vee \mu_{f2}\right), \tag{1.2}$$

- **Neutrosophic Subtraction:**

$$A_1 \ominus A_2 = \left(x_1 - x_2,\ \mu_{t1} \wedge \mu_{t2},\ \mu_{i1} \vee \mu_{i2},\ \mu_{f1} \vee \mu_{f2}\right), \tag{1.3}$$

- **Neutrosophic Division:**

$$\frac{A_1}{A_2} = \left(\frac{x_1}{x_2},\ \mu_{t1} \wedge \mu_{t2},\ \mu_{i1} \vee \mu_{i2},\ \mu_{f1} \vee \mu_{f2}\right), \quad \text{where } \mu_{t2} \neq 0,\ \mu_{i2} \neq 1,\ \mu_{f2} \neq 1, \tag{1.4}$$

here,

$$\mu_1 \wedge \mu_2 = \begin{cases} \mu_1, & \text{if } \mu_1 > \mu_2, \\ \mu_2, & \text{else} \end{cases} \quad \text{and} \quad \mu_1 \vee \mu_2 = \begin{cases} \mu_1, & \text{if } \mu_1 < \mu_2, \\ \mu_2, & \text{else} \end{cases} \tag{1.5}$$

## 2. Mathematical Formulation of Performance Measures in Neutrosophic Single Server Queueing Models

Overall system behavior characterized through Neutrosophic performance measures that capture uncertainty in prediction. Performance metrics in Neutrosophic queueing systems extend classical measures by explicitly incorporating uncertainty through Neutrosophic number representation. Expected waiting time before service begins, incorporating indeterminacy about arrival patterns and queue dynamics.

*Neutrosophic Arrival Process*

Customers arrive with Neutrosophic inter-arrival times characterized by $N_a = (x(N_a), \mu_t(N_a), \mu_l(N_a), \mu_f(N_a))$ representing the truth, indeterminacy, and falsity components of the arrival rate distribution.

*Neutrosophic Service Mechanism*

Service times are similarly Neutrosophic, reflecting inherent uncertainty in processing durations due to variable workload complexity or resource availability.

$$N_s = (x(N_s), \mu_t(N_s), \mu_l(N_s), \mu_f(N_s))$$

- **Neutrosophic Expected Number in Queue ($N_{Lq}$):** Quantifies queue length using Neutrosophic numbers that reflect indeterminate states. It represents the average total number of customers in the system, including those being served.
- **Neutrosophic Expected Number in System ($N_{Ls}$):** Quantifies total system occupancy using Neutrosophic numbers that reflect indeterminate states. It represents the average number of customers waiting for service at any given time.
- **Neutrosophic Mean Waiting Time in System ($N_{Ws}$):** Measures expected customer total system time, expressed as intervals that capture uncertainty rather than single point estimates. It represents the total time a customer spends in the system including both waiting and service duration.
- **Neutrosophic Mean Waiting Time in Queue ($N_{Wq}$):** Measures expected customer wait time in queue. It represents the waiting time a customer spends in the queue.

  **Classical Little's Law:**

  $$L_q = a \times W_q. \tag{2.1}$$

  It gives us the elegant relationship between average number in queue ($L_q$), arrival rate ($a$), and average time in queue ($W_q$) (Equation 2.1).
- **Neutrosophic Generalization:** If we denote average number in system ($N_{Lq}$), arrival rate ($N_a$), and average time in system ($N_{Wq}$) then relation between them will be as follows (Equation 2.2):

  $$N_{Lq} = N_a \otimes N_{Wq}, \quad N_{Ls} = N_a \otimes N_{Ws} \tag{2.2}$$

  Here $\otimes$ denotes neutrosophic multiplication. NLF enables deriving one performance measure from others despite inherent uncertainty, maintaining the fundamental relationship under neutrosophic conditions.
- **Performance Measure Computation:** Performance measures are expressed as neutrosophic numbers using interval arithmetic and $\alpha$-cut operations (Equation 2.3)

  $$N_{Wq} = f(N_a, N_s) = \left(\frac{N_\rho}{(1 - N_\rho)\, N_s}\right); N_\rho = \frac{N_a}{N_s}, \tag{2.3}$$

Here $N_\rho$ is the Neutrosophic utilization factor. Bounds and expected values computed through de fuzzification techniques incorporating indeterminacy terms. Overall system cost as a function of number of waiting customer and service rate is given below (Equation 2.4), here A and B are constants.

$$Total\, system\, cost(Crisp) = A * customers\, in\, queue + B * Service\, Rate; \tag{2.4}$$

- **Hausdorff Distance** Let $A_1 = (x_1, \mu_{t1}, \mu_{i1}, \mu_{f1})$ and $A_2 = (x_2, \mu_{t2}, \mu_{i2}, \mu_{f2})$ be two SVNs, then

$$d_H(A_1, A_2) = \max \{|\mu_{t1} - \mu_{t2}|, |\mu_{i1} - \mu_{i2}|, |\mu_{f1} - \mu_{f2}|\}. \tag{2.5}$$

  Between A and B the Hausdorff distance (d) is defined in Equation 2.5.

- **Linguistic Representation of SVNN:**

  Among the paper we will use the linguistic representation of SVNN $A_1 = (x_1, \mu_{t1}, \mu_{i1}, \mu_{f1})$ given in the following table 1 :

Table 1: Linguistic terms with corresponding SVNN values

| **Tag number** | **Linguistic Term** | **SVNN** |
|---|---|---|
| 1 | Extremely good | $(x, 1.000000, 0.000000, 0.000000)$ |
| 2 | Very Very good | $(x, 0.900000, 0.100000, 0.100000)$ |
| 3 | Very good | $(x, 0.800000, 0.150000, 0.200000)$ |
| 4 | Good | $(x, 0.700000, 0.250000, 0.300000)$ |
| 5 | Fairly good | $(x, 0.600000, 0.350000, 0.400000)$ |
| 6 | Average | $(x, 0.500000, 0.500000, 0.500000)$ |
| 7 | Moderately bad | $(x, 0.400000, 0.650000, 0.600000)$ |
| 8 | Bad | $(x, 0.300000, 0.750000, 0.700000)$ |
| 9 | Very bad | $(x, 0.200000, 0.850000, 0.800000)$ |
| 10 | Very Very bad | $(x, 0.100000, 0.900000, 0.900000)$ |
| 11 | Extremely bad | $(x, 0.000000, 1.000000, 1.000000)$ |

## 3. Numerical Examples

To understand the effect of various input parameters on the output measures especially on total system cost, in table 2 effect of various values of arrival rate and service rate on output measures like number of customer in system ($L_s$), number of customer in queue ($L_q$), waiting time in queue ($W_q$) ,waiting time in System ($W_s$), and total system Cost are presented. Here in Total cost $A$ is 200 and $B$ is taken 10.

In Fig 3 effect of rate of arrival on total cost on different service rate is exhibited. The increase in arrival rate leads to increase in waiting customer so finally increase in total system cost as visible in the figure below. In Fig 4 effect of rate of service on total cost on different arrival rate is displayed. In Fig 5 and Fig 6 combine effect of rate of arrival and service rate on total cost is demonstrated.

Table 2: Queue performance measures for different arrival and service rates

| Arrival Rate | Service Rate | $L_s$ | $L_q$ | $W_s$ | $W_q$ | Total Cost |
|---|---|---|---|---|---|---|
| 1 | 4 | 0.333333 | 0.083333 | 0.333333 | 0.083333 | 56.66667 |
| | 4.5 | 0.285714 | 0.063492 | 0.285714 | 0.063492 | 57.69841 |
| | 5 | 0.25 | 0.05 | 0.25 | 0.05 | 60 |
| | 5.5 | 0.222222 | 0.040404 | 0.222222 | 0.040404 | 63.08081 |
| | 6 | 0.2 | 0.033333 | 0.2 | 0.033333 | 66.66667 |
| 1.5 | 4 | 0.6 | 0.225 | 0.4 | 0.15 | 85 |
| | 4.5 | 0.5 | 0.166667 | 0.333333 | 0.111111 | 78.33333 |
| | 5 | 0.428571 | 0.128571 | 0.285714 | 0.085714 | 75.71429 |
| | 5.5 | 0.375 | 0.102273 | 0.25 | 0.068182 | 75.45455 |
| | 6 | 0.333333 | 0.083333 | 0.222222 | 0.055556 | 76.66667 |
| 2 | 4 | 1 | 0.5 | 0.5 | 0.25 | 140 |
| | 4.5 | 0.8 | 0.355556 | 0.4 | 0.177778 | 116.1111 |
| | 5 | 0.666667 | 0.266667 | 0.333333 | 0.133333 | 103.3333 |
| | 5.5 | 0.571429 | 0.207792 | 0.285714 | 0.103896 | 96.55844 |
| | 6 | 0.5 | 0.166667 | 0.25 | 0.083333 | 93.33333 |
| 2.5 | 4 | 1.666667 | 1.041667 | 0.666667 | 0.416667 | 248.3333 |
| | 4.5 | 1.25 | 0.694444 | 0.5 | 0.277778 | 183.8889 |
| | 5 | 1 | 0.5 | 0.4 | 0.2 | 150 |
| | 5.5 | 0.833333 | 0.378788 | 0.333333 | 0.151515 | 130.7576 |
| | 6 | 0.714286 | 0.297619 | 0.285714 | 0.119048 | 119.5238 |
| 3 | 4 | 3 | 2.25 | 1 | 0.75 | 490 |
| | 4.5 | 2 | 1.333333 | 0.666667 | 0.444444 | 311.6667 |
| | 5 | 1.5 | 0.9 | 0.5 | 0.3 | 230 |
| | 5.5 | 1.2 | 0.654545 | 0.4 | 0.218182 | 185.9091 |
| | 6 | 1 | 0.5 | 0.333333 | 0.166667 | 160 |

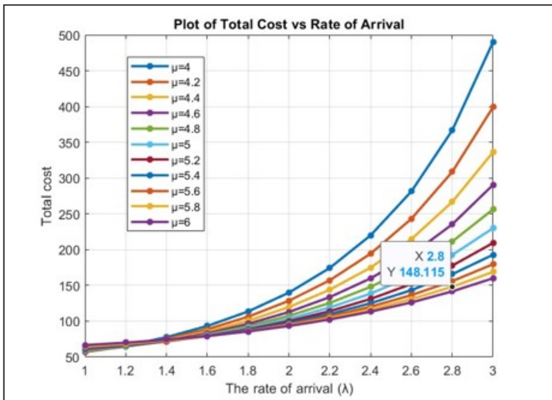


Fig 3 : effect of arrival rate on total cost on different service rate

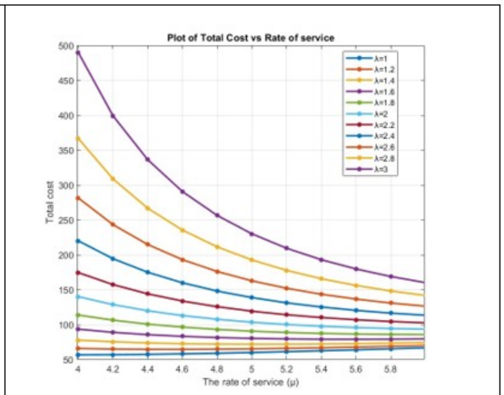


Fig 4: effect of service rate on total cost on different arrival rate

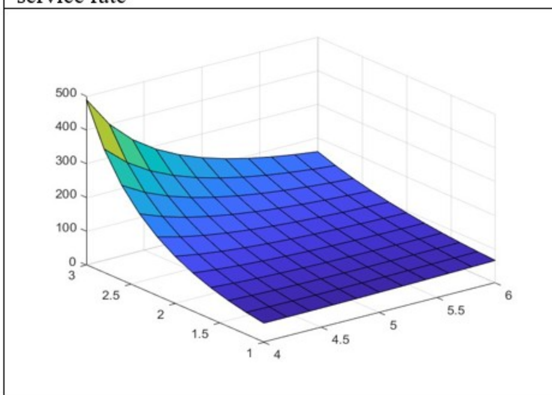

Fig 5: Combine effect of rate of arrival and service rate on total cost

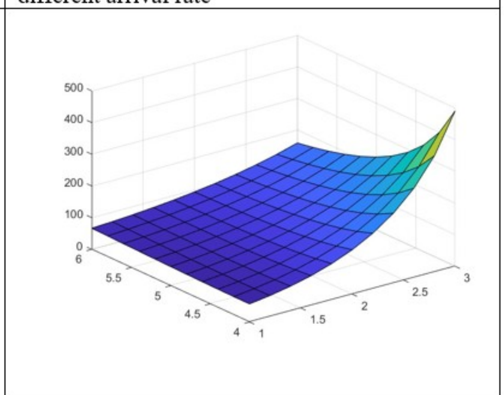

Fig 6 Combine effect of service rate and rate of arrival on total cost

Now to understand the effect in terms of linguistic value three sets of Arrival rate And Service rates as SVNN are considered and the performance measures are calculated. From example 1 it is clear that if we take arrival rate with tag 3 ("Very good") and service rate with tag 7("Moderately bad") then output measures comes with tag 7("Moderately bad") shown in table 3.

Table 3: Sensitivity analysis with input parameters as SVNN (example-1)

| Example 1 | | | | | |
|---|---|---|---|---|---|
| **Arrival Rate** | **Tag** | **Service Rate** | **Tag** | $L_s$ | **Tag** |
| $(3, 0.90, 0.20, 0.20)$ | 3 | $(8, 0.20, 0.30, 0.90)$ | 7 | $(0.6, 0.20, 0.30, 0.90)$ | 7 |
| $L_q$ | | $W_s$ | | $W_q$ | |
| $(0.22, 0.20, 0.30, 0.90)$ | 7 | $(0.2, 0.20, 0.30, 0.90)$ | 7 | $(0.07, 0.20, 0.30, 0.90)$ | 7 |

In example 2 if we take arrival rate and service rate with same tag 6 ("Average") then output measures comes with tag 6 ("Average") shown in table 4.

Table 4: Sensitivity analysis with input parameters as SVNN (example-2)

| Example 2 | | | | | |
|---|---|---|---|---|---|
| **Arrival Rate** | **Tag** | **Service Rate** | **Tag** | $L_s$ | **Tag** |
| $(3, 0.20, 0.20, 0.90)$ | 6 | $(8, 0.10, 0.10, 0.90)$ | 6 | $(0.6, 0.10, 0.20, 0.90)$ | 6 |
| $L_q$ | | $W_s$ | | $W_q$ | |
| $(0.22, 0.10, 0.20, 0.90)$ | 6 | $(0.2, 0.10, 0.20, 0.90)$ | 6 | $(0.07, 0.10, 0.20, 0.90)$ | 6 |

If we take arrival rate with tag 6("Average") and service rate with tag 2 ("Very Very good") then output measures comes with tag 6 ("Average") shown in table 5. Thus from these examples it is clear

Table 5: Sensitivity analysis with input parameters as SVNN (example-3)

| Example 3 | | | | | |
|---|---|---|---|---|---|
| **Arrival Rate** | **Tag** | **Service Rate** | **Tag** | $L_s$ | **Tag** |
| $(3, 0.90, 0.20, 0.20)$ | 6 | $(8, 0.90, 0.10, 0.10)$ | 2 | $(0.6, 0.10, 0.20, 0.90)$ | 3 |
| $L_q$ | | $W_s$ | | $W_q$ | |
| $(0.22, 0.10, 0.20, 0.90)$ | 6 | $(0.2, 0.10, 0.20, 0.90)$ | 6 | $(0.07, 0.10, 0.20, 0.90)$ | 6 |

that if quality of any of input parameter decrease the quality of output measures also decreases.

To understand the effect of Neutroshopic input parameters on output measures as Neutroshopic numbers graphically, here three examples are presented with different arrival rates and service rates with different membership's values of Truth, indeterminacy and false parameters. In Fig 7-24, all parameters are presented as Neutroshopic number with one crisp value and membership value of three components $(T, I, F)$. In Fig 7, 13 and 19 different values of Arrival rate and In Fig 8, 14 and 20 different service rate is displayed, and by using these values, various output measures are calculated, and presented as Neutrosophic number in following figures. Number of waiting customers in system and customers in queue are presented in Fig 10, 16, 22 and In Fig 9, 15 and 21 respectivelly. In Fig 11, 17 and 23 customer waiting time in system and In Fig 12, 18 and 24 customer waiting time in queue are presented.

**Example:1**

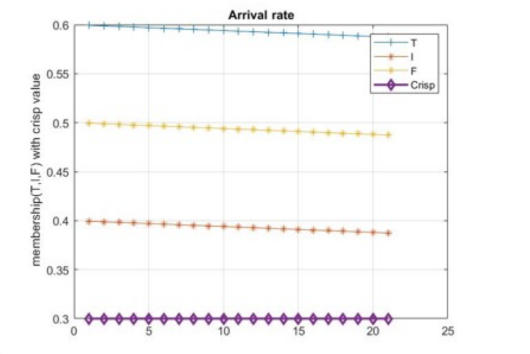


Fig 7 Arrival rate as Neutroshopic number with one crisp value and membership value of three components (T, I, F)

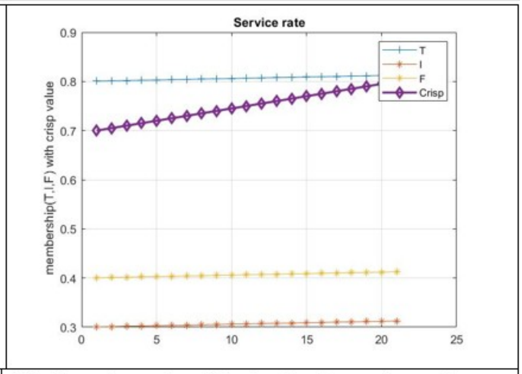


Fig 8 service rate as Neutroshopic number with one crisp value and membership value of three components (T, I, F)

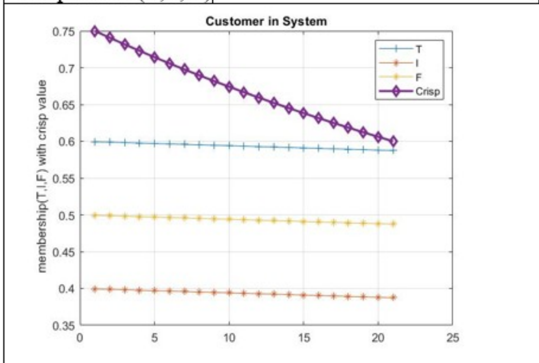


Fig 9 Customer in system as Neutroshopic number with one crisp value and membership value of three components (T, I, F)

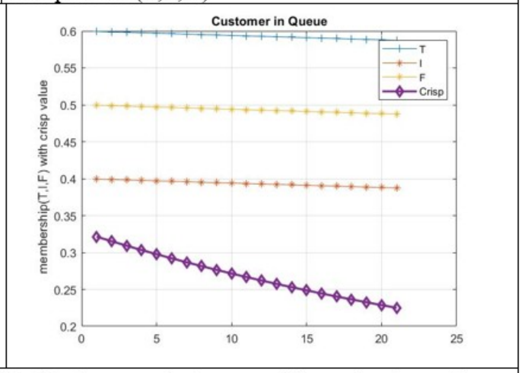


Fig 10 Customer in Queue as Neutroshopic number with one crisp value and membership value of three components (T, I, F)

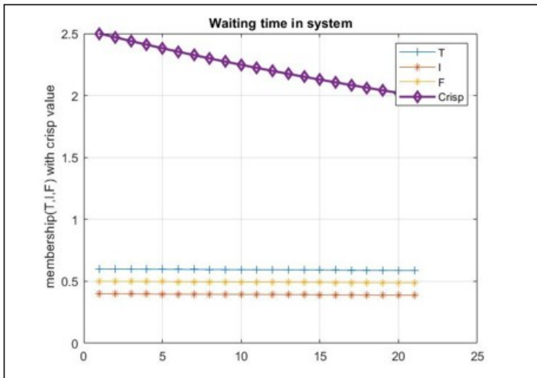


Fig 11Waiting time of customer in system as Neutroshopic number with one crisp value and membership value of three components (T, I, F)

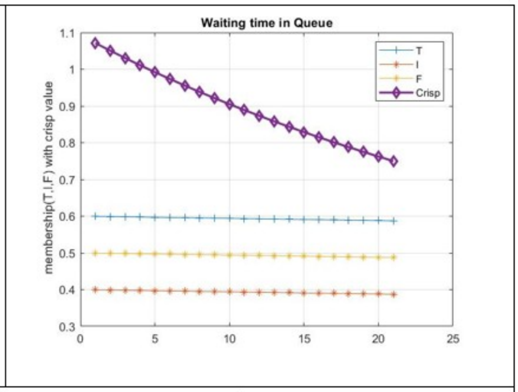


Fig 12 Waiting time of customer in Queue as Neutroshopic number with one crisp value and membership value of three components (T, I, F)

**Example:2**

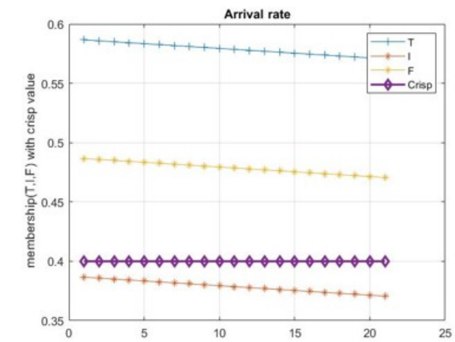


Fig 13 Arrival rate as Neutroshopic number with one crisp value and membership value of three components (T, I, F)

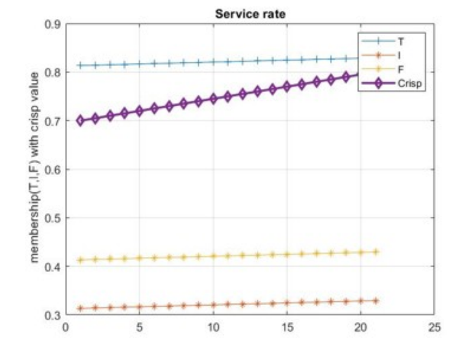


Fig 14 service rate as Neutroshopic number with one crisp value and membership value of three components (T, I, F)

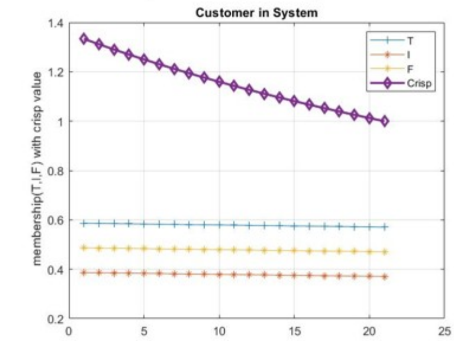


Fig 15 Customer in system as Neutroshopic number with one crisp value and membership value of three components (T, I, F)

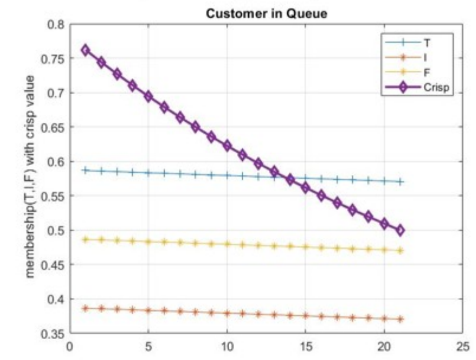


Fig 16 Customer in Queue as Neutroshopic number with one crisp value and membership value of three components (T, I, F)

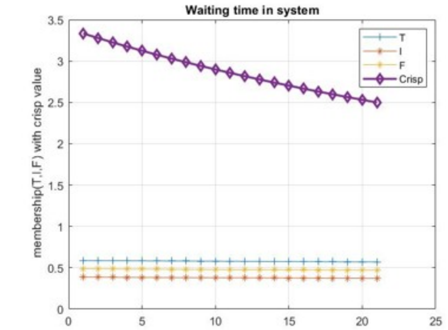


Fig 17Waiting time of customer in system as Neutroshopic number with one crisp value and membership value of three components (T, I, F)

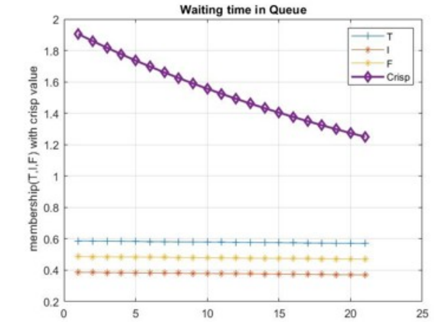


Fig 18 Waiting time of customer in Queue as Neutroshopic number with one crisp value and membership value of three components (T, I, F)

**Example:3**

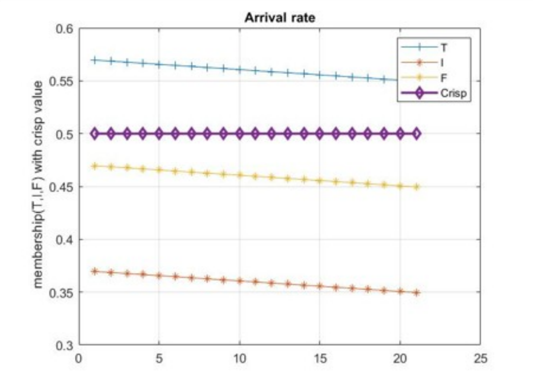


Fig 19 Arrival rate as Neutroshopic number with one crisp value and membership value of three components (T, I, F)

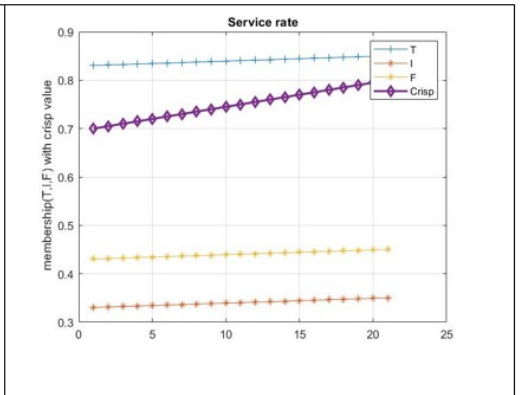


Fig 20 service rate as Neutroshopic number with one crisp value and membership value of three components (T, I, F)

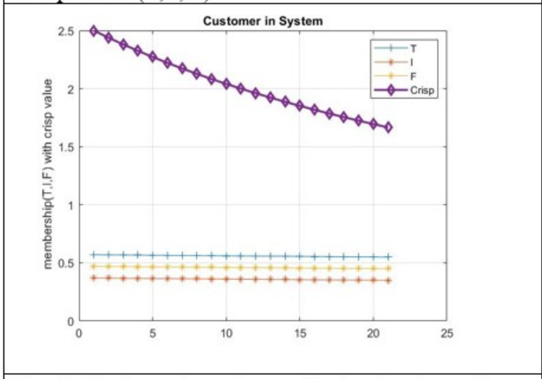


Fig 21 Customer in system as Neutroshopic number with one crisp value and membership value of three components (T, I, F)

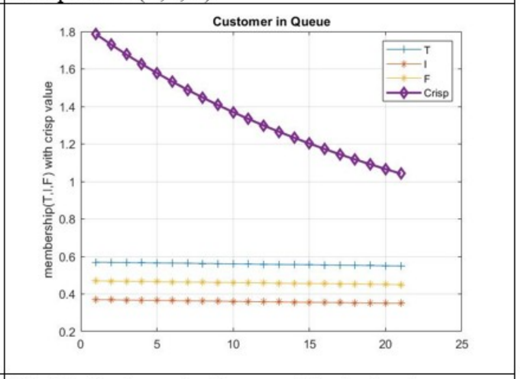


Fig 22 Customer in Queue as Neutroshopic number with one crisp value and membership value of three components (T, I, F)

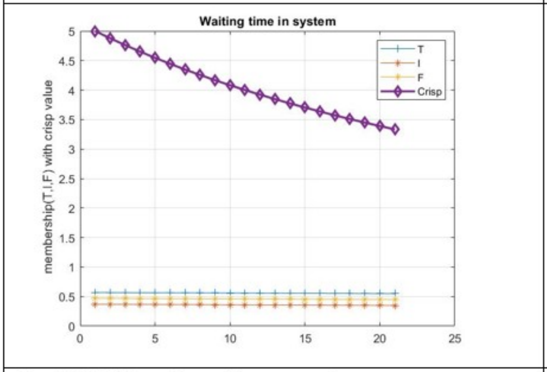


Fig 23 Waiting time of customer in system as Neutroshopic number with one crisp value and membership value of three components (T, I, F)

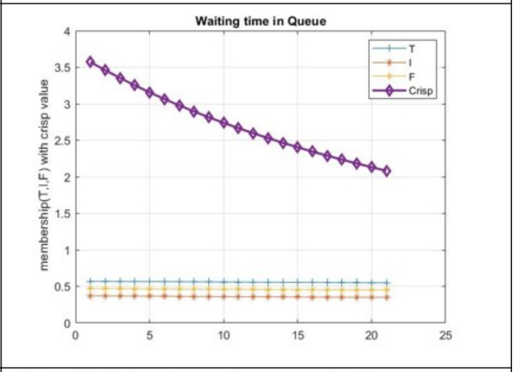


Fig 24 Waiting time of customer in Queue as Neutroshopic number with one crisp value and membership value of three components (T, I, F)

## 4. Genetic Algorithms

To achieve the optimized value of total cost, required arrival rates and service rates are obtained used genetic algorithm. The fitness function used is inverse of total cost obtained using following formula.

$$Total\, system\, cost(Crisp) = 200 * customers\, in\, queue + 10 * Service\, Rate;$$

The genetic algorithm can be summarized as follows:

- **Initialization:** A random population of potential solutions is created, where each solution is encoded as a "chromosome".

- **Fitness Evaluation:** An objective function calculates the "fitness" of each solution, measuring how good it is.
- **Selection:** Solutions are selected to become parents for the next generation.
- **Crossover:** The genetic material of selected parents is recombined to create new offspring solutions.
- **Mutation:** Small, random changes are introduced into the offspring to maintain genetic diversity.
- **Reproduction:** The new generation is formed by the offspring, and the process repeats.
- **Convergence:** The algorithm continues for several generations until the solutions converge to a satisfactory optimal or near-optimal state.

In Table 6, various combinations of arrival rate and corresponding service rates are considered to obtain the minimum total cost.

Table 6: Output Using Genetic Algorithm

| **Total Cost** | **Arrival Rate** | **Service Rate** |
|---|---|---|
| 74.7619 | 1 | 7 |
| 92.77778 | 1 | 9 |
| 92.77778 | 1 | 9 |
| 92.77778 | 1 | 9 |
| 92.77778 | 1 | 9 |
| 102.2222 | 1 | 10 |
| 102.2222 | 1 | 10 |
| 102.6984 | 2 | 9 |
| 102.6984 | 2 | 9 |
| 125.7143 | 3 | 10 |

## 5. Conclusion

The work is useful in managing internet packet arrivals where timing uncertainty affects routing decisions and congestion control in real-time communication systems. Also, in optimizing patient flow through emergency departments and clinics despite incomplete data on arrival patterns and variable treatment duration, neutrosophic sampling plans are useful to handle ambiguous defect rates and uncertain production timelines in complex manufacturing environments. Neutrosophic queueing systems provide the mathematical rigor needed to model complex environments where classical assumptions fail By explicitly modelling indeterminacy, organizations can make better-informed decisions with realistic uncertainty bounds rather than false precision Future innovations will unlock new applications wherever ambiguous data and uncertain timing challenge traditional optimization approaches The future of queueing theory lies not in eliminating uncertainty, but in embracing it with sophisticated mathematical tools that reflect the true complexity of the systems we seek to optimize. Calculating neutrosophic performance measures requires significantly more processing power than classical methods, demanding optimization algorithms Critical need for user-friendly simulation and analysis tools that make neutrosophic queueing accessible to practitioners without deep mathematical expertise Promising potential to combine neutrosophic models with machine learning for dynamic, adaptive decision-making that responds to uncertainty in real-time. It can be further refined and extended to encompass non-Markovian frameworks, including generalized and Erlang-distributed queueing models, as well as multi-server queueing systems.

## Statements and Declarations

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