



Generalized M -Closed Functions and Homeomorphisms in Fermatean Neutrosophic Topological Spaces

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ABSTRACT: In this paper, we undertake a detail study of Fermatean Neutrosophic generalized M -closed functions and Fermatean Neutrosophic generalized M -homeomorphisms by employing the framework of Fermatean Neutrosophic generalized M -open sets within Fermatean Neutrosophic topological spaces. The investigation not only introduces and formalizes these notions but also explores their fundamental characteristics, inter-relationships, and behavior under various topological operations. Furthermore, several essential properties, illustrative examples, and potential applications of these functions and mappings are presented to highlight their significance in the broader development of Fermatean Neutrosophic topology.

Keywords: Fermatean Neutrosophic generalized M -closed, Fermatean Neutrosophic generalized M -closed functions and Fermatean Neutrosophic generalized M -homeomorphism.

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1. Introduction

In 1965, Zadeh [21] introduced the concept of fuzzy sets to model ambiguity and uncertainty in real-world scenarios. Later, in 1986, Atanassov [1] extended this idea by proposing the concept of intuitionistic fuzzy sets (ifs's), which incorporated both membership and non-membership functions addressing a limitation in Zadeh's model that only considered the membership degree. However, in practical applications, the sum of membership and non-membership degrees can exceed one, indicating a need for more flexible models.

To address these challenges, Yager [20] introduced the Pythagorean fuzzy set (pfs), where the sum of the squares of the membership and non-membership degrees is constrained to be less than or equal to one. Building on this, Senapati and Yager [10] proposed the Fermatean fuzzy set ($\mathfrak{F}fs$), where the cube of the membership and non-membership degrees must sum to less than or equal to one. This extension provides greater flexibility and enhances the capability of managing uncertainty, making Fermatean fuzzy sets more efficient than ifs's and pfs's in decision-making and other real-life applications.

Smarandache [11] later introduced the concept of neutrosophic sets (NS's), representing a major advancement in the field of decision theory and beyond. Neutrosophic sets are built on the idea that any concept inherently involves degrees of truth (T), indeterminacy (I), and falsity (F). Unlike intuitionistic fuzzy sets where membership and non-membership are dependent neutrosophic sets allow these three components to be mutually independent, thereby offering greater modeling flexibility for incomplete, inconsistent, and indeterminate information.

This independence makes neutrosophic sets a powerful tool for representing uncertainty in a wide range of fields, including engineering, philosophy, economics, and information science. In particular, they

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support complex analyses involving both dependent and independent variables, contributing significantly to decision-making and strategic planning.

To leverage the strengths of both Fermatean fuzzy sets and neutrosophic sets, the Fermatean neutrosophic set was developed. This hybrid framework effectively handles uncertainty, imprecision, and indeterminacy in complex decision making contexts. For example, in evaluating a local restaurant, Fermatean fuzzy theory is applied to assess the importance of attributes such as quality, naturalness, freshness, taste, and presentation which are often subject to human hesitation and subjective judgment. By incorporating neutrosophic principles, the model better captures the vagueness and uncertainty in customer preferences.

The domain of local food service was chosen as a practical application due to the high degree of uncertainty in consumer decision making uncertainty that neutrosophic sets can represent more accurately than traditional methods.

From a topological perspective, Vadivel et al. [13] introduced the notion of δ -open sets within neutrosophic topological spaces. Prior to this, in 2008, Ekici [4] introduced e -open sets in general topology. Building on this foundation, Seenivasan et al. [9] in 2014 developed the concept of fuzzy e -open sets and their corresponding fuzzy e -continuity. Later, Vadivel et al. [3] extended these ideas into the realm of intuitionistic fuzzy topological spaces. Vadivel and his collaborators [14,5] have significantly contributed to the development of neutrosophic topology by investigating various classes of M -open sets in neutrosophic nano topological spaces as well as in neutrosophic soft topological spaces. More recently, their research has been extended to Fermatean neutrosophic settings, where they examined different types of open sets in Fermatean topological spaces [15,16,17], thereby advancing the theoretical foundations of both fuzzy and neutrosophic topology.

The primary purpose of this paper is to present a comprehensive discussion on Fermatean Neutrosophic generalized M -closed functions (Mcf) and Fermatean Neutrosophic generalized M -homeomorphisms ($M-Hom$) within the framework of Fermatean Neutrosophic generalized M -open sets. By employing these sets as the foundational tool, we systematically develop and analyze the structure of such functions and mappings, highlighting their role in extending classical topological concepts into the Fermatean Neutrosophic environment. In addition, the study aims to establish their essential properties, interconnections, and theoretical importance in the context of Fermatean Neutrosophic topological spaces.

2. Preliminaries

Definition 2.1 [10] Let X be a universe of discourse. A Fermatean fuzzy set ($\mathfrak{FF}s$) F in X is an object having the form $F = \{ \langle x, \alpha_F(x), \beta_F(x) \rangle : x \in X \}$ where $\alpha_F(x) : X \rightarrow [0, 1]$ and $\beta_F(x) : X \rightarrow [0, 1]$, including the condition $0 \leq (\alpha_F(x))^3 + (\beta_F(x))^3 \leq 1$, for all $x \in X$. The numbers $\alpha_F(x)$ and $\beta_F(x)$ denote, respectively, the degree of membership and the degree of non-membership of the element x in the set F . For any $\mathfrak{FF}s$ F and $x \in X$, $\pi_F(x) = \sqrt[3]{1 - [(\alpha_F(x))^3 - (\beta_F(x))^3]}$ is identified as the degree of indeterminacy of x to F . In the interest of simplicity, we shall mention the symbol $F = (\alpha_F, \beta_F)$ for the $\mathfrak{FF}s$ $F = \{ \langle x, \alpha_F(x), \beta_F(x) \rangle : x \in X \}$.

Definition 2.2 [8] Let X be a non-empty set. A neutrosophic set (briefly, Ns) L is an object having the form $L = \{ \langle x, \mu_L(x), \nu_L(x), \sigma_L(x) \rangle : x \in X \}$ where $\mu_L \rightarrow [0, 1]$ denote the degree of membership function, $\nu_L \rightarrow [0, 1]$ denote the degree of indeterminacy function and $\sigma_L \rightarrow [0, 1]$ denote the degree of non-membership function respectively of each element $x \in X$ to the set L and $0 \leq \mu_L(x) + \nu_L(x) + \sigma_L(x) \leq 3$ for each $x \in X$.

Definition 2.3 [7] A neutrosophic topology (briefly, Nt) on a non-empty set X is a family τ_N of neutrosophic subsets of X satisfying

- (i) $0_N, 1_N \in \tau_N$.
- (ii) $L_1 \cap L_2 \in \tau_N$ for any $L_1, L_2 \in \tau_N$.
- (iii) $\bigcup L_a \in \tau_N, \forall L_a : a \in A \subseteq \tau_N$.

Then (X, τ_N) is called a neutrosophic topological space (briefly, Nts) in X . The τ_N elements are called neutrosophic open sets (briefly, Nos) in X . A $Ns C$ is called a neutrosophic closed sets (briefly, Ncs) iff its complement C^c is Nos .

Definition 2.4 [12] Let X be a non-empty set. A Fermatean neutrosophic set (briefly, $\mathfrak{FN}s$) L is an object having the form $L = \{\langle x, \mu_L(x), \nu_L(x), \sigma_L(x) \rangle : x \in X\}$ where $\mu_L \rightarrow [0, 1]$ denote the degree of membership function, $\nu_L \rightarrow [0, 1]$ denote the degree of indeterminacy function and $\sigma_L \rightarrow [0, 1]$ denote the degree of non-membership function respectively of each element $x \in X$ to the set L such that $0 \leq (\mu_L(x))^3 + (\sigma_L(x))^3 \leq 1$ and $0 \leq (\nu_L(x))^3 \leq 1$. Then $0 \leq (\mu_L(x))^3 + (\nu_L(x))^3 + (\sigma_L(x))^3 \leq 2$ for all $x \in X$. Here $\mu_L(x)$ and $\sigma_L(x)$ are dependent components and $\nu_L(x)$ is an independent component.

The definitions of $1_{\mathfrak{FN}}$ and $0_{\mathfrak{FN}}$ that will be needed before proceeding to set operations will be given. In [7], possible definitions of $1_{\mathfrak{FN}}$ and $0_{\mathfrak{FN}}$ neutrosophic sets are given. In this paper, the theory will be constructed by defining $0_{\mathfrak{FN}}$ and $1_{\mathfrak{FN}}$ Fermatean neutrosophic sets in a single way. $0_{\mathfrak{FN}}$ and $1_{\mathfrak{FN}}$ are defined as $0_{\mathfrak{FN}} = \{\langle x, 0, 0, 1 \rangle : x \in X\}$ and $1_{\mathfrak{FN}} = \{\langle x, 1, 1, 0 \rangle : x \in X\}$ Now, the union, intersection and complement definitions necessary for the definition of the topological space will be given. These definitions are given in several different ways in classical neutrosophic spaces in [2]; to avoid confusion here, only one method will be given for sets with Fermatean structure, and this method is different from the method chosen in [7].

Definition 2.5 [12] Let X be a non-empty set & the $\mathfrak{FN}s$'s L & M in the form $L = \{\langle x, \mu_L(x), \nu_L(x), \sigma_L(x) \rangle : x \in X\}$, $M = \{\langle x, \mu_M(x), \nu_M(x), \sigma_M(x) \rangle : x \in X\}$, then

- (i) $0_{\mathfrak{FN}} = \langle x, 0, 0, 1 \rangle$ and $1_{\mathfrak{FN}} = \langle x, 1, 1, 0 \rangle$,
- (ii) $L \subseteq M$ iff $\mu_L(x) \leq \mu_M(x)$, $\nu_L(x) \leq \nu_M(x)$ & $\sigma_L(x) \geq \sigma_M(x) : x \in X$,
- (iii) $L = M$ iff $L \subseteq M$ and $M \subseteq L$,
- (iv) $1_{\mathfrak{FN}} - L = \{\langle x, \sigma_L(x), 1_{\mathfrak{FN}} - \nu_L(x), \mu_L(x) \rangle : x \in X\} = L^c$ or $C(L)$,
- (v) $L \cup M = \{\langle x, \max(\mu_L(x), \mu_M(x)), \max(\nu_L(x), \nu_M(x)), \min(\sigma_L(x), \sigma_M(x)) \rangle : x \in X\}$,
- (vi) $L \cap M = \{\langle x, \min(\mu_L(x), \mu_M(x)), \min(\nu_L(x), \nu_M(x)), \max(\sigma_L(x), \sigma_M(x)) \rangle : x \in X\}$.

Definition 2.6 [12] Let

$$A = \{\langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in X\}, \quad B = \{\langle x, T_B(x), I_B(x), F_B(x) \rangle : x \in X\}$$

be two Fermatean Neutrosophic Sets ($\mathfrak{FN}s$'s) on a universe X , where for all $x \in X$,

$$T_A(x), I_A(x), F_A(x), T_B(x), I_B(x), F_B(x) \in [0, 1]$$

and $T_A^3(x) + I_A^3(x) + F_A^3(x) \leq 1$, $T_B^3(x) + I_B^3(x) + F_B^3(x) \leq 1$.

Then the *difference* of A and B , denoted by $A - B$, is defined as a new $\mathfrak{FN}s$ C , where for each $x \in X$,

$$C(x) = A(x) - B(x) = \langle x, T_C(x), I_C(x), F_C(x) \rangle$$

with:

$$\begin{aligned} T_C(x) &= \max(0, T_A(x) - T_B(x)), \\ I_C(x) &= \max(1, I_A(x) + I_B(x)), \\ F_C(x) &= \min(1, F_A(x) + F_B(x)). \end{aligned}$$

To ensure that the Fermatean condition is preserved, if

$$T_C^3(x) + I_C^3(x) + F_C^3(x) > 1,$$

we normalize the components as follows:

$$T'_C(x) = \frac{T_C(x)}{M}, \quad I'_C(x) = \frac{I_C(x)}{M}, \quad F'_C(x) = \frac{F_C(x)}{M},$$

where

$$M = (T_C^3(x) + I_C^3(x) + F_C^3(x))^{1/3}.$$

Definition 2.7 [6] A Fermatean neutrosophic topology (briefly, $\mathfrak{F}\mathfrak{N}t$) on a non-empty set X is a family $\tau_{\mathfrak{F}\mathfrak{N}}$ of Fermatean neutrosophic subsets of X satisfying

- (i) $0_{\mathfrak{F}\mathfrak{N}}, 1_{\mathfrak{F}\mathfrak{N}} \in \tau_{\mathfrak{F}\mathfrak{N}}$,
- (ii) $L_1 \cap L_2 \in \tau_{\mathfrak{F}\mathfrak{N}}$ for any $L_1, L_2 \in \tau_{\mathfrak{F}\mathfrak{N}}$,
- (iii) $\bigcup L_a \in \tau_{\mathfrak{F}\mathfrak{N}}, \forall L_a : a \in A \subseteq \tau_{\mathfrak{F}\mathfrak{N}}$.

Then $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ is called a Fermatean neutrosophic topological space (briefly, $\mathfrak{F}\mathfrak{N}ts$) in X . The $\tau_{\mathfrak{F}\mathfrak{N}}$ elements are called Fermatean neutrosophic open sets (briefly, $\mathfrak{F}\mathfrak{N}os$) in X . A $\mathfrak{F}\mathfrak{N}s$ C is called a Fermatean neutrosophic closed sets (briefly, $\mathfrak{F}\mathfrak{N}cs$) iff its complement C^c is $\mathfrak{F}\mathfrak{N}os$.

Definition 2.8 [6] Let $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ be $\mathfrak{F}\mathfrak{N}ts$ on X and L be an $\mathfrak{F}\mathfrak{N}s$ on X , then the Fermatean neutrosophic interior of L (briefly, $\mathfrak{F}\mathfrak{N}int(L)$) and the Fermatean neutrosophic closure of L (briefly, $\mathfrak{F}\mathfrak{N}cl(L)$) are defined as

$$\mathfrak{F}\mathfrak{N}int(L) = \bigcup \{I : I \subseteq L \text{ \& } I \text{ is a } \mathfrak{F}\mathfrak{N}os \text{ in } X\}$$

$$\mathfrak{F}\mathfrak{N}cl(L) = \bigcap \{I : L \subseteq I \text{ \& } I \text{ is a } \mathfrak{F}\mathfrak{N}cs \text{ in } X\}.$$

Theorem 2.1 [6] Let L be an $\mathfrak{F}\mathfrak{N}s$ on X . In this case, the following four properties hold:

- (i) $\mathfrak{F}\mathfrak{N}cl(L)$ is a closed Fermatean neutrosophic set,
- (ii) $\mathfrak{F}\mathfrak{N}cl(1_{\mathfrak{F}\mathfrak{N}}) = 1_{\mathfrak{F}\mathfrak{N}}, \mathfrak{F}\mathfrak{N}cl(0_{\mathfrak{F}\mathfrak{N}}) = 0_{\mathfrak{F}\mathfrak{N}}$,
- (iii) $\mathfrak{F}\mathfrak{N}int(L)$ is an open Fermatean neutrosophic set.
- (iv) $\mathfrak{F}\mathfrak{N}int(1_{\mathfrak{F}\mathfrak{N}}) = 1_{\mathfrak{F}\mathfrak{N}}, \mathfrak{F}\mathfrak{N}int(0_{\mathfrak{F}\mathfrak{N}}) = 0_{\mathfrak{F}\mathfrak{N}}$.

Lemma 2.1 [6] For any Fermatean neutrosophic set A in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$, we have $C(\mathfrak{F}\mathfrak{N}int(A)) = \mathfrak{F}\mathfrak{N}cl(C(A))$ and $C(\mathfrak{F}\mathfrak{N}cl(A)) = \mathfrak{F}\mathfrak{N}int(C(A))$. Here $C(A)$ or \bar{A} denotes complement of A .

Definition 2.9 [19] Let $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ be an $\mathfrak{F}\mathfrak{N}ts$ and A be an $\mathfrak{F}\mathfrak{N}s$. Then A is said to be an Fermatean neutrosophic (i) regular open set ($\mathfrak{F}\mathfrak{N}ros$ in short) if $A = \mathfrak{F}\mathfrak{N}int(\mathfrak{F}\mathfrak{N}cl(A))$. (ii) regular closed set ($\mathfrak{F}\mathfrak{N}rcs$ in short) if $A = \mathfrak{F}\mathfrak{N}cl(\mathfrak{F}\mathfrak{N}int(A))$. By Lemma 2.1, it follows that A is an $\mathfrak{F}\mathfrak{N}ros$ iff \bar{A} is an $\mathfrak{F}\mathfrak{N}rcs$.

Definition 2.10 [19] Let $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ be an $\mathfrak{F}\mathfrak{N}ts$ and $A = \{ \langle a, \mu_A(a), \nu_A(a), \sigma_A(a) \rangle \mid a \in X \}$ be an $\mathfrak{F}\mathfrak{N}s$ in X . Then the δ -interior and the δ -closure of A are denoted by $\mathfrak{F}\mathfrak{N}\delta int(A)$ and $\mathfrak{F}\mathfrak{N}\delta cl(A)$ and are defined as follows. $\mathfrak{F}\mathfrak{N}\delta int(A) = \cup \{G \mid G \text{ is an } \mathfrak{F}\mathfrak{N}ros \text{ and } G \subseteq A\}$, $\mathfrak{F}\mathfrak{N}\delta cl(A) = \cap \{K \mid K \text{ is an } \mathfrak{F}\mathfrak{N}rcs \text{ and } A \subseteq K\}$.

Definition 2.11 [19] Let $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ be an $\mathfrak{F}\mathfrak{N}ts$ and $A = \{ \langle a, \mu_A(a), \nu_A(a), \sigma_A(a) \rangle \mid a \in X \}$ be an $\mathfrak{F}\mathfrak{N}s$ in X . A set A is said to be $\mathfrak{F}\mathfrak{N}$

- (i) δ -open set (briefly, $\mathfrak{F}\mathfrak{N}\delta os$) if $A = \mathfrak{F}\mathfrak{N}\delta int(A)$,
- (ii) δ -pre open set (briefly, $\mathfrak{F}\mathfrak{N}\delta Pos$) if $A \subseteq \mathfrak{F}\mathfrak{N}int(\mathfrak{F}\mathfrak{N}\delta cl(A))$.
- (iii) δ -semi open set (briefly, $\mathfrak{F}\mathfrak{N}\delta Sos$) if $A \subseteq \mathfrak{F}\mathfrak{N}cl(\mathfrak{F}\mathfrak{N}\delta int(A))$.
- (iv) δ (resp. δ -pre and δ -semi) dense if $\mathfrak{F}\mathfrak{N}\delta cl(A)$ (resp. $\mathfrak{F}\mathfrak{N}\delta Pcl(A)$, and $\mathfrak{F}\mathfrak{N}\delta Scl(A)$) = $1_{\mathfrak{F}\mathfrak{N}}$.

The complement of an $\mathfrak{F}\mathfrak{N}\delta os$ (resp. $\mathfrak{F}\mathfrak{N}\delta Pos$ and $\mathfrak{F}\mathfrak{N}\delta Sos$) is called an $\mathfrak{F}\mathfrak{N}\delta$ (resp. $\mathfrak{F}\mathfrak{N}\delta P$ and $\mathfrak{F}\mathfrak{N}\delta S$) closed set (briefly, $\mathfrak{F}\mathfrak{N}\delta cs$ (resp. $\mathfrak{F}\mathfrak{N}\delta Pcs$ and $\mathfrak{F}\mathfrak{N}\delta Scs$)) in X .

The family of all $\mathfrak{F}\mathfrak{N}\delta os$ (resp. $\mathfrak{F}\mathfrak{N}\delta cs$, $\mathfrak{F}\mathfrak{N}\delta Pos$, $\mathfrak{F}\mathfrak{N}\delta Pcs$, $\mathfrak{F}\mathfrak{N}\delta Sos$ and $\mathfrak{F}\mathfrak{N}\delta Scs$) of X is denoted by $\mathfrak{F}\mathfrak{N}\delta OS(X)$, (resp. $\mathfrak{F}\mathfrak{N}\delta CS(X)$, $\mathfrak{F}\mathfrak{N}\delta POS(X)$, $\mathfrak{F}\mathfrak{N}\delta PCS(X)$, $\mathfrak{F}\mathfrak{N}\delta SOS(X)$ and $\mathfrak{F}\mathfrak{N}\delta SCS(X)$).

Definition 2.12 [19] Let $(X, \tau_{\mathfrak{FN}})$ be an \mathfrak{FNts} and $A = \{ \langle a, \mu_A(a), \nu_A(a), \sigma_A(a) \rangle \mid a \in X \}$ be an \mathfrak{FNs} in X . Then the $\mathfrak{FN}\delta$ (resp. $\mathfrak{FN}\delta$ -pre and $\mathfrak{FN}\delta$ -semi)-interior and the $\mathfrak{FN}\delta$ (resp. $\mathfrak{FN}\delta$ -pre and $\mathfrak{FN}\delta$ -semi)-closure of A are denoted by $\mathfrak{FN}\delta int(A)$ (resp. $\mathfrak{FN}\delta P int(A)$, $\mathfrak{FN}\delta S int(A)$) and the $\mathfrak{FN}\delta cl(A)$ (resp. $\mathfrak{FN}\delta P cl(A)$ and $\mathfrak{FN}\delta S cl(A)$) and are defined as follows:

$\mathfrak{FN}\delta int(A)$ (resp. $\mathfrak{FN}\delta P int(A)$ and $\mathfrak{FN}\delta S int(A)$) = $\cup \{ G \mid G \text{ in a } \mathfrak{FN}\delta os \text{ (resp. } \mathfrak{FN}\delta Pos \text{ and } \mathfrak{FN}\delta Sos) \text{ and } G \subseteq A \}$ and $\mathfrak{FN}\delta cl(A)$ (resp. $\mathfrak{FN}\delta P cl(A)$ and $\mathfrak{FN}\delta S cl(A)$) = $\cap \{ K \mid K \text{ is an } \mathfrak{FN}\delta cs \text{ (resp. } \mathfrak{FN}\delta P cs \text{ and } \mathfrak{FN}\delta S cs) \text{ and } A \subseteq K \}$.

3. Fermatean Neutrosophic Generalized M -Closed Functions

In this section, we study the concepts of Fermatean neutrosophic generalized (resp. θ , θS , δP and M)-closed functions and some of their basic properties.

Definition 3.1 Let $(X, \tau_{\mathfrak{FN}})$ be a \mathfrak{FNts} and S be a \mathfrak{FNs} in X . A set S is said to be \mathfrak{FN}

- (i) θ -interior of S (briefly, $\mathfrak{FN}\theta int(S)$) is defined by $\mathfrak{FN}\theta int(S) = \cup \{ \mathfrak{FN} int(T) : T \subseteq S \text{ \& } T \text{ is a } \mathfrak{FNcs} \text{ in } X \}$.
- (ii) θ -open set (briefly, $\mathfrak{FN}\theta os$) if $S = \mathfrak{FN}\theta int(S)$.
- (iii) θ -semi open set (briefly, $\mathfrak{FN}\theta Sos$) if $S \subseteq \mathfrak{FN} cl(\mathfrak{FN}\theta int(S))$.
- (iv) M -open set (briefly, $\mathfrak{FN}Mos$) if $S \subseteq \mathfrak{FN} cl(\mathfrak{FN}\theta int(S)) \cup \mathfrak{FN} int(\mathfrak{FN}\delta cl(S))$.

The complement of a $\mathfrak{FN}Mos$ (resp. $\mathfrak{FN}\theta os$ & $\mathfrak{FN}\theta Sos$) is called an $\mathfrak{FN}M$ (resp. $\mathfrak{FN}\theta$ & $\mathfrak{FN}\theta S$) closed set (briefly, $\mathfrak{FN}Mcs$ (resp. $\mathfrak{FN}\theta cs$ & $\mathfrak{FN}\theta S cs$)) in X .

The family of all $\mathfrak{FN}\theta os$ (resp. $\mathfrak{FN}\theta cs$, $\mathfrak{FN}\theta Sos$, $\mathfrak{FN}\theta S cs$, $\mathfrak{FN}Mos$ and $\mathfrak{FN}Mcs$) of X is denoted by $\mathfrak{FN}\theta OS(X)$ (resp. $\mathfrak{FN}\theta CS(X)$, $\mathfrak{FN}\theta SOS(X)$, $\mathfrak{FN}\theta SCS(X)$, $\mathfrak{FN}MOS(X)$ and $\mathfrak{FN}MCS(X)$).

Definition 3.2 Let $(X, \tau_{\mathfrak{FN}})$ be a \mathfrak{FNts} and S be a \mathfrak{FNs} in X . Then the

- (i) $\mathfrak{FN}M$ -interior (resp. $\mathfrak{FN}\theta$ -interior and $\mathfrak{FN}\theta$ -semi interior) of S (briefly, $\mathfrak{FN}M int(S)$ (resp. $\mathfrak{FN}\theta int(S)$, $\mathfrak{FN}\theta S int(S)$)) is defined by $\mathfrak{FN}M int(S)$ (resp. $\mathfrak{FN}\theta int(S)$ and $\mathfrak{FN}\theta S int(S)$) = $\cup \{ T : T \subseteq S \text{ and } T \text{ is a } \mathfrak{FN}Mos \text{ (resp. } \mathfrak{FN}\theta os \text{ and } \mathfrak{FN}\theta Sos) \text{ in } X \}$.
- (ii) $\mathfrak{FN}M$ -closure (resp. $\mathfrak{FN}\theta$ -closure and $\mathfrak{FN}\theta$ -semi closure) of S (briefly, $\mathfrak{FN}M cl(S)$ (resp. $\mathfrak{FN}\theta cl(S)$ & $\mathfrak{FN}\theta S cl(S)$)) is defined by $\mathfrak{FN}M cl(S)$ (resp. $\mathfrak{FN}\theta cl(S)$ and $\mathfrak{FN}\theta S cl(S)$) = $\cap \{ T : S \subseteq T \text{ and } T \text{ is a } \mathfrak{FN}Mcs \text{ (resp. } \mathfrak{FN}\theta cs \text{ and } \mathfrak{FN}\theta S cs) \text{ in } X \}$.

Definition 3.3 Let $(X, \tau_{\mathfrak{FN}})$ be an \mathfrak{FNts} and K be an \mathfrak{FNs} . Then K is said to be an Fermatean neutrosophic generalized

- (i) closed, (briefly, $\mathfrak{FN}gc$) set if $\mathfrak{FN} cl(K) \subseteq V$ whenever $K \subseteq V$ and V is $\mathfrak{FN}o$ in X ,
- (ii) θ -closed, (briefly, $\mathfrak{FN}g\theta c$) set if $\mathfrak{FN}\theta cl(K) \subseteq V$ whenever $K \subseteq V$ and V is $\mathfrak{FN}o$ in X ,
- (iii) θS -closed, (briefly, $\mathfrak{FN}g\theta Sc$) set if $\mathfrak{FN}\theta S cl(K) \subseteq V$ whenever $K \subseteq V$ and V is $\mathfrak{FN}o$ in X ,
- (iv) δP -closed, (briefly, $\mathfrak{FN}g\delta Pc$) set if $\mathfrak{FN}\delta P cl(K) \subseteq V$ whenever $K \subseteq V$ and V is $\mathfrak{FN}o$ in X ,
- (v) M -closed, (briefly, $\mathfrak{FN}gMc$) set if $\mathfrak{FN}M cl(K) \subseteq V$ whenever $K \subseteq V$ and V is $\mathfrak{FN}o$ in X .

The collection of all $\mathfrak{FN}gc$ (resp. $\mathfrak{FN}g\theta c$, $\mathfrak{FN}g\theta Sc$, $\mathfrak{FN}g\delta Pc$ and $\mathfrak{FN}gMc$) sets of X is denoted by $\mathfrak{FN}GC(X)$ (resp. $\mathfrak{FN}G\theta C(X)$, $\mathfrak{FN}G\theta SC(X)$, $\mathfrak{FN}G\delta PC(X)$ and $\mathfrak{FN}GM C(X)$).

Definition 3.4 Let $(X, \tau_{\mathfrak{FN}})$ be an \mathfrak{FNts} and K be an \mathfrak{FNs} . Then the Fermatean neutrosophic generalized (resp. θ , θS , δP and M)-closure of K is the intersection of all $\mathfrak{FN}gc$ (resp. $\mathfrak{FN}g\theta c$, $\mathfrak{FN}g\theta Sc$, $\mathfrak{FN}g\delta Pc$ and $\mathfrak{FN}gMc$) sets containing K and denoted by $\mathfrak{FN}gcl(K)$ (resp. $\mathfrak{FN}g\theta cl(K)$, $\mathfrak{FN}g\theta S cl(K)$, $\mathfrak{FN}g\delta P cl(K)$ and $\mathfrak{FN}gM cl(K)$).

Definition 3.5 A map $f : (X, \tau_{\mathfrak{FN}}) \rightarrow (Y, \sigma_{\mathfrak{FN}})$ is called Fermatean neutrosophic generalized

- (i) continuous (briefly, $\mathfrak{N}gCts$) if the inverse image of every $\mathfrak{N}cs$ in $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is a $\mathfrak{N}gcs$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$,
- (ii) θ -continuous (briefly, $\mathfrak{N}g\theta Cts$) if the inverse image of every $\mathfrak{N}cs$ in $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is a $\mathfrak{N}g\theta cs$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$,
- (iii) $\theta\mathcal{S}$ -continuous (briefly, $\mathfrak{N}g\theta\mathcal{S}Cts$) if the inverse image of every $\mathfrak{N}cs$ in $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is a $\mathfrak{N}g\theta\mathcal{S}cs$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$,
- (iv) $\delta\mathcal{P}$ -continuous (briefly, $\mathfrak{N}g\delta\mathcal{P}Cts$) if the inverse image of every $\mathfrak{N}cs$ in $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is a $\mathfrak{N}g\delta\mathcal{P}cs$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$,
- (v) M -continuous (briefly, $\mathfrak{N}gMCts$) if the inverse image of every $\mathfrak{N}cs$ in $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is a $\mathfrak{N}gMcs$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$.

Definition 3.6 A map $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is called Fermatean neutrosophic generalized

- (i) closed function (briefly, $\mathfrak{N}gcf$) if the image of every $\mathfrak{N}cs$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ is a $\mathfrak{N}gcs$ in $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$,
- (ii) θ -closed function (briefly, $\mathfrak{N}g\theta cf$) if the image of every $\mathfrak{N}cs$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ is a $\mathfrak{N}g\theta cs$ in $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$,
- (iii) $\theta\mathcal{S}$ -closed function (briefly, $\mathfrak{N}g\theta\mathcal{S}cf$) if the image of every $\mathfrak{N}cs$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ is a $\mathfrak{N}g\theta\mathcal{S}cs$ in $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$,
- (iv) $\delta\mathcal{P}$ -closed function (briefly, $\mathfrak{N}g\delta\mathcal{P}cf$) if the image of every $\mathfrak{N}cs$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ is a $\mathfrak{N}g\delta\mathcal{P}cs$ in $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$,
- (v) M -closed function (briefly, $\mathfrak{N}gMcf$) if the image of every $\mathfrak{N}cs$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ is a $\mathfrak{N}gMcs$ in $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$.

Theorem 3.1 Let $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ be a mapping. Then

- (i) Every $\mathfrak{N}g\theta cf$ is $\mathfrak{N}gcf$,
- (ii) Every $\mathfrak{N}g\theta cf$ is $\mathfrak{N}g\delta\mathcal{P}cf$,
- (iii) Every $\mathfrak{N}g\theta cf$ is $\mathfrak{N}g\theta\mathcal{S}cf$,
- (iv) Every $\mathfrak{N}g\delta\mathcal{P}cf$ is $\mathfrak{N}gMcf$,
- (v) Every $\mathfrak{N}g\theta\mathcal{S}cf$ is $\mathfrak{N}gMcf$.

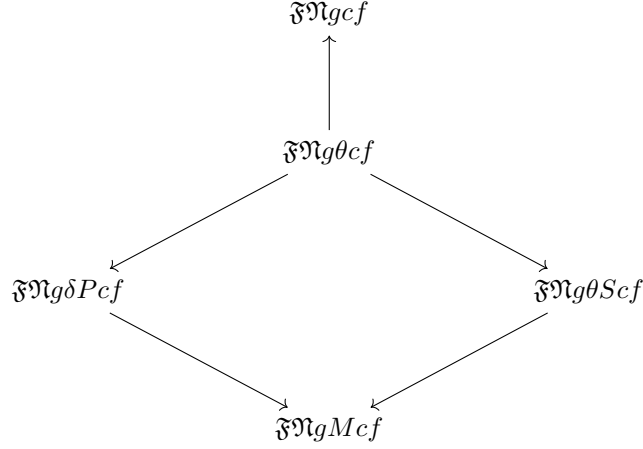
But the converse does not true.

Proof: We prove only (iv) and (v), the others are similar.

- (iv) Let λ be a $\mathfrak{N}cs$ in X . Since f is $\mathfrak{N}g\delta\mathcal{P}cf$, $f(\lambda)$ is a $\mathfrak{N}g\delta\mathcal{P}cs$ in Y . Since every $\mathfrak{N}g\delta\mathcal{P}cs$ is a $\mathfrak{N}gMcs$, $f(\lambda)$ is a $\mathfrak{N}gMcs$ in Y . Hence f is a $\mathfrak{N}gMcf$.
- (v) Let λ be a $\mathfrak{N}cs$ in X . Since f is $\mathfrak{N}g\theta\mathcal{S}cf$, $f(\lambda)$ is a $\mathfrak{N}g\theta\mathcal{S}cs$ in Y . Since every $\mathfrak{N}g\theta\mathcal{S}cs$ is a $\mathfrak{N}gMcs$, $f(\lambda)$ is a $\mathfrak{N}gMcs$ in Y . Hence f is a $\mathfrak{N}gMcf$.

□

Remark 3.1 *The implication diagram is obtained and none of the implications in the diagram are reversible as seen from the example.*



Note: $A \rightarrow B$ denotes A implies B , but not conversely.

Example 3.1 *Let $X = Y = \{a, b\}$ and the $\mathfrak{FN}s$'s A_1, A_2 and A_3 are defined as*

$$\begin{aligned}
 \mu_{A_1}(a) &= 0.8, \nu_{A_1}(a) = 0.8, \sigma_{A_1}(a) = 0.1, \\
 \mu_{A_1}(b) &= 0.9, \nu_{A_1}(b) = 0.8, \sigma_{A_1}(b) = 0.2; \\
 \mu_{A_2}(a) &= 0.6, \nu_{A_2}(a) = 0.7, \sigma_{A_2}(a) = 0.2, \\
 \mu_{A_2}(b) &= 0.5, \nu_{A_2}(b) = 0.7, \sigma_{A_2}(b) = 0.6; \\
 \mu_{A_3}(a) &= 0.1, \nu_{A_3}(a) = 0.2, \sigma_{A_3}(a) = 0.7, \\
 \mu_{A_3}(b) &= 0.2, \nu_{A_3}(b) = 0.3, \sigma_{A_3}(b) = 0.8.
 \end{aligned}$$

Let $\tau_{\mathfrak{FN}} = \sigma_{\mathfrak{FN}} = \{0_{\mathfrak{FN}}, 1_{\mathfrak{FN}}, A_1, A_2, A_3\}$ be a $\mathfrak{FN}s$ on X . Let $f : (X, \tau_{\mathfrak{FN}}) \rightarrow (Y, \sigma_{\mathfrak{FN}})$ be an identity mapping, then f is (i) \mathfrak{FNgcf} but not $\mathfrak{FNg\theta cf}$, the set A_1^c is a $\mathfrak{FN}s$ in X but $f(A_1^c)$ is not $\mathfrak{FNg\theta cs}$ in Y , (ii) \mathfrak{FNgMcf} but not $\mathfrak{FNg\theta Scf}$, the set A_2^c is a $\mathfrak{FN}s$ in X but $f(A_2^c)$ is not $\mathfrak{FNg\theta Scs}$ in Y , (iii) $\mathfrak{FNg\delta Pcf}$ but not $\mathfrak{FNg\theta cf}$, the set A_3^c is a $\mathfrak{FN}s$ in X but $f(A_3^c)$ is not $\mathfrak{FNg\theta cs}$ in Y .

Example 3.2 *Let $X = Y = \{a, b\}$ and the $\mathfrak{FN}s$'s A_1, A_2 and A_3 are defined as*

$$\begin{aligned}
 \mu_{A_1}(a) &= 0.8, \nu_{A_1}(a) = 0.8, \sigma_{A_1}(a) = 0.1, \\
 \mu_{A_1}(b) &= 0.9, \nu_{A_1}(b) = 0.8, \sigma_{A_1}(b) = 0.2; \\
 \mu_{A_2}(a) &= 0.6, \nu_{A_2}(a) = 0.7, \sigma_{A_2}(a) = 0.2, \\
 \mu_{A_2}(b) &= 0.5, \nu_{A_2}(b) = 0.7, \sigma_{A_2}(b) = 0.6; \\
 \mu_{A_3}(a) &= 0.1, \nu_{A_3}(a) = 0.2, \sigma_{A_3}(a) = 0.7, \\
 \mu_{A_3}(b) &= 0.2, \nu_{A_3}(b) = 0.3, \sigma_{A_3}(b) = 0.8.
 \end{aligned}$$

Let $\sigma_{\mathfrak{FN}} = \{0_{\mathfrak{FN}}, 1_{\mathfrak{FN}}, A_1, A_2, A_3\}$, $\tau_{\mathfrak{FN}} = \{0_{\mathfrak{FN}}, 1_{\mathfrak{FN}}, A_1\}$ be a $\mathfrak{FN}s$ on X . Let $f : (X, \tau_{\mathfrak{FN}}) \rightarrow (Y, \sigma_{\mathfrak{FN}})$ be an identity mapping, then f is $\mathfrak{FNg\theta Scf}$ but not $\mathfrak{FNg\theta cf}$, the set A_1^c is a $\mathfrak{FN}s$ in X but $f(A_1^c)$ is not $\mathfrak{FNg\theta cs}$ in Y .

Example 3.3 *Let $X = Y = \{a, b\}$ and the $\mathfrak{FN}s$'s A_1, A_2, A_3 and A_4 are defined as*

$$\begin{aligned}
 \mu_{A_1}(a) &= 0.8, \nu_{A_1}(a) = 0.8, \sigma_{A_1}(a) = 0.1, \\
 \mu_{A_1}(b) &= 0.9, \nu_{A_1}(b) = 0.8, \sigma_{A_1}(b) = 0.2; \\
 \mu_{A_2}(a) &= 0.6, \nu_{A_2}(a) = 0.7, \sigma_{A_2}(a) = 0.2, \\
 \mu_{A_2}(b) &= 0.5, \nu_{A_2}(b) = 0.7, \sigma_{A_2}(b) = 0.6; \\
 \mu_{A_3}(a) &= 0.1, \nu_{A_3}(a) = 0.2, \sigma_{A_3}(a) = 0.7, \\
 \mu_{A_3}(b) &= 0.2, \nu_{A_3}(b) = 0.3, \sigma_{A_3}(b) = 0.8;
 \end{aligned}$$

$$\begin{aligned}\mu_{A_4}(a) &= 0.2, \nu_{A_4}(a) = 0.3, \sigma_{A_4}(a) = 0.6, \\ \mu_{A_4}(b) &= 0.6, \nu_{A_4}(b) = 0.3, \sigma_{A_4}(b) = 0.5.\end{aligned}$$

Let $\sigma_{\mathfrak{F}\mathfrak{N}} = \{0_{\mathfrak{F}\mathfrak{N}}, 1_{\mathfrak{F}\mathfrak{N}}, A_1, A_2, A_3\}$, $\tau_{\mathfrak{F}\mathfrak{N}} = \{0_{\mathfrak{F}\mathfrak{N}}, 1_{\mathfrak{F}\mathfrak{N}}, A_4\}$ be a $\mathfrak{F}\mathfrak{N}$ ts on X . Let $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ be an identity mapping, then f is $\mathfrak{F}\mathfrak{N}gMcf$ but not $\mathfrak{F}\mathfrak{N}g\delta Pcf$, the set A_4^c is a $\mathfrak{F}\mathfrak{N}cs$ in X but $f(A_4^c)$ is not $\mathfrak{F}\mathfrak{N}g\delta Pcs$ in Y .

Theorem 3.2 A map $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is $\mathfrak{F}\mathfrak{N}gMcf$ iff the image of each $\mathfrak{F}\mathfrak{N}os$ in X is $\mathfrak{F}\mathfrak{N}gMos$ in Y .

Proof: Let λ be a $\mathfrak{F}\mathfrak{N}os$ in X . This implies λ^c is $\mathfrak{F}\mathfrak{N}cs$ in X . Since f is $\mathfrak{F}\mathfrak{N}gMcf$, $f(\lambda^c)$ is $\mathfrak{F}\mathfrak{N}gMcs$ in Y . Since $f(\lambda^c) = (f(\lambda))^c$, $f(\lambda)$ is a $\mathfrak{F}\mathfrak{N}gMos$ in Y .

Conversely, let λ be a $\mathfrak{F}\mathfrak{N}os$ in X . Then λ^c is a $\mathfrak{F}\mathfrak{N}cs$ in X . By hypothesis $f(\lambda^c)$ is $\mathfrak{F}\mathfrak{N}gMcs$ in Y . Since $f(\lambda^c) = (f(\lambda))^c$, $(f(\lambda))^c$ is a $\mathfrak{F}\mathfrak{N}gMcs$ in Y . Therefore $f(\lambda)$ is a $\mathfrak{F}\mathfrak{N}gMos$ in Y . Hence f is $\mathfrak{F}\mathfrak{N}gMcf$. \square

Theorem 3.3 A function $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is a $\mathfrak{F}\mathfrak{N}gMcf$ iff $\mathfrak{F}\mathfrak{N}gMcl(f(\lambda)) \subseteq f(\mathfrak{F}\mathfrak{N}cl(\lambda))$ for every $\mathfrak{F}\mathfrak{N}s$ λ in X .

Proof: Suppose f is $\mathfrak{F}\mathfrak{N}gMcf$ and $\lambda \subseteq 1_{\mathfrak{F}\mathfrak{N}}$. Then $f(\mathfrak{F}\mathfrak{N}cl(\lambda))$ is $\mathfrak{F}\mathfrak{N}gMcf$ in Y . Since $f(\lambda) \subseteq f(\mathfrak{F}\mathfrak{N}cl(\lambda))$, we get $\mathfrak{F}\mathfrak{N}gMcl(f(\lambda)) \subseteq \mathfrak{F}\mathfrak{N}gMcl(f(\mathfrak{F}\mathfrak{N}cl(\lambda))) = f(\mathfrak{F}\mathfrak{N}cl(\lambda))$. Hence $\mathfrak{F}\mathfrak{N}gMcl(f(\lambda)) \subseteq f(\mathfrak{F}\mathfrak{N}cl(\lambda))$.

Conversely, let μ is any $\mathfrak{F}\mathfrak{N}c$ set in X . Then $\mathfrak{F}\mathfrak{N}cl(\mu) = \mu$. Therefore $f(\mu) = f(\mathfrak{F}\mathfrak{N}cl(\mu))$. By hypothesis $\mathfrak{F}\mathfrak{N}gMcl(f(\mu)) \subseteq f(\mathfrak{F}\mathfrak{N}cl(\mu)) = f(\mu)$, implies $\mathfrak{F}\mathfrak{N}gMcl(f(\mu)) \subseteq f(\mu)$. But $f(\mu) \subseteq \mathfrak{F}\mathfrak{N}gMcl(f(\mu))$ is always true. This shows $\mathfrak{F}\mathfrak{N}gMcl(f(\mu)) = f(\mu)$. Therefore $f(\mu)$ is $\mathfrak{F}\mathfrak{N}gMcf$ in Y and hence f is $\mathfrak{F}\mathfrak{N}gMcf$. \square

Theorem 3.4 If f is $\mathfrak{F}\mathfrak{N}gMcf$, then $f(\mathfrak{F}\mathfrak{N}int(\mu)) \subseteq \mathfrak{F}\mathfrak{N}gMint(f(\mu))$, for all $\mathfrak{F}\mathfrak{N}cs$ μ in X .

Proof: If f is $\mathfrak{F}\mathfrak{N}gMcf$ and $\mu \in \tau_{\mathfrak{F}\mathfrak{N}}$. $\mathfrak{F}\mathfrak{N}int(\mu)$ is $\mathfrak{F}\mathfrak{N}o$ in X and hence, $f(\mathfrak{F}\mathfrak{N}int(\mu))$ is $\mathfrak{F}\mathfrak{N}gMo$ in Y . Therefore $\mathfrak{F}\mathfrak{N}gMint(f(\mathfrak{F}\mathfrak{N}gMint(\mu))) = f(\mathfrak{F}\mathfrak{N}int(\mu))$. Also, $\mathfrak{F}\mathfrak{N}int(\mu) \subseteq \mu$, implies that $f(\mathfrak{F}\mathfrak{N}int(\mu)) \subseteq f(\mu)$. Therefore $\mathfrak{F}\mathfrak{N}gMint(f(\mathfrak{F}\mathfrak{N}int(\mu))) \subseteq \mathfrak{F}\mathfrak{N}gMint(f(\mu))$. That is $f(\mathfrak{F}\mathfrak{N}int(\mu)) \subseteq \mathfrak{F}\mathfrak{N}gMint(f(\mu))$.

Conversely, let $f(\mathfrak{F}\mathfrak{N}int(\mu)) \subseteq \mathfrak{F}\mathfrak{N}gMint(f(\mu))$ for all subset μ of X . If μ is $\mathfrak{F}\mathfrak{N}o$ in X , then $\mathfrak{F}\mathfrak{N}int(\mu) = \mu$. By assumption, $f(\mathfrak{F}\mathfrak{N}int(\mu)) \subseteq \mathfrak{F}\mathfrak{N}gMint(f(\mu))$. Thus $f(\mu) \subseteq \mathfrak{F}\mathfrak{N}gMint(f(\mu))$. But $\mathfrak{F}\mathfrak{N}gMint(f(\mu)) \subseteq f(\mu)$. Therefore $\mathfrak{F}\mathfrak{N}gMint(f(\mu)) = f(\mu)$. That is, $f(\mu)$ is $\mathfrak{F}\mathfrak{N}gMo$ in Y , for all $\mathfrak{F}\mathfrak{N}os$ μ in X . Therefore f is $\mathfrak{F}\mathfrak{N}gMcf$ on Y . \square

Remark 3.2 If f is $\mathfrak{F}\mathfrak{N}gMcf$, then $\mathfrak{F}\mathfrak{N}gMint(f(\mu))$ is not necessarily equal to $f(\mathfrak{F}\mathfrak{N}int(\mu))$ where μ in X .

Example 3.4 In Example 3.1, f is a $\mathfrak{F}\mathfrak{N}gMcf$. Let $\eta = A_1^c$. Then $f(\mathfrak{F}\mathfrak{N}int(\eta)) = 0_{\mathfrak{F}\mathfrak{N}}$. But $\mathfrak{F}\mathfrak{N}gMint(f(\eta)) = A_3$. Thus $f(\mathfrak{F}\mathfrak{N}int(\eta)) \neq \mathfrak{F}\mathfrak{N}gMint(f(\eta))$.

Remark 3.3 The composition of two $\mathfrak{F}\mathfrak{N}gMcf$ functions need not be $\mathfrak{F}\mathfrak{N}gMcf$ as seen from the following example.

Example 3.5 Let $X = Y = Z = \{a, b\}$ and the $\mathfrak{F}\mathfrak{N}s$'s $A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4$ and C_1 are defined as

$$\begin{aligned}\mu_{A_1}(a) &= 0.2, \nu_{A_1}(a) = 0.5, \sigma_{A_1}(a) = 0.8, \\ \mu_{A_1}(b) &= 0.4, \nu_{A_1}(b) = 0.5, \sigma_{A_1}(b) = 0.6; \\ \mu_{A_2}(a) &= 0.1, \nu_{A_2}(a) = 0.5, \sigma_{A_2}(a) = 0.9, \\ \mu_{A_2}(b) &= 0.3, \nu_{A_2}(b) = 0.5, \sigma_{A_2}(b) = 0.7; \\ \mu_{A_3}(a) &= 0.9, \nu_{A_3}(a) = 0.5, \sigma_{A_3}(a) = 0.1, \\ \mu_{A_3}(b) &= 0.7, \nu_{A_3}(b) = 0.5, \sigma_{A_3}(b) = 0.3; \\ \mu_{A_4}(a) &= 0.2, \nu_{A_4}(a) = 0.5, \sigma_{A_4}(a) = 0.8,\end{aligned}$$

$$\begin{aligned}
\mu_{A_4}(b) &= 0.3, \nu_{A_4}(b) = 0.5, \sigma_{A_4}(b) = 0.7; \\
\mu_{B_1}(a) &= 0.6, \nu_{B_1}(a) = 0.5, \sigma_{B_1}(a) = 0.4, \\
\mu_{B_1}(b) &= 0.5, \nu_{B_1}(b) = 0.5, \sigma_{B_1}(b) = 0.5; \\
\mu_{B_2}(a) &= 0.4, \nu_{B_2}(a) = 0.5, \sigma_{B_2}(a) = 0.6, \\
\mu_{B_2}(b) &= 0.4, \nu_{B_2}(b) = 0.5, \sigma_{B_2}(b) = 0.6; \\
\mu_{B_3}(a) &= 0.3, \nu_{B_3}(a) = 0.5, \sigma_{B_3}(a) = 0.7, \\
\mu_{B_3}(b) &= 0.4, \nu_{B_3}(b) = 0.5, \sigma_{B_3}(b) = 0.6; \\
\mu_{B_4}(a) &= 0.6, \nu_{B_4}(a) = 0.5, \sigma_{B_4}(a) = 0.4, \\
\mu_{B_4}(b) &= 0.6, \nu_{B_4}(b) = 0.5, \sigma_{B_4}(b) = 0.4; \\
\mu_{C_1}(a) &= 0.8, \nu_{C_1}(a) = 0.5, \sigma_{C_1}(a) = 0.2, \\
\mu_{C_1}(b) &= 0.7, \nu_{C_1}(b) = 0.5, \sigma_{C_1}(b) = 0.3.
\end{aligned}$$

Then we have $\tau_{\mathfrak{F}\mathfrak{N}} = \{0_{\mathfrak{F}\mathfrak{N}}, A_1, A_2, A_3, A_4, 1_{\mathfrak{F}\mathfrak{N}}\}$, $\sigma_{\mathfrak{F}\mathfrak{N}} = \{0_{\mathfrak{F}\mathfrak{N}}, B_1, B_2, B_3, B_4, 1_{\mathfrak{F}\mathfrak{N}}\}$ and $\eta_{\mathfrak{F}\mathfrak{N}} = \{0_{\mathfrak{F}\mathfrak{N}}, C_1, 1_{\mathfrak{F}\mathfrak{N}}\}$. Then, the identity mappings $f : (X, \eta_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ and $g : (Y, \sigma_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Z, \tau_{\mathfrak{F}\mathfrak{N}})$ are $\mathfrak{F}\mathfrak{N}gMcf$ but the composition $f \circ g$ is not $\mathfrak{F}\mathfrak{N}gMcf$, the set C_1^c is $\mathfrak{F}\mathfrak{N}c$ in X but $(f \circ g)(C_1^c) = C_1^c$ is not $\mathfrak{F}\mathfrak{N}gMcs$ in Z .

Theorem 3.5 A function $f : (X, \tau_X) \rightarrow (Y, \sigma_Y)$ is $\mathfrak{F}\mathfrak{N}gMcf$ iff for each $\mathfrak{F}\mathfrak{N}s$ S of Y and for each $\mathfrak{F}\mathfrak{N}o$ set G in τ_X containing $f^{-1}(S)$, there exists a $\mathfrak{F}\mathfrak{N}gMo$ set H of σ_Y such that $S \subseteq H$ and $f^{-1}(H) \subseteq G$.

Proof: Let $S \subseteq 1_Y$ be a $\mathfrak{F}\mathfrak{N}o$ set of 1_X containing $f^{-1}(S)$. Let f be a $\mathfrak{F}\mathfrak{N}gMcf$ and $1_X - G$ is $\mathfrak{F}\mathfrak{N}cs$ in 1_X , therefore $f(1_X - G)$ is a $\mathfrak{F}\mathfrak{N}gMc$ set in 1_Y . Then take $H = 1_Y - f(1_X - G)$, implies $H = f(G)$ where H is $\mathfrak{F}\mathfrak{N}gMo$ set in 1_Y . Since $f^{-1}(S) \subseteq G$, $S \subseteq f(G)$, $S \subseteq H$. Therefore $f(1_X - G) = 1_Y - H \Rightarrow f(1_X - G) \subseteq 1_Y - S$ and $f^{-1}(H) \subseteq f^{-1}(1_Y - f(1_X - G)) \subseteq 1_X - (1_X - G) = G$. Thus H is $\mathfrak{F}\mathfrak{N}gMo$ set in 1_Y such that $S \subseteq H$ and $f^{-1}(H) \subseteq G$.

Conversely, let G be a $\mathfrak{F}\mathfrak{N}c$ set in 1_X , then $1_X - G$ is a $\mathfrak{F}\mathfrak{N}o$ set in 1_X . Take $S = 1_Y - f(G)$ be a $\mathfrak{F}\mathfrak{N}s$ of 1_Y , $f^{-1}(S) = f^{-1}(1_Y - f(G)) \subseteq 1_X - G$. By hypothesis, there is a $\mathfrak{F}\mathfrak{N}gMo$ set H of 1_Y such that $1_Y - f(G) \subseteq H$ and $f^{-1}(H) \subseteq 1_X - G$. Therefore $1_Y - H \subseteq f(G) \subseteq f(1_X - f^{-1}(H)) \subseteq 1_Y - H$, that is $f(G) = 1_Y - H$. Since H is $\mathfrak{F}\mathfrak{N}gMo$ set in 1_Y and so $f(G)$ is $\mathfrak{F}\mathfrak{N}gMcs$ in 1_Y . Hence f is $\mathfrak{F}\mathfrak{N}gMcf$. \square

Theorem 3.6 If $f : (X, \tau_X) \rightarrow (Y, \sigma_Y)$ is a $\mathfrak{F}\mathfrak{N}gMcf$, then for each $\mathfrak{F}\mathfrak{N}c$ set K of 1_Y and each $\mathfrak{F}\mathfrak{N}o$ set G of 1_X containing $f^{-1}(K)$, there exists $H \in \mathfrak{F}\mathfrak{N}GMO(Y)$ containing K such that $f^{-1}(H) \subseteq G$.

Proof: Suppose f is $\mathfrak{F}\mathfrak{N}gMcf$. Let K be any $\mathfrak{F}\mathfrak{N}c$ set of 1_Y and G is a $\mathfrak{F}\mathfrak{N}o$ set in 1_X containing $f^{-1}(K)$. By Theorem 3.5, there exists a $\mathfrak{F}\mathfrak{N}gMo$ set F of 1_Y such that $K \subseteq F$ and $f^{-1}(F) \subseteq G$. Since K is $\mathfrak{F}\mathfrak{N}c$ and F is a $\mathfrak{F}\mathfrak{N}gMo$ set containing K , then $K \subseteq \mathfrak{F}\mathfrak{N}gMint(F)$. Put $H = \mathfrak{F}\mathfrak{N}gMint(F)$, then $K \subseteq H \in \mathfrak{F}\mathfrak{N}GMO(Y)$ and $f^{-1}(H) \subseteq G$. \square

Theorem 3.7 Let $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ and $g : (Y, \sigma_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Z, \nu_{\mathfrak{F}\mathfrak{N}})$ be any two functions. If f is a $\mathfrak{F}\mathfrak{N}cf$ and g is $\mathfrak{F}\mathfrak{N}gMcf$ functions, then $f \circ g$ is $\mathfrak{F}\mathfrak{N}gMcf$.

Proof: Let λ be any $\mathfrak{F}\mathfrak{N}c$ set in X . As f is $\mathfrak{F}\mathfrak{N}Cf$, $f(\lambda)$ is $\mathfrak{F}\mathfrak{N}c$ in Y . Since g is $\mathfrak{F}\mathfrak{N}gMcf$, implies $f(g(\lambda)) = (f \circ g)(\lambda)$ is $\mathfrak{F}\mathfrak{N}gMc$ in Z . Therefore $f \circ g$ is $\mathfrak{F}\mathfrak{N}gMcf$. \square

Theorem 3.8 Let $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ and $g : (Y, \sigma_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Z, \nu_{\mathfrak{F}\mathfrak{N}})$ be any two functions such that $g \circ f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Z, \nu_{\mathfrak{F}\mathfrak{N}})$ be a $\mathfrak{F}\mathfrak{N}gMcf$. Then following results hold.

- (i) If f is $\mathfrak{F}\mathfrak{N}Cts$ surjection, then g is a $\mathfrak{F}\mathfrak{N}gMcf$.
- (ii) If g is $\mathfrak{F}\mathfrak{N}gMIrr$ and injective, then f is a $\mathfrak{F}\mathfrak{N}gMcf$.

Proof: (i) Suppose λ is a $\mathfrak{F}\mathfrak{N}c$ set in Y . Since f is a $\mathfrak{F}\mathfrak{N}Cts$ function, $f^{-1}(\lambda)$ is a $\mathfrak{F}\mathfrak{N}c$ set in X . Therefore, $(g \circ f)(f^{-1}(\lambda)) = g(\lambda)$ is a $\mathfrak{F}\mathfrak{N}gMc$ set in Z . Hence g is a $\mathfrak{F}\mathfrak{N}gMcf$.

(ii) Suppose λ is $\mathfrak{F}\mathfrak{N}c$ set in X . Then $(g \circ f)(\lambda)$ is a $\mathfrak{F}\mathfrak{N}gMc$ set in Z . Since g is a $\mathfrak{F}\mathfrak{N}gMIrr$ function, implies $g^{-1}((g \circ f)(\lambda)) = f(\lambda)$ is a $\mathfrak{F}\mathfrak{N}gMc$ set in Y . Hence f is a $\mathfrak{F}\mathfrak{N}gMcf$. \square

Definition 3.7 A function $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is Fermatean neutrosophic generalized M^* closed function (briefly, $\mathfrak{F}\mathfrak{N}gM^*cf$), if for each $\mathfrak{F}\mathfrak{N}Mc$ set λ of X , the set $f(\lambda)$ is $\mathfrak{F}\mathfrak{N}gMc$ set in Y .

Theorem 3.9 A function $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is $\mathfrak{F}\mathfrak{N}gM^*cf$, iff $f(\lambda)$ is $\mathfrak{F}\mathfrak{N}gMo$ in Y for every $\mathfrak{F}\mathfrak{N}Mo$ set λ in X .

Proof: The proof is similar in the Theorem 3.2. □

Theorem 3.10 If $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is $\mathfrak{F}\mathfrak{N}gM^*$ Cts and $\mathfrak{F}\mathfrak{N}cf$ and K is $\mathfrak{F}\mathfrak{N}gMo$ s in Y , then $f^{-1}(K)$ is a $\mathfrak{F}\mathfrak{N}gMo$ set in X .

Proof: Let K be $\mathfrak{F}\mathfrak{N}gMo$ set in Y and F be a $\mathfrak{F}\mathfrak{N}cs$ in X such that $F \subset f^{-1}(K)$, then $f(F) \subset K$. Since f is $\mathfrak{F}\mathfrak{N}cf$, implies $f(F)$ is $\mathfrak{F}\mathfrak{N}c$ in Y . Since K is $\mathfrak{F}\mathfrak{N}gMo$ set in Y , implies $f(F) \subset \mathfrak{F}\mathfrak{N}Mint(K)$, This implies $F \subset f^{-1}(\mathfrak{F}\mathfrak{N}Mint(K))$. Since f is $\mathfrak{F}\mathfrak{N}gM^*$ Cts and $\mathfrak{F}\mathfrak{N}Mint(K)$ is $\mathfrak{F}\mathfrak{N}Mo$ set in Y , $f^{-1}(\mathfrak{F}\mathfrak{N}Mint(K))$ is $\mathfrak{F}\mathfrak{N}Mo$ set in X . Therefore $F \subset \mathfrak{F}\mathfrak{N}gMint(f^{-1}(\mathfrak{F}\mathfrak{N}Mint(K))) \subset f^{-1}(\mathfrak{F}\mathfrak{N}Mint(K)) \subset \mathfrak{F}\mathfrak{N}Mint(f^{-1}(K))$. Thus $F \subset \mathfrak{F}\mathfrak{N}Mint(f^{-1}(K))$. This shows that $f^{-1}(K)$ is $\mathfrak{F}\mathfrak{N}gMo$ set in X . □

Corollary 3.1 If $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is $\mathfrak{F}\mathfrak{N}gM^*$ Cts and $\mathfrak{F}\mathfrak{N}of$ and K is $\mathfrak{F}\mathfrak{N}gMcs$ in Y , then $f^{-1}(K)$ is a $\mathfrak{F}\mathfrak{N}gMc$ set in X .

Theorem 3.11 If $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is $\mathfrak{F}\mathfrak{N}gM^*Cts$ and $\mathfrak{F}\mathfrak{N}cf$ and $g : (Y, \sigma_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Z, \eta_{\mathfrak{F}\mathfrak{N}})$ is $\mathfrak{F}\mathfrak{N}gM^*$ Cts, then $g \circ f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Z, \eta_{\mathfrak{F}\mathfrak{N}})$ is $\mathfrak{F}\mathfrak{N}gM^*Cts$.

Proof: Suppose F is a $\mathfrak{F}\mathfrak{N}Mc$ in Z . As g is $\mathfrak{F}\mathfrak{N}gM^*Cts$, implies $g^{-1}(F)$ is $\mathfrak{F}\mathfrak{N}gMc$ in Y . Now f is $\mathfrak{F}\mathfrak{N}gM^*Cts$ and $\mathfrak{F}\mathfrak{N}cf$, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is $\mathfrak{F}\mathfrak{N}gMc$ in X . Therefore $g \circ f$ is a $\mathfrak{F}\mathfrak{N}gM^*$ Cts function. □

Theorem 3.12 A function $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is $\mathfrak{F}\mathfrak{N}gM^*c$ iff for each subset S of Y and for each $\mathfrak{F}\mathfrak{N}Mo$ set G in X containing $f^{-1}(S)$, there exists a $\mathfrak{F}\mathfrak{N}gMo$ set H of Y such that $S \subseteq H$ and $f^{-1}(H) \subseteq G$.

Proof: The proof is similar to Theorem 3.5. □

Theorem 3.13 If $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is $\mathfrak{F}\mathfrak{N}gM^*c$ then for each $\mathfrak{F}\mathfrak{N}c$ set K of Y and each $G \in \mathfrak{F}\mathfrak{N}MO(X)$ of X containing $f^{-1}(K)$, there exists $H \in \mathfrak{F}\mathfrak{N}MO(Y)$ containing K such that $f^{-1}(H) \subseteq G$.

Proof: The proof is similar to Theorem 3.6. □

Theorem 3.14 If f is $\mathfrak{F}\mathfrak{N}Cts$ and $\mathfrak{F}\mathfrak{N}gM^*cf$ then $f(H)$ is $\mathfrak{F}\mathfrak{N}gMc$ set in Y for each $\mathfrak{F}\mathfrak{N}gMc$ set H in X .

Proof: Let f be a $\mathfrak{F}\mathfrak{N}Cts$ and $\mathfrak{F}\mathfrak{N}gM^*cf$ from $\mathfrak{F}\mathfrak{N}ts (X, \tau_{\mathfrak{F}\mathfrak{N}})$ to $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$. Suppose H is any $\mathfrak{F}\mathfrak{N}gMc$ set of X and G is $\mathfrak{F}\mathfrak{N}o$ set of Y containing $f(H)$. This implies $H \subseteq f^{-1}(G)$, which is $\mathfrak{F}\mathfrak{N}o$ in X , since f is $\mathfrak{F}\mathfrak{N}Cts$. Therefore $\mathfrak{F}\mathfrak{N}Mcl(H) \subseteq f^{-1}(G)$ and hence $f(\mathfrak{F}\mathfrak{N}Mcl(H)) \subseteq G$. Since f is $\mathfrak{F}\mathfrak{N}gM^*c$ and $\mathfrak{F}\mathfrak{N}Mcl(H) \in \mathfrak{F}\mathfrak{N}MC(X)$ implies $f(\mathfrak{F}\mathfrak{N}Mcl(H))$ is $\mathfrak{F}\mathfrak{N}gMc$ set contained in G . Therefore $\mathfrak{F}\mathfrak{N}Mcl(f(\mathfrak{F}\mathfrak{N}Mcl(H))) \subseteq G$. Thus $\mathfrak{F}\mathfrak{N}Mcl(f(H)) \subseteq \mathfrak{F}\mathfrak{N}Mcl(f(\mathfrak{F}\mathfrak{N}Mcl(H))) \subseteq G$. Therefore $\mathfrak{F}\mathfrak{N}Mcl(f(H)) \subseteq G$. This shows that $f(H)$ is $\mathfrak{F}\mathfrak{N}gMc$ in Y . □

Theorem 3.15 Let $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ and $g : (Y, \sigma_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Z, \eta_{\mathfrak{F}\mathfrak{N}})$ be any two functions. Then $g \circ f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Z, \eta_{\mathfrak{F}\mathfrak{N}})$ is a $\mathfrak{F}\mathfrak{N}gM^*cf$ if f and g satisfy any one of the following conditions.

(i) f is \mathfrak{FNgM}^*c and g is a $\mathfrak{FN}Cts$, \mathfrak{FNgM}^*cf .

(ii) f is \mathfrak{FNgM}^*c and g is a $\mathfrak{FN}Cts$, $\mathfrak{FNgM}cf$.

Proof: (i) Suppose F is a $\mathfrak{FNgM}c$ set in X . Since f is a \mathfrak{FNgM}^*cf , $f(F)$ is a $\mathfrak{FNgM}c$ set in Y . Since g is $\mathfrak{FN}Cts$, \mathfrak{FNgM}^*cf , by Theorem 3.14, $g(f(F)) = (g \circ f)(F)$ is a $\mathfrak{FNgM}c$ set in Z . Hence $g \circ f$ is a \mathfrak{FNgM}^*cf .

(ii) Suppose F is a $\mathfrak{FNgM}c$ set in X . Since f is a \mathfrak{FNgM}^*cf , $f(F)$ is a $\mathfrak{FNgM}c$ set in Y . Since g is $\mathfrak{FN}Cts$, $\mathfrak{FNgM}cf$, $g(f(F)) = (g \circ f)(F)$ is a $\mathfrak{FNgM}c$ set in Z . Hence $g \circ f$ is a \mathfrak{FNgM}^*cf . \square

Theorem 3.16 Let $f : (X, \tau_{\mathfrak{FN}}) \rightarrow (Y, \sigma_{\mathfrak{FN}})$ and $g : (Y, \sigma_{\mathfrak{FN}}) \rightarrow (Z, \eta_{\mathfrak{FN}})$ be any two functions such that $g \circ f : (X, \tau_{\mathfrak{FN}}) \rightarrow (Z, \eta_{\mathfrak{FN}})$ be a $\mathfrak{FNgM}cf$. Then following results hold.

(i) If f is \mathfrak{FNMIrr} surjection, then g is a \mathfrak{FNgM}^*cf .

(ii) If f is $\mathfrak{FN}MCts$ surjection, then g is a \mathfrak{FNgM}^*cf .

(iii) If g is $\mathfrak{FNgMIrr}$ and injective, then f is a \mathfrak{FNgM}^*cf .

Proof: (i) Suppose F is a $\mathfrak{FNgM}c$ set in Y . Since f is a \mathfrak{FNMIrr} and surjective, $f^{-1}(F)$ is a $\mathfrak{FN}c$ set in X . Since $g \circ f$ is a \mathfrak{FNgM}^*cf , implies $(g \circ f)(f^{-1}(F)) = g(F)$ is a $\mathfrak{FNgM}c$ set in Z . Therefore g is a \mathfrak{FNgM}^*cf .

(ii) Suppose F is $\mathfrak{FN}c$ set in Y . Since f is $\mathfrak{FN}MCts$ and surjective, $f^{-1}(F)$ is $\mathfrak{FNgM}c$ in X . Now $g \circ f$ is a \mathfrak{FNgM}^*cf , implies $(g \circ f)(f^{-1}(F)) = g(F)$ is a $\mathfrak{FNgM}c$ set in Z . Therefore g is a \mathfrak{FNgM}^*cf .

(iii) Suppose F is $\mathfrak{FN}c$ set in X . Now $g \circ f$ is a \mathfrak{FNgM}^*cf , implies $(g \circ f)(F)$ is a $\mathfrak{FNgM}c$ set in Z . Since g is a $\mathfrak{FNgMIrr}$, implies $g^{-1}((g \circ f)(F)) = f(F)$ is a $\mathfrak{FNgM}c$ set in Y . Hence f is a \mathfrak{FNgM}^*cf . \square

Theorem 3.17 If $f : (X, \tau_{\mathfrak{FN}}) \rightarrow (Y, \sigma_{\mathfrak{FN}})$ is bijective, $\mathfrak{FN}o$ and \mathfrak{FNgM}^*Cts function, then f is $\mathfrak{FNgMIrr}$.

Proof: Let K be $\mathfrak{FNgM}c$ in Y and G be any $\mathfrak{FN}o$ set in X such that $K \subset f(G)$. Since K is $\mathfrak{FNgM}c$ and $f(G)$ is $\mathfrak{FN}o$ in Y , implies $\mathfrak{FNgM}cl(K) \subset f(G)$. This implies $f^{-1}(\mathfrak{FNgM}cl(K)) \subset G$. Since f is \mathfrak{FNgM}^*Cts and $\mathfrak{FNgM}cl(K)$ is $\mathfrak{FNgM}c$ in Y , implies $f^{-1}(\mathfrak{FNgM}cl(K))$ is $\mathfrak{FNgM}c$ in X . Therefore $\mathfrak{FNgM}cl(f^{-1}(\mathfrak{FNgM}cl(K))) \subset G$. This implies $\mathfrak{FNgM}cl(f^{-1}(K)) \subset \mathfrak{FNgM}cl(f^{-1}(\mathfrak{FNgM}cl(K))) \subset G$. That is $\mathfrak{FNgM}cl(f^{-1}(K)) \subset G$. Therefore $f^{-1}(K)$ is $\mathfrak{FNgM}c$ in X . Hence f is $\mathfrak{FNgMIrr}$. \square

Corollary 3.2 If $f : (X, \tau_{\mathfrak{FN}}) \rightarrow (Y, \sigma_{\mathfrak{FN}})$ is bijective, $\mathfrak{FN}o$ and $\mathfrak{FNgMIrr}$ function, then f is $\mathfrak{FNgMIrr}$.

4. Fermatean Neutrosophic Generalized M Homeomorphisms

Definition 4.1 Let $(X, \tau_{\mathfrak{FN}})$ and $(Y, \sigma_{\mathfrak{FN}})$ be $\mathfrak{FN}ts$ and let $f : (X, \tau_{\mathfrak{FN}}) \rightarrow (Y, \sigma_{\mathfrak{FN}})$ be a bijective function. If both the function f and the inverse function f^{-1} are $\mathfrak{FN}MCts$ (resp. $\mathfrak{FN}Cts$, $\mathfrak{FN}\theta Cts$, $\mathfrak{FN}\theta SCts$ and $\mathfrak{FN}\delta PCts$), then f is called Fermatean neutrosophic M homeomorphism (resp. homeomorphism, θ homeomorphism, θS homeomorphism and δP homeomorphism) (briefly, $\mathfrak{FN}MHom$ (resp. $\mathfrak{FN}Hom$, $\mathfrak{FN}\theta Hom$, $\mathfrak{FN}\theta SHom$ and $\mathfrak{FN}\delta PHom$)). Equivalently, if f both $\mathfrak{FN}MCts$ (resp. $\mathfrak{FN}Cts$, $\mathfrak{FN}\theta Cts$, $\mathfrak{FN}\theta SCts$ and $\mathfrak{FN}\delta PCts$) and $\mathfrak{FN}Mof$ (resp. $\mathfrak{FN}of$, $\mathfrak{FN}\theta of$, $\mathfrak{FN}\theta Sof$ and $\mathfrak{FN}\delta Pof$) then f is called $\mathfrak{FN}MHom$ (resp. $\mathfrak{FN}Hom$, $\mathfrak{FN}\theta Hom$, $\mathfrak{FN}\theta SHom$ and $\mathfrak{FN}\delta PHom$).

The family of all $\mathfrak{FN}MHom$ (resp. $\mathfrak{FN}Hom$, $\mathfrak{FN}\theta Hom$, $\mathfrak{FN}\theta SHom$ and $\mathfrak{FN}\delta PHom$) in X is denoted by $\mathfrak{FN}MH(X)$ (resp. $\mathfrak{FN}H(X)$, $\mathfrak{FN}\theta H(X)$, $\mathfrak{FN}\theta SH(X)$ and $\mathfrak{FN}\delta PH(X)$).

Definition 4.2 Let $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ and $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ be $\mathfrak{F}\mathfrak{N}ts$ and let $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ be a bijective function. If both the function f and the inverse function f^{-1} are $\mathfrak{F}\mathfrak{N}gM\mathfrak{C}ts$ (resp. $\mathfrak{F}\mathfrak{N}g\mathfrak{C}ts$, $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{C}ts$, $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{S}\mathfrak{C}ts$ and $\mathfrak{F}\mathfrak{N}g\delta\mathfrak{P}\mathfrak{C}ts$), then f is called Fermatean neutrosophic generalized M homeomorphism (resp. homeomorphism, θ homeomorphism, $\theta\mathfrak{S}$ homeomorphism and $\delta\mathfrak{P}$ homeomorphism) (briefly, $\mathfrak{F}\mathfrak{N}gM\mathfrak{H}om$ (resp. $\mathfrak{F}\mathfrak{N}g\mathfrak{H}om$, $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{H}om$, $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{S}\mathfrak{H}om$ and $\mathfrak{F}\mathfrak{N}g\delta\mathfrak{P}\mathfrak{H}om$)). Equivalently, if f both $\mathfrak{F}\mathfrak{N}gM\mathfrak{C}ts$ (resp. $\mathfrak{F}\mathfrak{N}g\mathfrak{C}ts$, $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{C}ts$, $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{S}\mathfrak{C}ts$ and $\mathfrak{F}\mathfrak{N}g\delta\mathfrak{P}\mathfrak{C}ts$) and $\mathfrak{F}\mathfrak{N}gM\mathfrak{o}f$ (resp. $\mathfrak{F}\mathfrak{N}g\mathfrak{o}f$, $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{o}f$, $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{S}\mathfrak{o}f$ and $\mathfrak{F}\mathfrak{N}g\delta\mathfrak{P}\mathfrak{o}f$) then f is called $\mathfrak{F}\mathfrak{N}gM\mathfrak{H}om$ (resp. $\mathfrak{F}\mathfrak{N}g\mathfrak{H}om$, $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{H}om$, $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{S}\mathfrak{H}om$ and $\mathfrak{F}\mathfrak{N}g\delta\mathfrak{P}\mathfrak{H}om$).

The family of all $\mathfrak{F}\mathfrak{N}gM\mathfrak{H}om$ (resp. $\mathfrak{F}\mathfrak{N}g\mathfrak{H}om$, $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{H}om$, $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{S}\mathfrak{H}om$ and $\mathfrak{F}\mathfrak{N}g\delta\mathfrak{P}\mathfrak{H}om$) in X is denoted by $\mathfrak{F}\mathfrak{N}GM\mathfrak{H}(X)$ (resp. $\mathfrak{F}\mathfrak{N}G\mathfrak{H}(X)$, $\mathfrak{F}\mathfrak{N}G\theta\mathfrak{H}(X)$, $\mathfrak{F}\mathfrak{N}G\theta\mathfrak{S}\mathfrak{H}(X)$ and $\mathfrak{F}\mathfrak{N}G\delta\mathfrak{P}\mathfrak{H}(X)$).

Definition 4.3 Let $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ and $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ be $\mathfrak{F}\mathfrak{N}ts$ and let $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ be a bijective function. If both the function f and the inverse function f^{-1} are $\mathfrak{F}\mathfrak{N}gM\mathfrak{I}rr$ (resp. $\mathfrak{F}\mathfrak{N}g\mathfrak{I}rr$, $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{I}rr$, $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{S}\mathfrak{I}rr$ and $\mathfrak{F}\mathfrak{N}g\delta\mathfrak{P}\mathfrak{I}rr$), then f is called Fermatean neutrosophic generalized M^* homeomorphism (resp. $*$ homeomorphism, θ^* homeomorphism, $\theta\mathfrak{S}^*$ homeomorphism and $\delta\mathfrak{P}^*$ homeomorphism) (briefly, $\mathfrak{F}\mathfrak{N}gM^*\mathfrak{H}om$ (resp. $\mathfrak{F}\mathfrak{N}g^*\mathfrak{H}om$, $\mathfrak{F}\mathfrak{N}g\theta^*\mathfrak{H}om$, $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{S}^*\mathfrak{H}om$ and $\mathfrak{F}\mathfrak{N}g\delta\mathfrak{P}^*\mathfrak{H}om$)). Equivalently, if f both $\mathfrak{F}\mathfrak{N}gM\mathfrak{I}rr$ (resp. $\mathfrak{F}\mathfrak{N}g\mathfrak{I}rr$, $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{I}rr$, $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{S}\mathfrak{I}rr$ and $\mathfrak{F}\mathfrak{N}g\delta\mathfrak{P}\mathfrak{I}rr$) and $\mathfrak{F}\mathfrak{N}gM^*\mathfrak{o}f$ (resp. $\mathfrak{F}\mathfrak{N}g^*\mathfrak{o}f$, $\mathfrak{F}\mathfrak{N}g\theta^*\mathfrak{o}f$, $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{S}^*\mathfrak{o}f$ and $\mathfrak{F}\mathfrak{N}g\delta\mathfrak{P}^*\mathfrak{o}f$) then f is called $\mathfrak{F}\mathfrak{N}gM^*\mathfrak{H}om$ (resp. $\mathfrak{F}\mathfrak{N}g^*\mathfrak{H}om$, $\mathfrak{F}\mathfrak{N}g\theta^*\mathfrak{H}om$, $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{S}^*\mathfrak{H}om$ and $\mathfrak{F}\mathfrak{N}g\delta\mathfrak{P}^*\mathfrak{H}om$).

The family of all $\mathfrak{F}\mathfrak{N}gM^*\mathfrak{H}om$ (resp. $\mathfrak{F}\mathfrak{N}g^*\mathfrak{H}om$, $\mathfrak{F}\mathfrak{N}g\theta^*\mathfrak{H}om$, $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{S}^*\mathfrak{H}om$ and $\mathfrak{F}\mathfrak{N}g\delta\mathfrak{P}^*\mathfrak{H}om$) in X is denoted by $\mathfrak{F}\mathfrak{N}GM^*\mathfrak{H}(X)$ (resp. $\mathfrak{F}\mathfrak{N}G^*\mathfrak{H}(X)$, $\mathfrak{F}\mathfrak{N}G\theta^*\mathfrak{H}(X)$, $\mathfrak{F}\mathfrak{N}G\theta\mathfrak{S}^*\mathfrak{H}(X)$ and $\mathfrak{F}\mathfrak{N}G\delta\mathfrak{P}^*\mathfrak{H}(X)$).

Theorem 4.1 Let $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ be a mapping. Then

- (i) Every $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{H}om$ is $\mathfrak{F}\mathfrak{N}g\mathfrak{H}om$,
- (ii) Every $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{H}om$ is $\mathfrak{F}\mathfrak{N}g\delta\mathfrak{P}\mathfrak{H}om$,
- (iii) Every $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{H}om$ is $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{S}\mathfrak{H}om$,
- (iv) Every $\mathfrak{F}\mathfrak{N}g\delta\mathfrak{P}\mathfrak{H}om$ is $\mathfrak{F}\mathfrak{N}gM\mathfrak{H}om$,
- (v) Every $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{S}\mathfrak{H}om$ is $\mathfrak{F}\mathfrak{N}gM\mathfrak{H}om$.
- (vi) Every $\mathfrak{F}\mathfrak{N}M\mathfrak{H}om$ is $\mathfrak{F}\mathfrak{N}gM\mathfrak{H}om$.
- (vii) Every $\mathfrak{F}\mathfrak{N}gM^*\mathfrak{H}om$ is $\mathfrak{F}\mathfrak{N}gM\mathfrak{H}om$.

But the converse does not true.

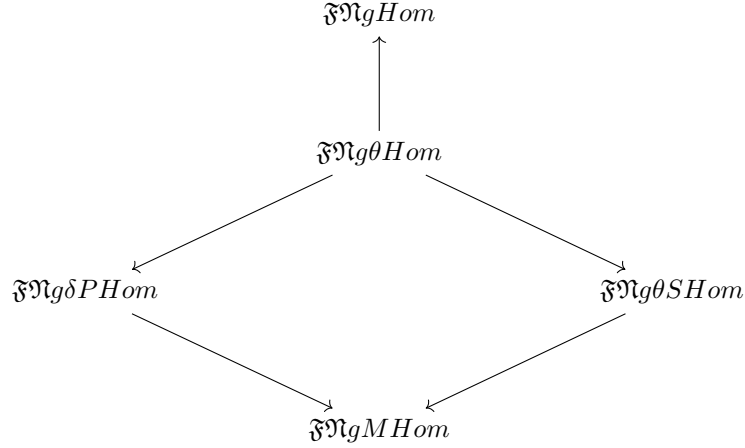
Proof: (vi) Every $\mathfrak{F}\mathfrak{N}M\mathfrak{o}$ is $\mathfrak{F}\mathfrak{N}gM\mathfrak{o}$ proof follows. Similarly the other results can be proved. \square

Remark 4.1 The implication diagram is obtained and none of the implications in the diagram are reversible as seen from the example.

Example 4.1 Let $X = Y = \{a, b\}$ and the $\mathfrak{F}\mathfrak{N}ts$'s A_1, A_2 and A_3 are defined as

$$\begin{aligned} \mu_{A_1}(a) &= 0.8, \nu_{A_1}(a) = 0.8, \sigma_{A_1}(a) = 0.1, \\ \mu_{A_1}(b) &= 0.9, \nu_{A_1}(b) = 0.8, \sigma_{A_1}(b) = 0.2; \\ \mu_{A_2}(a) &= 0.6, \nu_{A_2}(a) = 0.7, \sigma_{A_2}(a) = 0.2, \\ \mu_{A_2}(b) &= 0.5, \nu_{A_2}(b) = 0.7, \sigma_{A_2}(b) = 0.6; \\ \mu_{A_3}(a) &= 0.1, \nu_{A_3}(a) = 0.2, \sigma_{A_3}(a) = 0.7, \\ \mu_{A_3}(b) &= 0.2, \nu_{A_3}(b) = 0.3, \sigma_{A_3}(b) = 0.8. \end{aligned}$$

Let $\tau_{\mathfrak{F}\mathfrak{N}} = \sigma_{\mathfrak{F}\mathfrak{N}} = \{0_{\mathfrak{F}\mathfrak{N}}, 1_{\mathfrak{F}\mathfrak{N}}, A_1, A_2, A_3\}$ be a $\mathfrak{F}\mathfrak{N}ts$ on X . Let $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \rightarrow (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ be an identity mapping, then (i) f is $\mathfrak{F}\mathfrak{N}g\mathfrak{H}om$ (resp. $\mathfrak{F}\mathfrak{N}g\delta\mathfrak{P}\mathfrak{H}om$) but not $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{H}om$, the set A_1^c is a $\mathfrak{F}\mathfrak{N}ts$ in X but $f(A_1^c)$ is not $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{ts}$ in Y , (ii) f is $\mathfrak{F}\mathfrak{N}gM\mathfrak{H}om$ but not $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{S}\mathfrak{H}om$, the set A_2^c is a $\mathfrak{F}\mathfrak{N}ts$ in X but $f(A_2^c)$ is not $\mathfrak{F}\mathfrak{N}g\theta\mathfrak{S}\mathfrak{ts}$ in Y .



Note: $A \rightarrow B$ denotes A implies B , but not conversely.

Example 4.2 Let $X = Y = \{a, b\}$ and the $\mathfrak{FN}s$'s A_1, A_2, A_3 and A_4 are defined as

$$\begin{aligned}
 \mu_{A_1}(a) &= 0.8, \nu_{A_1}(a) = 0.8, \sigma_{A_1}(a) = 0.1, \\
 \mu_{A_1}(b) &= 0.9, \nu_{A_1}(b) = 0.8, \sigma_{A_1}(b) = 0.2; \\
 \mu_{A_2}(a) &= 0.6, \nu_{A_2}(a) = 0.7, \sigma_{A_2}(a) = 0.2, \\
 \mu_{A_2}(b) &= 0.5, \nu_{A_2}(b) = 0.7, \sigma_{A_2}(b) = 0.6; \\
 \mu_{A_3}(a) &= 0.1, \nu_{A_3}(a) = 0.2, \sigma_{A_3}(a) = 0.7, \\
 \mu_{A_3}(b) &= 0.2, \nu_{A_3}(b) = 0.3, \sigma_{A_3}(b) = 0.8; \\
 \mu_{A_4}(a) &= 0.2, \nu_{A_4}(a) = 0.3, \sigma_{A_4}(a) = 0.6, \\
 \mu_{A_4}(b) &= 0.6, \nu_{A_4}(b) = 0.3, \sigma_{A_4}(b) = 0.5.
 \end{aligned}$$

Let $\sigma_{\mathfrak{FN}} = \{0_{\mathfrak{FN}}, 1_{\mathfrak{FN}}, A_1, A_2, A_3\}$, $\tau_{\mathfrak{FN}} = \{0_{\mathfrak{FN}}, 1_{\mathfrak{FN}}, A_4\}$ be a $\mathfrak{FN}s$ on X . Let $f : (X, \tau_{\mathfrak{FN}}) \rightarrow (Y, \sigma_{\mathfrak{FN}})$ be an identity mapping, then f is $\mathfrak{FNgMHom}$ but not $\mathfrak{FNg\delta PHom}$, the set A_2 is a \mathfrak{FNcs} in X but $f(A_2)$ is not $\mathfrak{FNg\delta PCs}$ in Y .

Example 4.3 Let $X = Y = \{a, b\}$ and the $\mathfrak{FN}s$'s $A_1, A_2, A_3, A_4, B_1, B_2, B_3$ and B_4 are defined as

$$\begin{aligned}
 \mu_{A_1}(a) &= 0.2, \nu_{A_1}(a) = 0.5, \sigma_{A_1}(a) = 0.8, \\
 \mu_{A_1}(b) &= 0.4, \nu_{A_1}(b) = 0.5, \sigma_{A_1}(b) = 0.6; \\
 \mu_{A_2}(a) &= 0.1, \nu_{A_2}(a) = 0.5, \sigma_{A_2}(a) = 0.9, \\
 \mu_{A_2}(b) &= 0.3, \nu_{A_2}(b) = 0.5, \sigma_{A_2}(b) = 0.7; \\
 \mu_{A_3}(a) &= 0.9, \nu_{A_3}(a) = 0.5, \sigma_{A_3}(a) = 0.1, \\
 \mu_{A_3}(b) &= 0.7, \nu_{A_3}(b) = 0.5, \sigma_{A_3}(b) = 0.3; \\
 \mu_{A_4}(a) &= 0.2, \nu_{A_4}(a) = 0.5, \sigma_{A_4}(a) = 0.8, \\
 \mu_{A_4}(b) &= 0.3, \nu_{A_4}(b) = 0.5, \sigma_{A_4}(b) = 0.7; \\
 \mu_{B_1}(a) &= 0.4, \nu_{B_1}(a) = 0.5, \sigma_{B_1}(a) = 0.6, \\
 \mu_{B_1}(b) &= 0.5, \nu_{B_1}(b) = 0.5, \sigma_{B_1}(b) = 0.5; \\
 \mu_{B_2}(a) &= 0.6, \nu_{B_2}(a) = 0.5, \sigma_{B_2}(a) = 0.4, \\
 \mu_{B_2}(b) &= 0.6, \nu_{B_2}(b) = 0.5, \sigma_{B_2}(b) = 0.4; \\
 \mu_{B_3}(a) &= 0.7, \nu_{B_3}(a) = 0.5, \sigma_{B_3}(a) = 0.3, \\
 \mu_{B_3}(b) &= 0.6, \nu_{B_3}(b) = 0.5, \sigma_{B_3}(b) = 0.4; \\
 \mu_{B_4}(a) &= 0.4, \nu_{B_4}(a) = 0.5, \sigma_{B_4}(a) = 0.6, \\
 \mu_{B_4}(b) &= 0.4, \nu_{B_4}(b) = 0.5, \sigma_{B_4}(b) = 0.6.
 \end{aligned}$$

Then we have $\sigma_{\mathfrak{FN}} = \{0_{\mathfrak{FN}}, A_1, A_2, A_3, A_4, 1_{\mathfrak{FN}}\}$, $\tau_{\mathfrak{FN}} = \{0_{\mathfrak{FN}}, B_1, B_2, B_3, B_4, 1_{\mathfrak{FN}}\}$. Then, the identity mappings $f : (X, \tau_{\mathfrak{FN}}) \rightarrow (Y, \sigma_{\mathfrak{FN}})$ is (i) $\mathfrak{FNgMHom}$ but not $\mathfrak{FNgMHom}$, the set B_1^c is a \mathfrak{FNcs} in X but $f(B_1^c)$ is not \mathfrak{FNgMcs} in Y , (ii) $\mathfrak{FNgMHom}$ but not $\mathfrak{FNgM} * \text{Hom}$, the set A_4 is a \mathfrak{FNMc} s in X but $f(A_4)$ is not \mathfrak{FNgMcs} in Y .

Theorem 4.2 For any bijection $f : (X, \tau_{\mathfrak{FN}}) \rightarrow (Y, \sigma_{\mathfrak{FN}})$ the following statements are equivalent.

- (i) Inverse of f is $\mathfrak{FN}gM$ Cts.
- (ii) f is a $\mathfrak{FN}gMof$.
- (iii) f is a $\mathfrak{FN}gMcf$.

Proof: (i) \Rightarrow (ii): Suppose G is a $\mathfrak{FN}o$ set in X , then by (i), $(f^{-1})^{-1}(G) = f(G)$ is a $\mathfrak{FN}gMo$ set in Y and hence f is a $\mathfrak{FN}gMof$.

(ii) \Rightarrow (iii): Suppose F is $\mathfrak{FN}c$ in X , then $1_X - F$ is $\mathfrak{FN}o$ in X . By (ii), $f(1_X - F) = 1_Y - f(F)$ is a $\mathfrak{FN}gMo$ set in Y , implies $f(F)$ is a $\mathfrak{FN}gMc$ set in Y . Therefore f is $\mathfrak{FN}gMcf$.

(iii) \Rightarrow (i): Let F be a $\mathfrak{FN}c$ set in X . By (iii), $f(F) = (f^{-1})^{-1}(F)$ is a $\mathfrak{FN}gMc$ set in Y and hence the inverse of f is a $\mathfrak{FN}gMCts$ function. \square

Theorem 4.3 If $f : (X, \tau_{\mathfrak{FN}}) \rightarrow (Y, \sigma_{\mathfrak{FN}})$ is bijective and $\mathfrak{FN}gMCts$ then the following statements are equivalent.

- (i) f is $\mathfrak{FN}gMof$.
- (ii) f is a $\mathfrak{FN}gMHom$.
- (iii) f is a $\mathfrak{FN}gMcf$.

Proof: (i) \Rightarrow (ii): By the assumption f is bijective, $\mathfrak{FN}gMCts$ and $\mathfrak{FN}gMof$. Then by definition, f is $\mathfrak{FN}gMHom$.

(ii) \Rightarrow (iii): By the assumption f is bijective and $\mathfrak{FN}gMof$. Then by Theorem 4.2, f is $\mathfrak{FN}gMcf$.

(iii) \Rightarrow (i): By the assumption f is bijective and $\mathfrak{FN}gMcf$. Then by Theorem 4.2, f is $\mathfrak{FN}gMof$. \square

Theorem 4.4 If $f : (X, \tau_{\mathfrak{FN}}) \rightarrow (Y, \sigma_{\mathfrak{FN}})$ is bijective and $\mathfrak{FN}gMIrr$ then the following statements are equivalent.

- (i) f is $\mathfrak{FN}gM^*of$.
- (ii) f is a $\mathfrak{FN}gM^*Hom$.
- (iii) f is a $\mathfrak{FN}gM^*cf$.

Proof: Similar to the proof of Theorem 4.3. \square

Theorem 4.5 If $f : (X, \tau_{\mathfrak{FN}}) \rightarrow (Y, \sigma_{\mathfrak{FN}})$ is $\mathfrak{FN}gM^*Hom$ then $\mathfrak{FN}gMcl(f^{-1}(K)) \subseteq f^{-1}(\mathfrak{FN}Mcl(K))$, for every subset K of Y .

Proof: Suppose f is $\mathfrak{FN}gM^*Hom$ then if f is both $\mathfrak{FN}gMIrr$ and $\mathfrak{FN}gMo$. Since $\mathfrak{FN}Mcl(K)$ is a $\mathfrak{FN}gMc$ set in Y , $f^{-1}(\mathfrak{FN}Mcl(K))$ is a $\mathfrak{FN}gMc$ set in X . This implies $\mathfrak{FN}gMcl(f^{-1}(\mathfrak{FN}Mcl(K))) = f^{-1}(\mathfrak{FN}Mcl(K))$. Now $f^{-1}(K) \subseteq f^{-1}(\mathfrak{FN}Mcl(K))$, $\mathfrak{FN}gMcl(f^{-1}(K)) \subseteq \mathfrak{FN}gMcl(f^{-1}(\mathfrak{FN}Mcl(K)))$. This implies $\mathfrak{FN}gMcl(f^{-1}(K)) \subseteq f^{-1}(\mathfrak{FN}Mcl(K))$. \square

5. Conclusion

In this paper, we have undertaken a systematic study of several interesting notions concerning various forms of Fermatean Neutrosophic generalized M -closed functions ($\mathfrak{FN}gMcf$), Fermatean Neutrosophic generalized \mathcal{M}^* -closed functions ($\mathfrak{FN}g\mathcal{M}^*cf$), and their corresponding extensions. In particular, we investigate their associated concepts of irresoluteness and homeomorphisms, thereby providing a deeper understanding of how these function classes behave under the framework of Fermatean Neutrosophic topology. The exploration not only formalizes the definitions of these mappings but also examines their structural properties, mutual relationships, and significant roles in generalizing classical topological results to the Fermatean Neutrosophic setting.

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