Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2026 (44) 11** : 1–17.
ISSN-0037-8712
doi:10.5269/bspm.82986

Stronger and Weaker Forms of Homeomorphisms Mappings via Fermatean Fuzzy M-Open Sets

G. Bhuvaneswari and S. Mehar Banu*

ABSTRACT: In this paper, we introduce the concept of Fermatean fuzzy M open and Fermatean fuzzy M closed mappings in Fermatean fuzzy topological spaces. Also, we study about Fermatean fuzzy M Homeomorphism, almost Fermatean fuzzy M totally mappings, almost Fermatean fuzzy M totally continuous mappings and super Fermatean fuzzy M clopen continuous functions and their properties in Fermatean fuzzy topological spaces.

Keywords: $\mathfrak{F}\mathcal{F}Mo$ set, $\mathfrak{F}\mathcal{F}MO$ map, $\mathfrak{F}\mathcal{F}MHom$, $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}$ map, $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}Cts$ map, $\mathcal{SU}\mathfrak{F}\mathcal{F}MceloCts$ map.

Contents

1 Introduction 1
2 Preliminaries 2
3 Fermatean Fuzzy M homeomorphism 5
4 Fermatean fuzzy M-C homeomorphism 9
5 Almost Fermatean fuzzy M totally mappings 11
6 Almost Fermatean fuzzy M totally continuous functions 13
7 Super Fermatean fuzzy M clopen continuous functions 14
8 Application 15
9 Conclusion 15

1. Introduction

Fuzzy sets were introduced by Zadeh [29] in 1965. The fuzzy set concept was the basis of mathematical testing of the fuzzy concept that exists in our real world and the formation of new branches in mathematics. The fuzzy set concept corresponding to unexplained physical situations gives useful applications on many topics such as statistics, data processing and linguistics. A lot of research has been done on this subject since 1965. In 1968, Chang [7] defined the concept of fuzzy topological space and generalized some basic notions of topology such as open set, closed set, continuity and compactness to fuzzy topological spaces. The idea of intuitionistic fuzzy set was first published by Atanassov [1] and many works by the same author and his colleagues appeared in the literature [2,6]. Coker [8] initiated a study of intuitionistic fuzzy topological spaces. Later Yager [27] launched a non standard fuzzy set referred to as Phythagorean fuzzy set. Olgun et al., [13] defined a Phythagorean fuzzy topological spaces. Fermatean fuzzy sets proposed by Senapati and Yager in 2020 [17], can handle uncertain information more easily in the process of decision making. They defined basic operations over the Fermatean fuzzy sets. Hariwan Z. Ibrahim defined a Fermatean fuzzy topological spaces and the continuity of a function defind among Fermatean fuzzy topological spaces. we developed the concept of some stronger and weaker forms of Fermatean fuzzy open sets in Fermatean fuzzy topological spaces and also specialized some of their basic

* Corresponding author.
2020 *Mathematics Subject Classification:* 54A05, 54A40.
Submitted March 25, 2026. Published June 03, 2026.

properties with examples. S. Saha [16] defined δ-open sets in fuzzy topological spaces, topological space by Pankajam et al. [15] and neutrosophic topological space by Vadivel et al. [20]. Lellis Thivagar et.al [12] explored a new concept of neutrosophic topology, intuitionistic topology and fuzzy topology. El-Maghrabi and Al-Juhani [9] proposed the concept of M-open sets in topological spaces in 2011 and examined some of their features. Padma et al. [14] also found M-open sets in topological spaces. Vadivel et al. [18,19,21] discussed some open sets in fuzzy and neutrosophic topological spaces. Kalaiyarsan et al. [?] and Vadivel et al. [22] introduced M-open sets in fuzzy and neutrosophic topological spaces.

Research Gap: No investigation on some new mappings such as Fermatean fuzzy M open, Fermatean fuzzy M closed mappings, Fermatean fuzzy M-Homeomorphism, almost Fermatean fuzzy M totally mappings, almost Fermatean fuzzy M totally continuous mappings and super Fermatean fuzzy M clopen continuous functions on Fermatean fuzzy topological space has been reported in the Fermatean fuzzy literature. In section 2 of this paper is some basic definitions of fs's, IFS's $\mathcal{PF}'s$ and $\mathfrak{F}\mathcal{F}s$'s are briefly reviewed. In section 3 and 4 we introduce Fermatean fuzzy M open and Fermatean fuzzy M closed mappings in Fermatean fuzzy topological spaces. Also, we study about Fermatean fuzzy M Homeomorphism, almost Fermatean fuzzy M totally mappings, almost Fermatean fuzzy M totally continuous mappings and super Fermatean fuzzy M clopen continuous functions and discuss their properties in $\mathfrak{F}\mathcal{F}ts$'s.

2. Preliminaries

We recall some basic notions of fuzzy sets, IFS's, pfs's and $\mathfrak{F}\mathcal{F}s$'s.

Definition 2.1 [29] Let X be a nonempty set. A fuzzy set A in X is characterized by a membership function $\mu_A : X \to [0,1]$. That is:

$$\mu_A(x) = \begin{cases} 1, & \text{if} \quad x \in X \\ 0, & \text{if} \quad x \notin X \\ (0,1) & \text{if } x \text{ is partly in } X. \end{cases}$$

Alternatively, a fuzzy set A in X is an object having the form $A = \{< x, \mu_A(x) > | x \in X\}$ or $A = \left\{ \left\langle \frac{\mu_A(x)}{x} \right\rangle | x \in X \right\}$, where the function $\mu_A(x) : X \to [0,1]$ defines the degree of membership of the element, $x \in X$.

The closer the membership value $\mu_A(x)$ to 1, the more x belongs to A, where the grades 1 and 0 represent full membership and full nonmembership. Fuzzy set is a collection of objects with graded membership, that is, having degree of membership. Fuzzy set is an extension of the classical notion of set. In classical set theory, the membership of elements in a set is assessed in a binary terms according to a bivalent condition; an element either belongs or does not belong to the set. Classical bivalent sets are in fuzzy set theory called crisp sets. Fuzzy sets are generalized classical sets, since the indicator function of classical sets is special cases of the membership functions of fuzzy sets, if the latter only take values 0 or 1. Fuzzy sets theory permits the gradual assessment of the membership of element in a set; this is described with the aid of a membership function valued in the real unit interval $[0,1]$.

Let us consider two examples:

(i) all employees of XYZ who are over $1.8m$ in height; (ii) all employees of XYZ who are tall. The first example is a classical set with a universe (all XYZ employees) and a membership rule that divides the universe into members (those over $1.8m$) and nonmembers. The second example is a fuzzy set, because some employees are definitely in the set and some are definitely not in the set, but some are borderline.

This distinction between the ins, the outs, and the borderline is made more exact by the membership function, μ. If we return to our second example and let A represent the fuzzy set of all tall employees and x represent a member of the universe X (i.e. all employees), then $\mu_A(x)$ would be $\mu_A(x) = 1$ if x is definitely tall or $\mu_A(x) = 0$ if x is definitely not tall or $0 < \mu_A(x) < 1$ for borderline cases.

Definition 2.2 [1] The intuitionistic fuzzy sets are defined on a non-empty sets X as objects having the form $I = \{\langle x, \mu_I(x), \lambda_I(x)\rangle : x \in X\}$, where $\mu_I(x) : X \to [0,1]$ and $\lambda_I(x) : X \to [0,1]$ denote the degree of memebership and the degree of non-memebership of each element $x \in X$ to the set I, respectively, and $0 \leq \mu_I(x) + \lambda_I(x) \leq 1$, for all $x \in X$.

Definition 2.3 [1,2,4,5] Let a nonempty set X be fixed. An IFS A in X is an object having the form: $A = \{< x, \mu_A(x), \lambda_A(x) > | x \in X\}$ or $A = \left\{\left\langle \frac{\mu_A(x), \lambda_A(x)}{x} \right\rangle | x \in X\right\}$, where the functions $\mu_A(x) : X \to [0,1]$ and $\lambda_A(x) : X \to [0,1]$ define the degree of membership and the degree of nonmembership, respectively, of the element $x \in X$ to A, which is a subset of X, and for every $x \in X$: $0 \le \mu_A(x) + \lambda_A(x) \le 1$. For each A in X: $\pi_A(x) = 1 - \mu_A(x) - \lambda_A(x)$ is the intuitionistic fuzzy set index or hesitation margin of x in X. The hesitation margin $\pi_A(x)$ is the degree of nondeterminacy of $x \in X$ to the set A and $\pi_A(x) \in [0,1]$. The hesitation margin is the function that expresses lack of knowledge of whether $x \in X$ or $x \notin X$. Thus: $\mu_A(x) + \lambda_A(x) + \pi_A(x) = 1$.

Example 2.1 *Let $X = \{x, y, z\}$ be a fixed universe of discourse and $A = \left\{\left\langle \frac{0.6, 0.1}{x} \right\rangle, \left\langle \frac{0.8, 0.1}{y} \right\rangle, \left\langle \frac{0.5, 0.3}{z} \right\rangle\right\}$, be the intuitionistic fuzzy set in X. The hesitation margins of the elements x, y, z to A are as follows: $\pi_A(x) = 0.3$, $\pi_A(y) = 0.1$ and $\pi_A(z) = 0.2$.*

Definition 2.4 [26,27,28] Let X be a universal set. Then, a Pythagorean fuzzy set A, which is a set of ordered pairs over X, is defined by the following: $A = \{< x, \mu_A(x), \lambda_A(x) | x \in X\}$ or $A = \left\{\left\langle \frac{\mu_A(x), \lambda_A(x)}{x} \right\rangle | x \in X\right\}$, where the functions $\mu_A(x) : X \to [0,1]$ and $\lambda_A(x) : X \to [0,1]$ define the degree of membership and the degree of nonmembership, respectively, of the element $x \in X$ to A, which is a subset of X, and for every $x \in X$, $0 \le (\mu_A(x))^2 + (\lambda_A(x))^2 \le 1$. Supposing $(\mu_A(x))^2 + (\lambda_A(x))^2 \le 1$, then there is a degree of indeterminacy of $x \in X$ to A defined by $\pi_A(x) = \sqrt{1 - [(\mu_A(x))^2 + (\lambda_A(x))^2]}$ and $\pi_A(x) \in [0,1]$. In what follows, $(\mu_A(x))^2 + (\lambda_A(x))^2 + (\pi_A(x))^2 = 1$. Otherwise, $\pi_A(x) = 0$ whenever $(\mu_A(x))^2 + (\lambda_A(x))^2 = 1$. We denote the set of all PFS's over X by $pfs(X)$.

Definition 2.5 [17] Let X be a universe of discourse. A Fermatean fuzzy set ($\mathfrak{F}\mathcal{F}s$) F in X is an object having the form $F = \{< x, \mu_F(x), \lambda_F(x) >: x \in X\}$ where $\mu_F(x) : X \to [0,1]$ and $\lambda_F(x) : X \to [0,1]$, including the condition $0 \le (\mu_F(x))^3 + (\lambda_F(x))^3 \le 1$, for all $x \in X$. The numbers $\mu_F(x)$ and $\lambda_F(x)$ denote, respectively, the degree of memebership and the degree of non-memebership of the element x in the set F. For any $\mathfrak{F}\mathcal{F}s$ F and $x \in X$, $\pi_F(x) = \sqrt[3]{1 - [(\mu_F(x))^3 - (\lambda_F(x))^3]}$ is identified as the degree of interminancy of x to F. In the interest of simplicity, we shall mention the symbol $F = (\mu_F, \lambda_F)$ for the $\mathfrak{F}\mathcal{F}s$ $F = \{< x, \mu_F(x), \lambda_F(x) : x \in X\}$.

Definition 2.6 [17] Let $F = (\mu_F, \lambda_F)$, $F_1 = (\mu_{F_1}, \lambda_{F_1})$ and $F_2 = (\mu_{F_2}, \lambda_{F_2})$, be three Fermatean fuzzy sets ($\mathfrak{F}\mathcal{F}s$'s), then their operations are defined as follows:

(i) $F_1 \cap F_2 = (min\{\mu_{F_1}, \mu_{F_2}\}, max\{\lambda_{F_1}, \lambda_{F_2}\})$.

(ii) $F_1 \cup F_2 = (max\{\mu_{F_1}, \mu_{F_2}\}, min\{\lambda_{F_1}, \lambda_{F_2}\})$.

(iii) $F^c = (\lambda_F, \mu_F)$.

Remark 2.1 *If $\mu_{F_1} = \mu_{F_2}$ and $\lambda_{F_1} = \lambda_{F_2}$, then $F_1 = F_2$*

For understanding the Fermatean fuzzy set better, we give an instance to illuminate the understandability of the Fermatean fuzzy set. The point when someone needs will plan as much craving for the level for an alternative s_i on a criterion C_j, he might provide for the degree on which that alternative s_i fulfils those criteria C_j likewise 0.85, what is more correspondingly the elective s_i dissatisfies the criterion C_j similarly as 0.65. We can definitely get $0.85 + 0.65 = 1.5 > 1$, and, therefore, it does not follow the condition of intuitionistic fuzzy sets. Also, we can get $(0.85)^2 + (0.65)^2 = 0.7225 + 0.4225 = 1.145 > 1$, which does not obey the constraint condition of Pythagorean fuzzy set. However, we can get $(0.85)^3 + (0.65)^3 = 0.614125 + 0.274625 = 0.88875 \le 1$, which is good enough to apply the Fermatean fuzzy set to control it [17]. Throughout this paper, we use the notation $1_{\mathfrak{F}}$ for the Fermatean fuzzy subset $(1, 0)$ and we use the notation $0_{\mathfrak{F}}$ for the Fermatean fuzzy subset $(0, 1)$, that is, $\mu_{1_{\mathfrak{F}}} = 1$, $\lambda_{1_{\mathfrak{F}}} = 0$, $\mu_{0_{\mathfrak{F}}} = 0$, $\lambda_{0_{\mathfrak{F}}} = 1$. A Fermatean fuzzy subset $\mathfrak{F}$ of a non-empty set X is a pair $(\mu_{\mathfrak{F}}, \lambda_{\mathfrak{F}})$ of a membership function $(\mu_{\mathfrak{F}}(x) : X \to [0,1]$ and a non-membership function $(\lambda_{\mathfrak{F}}(x) : X \to [0,1]$ with $(\mu_{\mathfrak{F}}(x))^3 + (\lambda_{\mathfrak{F}}(x))^3 = (\gamma_{\mathfrak{F}}(x))^3$ for any $x \in X$ where $\gamma_{\mathfrak{F}}(x) : X \to [0,1]$ is a function which is called the strength of commitment at point x.

Definition 2.7 [10] Let X be a non empty set and τ be a family of Fermatean fuzzy subsets of X. If

(i) $1_F, 0_F \in \tau$

(ii) for any $F_1, F_2 \in \tau$, we have $F_1 \cap F_2 \in \tau$,

(iii) for any $\{F_i\}_{i\in I} \subset \tau$, we have $\bigcup_{i\in I} F_i \in \tau$ where I is an arbitrary index set then τ is called a Fermatean fuzzy topology on X.
The pair (X, τ) is said to be a Fermatean fuzzy topological space. Each member of τ is called an Fermatean fuzzy oprn set. The complement of an Fermatean fuzzy open set is called a Fermatean fuzzy closed set.

Remark 2.2 *[10] As any Intuitionistic fuzzy subset or Phythagorean fuzzy subset of a set can be considered as Fermatean fuzzy subset, we observe that any Intuitionstic fuzzy topological space or Phythagorean fuzzy topological space is a Fermatean fuzzy topological space as well. On the other hand, it is obvious that a Fermatean fuzzy topological space need not be Intuitionistic fuzzy topological space and Phythagorean fuzzy topological space. Even an Fermatean fuzzy open set maybe neither an Intuitionistic fuzzy set nor Phythagorean fuzzy set.*

Example 2.2 *[10] Let $X = \{c_1, c_2\}$. Consider the following family Fermatean fuzzy subsets $\tau = \{1_F, 0_F, F_1, F2\}$ where $F_1 = \{\langle c_1, \mu_{F_1}(c_1) = 0.4, \lambda_{F_1}(c_1) = 0.6\rangle, \langle c_2, \mu_{F_1}(c_2) = 0.1, \lambda_{F_1}(c_2) = 0.3\rangle\}$ and $F_2 = \{\langle c_1, \mu_{F_2}(c_1) = 0.9, \lambda_{F_2}(c_1) = 0.6\rangle, \langle c_2, \mu_{F_2}(c_2) = 0.2, \lambda_{F_2}(c_2) = 0.3\rangle\}$. Observe that (X, τ) is a Fermatean fuzzy topological space but (X, τ) is neither Intuitionistic fuzzy topological space nor Phythagorean fuzzy topological space.*

Definition 2.8 [10] Let (X, τ) be an $\mathfrak{F}\mathcal{F}ts$ and $A = \{< a, \mu_A(a), \lambda_A(a) > |a \in X\}$ be an $\mathfrak{F}\mathcal{F}s$ in X. Then the Fermatean fuzzy interior and the Fermatean fuzzy closure of A are denoted by $\mathfrak{F}\mathcal{F}int(A)$ and $\mathfrak{F}\mathcal{F}cl(A)$ and are defined as follows: $\mathfrak{F}\mathcal{F}int(A) = \cup\{G|G$ is a $\mathfrak{F}\mathcal{F}os$ and $G \subseteq A\}$ and $\mathfrak{F}\mathcal{F}cl(A) = \cap\{K|K$ is a $\mathfrak{F}\mathcal{F}cs$ and $A \subseteq K\}$. Also, it can be established that $\mathfrak{F}\mathcal{F}cl(A)$ is an $\mathfrak{F}\mathcal{F}cs$ and $\mathfrak{F}\mathcal{F}int(A)$ is an $\mathfrak{F}\mathcal{F}os$, A is an $\mathfrak{F}\mathcal{F}cs$ if and only if $\mathfrak{F}\mathcal{F}cl(A) = A$ and A is an $\mathfrak{F}\mathcal{F}os$ if and only if $\mathfrak{F}\mathcal{F}int(A) = A$. We say that A is $\mathfrak{F}\mathcal{F}$-dense if $\mathfrak{F}\mathcal{F}cl(A) = 1_{\mathfrak{F}}$.

Lemma 2.1 *[10] For any Fermatean fuzzy set A in (X, τ), we have $1_F - \mathfrak{F}\mathcal{F}int(A) = \mathfrak{F}\mathcal{F}cl(1_{\mathfrak{F}} - A)$ and $1_{\mathfrak{F}} - \mathfrak{F}\mathcal{F}cl(A) = \mathfrak{F}\mathcal{F}int(1_{\mathfrak{F}} - A)$.*

Definition 2.9 [23] Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$ and $S = \{< s, \mu_S(s), \lambda_S(s) > |s \in X\}$ be an $\mathfrak{F}\mathcal{F}s$ in X. A set S is said to be $\mathfrak{F}\mathcal{F}$

(i) regular open (briefly, $\mathfrak{F}\mathcal{F}ro$) set if $S = \mathfrak{F}\mathcal{F}int(\mathfrak{F}\mathcal{F}cl(S))$.

(ii) regular closed (briefly, $\mathfrak{F}\mathcal{F}rc$) set if $S = \mathfrak{F}\mathcal{F}cl(\mathfrak{F}\mathcal{F}int(S))$.

Definition 2.10 [23] Let (X, τ) be an $\mathfrak{F}\mathcal{F}ts$ and $S = \{< s, \mu_S(s), \lambda_S(s) > |s \in X\}$ be an $\mathfrak{F}\mathcal{F}s$ in X. Then the $\mathfrak{F}\mathcal{F}\delta$-interior and the $\mathfrak{F}\mathcal{F}\delta$-closure of S are denoted by $\mathfrak{F}\mathcal{F}\delta int(S)$ and $\mathfrak{F}\mathcal{F}\delta cl(S)$ and are defined as follows. $\mathfrak{F}\mathcal{F}\delta int(S) = \bigcup\{G|G$ is an $\mathfrak{F}\mathcal{F}ros$ and $G \subseteq S\}$, $\mathfrak{F}\mathcal{F}\delta cl(S) = \bigcap\{K|K$ is an $\mathfrak{F}\mathcal{F}rcs$ and $S \subseteq K\}$.

Definition 2.11 [23] Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$ and $S = \{< s, \mu_S(s), \lambda_S(s) > |s \in X\}$ be an $\mathfrak{F}\mathcal{F}s$ in X. A set S is said to be $\mathfrak{F}\mathcal{F}$

(i) δ-open set (briefly, $\mathfrak{F}\mathcal{F}\delta os$) if $S = \mathfrak{F}\mathcal{F}\delta int(S)$,

(ii) δ-pre open set (briefly, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}os$) if $S \subseteq \mathfrak{F}\mathcal{F}int(\mathfrak{F}\mathcal{F}\delta cl(S))$.

(iii) δ-semi open set (briefly, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}os$) if $S \subseteq \mathfrak{F}\mathcal{F}cl(\mathfrak{F}\mathcal{F}\delta int(S))$.

(iv) e open set (briefly, $\mathfrak{F}\mathcal{F}eos$) if $S \subseteq \mathfrak{F}\mathcal{F}cl(\mathfrak{F}\mathcal{F}\delta int(S)) \cup \mathfrak{F}\mathcal{F}int(\mathfrak{F}\mathcal{F}\delta cl(S))$.

(v) δ (resp. δ-pre, δ-semi and e) dense if $\mathfrak{F}\mathcal{F}\delta cl(S)$ (resp.$\mathfrak{F}\mathcal{F}\delta\mathcal{P}cl(S)$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}cl(S)$ and $\mathfrak{F}\mathcal{F}ecl(S)) = 1_{\mathfrak{F}}$.

The complement of an $\mathfrak{F}\mathcal{F}\delta os$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{P}os$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}os$ and $\mathfrak{F}\mathcal{F}eos$) is called an $\mathfrak{F}\mathcal{F}\delta$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{P}$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}$ and $\mathfrak{F}\mathcal{F}e$) closed set (briefly, $\mathfrak{F}\mathcal{F}\delta cs$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{P}cs$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}cs$ and $\mathfrak{F}\mathcal{F}ecs$)) in X.

The family of all $\mathfrak{F}\mathcal{F}\delta os$ (resp. $\mathfrak{F}\mathcal{F}\delta cs$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}os$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}cs$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}os$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}cs$, $\mathfrak{F}\mathcal{F}eos$ and $\mathfrak{F}\mathcal{F}ecs$) of X is denoted by $\mathfrak{F}\mathcal{F}\delta OS(X)$, (resp. $\mathfrak{F}\mathcal{F}\delta CS(X)$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}OS(X)$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}CS(X), \mathfrak{F}\mathcal{F}\delta\mathcal{S}OS(X), \mathfrak{F}\mathcal{F}\delta\mathcal{S}CS(X)$, $\mathfrak{F}\mathcal{F}eOS(X)$ and $\mathfrak{F}\mathcal{F}eCS(X)$).

Definition 2.12 [23] Let (X,τ) be an $\mathfrak{F}\mathcal{F}ts$ and $S = \{< s, \mu_S(s), \lambda_S(s) > | s \in X\}$ be an $\mathfrak{F}\mathcal{F}s$ in X. Then the $\mathfrak{F}\mathcal{F}\delta$-pre (resp. $\mathfrak{F}\mathcal{F}\delta$-semi and $\mathfrak{F}\mathcal{F}e$)-interior and the $\mathfrak{F}\mathcal{F}\delta$-pre (resp. $\mathfrak{F}\mathcal{F}\delta$-semi and $\mathfrak{F}\mathcal{F}e$)-closure of S are denoted by $\mathfrak{F}\mathcal{F}\delta\mathcal{P}int(S)$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S}int(S)$ and $\mathfrak{F}\mathcal{F}eint(S)$) and the $\mathfrak{F}\mathcal{F}\delta\mathcal{P}cl(S)$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S}cl(S)$ and $\mathfrak{F}\mathcal{F}ecl(S)$) and are defined as follows:

$\mathfrak{F}\mathcal{F}\delta\mathcal{P}int(S)$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S}int(S)$ and $\mathfrak{F}\mathcal{F}eint(S)$) $= \bigcup\{G|G$ is a $\mathfrak{F}\mathcal{F}\delta\mathcal{P}os$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S}os$ and $\mathfrak{F}\mathcal{F}eos$) and $G \subseteq S\}$ and $\mathfrak{F}\mathcal{F}\delta\mathcal{P}cl(S)$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S}cl(S)$ and $\mathfrak{F}\mathcal{F}ecl(S)$) $= \bigcap\{K|K$ is an $\mathfrak{F}\mathcal{F}\delta\mathcal{P}cs$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{S}cs$ *and* $\mathfrak{F}\mathcal{F}ecs$) and $S \subseteq K\}$.

Definition 2.13 [24] Let (X_1,τ_1) and (X_2,τ_2) be any two $\mathfrak{F}\mathcal{F}ts$'s. A mapping $h_{\mathfrak{F}} : (X_1,\tau_1) \to (X_2,\tau_2)$ is said to be a Fermatean fuzzy (resp. δ, $\delta\mathcal{P}$ and $\delta\mathcal{S}$)-continuous (briefly, $\mathfrak{F}\mathcal{F}Cts$ (resp. $\mathfrak{F}\mathcal{F}\delta Cts$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Cts$ and $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$)) if the inverse image of every $\mathfrak{F}\mathcal{F}os$ in (X_2,τ_2) is a $\mathfrak{F}\mathcal{F}os$ (resp. $\mathfrak{F}\mathcal{F}\delta os$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}os$ and $\mathfrak{F}\mathcal{F}\delta\mathcal{S}os$) in (X_1,τ_1).

Definition 2.14 [25]Let (X_1,τ_1) and (X_2,τ_2) be two $\mathfrak{F}\mathcal{F}ts$. A function $h_{\mathfrak{F}} : (X_1,\tau_1) \to (X_2,\tau_2)$ is said to be Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$ and $\delta\mathcal{P}$) open map (briefly, $\mathfrak{F}\mathcal{F}O$ (resp. $\mathfrak{F}\mathcal{F}\delta O$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}O$ and $\mathfrak{F}\mathcal{F}\delta\mathcal{P}O$)) if the image of each $\mathfrak{F}\mathcal{F}c$ set in X_1 is $\mathfrak{F}\mathcal{F}o$ (resp. $\mathfrak{F}\mathcal{F}\delta o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ and $\mathfrak{F}\mathcal{F}\delta\mathcal{P}o$)-set in X_2.

Definition 2.15 [25] Let (X_1,τ_1) and (X_2,τ_2) be two $\mathfrak{F}\mathcal{F}ts$. A function $h_{\mathfrak{F}} : (X_1,\tau_1) \to (X_2,\tau_2)$ is said to be Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$ and $\delta\mathcal{P}$) closed map (briefly, $\mathfrak{F}\mathcal{F}C$ (resp. $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\delta\mathcal{P}C$)) if the image of each $\mathfrak{F}\mathcal{F}c$ set in X_1 is $\mathfrak{F}\mathcal{F}c$ (resp. $\mathfrak{F}\mathcal{F}\delta c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$ and $\mathfrak{F}\mathcal{F}\delta\mathcal{P}c$)-set in X_2.

3. Fermatean Fuzzy M homeomorphism

The purpose of this section is to introduces the idea of Fermatean fuzzy M homeomorphism in $\mathfrak{F}\mathcal{F}ts$ and establish some of their attributes.

Definition 3.1 Let (X,τ) be a $\mathfrak{F}\mathcal{F}ts$ and S be a $\mathfrak{F}\mathcal{F}s$ in X. A set S is said to be $\mathfrak{F}\mathcal{F}$

(i) θ-interior of S (briefly, $\mathfrak{F}\mathcal{F}\theta int(S)$) is defined by
$\mathfrak{F}\mathcal{F}\theta int(S) = \bigcup\{\mathfrak{F}\mathcal{F}int(T) : T \subseteq S \;\&\; T$ is a $\mathfrak{F}\mathcal{F}cs$ in $X\}$.

(ii) θ-open set (briefly, $\mathfrak{F}\mathcal{F}\theta os$) if $S = \mathfrak{F}\mathcal{F}\theta int(S)$.

(iii) θ -semi open set (briefly, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}os$) if $S \subseteq \mathfrak{F}\mathcal{F}cl(\mathfrak{F}\mathcal{F}\theta int(S))$.

(iv) M-open set (briefly, $\mathfrak{F}\mathcal{F}Mos$) if $S \subseteq \mathfrak{F}\mathcal{F}cl(\mathfrak{F}\mathcal{F}\theta int(S)) \cup \mathfrak{F}\mathcal{F}int(\mathfrak{F}\mathcal{F}\delta cl(S))$.

The complement of a $\mathfrak{F}\mathcal{F}Mos$ (resp. $\mathfrak{F}\mathcal{F}\theta os$ & $\mathfrak{F}\mathcal{F}\theta\mathcal{S}os$) is called an $\mathfrak{F}\mathcal{F}M$ (resp. $\mathfrak{F}\mathcal{F}\theta$ & $\mathfrak{F}\mathcal{F}\theta\mathcal{S}$) closed set (briefly, $\mathfrak{F}\mathcal{F}Mcs$ (resp. $\mathfrak{F}\mathcal{F}\theta cs$ & $\mathfrak{F}\mathcal{F}\theta\mathcal{S}cs$)) in X.

The family of all $\mathfrak{F}\mathcal{F}\theta os$ (resp. $\mathfrak{F}\mathcal{F}\theta cs$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}os$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}cs$, $\mathfrak{F}\mathcal{F}Mos$ and $\mathfrak{F}\mathcal{F}Mcs$) of X is denoted by $\mathfrak{F}\mathcal{F}\theta OS(X)$ (resp. $\mathfrak{F}\mathcal{F}\theta CS(X)$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}OS(X)$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}CS(X)$, $\mathfrak{F}\mathcal{F}MOS(X)$ and $\mathfrak{F}\mathcal{F}MCS(X)$).

Definition 3.2 Let (X,τ) be a $\mathfrak{F}\mathcal{F}ts$ and S be a $\mathfrak{F}\mathcal{F}s$ in X. Then the $\mathfrak{F}\mathcal{F}$

(i) M-interior (resp. $\mathfrak{F}\mathcal{F}\theta$-interior and $\mathfrak{F}\mathcal{F}\theta$-semi interior) of S (briefly, $\mathfrak{F}\mathcal{F}Mint(S)$ (resp. $\mathfrak{F}\mathcal{F}\theta int(S)$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}int(S)$) is defined by $\mathfrak{F}\mathcal{F}Mint(S)$ (resp. $\mathfrak{F}\mathcal{F}\theta int(S)$ and $\mathfrak{F}\mathcal{F}\theta\mathcal{S}int(S)$) $= \bigcup\{T : T \subseteq S$ and T is a $\mathfrak{F}\mathcal{F}Mos$ (resp. $\mathfrak{F}\mathcal{F}\theta os$ and $\mathfrak{F}\mathcal{F}\theta\mathcal{S}os$) in $X\}$.

(ii) M-closure (resp. θ-closure and θ-semi closure) of S (briefly, $\mathfrak{F}\mathcal{F}Mcl(S)$ (resp. $\mathfrak{F}\mathcal{F}\theta cl(S)$ & $\mathfrak{F}\mathcal{F}\theta\mathcal{S}cl(S)$) is defined by $\mathfrak{F}\mathcal{F}Mcl(S)$ (resp. $\mathfrak{F}\mathcal{F}\theta cl(S)$ and $\mathfrak{F}\mathcal{F}\theta\mathcal{S}cl(S)$) $= \bigcap\{T : S \subseteq T$ and T is a $\mathfrak{F}\mathcal{F}Mcs$ (resp. $\mathfrak{F}\mathcal{F}\theta cs$ and $\mathfrak{F}\mathcal{F}\theta\mathcal{S}cs$) in $X\}$.

Theorem 3.1 *Let K be a Fermatean fuzzy subset of a space (X, τ) Then*

(i) K is a $\mathfrak{F}\mathcal{F}Mo$ set iff $K = \mathfrak{F}\mathcal{F}Mint(K)$,

(ii) K is a $\mathfrak{F}\mathcal{F}Mc$ set iff $K = \mathfrak{F}\mathcal{F}Mcl(K)$.

Definition 3.3 Let (X_1, τ_1) and (X_2, τ_2) be any two $\mathfrak{F}\mathcal{F}ts$'s. A mapping $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is said to be a Fermatean fuzzy M (resp. θ, $\theta\mathcal{S}$ and e)-continuous (briefly, $\mathfrak{F}\mathcal{F}MCts$ (resp. $\mathfrak{F}\mathcal{F}\theta Cts$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}Cts$ and $\mathfrak{F}\mathcal{F}eCts$)) if the inverse image of every $\mathfrak{F}\mathcal{F}os$ in (X_2, τ_2) is a $\mathfrak{F}\mathcal{F}Mos$ (resp. $\mathfrak{F}\mathcal{F}\theta os$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}os$ and $\mathfrak{F}\mathcal{F}eos$) in (X_1, τ_1).

Definition 3.4 Let (X_1, τ_1) and (X_2, τ_2) be two $\mathfrak{F}\mathcal{F}ts$. A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is said to be Fermatean fuzzy M (resp. θ, $\theta\mathcal{S}$ and e) open map (briefly, $\mathfrak{F}\mathcal{F}MO$ (resp. $\mathfrak{F}\mathcal{F}\theta O$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}O$ and $\mathfrak{F}\mathcal{F}eO$)) if the image of each $\mathfrak{F}\mathcal{F}o$ set in X_1 is $\mathfrak{F}\mathcal{F}Mo$ (resp. $\mathfrak{F}\mathcal{F}\theta o$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}o$ and $\mathfrak{F}\mathcal{F}eo$)-set in X_2.

Definition 3.5 Let (X_1, τ_1) and (X_2, τ_2) be two $\mathfrak{F}\mathcal{F}ts$. A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is said to be Fermatean fuzzy M (resp. θ, $\theta\mathcal{S}$ and e) closed map (briefly, $\mathfrak{F}\mathcal{F}MC$ (resp. $\mathfrak{F}\mathcal{F}\theta C$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}eC$)) if the image of each $\mathfrak{F}\mathcal{F}c$ set in X_1 is $\mathfrak{F}\mathcal{F}Mc$ (resp. $\mathfrak{F}\mathcal{F}\theta c$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}c$ and $\mathfrak{F}\mathcal{F}ec$)-set in X_2.

Definition 3.6 Let $(X_1, \tau_{\mathfrak{F}}(F_1))$ and $(X_2, \tau_{\mathfrak{F}}(F_2))$ be $\mathfrak{F}\mathcal{F}ts$. A mapping $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is said to be a Fermatean fuzzy (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$, δ, $\delta\mathcal{S}$, e and M) homeomorphism (briefly, $\mathfrak{F}\mathcal{F}Hom$ (resp. $\mathfrak{F}\mathcal{F}\theta Hom$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}Hom$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Hom$, $\mathfrak{F}\mathcal{F}\delta Hom$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Hom$, $\mathfrak{F}\mathcal{F}eHom$ and $\mathfrak{F}\mathcal{F}MHom$)) if $h_{\mathfrak{F}}$ is bijective, $\mathfrak{F}\mathcal{F}Cts$ (resp. $\mathfrak{F}\mathcal{F}\theta Cts$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}Cts$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Cts$, $\mathfrak{F}\mathcal{F}\delta Cts$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$, $\mathfrak{F}\mathcal{F}eCts$ and $\mathfrak{F}\mathcal{F}MCts$) function and $\mathfrak{F}\mathcal{F}Cts$ (resp. $\mathfrak{F}\mathcal{F}\theta Cts$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}Cts$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Cts$, $\mathfrak{F}\mathcal{F}\delta Cts$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$, $\mathfrak{F}\mathcal{F}eCts$ and $\mathfrak{F}\mathcal{F}MCts$) mapping.

Proposition 3.1 *Let (X_1, τ_1) & (X_2, τ_2) be a $\mathfrak{F}\mathcal{F}ts$'s. Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be a mapping. Then the following statements are hold for $\mathfrak{F}\mathcal{F}ts$, but not conversely.*

(i) Every $\mathfrak{F}\mathcal{F}\theta Hom$ is a $\mathfrak{F}\mathcal{F}Hom$.

(ii) Every $\mathfrak{F}\mathcal{F}\theta Hom$ is a $\mathfrak{F}\mathcal{F}\theta\mathcal{S}Hom$.

(iii) Every $\mathfrak{F}\mathcal{F}\theta\mathcal{S}Hom$ is a $\mathfrak{F}\mathcal{F}MHom$.

(iv) Every $\mathfrak{F}\mathcal{F}\delta Hom$ is a $\mathfrak{F}\mathcal{F}Hom$.

(v) Every $\mathfrak{F}\mathcal{F}\delta Hom$ is a $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Hom$.

(vi) Every $\mathfrak{F}\mathcal{F}\delta Hom$ is a $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Hom$.

(vii) Every $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Hom$ is a $\mathfrak{F}\mathcal{F}eHom$.

(viii) Every $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Hom$ is a $\mathfrak{F}\mathcal{F}MHom$.

(ix) Every $\mathfrak{F}\mathcal{F}MHom$ is a $\mathfrak{F}\mathcal{F}eHom$.

Proof: (i) Let $h_{\mathfrak{F}}$ be $\mathfrak{F}\mathcal{F}\theta Hom$, then $h_{\mathfrak{F}}$ and $h_{\mathfrak{F}}^{-1}$ are $\mathfrak{F}\mathcal{F}\theta Cts$. But every $\mathfrak{F}\mathcal{F}\theta Cts$ function is $\mathfrak{F}\mathcal{F}Cts$. Hence, $h_{\mathfrak{F}}$ and $h_{\mathfrak{F}}^{-1}$ are $\mathfrak{F}\mathcal{F}Cts$. Therefore, $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}Hom$.

(ii) Let $h_{\mathfrak{F}}$ be $\mathfrak{F}\mathcal{F}\theta Hom$, then $h_{\mathfrak{F}}$ and $h_{\mathfrak{F}}^{-1}$ are $\mathfrak{F}\mathcal{F}\theta Cts$. But every $\mathfrak{F}\mathcal{F}\theta Cts$ function is $\mathfrak{F}\mathcal{F}\theta\mathcal{S}Cts$. Hence, $h_{\mathfrak{F}}$ and $h_{\mathfrak{F}}^{-1}$ are $\mathfrak{F}\mathcal{F}\theta\mathcal{S}Cts$. Therefore, $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}\theta\mathcal{S}Hom$.

(iii) Let $h_{\mathfrak{F}}$ be $\mathfrak{F}\mathcal{F}\theta\mathcal{S}Hom$, then $h_{\mathfrak{F}}$ and $h_{\mathfrak{F}}^{-1}$ are $\mathfrak{F}\mathcal{F}\theta\mathcal{S}Cts$. But every $\mathfrak{F}\mathcal{F}\theta\mathcal{S}Cts$ function is $\mathfrak{F}\mathcal{F}MCts$. Hence, $h_{\mathfrak{F}}$ and $h_{\mathfrak{F}}^{-1}$ are $\mathfrak{F}\mathcal{F}MCts$. Therefore, $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}MHom$.

(iv) Let $h_{\mathfrak{F}}$ be $\mathfrak{F}\mathcal{F}\delta Hom$, then $h_{\mathfrak{F}}$ and $h_{\mathfrak{F}}^{-1}$ are $\mathfrak{F}\mathcal{F}\delta Cts$. But every $\mathfrak{F}\mathcal{F}\delta Cts$ function is $\mathfrak{F}\mathcal{F}Cts$. Hence, $h_{\mathfrak{F}}$ and $h_{\mathfrak{F}}^{-1}$ are $\mathfrak{F}\mathcal{F}Cts$. Therefore, $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}Hom$.

(v) Let $h_{\mathfrak{F}}$ be $\mathfrak{F}\mathcal{F}\delta Hom$, then $h_{\mathfrak{F}}$ and $h_{\mathfrak{F}}^{-1}$ are $\mathfrak{F}\mathcal{F}\delta Cts$. But every $\mathfrak{F}\mathcal{F}\delta Cts$ function is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$. Hence, $h_{\mathfrak{F}}$ and $h_{\mathfrak{F}}^{-1}$ are $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$. Therefore, $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Hom$.

(vi) Let $h_{\mathfrak{F}}$ be $\mathfrak{F}\mathcal{F}\delta Hom$, then $h_{\mathfrak{F}}$ and $h_{\mathfrak{F}}^{-1}$ are $\mathfrak{F}\mathcal{F}\delta Cts$. But every $\mathfrak{F}\mathcal{F}\delta Cts$ function is $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Cts$. Hence, $h_{\mathfrak{F}}$ and $h_{\mathfrak{F}}^{-1}$ are $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Cts$. Therefore, $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Hom$.

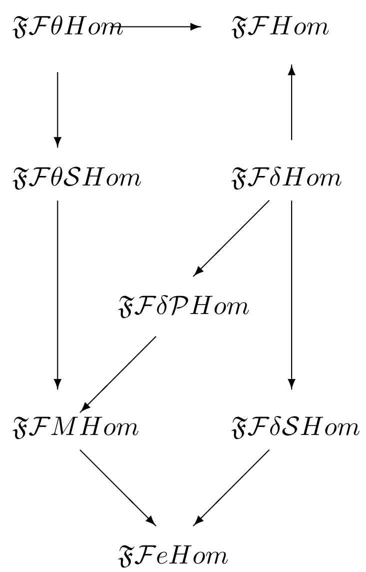
(vii) Let $h_{\mathfrak{F}}$ be $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Hom$, then $h_{\mathfrak{F}}$ and $h_{\mathfrak{F}}^{-1}$ are $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$. But every $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$ function is $\mathfrak{F}\mathcal{F}eCts$. Hence, $h_{\mathfrak{F}}$ and $h_{\mathfrak{F}}^{-1}$ are $\mathfrak{F}\mathcal{F}eCts$. Therefore, $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}eHom$.

(viii) Let $h_{\mathfrak{F}}$ be $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Hom$, then $h_{\mathfrak{F}}$ and $h_{\mathfrak{F}}^{-1}$ are $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Cts$. But every $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Cts$ function is $\mathfrak{F}\mathcal{F}MCts$. Hence, $h_{\mathfrak{F}}$ and $h_{\mathfrak{F}}^{-1}$ are $\mathfrak{F}\mathcal{F}MCts$. Therefore, $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}MHom$.

(ix) Let $h_{\mathfrak{F}}$ be $\mathfrak{F}\mathcal{F}MHom$, then $h_{\mathfrak{F}}$ and $h_{\mathfrak{F}}^{-1}$ are $\mathfrak{F}\mathcal{F}MCts$. But every $\mathfrak{F}\mathcal{F}MCts$ function is $\mathfrak{F}\mathcal{F}eCts$. Hence, $h_{\mathfrak{F}}$ and $h_{\mathfrak{F}}^{-1}$ are $\mathfrak{F}\mathcal{F}eCts$. Therefore, $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}eHom$.

□

Remark 3.1 *We obtain the following diagram from the results are discussed above.*



Note: $A \to B$ denotes A implies B, but not conversely.

Example 3.1 *Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1, A_2, A_3 and A_4 are defined as*
$\alpha_{A_1}(a) = 0.2,\ \beta_{A_1}(a) = 0.8,$
$\alpha_{A_1}(b) = 0.4,\ \beta_{A_1}(b) = 0.6;$
$\alpha_{A_2}(a) = 0.1,\ \beta_{A_2}(a) = 0.9,$
$\alpha_{A_2}(b) = 0.3,\ \beta_{A_2}(b) = 0.7;$
$\alpha_{A_3}(a) = 0.9,\ \beta_{A_3}(a) = 0.1,$
$\alpha_{A_3}(b) = 0.7,\ \beta_{A_3}(b) = 0.7;$
$\alpha_{A_4}(a) = 0.2,\ \beta_{A_4}(a) = 0.8,$
$\alpha_{A_4}(b) = 0.3,\ \beta_{A_4}(b) = 0.7;$
Let $\tau_1 = \tau_2 = \tau = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2,\ A_3,\ A_4\}$ be a $\mathfrak{F}\mathcal{F}ts$ on X and let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}Hom$ (resp. $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Hom$) but not $\mathfrak{F}\mathcal{F}\delta Hom$ (resp. $\mathfrak{F}\mathcal{F}\delta Hom$). Since, A_4 is a $\mathfrak{F}\mathcal{F}o$ set in X_2 but $h_{\mathfrak{F}}^{-1}(A_4) = A_4$ is not $\mathfrak{F}\mathcal{F}\delta o$ (resp. $\mathfrak{F}\mathcal{F}\delta o$) set in X_1.

Example 3.2 *Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1, A_2, A_3 and A_4 are defined as*
$\alpha_{A_1}(a) = 0.4,\ \beta_{A_1}(a) = 0.6,$
$\alpha_{A_1}(b) = 0.5,\ \beta_{A_1}(b) = 0.5;$
$\alpha_{A_2}(a) = 0.6,\ \beta_{A_2}(a) = 0.4,$
$\alpha_{A_2}(b) = 0.6,\ \beta_{A_2}(b) = 0.4;$
$\alpha_{A_3}(a) = 0.7,\ \beta_{A_3}(a) = 0.3,$

$\alpha_{A_3}(b) = 0.6,\ \beta_{A_3}(b) = 0.4;$
$\alpha_{A_4}(a) = 0.4,\ \beta_{A_4}(a) = 0.6,$
$\alpha_{A_4}(b) = 0.4,\ \beta_{A_4}(b) = 0.6;$
Let $\tau_1 = \tau_2 = \tau = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2,\ A_3,\ A_4\}$ *be a* $\mathfrak{F}\mathcal{F}ts$ *on* X *and let* $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ *be an identity function, Then* $h_{\mathfrak{F}}$ *is* $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Hom$ *but not* $\mathfrak{F}\mathcal{F}\delta Hom$*. Since,* A_3 *is a* $\mathfrak{F}\mathcal{F}o$ *set in* X_2 *but* $h_{\mathfrak{F}}^{-1}(A_3) = A_3$ *is not* $\mathfrak{F}\mathcal{F}\delta o$ *set in* X_1*.*

Example 3.3 *Let* $X_1 = X_2 = X = \{a, b\}$ *and the* $\mathfrak{F}\mathcal{F}s$*'s* A_1 *and* A_2 *are defined as*

$$A_1 = \{< x_1, 0.20, 0.70 >, < x_2, 0.10, 0.80 >\}$$
$$A_2 = \{< x_1, 0.30, 0.60 >, < x_2, 0.40, 0.50 >\}.$$

Let $\tau_1 = \tau_2 = \tau = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2\}$ *be a* $\mathfrak{F}\mathcal{F}ts$ *on* X *and let* $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ *be an identity function, Then* $h_{\mathfrak{F}}$ *is* $\mathfrak{F}\mathcal{F}Hom$ *but not* $\mathfrak{F}\mathcal{F}\theta Hom$*. Since,* A_1 *is a* $\mathfrak{F}\mathcal{F}o$ *set in* X_2 *but* $h_{\mathfrak{F}}^{-1}(A_1) = A_1$ *is not* $\mathfrak{F}\mathcal{F}\theta o$ *set in* X_1*.*

Example 3.4 *Let* $X_1 = X_2 = X = \{a, b\}$ *and the* $\mathfrak{F}\mathcal{F}s$*'s* A_1*,* A_2 *and* A_3 *are defined as*

$$A_1 = \{< x_1, 0.90, 0.10 >, < x_2, 0.70, 0.30 >\}$$
$$A_2 = \{< x_1, 0.10, 0.90 >, < x_2, 0.30, 0.70 >\}$$
$$A_3 = \{< x_1, 0.20, 0.80 >, < x_2, 0.40, 0.60 >\}.$$

Let $\tau_1 = \tau_2 = \tau = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2,\ A_3\}$ *be a* $\mathfrak{F}\mathcal{F}ts$ *on* X *and let* $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ *be an identity function, Then* $h_{\mathfrak{F}}$ *is* $\mathfrak{F}\mathcal{F}MHom$ *but not* $\mathfrak{F}\mathcal{F}\theta\mathcal{S}Hom$*. Since,* A_3 *is a* $\mathfrak{F}\mathcal{F}o$ *set in* X_2 *but* $h_{\mathfrak{F}}^{-1}(A_3) = A_3$ *is not* $\mathfrak{F}\mathcal{F}\theta\mathcal{S}o$ *set in* X_1*.*

Example 3.5 *Let* $X_1 = X_2 = X = \{a, b\}$ *and the* $\mathfrak{F}\mathcal{F}s$*'s* A_1*,* B_1 *and* B_2 *are defined as*

$$A_1 = \{< x_1, 0.10, 0.90 >, < x_2, 0.30, 0.70 >\}$$
$$B_1 = \{< x_1, 0.30, 0.70 >, < x_2, 0.40, 0.60 >\}$$
$$B_2 = \{< x_1, 0.20, 0.80 >, < x_2, 0.30, 0.70 >\}.$$

Let $\tau_1 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1\}$ *and* $\tau_2 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ B_1,\ B_2\}$ *be a* $\mathfrak{F}\mathcal{F}ts$ *on* X_1 *and* X_2*. Let* $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ *be an identity function, Then* $h_{\mathfrak{F}}$ *is* $\mathfrak{F}\mathcal{F}eHom$ *(resp.* $\mathfrak{F}\mathcal{F}eHom$*) but not* $\mathfrak{F}\mathcal{F}MHom$ *(resp.* $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Hom$*). Since, (i)* B_1 *is a* $\mathfrak{F}\mathcal{F}o$ *set in* X_2 *but* $h_{\mathfrak{F}}^{-1}(B_1) = B_1$ *is not* $\mathfrak{F}\mathcal{F}Mo$ *set in* X_1*. (ii)* A_1 *is a* $\mathfrak{F}\mathcal{F}o$ *set in* X_2 *but* $h_{\mathfrak{F}}^{-1}(A_1) = A_1$ *is not* $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ *set in* X_1*.*

Theorem 3.2 *Let* (X_1, τ_1) *and* (X_2, τ_2) *be two* $\mathfrak{F}\mathcal{F}ts$ *and* $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ *be a bijective function. Then* $h_{\mathfrak{F}}$ *is a* $\mathfrak{F}\mathcal{F}MHom$ *if and only if* $h_{\mathfrak{F}}$ *is a* $\mathfrak{F}\mathcal{F}MCts$ *function and* $\mathfrak{F}\mathcal{F}MC$ *mapping.*

Proof: Let $h_{\mathfrak{F}}$ be a $\mathfrak{F}\mathcal{F}MHom$. From Definition 3.6 $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}MCts$ function. From Theorem **??**, we have $h_{\mathfrak{F}}^{-1}$ is a $\mathfrak{F}\mathcal{F}MC$ function. So, $(h_{\mathfrak{F}}^{-1})^{-1} = h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}MC$ function. □

Theorem 3.3 *Let* $g_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ *be a bijective mapping. If* $g_{\mathfrak{F}}$ *is* $\mathfrak{F}\mathcal{F}MCts$*, then the following statements are equivalent:*

(a) $g_{\mathfrak{F}}$ *is a* $\mathfrak{F}\mathcal{F}MC$ *mapping.*

(b) $g_{\mathfrak{F}}$ *is a* $\mathfrak{F}\mathcal{F}MO$ *mapping.*

(c) $g_{\mathfrak{F}}^{-1}$ *is a* $\mathfrak{F}\mathcal{F}MHom$*.*

Proof: (a) $\Rightarrow$ (b) Let us assume that $g_{\mathfrak{F}}$ is a bijective mapping and a $\mathfrak{F}\mathcal{F}MC$ mapping. Hence, $g_{\mathfrak{F}}^{-1}$ is a $\mathfrak{F}\mathcal{F}MCts$ mapping. Since each $\mathfrak{F}\mathcal{F}o$ set is a $\mathfrak{F}\mathcal{F}Mo$ set, $g_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}MO$ mapping.

(b) $\Rightarrow$ (c) Let $g_{\mathfrak{F}}$ be a bijective and $\mathfrak{F}\mathcal{F}MO$ mapping. Furthermore, $g_{\mathfrak{F}}^{-1}$ is a $\mathfrak{F}\mathcal{F}MCts$ mapping. Hence, $g_{\mathfrak{F}}$ and $g_{\mathfrak{F}}^{-1}$ are $\mathfrak{F}\mathcal{F}MCts$. Therefore, $g_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}MHom$.

(c) $\Rightarrow$ (a) Let $g_{\mathfrak{F}}$ be a $\mathfrak{F}\mathcal{F}MHom$. Then $g_{\mathfrak{F}}$ and $g_{\mathfrak{F}}^{-1}$ are $\mathfrak{F}\mathcal{F}MCts$. Since each $\mathfrak{F}\mathcal{F}c$ set in X_1 is a $\mathfrak{F}\mathcal{F}Mc$ set in X_2, hence $g_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}MC$ mapping. □

Remark 3.2 *Theorems 3.2 and 3.3 are holds for $\mathfrak{F}\mathcal{F}o$, $\mathfrak{F}\mathcal{F}\theta o$, $\mathfrak{F}\mathcal{F}\theta \mathcal{S}o$ & $\mathfrak{F}\mathcal{F}\delta\mathcal{P}o$ sets.*

4. Fermatean fuzzy M-C homeomorphism

Definition 4.1 Let $(X_1, \tau_{\mathfrak{F}}(F_1))$ and $(X_2, \tau_{\mathfrak{F}}(F_2))$ be $\mathfrak{F}\mathcal{F}ts$. A mapping $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is said to be a Fermatean fuzzy (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$, δ, $\delta\mathcal{S}$, e and M) C homeomorphism (briefly, $\mathfrak{F}\mathcal{F}CHom$ (resp. $\mathfrak{F}\mathcal{F}\theta CHom$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}CHom$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}CHom$, $\mathfrak{F}\mathcal{F}\delta CHom$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}CHom$, $\mathfrak{F}\mathcal{F}eCHom$ and $\mathfrak{F}\mathcal{F}MCHom$)) if $h_{\mathfrak{F}}$ is bijective, $\mathfrak{F}\mathcal{F}Irr$ (resp. $\mathfrak{F}\mathcal{F}\theta Irr$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}Irr$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Irr$, $\mathfrak{F}\mathcal{F}\delta Irr$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Irr$, $\mathfrak{F}\mathcal{F}eIrr$ and $\mathfrak{F}\mathcal{F}MIrr$) function and $\mathfrak{F}\mathcal{F}Irr$ (resp. $\mathfrak{F}\mathcal{F}\theta Irr$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}Irr$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Irr$, $\mathfrak{F}\mathcal{F}\delta Irr$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Irr$, $\mathfrak{F}\mathcal{F}eIrr$ and $\mathfrak{F}\mathcal{F}MIrr$) mapping.

Proposition 4.1 *Let (X_1, τ_1) & (X_2, τ_2) be a $\mathfrak{F}\mathcal{F}ts$'s. Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be a mapping. Then the following statements are hold for $\mathfrak{F}\mathcal{F}ts$, but not conversely.*

(i) Every $\mathfrak{F}\mathcal{F}Hom$ is a $\mathfrak{F}\mathcal{F}CHom$.

(ii) Every $\mathfrak{F}\mathcal{F}\theta Hom$ is a $\mathfrak{F}\mathcal{F}\theta CHom$.

(iii) Every $\mathfrak{F}\mathcal{F}\delta Hom$ is a $\mathfrak{F}\mathcal{F}\delta CHom$.

(iv) Every $\mathfrak{F}\mathcal{F}\delta\mathcal{S}CHom$ is a $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Hom$.

(v) Every $\mathfrak{F}\mathcal{F}\delta\mathcal{P}CHom$ is a $\mathfrak{F}\mathcal{F}\delta\mathcal{P}Hom$.

(vi) Every $\mathfrak{F}\mathcal{F}\theta\mathcal{S}CHom$ is a $\mathfrak{F}\mathcal{F}\theta\mathcal{S}Hom$.

(vii) Every $\mathfrak{F}\mathcal{F}MCHom$ is a $\mathfrak{F}\mathcal{F}MHom$.

(viii) Every $\mathfrak{F}\mathcal{F}eCHom$ is a $\mathfrak{F}\mathcal{F}eHom$.

Proof: (i) Let us assume that K be a $\mathfrak{F}\mathcal{F}cs$ in (X_2, τ_2). This shows that K is a $\mathfrak{F}\mathcal{F}Mcs$ in (X_2, τ_2). By assumption, $h_{\mathfrak{F}}^{-1}(K)$ is a $\mathfrak{F}\mathcal{F}Mcs$ in (X_1, τ_1). Hence, $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}MCts$ mapping. Hence, $h_{\mathfrak{F}}$ and $h_{\mathfrak{F}}^{-1}$ are $\mathfrak{F}\mathcal{F}MCts$ mappings. Hence $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}MHom$. The proof of other cases are similar. □

Example 4.1 *Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1, A_2, A_3 and A_4 are defined as*
$\alpha_{A_1}(a) = 0.2,\ \beta_{A_1}(a) = 0.8,$
$\alpha_{A_1}(b) = 0.4,\ \beta_{A_1}(b) = 0.6;$
$\alpha_{A_2}(a) = 0.1,\ \beta_{A_2}(a) = 0.9,$
$\alpha_{A_2}(b) = 0.3,\ \beta_{A_2}(b) = 0.7;$
$\alpha_{A_3}(a) = 0.9,\ \beta_{A_3}(a) = 0.1,$
$\alpha_{A_3}(b) = 0.7,\ \beta_{A_3}(b) = 0.3;$
$\alpha_{A_4}(a) = 0.2,\ \beta_{A_4}(a) = 0.8,$
$\alpha_{A_4}(b) = 0.3,\ \beta_{A_4}(b) = 0.7;$
Let $\tau_1 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2,\ A_3,\ A_4\}$, $\tau_2 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2,\ A_3\}$ be a $\mathfrak{F}\mathcal{F}ts$ on X_1 and X_2; and let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}CHom$ (resp. $\mathfrak{F}\mathcal{F}\delta CHom$) but not $\mathfrak{F}\mathcal{F}Hom$ (resp. $\mathfrak{F}\mathcal{F}\delta Hom$). Since, A_4 is a $\mathfrak{F}\mathcal{F}o$ set in X_2 but $(h_{\mathfrak{F}}^{-1})^{-1}(A_4) = A_4$ is not $\mathfrak{F}\mathcal{F}o$ (resp. $\mathfrak{F}\mathcal{F}\delta o$) set in X_1.

Example 4.2 *Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1, A_2, A_3, A_4, B_1, B_2 and B_3 are defined as*

$$A_1 = \{< x_1, 0.20, 0.80 >, < x_2, 0.40, 0.60 >\}$$
$$A_2 = \{< x_1, 0.10, 0.90 >, < x_2, 0.30, 0.70 >\}$$
$$A_3 = \{< x_1, 0.90, 0.10 >, < x_2, 0.70, 0.30 >\}$$
$$A_4 = \{< x_1, 0.20, 0.80 >, < x_2, 0.30, 0.70 >\}$$
$$B_1 = \{< x_1, 0.90, 0.10 >, < x_2, 0.70, 0.30 >\}$$
$$B_2 = \{< x_1, 0.10, 0.90 >, < x_2, 0.30, 0.70 >\}$$
$$B_3 = \{< x_1, 0.20, 0.80 >, < x_2, 0.40, 0.60 >\}.$$

Let $\tau_1 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2,\ A_3,\ A_4\}$, $\tau_2 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ B_1,\ B_2,\ B_3\}$ be a $\mathfrak{F}\mathcal{F}ts$ on X_1 and X_2; and let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\theta CHom$ but not $\mathfrak{F}\mathcal{F}\theta Hom$. Since, A_4 is a $\mathfrak{F}\mathcal{F}o$ set in X_2 but $(h_{\mathfrak{F}}^{-1})^{-1}(A_4) = A_4$ is not $\mathfrak{F}\mathcal{F}\theta o$ set in X_1.

Example 4.3 *Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1, A_2, B_1 and B_2 are defined as*

$$A_1 = \{< x_1, 0.20, 0.70 >, < x_2, 0.10, 0.80 >\}$$
$$A_2 = \{< x_1, 0.30, 0.60 >, < x_2, 0.40, 0.50 >\}$$
$$B_1 = \{< x_1, 0.10, 0.90 >, < x_2, 0.20, 0.90 >\}$$
$$B_2 = \{< x_1, 0.20, 0.90 >, < x_2, 0.40, 0.70 >\}.$$

Let $\tau_1 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2\}$, $\tau_2 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ B_1,\ B_2\}$ be a $\mathfrak{F}\mathcal{F}ts$ on X_1 and X_2; and let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}MHom$ but not $\mathfrak{F}\mathcal{F}MCHom$. Since, A_2 is a $\mathfrak{F}\mathcal{F}o$ set in X_2 but $(h_{\mathfrak{F}}^{-1})^{-1}(A_2) = A_2$ is not $\mathfrak{F}\mathcal{F}Mo$ set in X_1.

Example 4.4 *Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1, B_1 and B_2 are defined as*

$$A_1 = \{< x_1, 0.10, 0.90 >, < x_2, 0.30, 0.70 >\}$$
$$A_2 = \{< x_1, 0.20, 0.80 >, < x_2, 0.40, 0.60 >\}$$
$$B_1 = \{< x_1, 0.30, 0.70 >, < x_2, 0.40, 0.60 >\}$$
$$B_2 = \{< x_1, 0.20, 0.80 >, < x_2, 0.30, 0.70 >\}.$$

Let $\tau_1 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1\}$, $\tau_2 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ B_1,\ B_2\}$ be a $\mathfrak{F}\mathcal{F}ts$ on X_1 and X_2; and let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}eHom$ but not $\mathfrak{F}\mathcal{F}eCHom$. Since, A_2^c is a $\mathfrak{F}\mathcal{F}eo$ set in X_2 but $(h_{\mathfrak{F}}^{-1})^{-1}(A_2^c) = A_2^c$ is not $\mathfrak{F}\mathcal{F}eo$ set in X_1.

Theorem 4.1 *If $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is a $\mathfrak{F}\mathcal{F}MHom$, then $\mathfrak{F}\mathcal{F}Mcl(h_{\mathfrak{F}}^{-1}(K)) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(K))$ for each $\mathfrak{F}\mathcal{F}s$ K in (X_2, τ_2).*

Proof: Let K be a $\mathfrak{F}\mathcal{F}s$ in (X_2, τ_2). Then, $\mathfrak{F}\mathcal{F}cl(K)$ is a $\mathfrak{F}\mathcal{F}cs$ in (X_2, τ_2), and every $\mathfrak{F}\mathcal{F}cs$ is a $\mathfrak{F}\mathcal{F}Mcs$ in (X_2, τ_2). Assume $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}MIrr$ and $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(K))$ is a $\mathfrak{F}\mathcal{F}Mcs$ in (X_1, τ_1). Then, $\mathfrak{F}\mathcal{F}cl(h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(K)))$ $= h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(K))$. Here, $\mathfrak{F}\mathcal{F}Mcl(h_{\mathfrak{F}}^{-1}(K)) \subseteq \mathfrak{F}\mathcal{F}Mcl(h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(K))) = h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}\ cl(K))$. Therefore, $\mathfrak{F}\mathcal{F}Mcl\ (h_{\mathfrak{F}}^{-1}(K)) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}cl(K))$ for every $\mathfrak{F}\mathcal{F}s$ K in (X_2, τ_2). □

Theorem 4.2 *Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be a $\mathfrak{F}\mathcal{F}MCHom$. Then $\mathfrak{F}\mathcal{F}Mcl(h_{\mathfrak{F}}^{-1}(K)) = h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Mcl(K))$ for each $\mathfrak{F}\mathcal{F}s$ K in (X_2, τ_2).*

Proof: Since $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}MCHom$, $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}MIrr$ mapping. Let K be a $\mathfrak{F}\mathcal{F}s$ in (X_2, τ_2). Clearly, $\mathfrak{F}\mathcal{F}Mcl(K)$ is a $\mathfrak{F}\mathcal{F}Mcs$ in (X_2, τ_2). Then $\mathfrak{F}\mathcal{F}Mcl(K)$ is a $\mathfrak{F}\mathcal{F}Mcs$ in (X_2, τ_2). Since $h_{\mathfrak{F}}^{-1}(K) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Mcl(K))$, then $\mathfrak{F}\mathcal{F}Mcl(h_{\mathfrak{F}}^{-1}(K)) \subseteq \mathfrak{F}\mathcal{F}Mcl(h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Mcl(K))) = h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Mcl(K))$. Therefore, $\mathfrak{F}\mathcal{F}\ Mcl(h_{\mathfrak{F}}^{-1}(K)) \subseteq h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Mcl(K))$. Let $h_{\mathfrak{F}}$ be a $\mathfrak{F}\mathcal{F}MC\ Hom$. $h_{\mathfrak{F}}^{-1}$ is a $\mathfrak{F}\mathcal{F}MIrr$ mapping. Let us consider $\mathfrak{F}\mathcal{F}s$ $h_{\mathfrak{F}}^{-1}(K)$ in (X_1, τ_1), which implies $\mathfrak{F}\mathcal{F}Mcl(h_{\mathfrak{F}}^{-1}(K))$ is a $\mathfrak{F}\mathcal{F}Mcs$ in (X_1, τ_1). Hence, $\mathfrak{F}\mathcal{F}Mcl(h_{\mathfrak{F}}^{-1}(K))$ is a $\mathfrak{F}\mathcal{F}Mcs$ in (X_1, τ_1). This implies that $(h_{\mathfrak{F}}^{-1})^{-1}(\mathfrak{F}\mathcal{F}Mcl(h_{\mathfrak{F}}^{-1}(K))) = h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}Mcl(h_{\mathfrak{F}}^{-1}(K)))$ is a $\mathfrak{F}\mathcal{F}Mcs$ in (X_2, τ_2). This proves $K = (h_{\mathfrak{F}}^{-1})^{-1}(h_{\mathfrak{F}}^{-1}(K)) \subseteq (h_{\mathfrak{F}}^{-1})^{-1}(\mathfrak{F}\mathcal{F}Mcl(h_{\mathfrak{F}}^{-1}(K))) = h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}Mcl(h_{\mathfrak{F}}^{-1}(K)))$. Therefore, $\mathfrak{F}\mathcal{F}Mcl(K) \subseteq \mathfrak{F}\mathcal{F}Mcl(h_p(\mathfrak{F}\mathcal{F}Mcl(h_{\mathfrak{F}}^{-1}(K)))) = h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}Mcl(h_{\mathfrak{F}}^{-1}(K)))$, since $h_{\mathfrak{F}}^{-1}$ is a $\mathfrak{F}\mathcal{F}MIrr$ mapping. Hence, $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Mcl(K)) \subseteq h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}Mcl(h_{\mathfrak{F}}^{-1}(K)))) = \mathfrak{F}\mathcal{F}Mcl(h_{\mathfrak{F}}^{-1}(K))$. That is, $h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Mcl(K)) \subseteq \mathfrak{F}\mathcal{F}Mcl(h_{\mathfrak{F}}^{-1}(K))$. Hence, $\mathfrak{F}\mathcal{F}Mcl(h_{\mathfrak{F}}^{-1}(K)) = h_{\mathfrak{F}}^{-1}(\mathfrak{F}\mathcal{F}Mcl(K))$. □

Remark 4.1 *Theorems 4.1 and 4.2 are also true if $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}CHom$ (resp. $\mathfrak{F}\mathcal{F}\delta CHom$, $\mathfrak{F}\mathcal{F}\theta CHom$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}CHom$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}CHom$, $\mathfrak{F}\mathcal{F}MCHom$, $\mathfrak{F}\mathcal{F}eCHom$ & $\mathfrak{F}\mathcal{F}\delta\mathcal{P}CHom$.)*

Theorem 4.3 *If $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ and $g_{\mathfrak{F}} : (X_2, \tau_2) \to (X_3, \tau_3)$ are $\mathfrak{F}\mathcal{F}CHom$ (resp. $\mathfrak{F}\mathcal{F}\theta CHom$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}CHom$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}CHom$, $\mathfrak{F}\mathcal{F}\delta CHom$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}CHom$, $\mathfrak{F}\mathcal{F}eCHom$ and $\mathfrak{F}\mathcal{F}MCHom$)'s, then $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}CHom$ (resp. $\mathfrak{F}\mathcal{F}\theta CHom$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}CHom$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}CHom$, $\mathfrak{F}\mathcal{F}\delta CHom$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}CHom$, $\mathfrak{F}\mathcal{F}eCHom$ and $\mathfrak{F}\mathcal{F}MCHom$).*

Proof: Let $h_{\mathfrak{F}}$ and $g_{\mathfrak{F}}$ be two $\mathfrak{F}\mathcal{F}MCHom$'s. Assume K is a $\mathfrak{F}\mathcal{F}Mcs$ in (X_3, τ_3). Then, $g_{\mathfrak{F}}^{-1}(K)$ is a $\mathfrak{F}\mathcal{F}Mcs$ in (X_2, τ_2). Then, by hypothesis, $h_{\mathfrak{F}}^{-1}(g_{\mathfrak{F}}^{-1}(K))$ is a $\mathfrak{F}\mathcal{F}Mcs$ in (X_1, τ_1). Hence, $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}MIrr$ mapping. Now, let K be a $\mathfrak{F}\mathcal{F}Mcs$ in (X_1, τ_1). Then, by presumption, $h_{\mathfrak{F}}(K)$ is a $\mathfrak{F}\mathcal{F}Mcs$ in (X_2, τ_2). Then, by hypothesis, $g_{\mathfrak{F}}(h_{\mathfrak{F}}(K))$ is a $\mathfrak{F}\mathcal{F}Mcs$ in (X_3, τ_3). This implies that $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}MIrr$ mapping. Hence, $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}MCHom$. The proof of other cases are similar. □

5. Almost Fermatean fuzzy M totally mappings

In this section, we introduce almost Fermatean fuzzy M totally mappings and we discuss some basic properties.

Definition 5.1 A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is said to be

(i) Almost Fermatean fuzzy (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) open map (briefly, $\mathcal{A}\mathfrak{F}\mathcal{F}O$ (resp. $\mathcal{A}\mathfrak{F}\mathcal{F}\theta O$, $\mathcal{A}\mathfrak{F}\mathcal{F}\theta\mathcal{S}O$, $\mathcal{A}\mathfrak{F}\mathcal{F}\delta\mathcal{P}O$ and $\mathcal{A}\mathfrak{F}\mathcal{F}MO$)) if the image of each $\mathfrak{F}\mathcal{F}ro$ set in X_1 is $\mathfrak{F}\mathcal{F}o$ (resp. $\mathfrak{F}\mathcal{F}\theta o$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}o$ and $\mathfrak{F}\mathcal{F}Mo$)-set in X_2.

(ii) Almost Fermatean fuzzy (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) closed map (briefly, $\mathcal{A}\mathfrak{F}\mathcal{F}C$ (resp. $\mathcal{A}\mathfrak{F}\mathcal{F}\theta C$, $\mathcal{A}\mathfrak{F}\mathcal{F}\theta\mathcal{S}C$, $\mathcal{A}\mathfrak{F}\mathcal{F}\delta\mathcal{P}C$ and $\mathcal{A}\mathfrak{F}\mathcal{F}MC$)) if the image of each $\mathfrak{F}\mathcal{F}rc$ set in X_1 is $\mathfrak{F}\mathcal{F}c$ (resp. $\mathfrak{F}\mathcal{F}\theta c$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}c$ and $\mathfrak{F}\mathcal{F}Mc$)-set in X_2.

(iii) Almost Fermatean fuzzy (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) clopen map (briefly, $\mathcal{A}\mathfrak{F}\mathcal{F}clO$ (resp. $\mathcal{A}\mathfrak{F}\mathcal{F}\theta clO$, $\mathcal{A}\mathfrak{F}\mathcal{F}\theta\mathcal{S}clO$, $\mathcal{A}\mathfrak{F}\mathcal{F}\delta\mathcal{P}clO$ and $\mathcal{A}\mathfrak{F}\mathcal{F}MclO$)) if the image of each $\mathfrak{F}\mathcal{F}rclo$ set in X_1 is $\mathfrak{F}\mathcal{F}clo$ (resp. $\mathfrak{F}\mathcal{F}\theta clo$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}clo$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}clo$ and $\mathfrak{F}\mathcal{F}Mclo$)-set in X_2.

(iv) Fermatean fuzzy (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) totally open map (briefly, $\mathfrak{F}\mathcal{F}\mathcal{T}O$ (resp. $\mathfrak{F}\mathcal{F}\theta\mathcal{T}O$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}\mathcal{T}O$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}\mathcal{T}O$ and $\mathfrak{F}\mathcal{F}M\mathcal{T}O$)) if the image of each $\mathfrak{F}\mathcal{F}o$ (resp. $\mathfrak{F}\mathcal{F}\theta o$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}o$ and $\mathfrak{F}\mathcal{F}Mo$) set in X_1 is $\mathfrak{F}\mathcal{F}clo$ (resp. $\mathfrak{F}\mathcal{F}\theta clo$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}clo$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}clo$ and $\mathfrak{F}\mathcal{F}Mclo$)-set in X_2.

(v) Fermatean fuzzy (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) totally closed map (briefly, $\mathfrak{F}\mathcal{F}\mathcal{T}C$ (resp. $\mathfrak{F}\mathcal{F}\theta\mathcal{T}C$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}\mathcal{T}C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}\mathcal{T}C$ and $\mathfrak{F}\mathcal{F}M\mathcal{T}C$)) if the image of each $\mathfrak{F}\mathcal{F}c$ (resp. $\mathfrak{F}\mathcal{F}\theta c$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}c$ and $\mathfrak{F}\mathcal{F}Mc$) set in X_1 is $\mathfrak{F}\mathcal{F}clo$ (resp. $\mathfrak{F}\mathcal{F}\theta clo$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}clo$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}clo$ and $\mathfrak{F}\mathcal{F}Mclo$)-set in X_2.

(vi) Almost Fermatean fuzzy (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) totally open map (briefly, $\mathcal{A}\mathfrak{F}\mathcal{F}\mathcal{T}O$ (resp. $\mathcal{A}\mathfrak{F}\mathcal{F}\theta\mathcal{T}O$, $\mathcal{A}\mathfrak{F}\mathcal{F}\theta\mathcal{S}\mathcal{T}O$, $\mathcal{A}\mathfrak{F}\mathcal{F}\delta\mathcal{P}\mathcal{T}O$ and $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}O$)) if the image of each $\mathfrak{F}\mathcal{F}ro$ set in X_1 is $\mathfrak{F}\mathcal{F}clo$ (resp. $\mathfrak{F}\mathcal{F}\theta clo$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}clo$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}clo$ and $\mathfrak{F}\mathcal{F}Mclo$)-set in X_2.

(vii) Almost Fermatean fuzzy (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) totally closed map (briefly, $\mathcal{AFF}\mathcal{TC}$ (resp. $\mathcal{AFF}\theta\mathcal{TC}$, $\mathcal{AFF}\theta\mathcal{STC}$, $\mathcal{AFF}\delta\mathcal{PTC}$ and $\mathcal{AFF}M\mathcal{TC}$)) if the image of each $\mathfrak{F}\mathcal{F}rc$ set in X_1 is $\mathfrak{F}\mathcal{F}clo$ (resp. $\mathfrak{F}\mathcal{F}\theta clo$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}clo$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}clo$ and $\mathfrak{F}\mathcal{F}Mclo$)-set in X_2.

(viii) Almost Fermatean fuzzy (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) totally clopen map (briefly, $\mathcal{AFF}\mathcal{T}clO$ (resp. $\mathcal{AFF}\theta\mathcal{T}clO$, $\mathcal{AFF}\theta\mathcal{ST}clO$, $\mathcal{AFF}\delta\mathcal{PT}clO$ and $\mathcal{AFF}M\mathcal{T}clO$)) if the image of each $\mathfrak{F}\mathcal{F}rclo$ set in X_1 is $\mathfrak{F}\mathcal{F}clo$ (resp. $\mathfrak{F}\mathcal{F}\theta clo$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}clo$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}clo$ and $\mathfrak{F}\mathcal{F}Mclo$)-set in X_2.

Theorem 5.1 *Every $\mathcal{AFF}M\mathcal{TC}$ map is $\mathcal{AFF}MC$.*

Proof: Let X_1 and X_2 be $\mathfrak{F}\mathcal{F}ts$. Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be an $\mathcal{AFF}M\mathcal{TC}$ mapping. To prove $h_{\mathfrak{F}}$ is $\mathcal{AFF}MC$, let H be any $\mathfrak{F}\mathcal{F}rc$ subset of X_1. Since $h_{\mathfrak{F}}$ is $\mathcal{AFF}M\mathcal{TC}$ mapping, $h_{\mathfrak{F}}(H)$ is $\mathcal{AFF}Mclo$ in X_2. This implies that $h_{\mathfrak{F}}(H)$ is $\mathfrak{F}\mathcal{F}c$ in X_2. Therefore $h_{\mathfrak{F}}$ is $\mathcal{AFF}MC$. □

Corollary 5.1 *Every $\mathcal{AFF}M\mathcal{TO}$ map is $\mathcal{AFF}MO$.*

Theorem 5.2 *If a bijective function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathcal{AFF}M\mathcal{TO}$, then the image of each $\mathfrak{F}\mathcal{F}rc$ set in X_1 is $\mathcal{AFF}Mclo$ set in X_2.*

Proof: Let F be a $\mathfrak{F}\mathcal{F}rc$ set in X_1. Then F^c is $\mathfrak{F}\mathcal{F}ro$ in X_1. Since $h_{\mathfrak{F}}$ is $\mathcal{AFF}M\mathcal{TO}$, $h^c_{\mathfrak{F}} = [f(F)]^c$ is $\mathcal{AFF}Mclo$ in X_2. This implies that $h_{\mathfrak{F}}(F)$ is $\mathcal{AFF}Mclo$ set in X_2. □

Theorem 5.3 *A surjective function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathcal{AFF}M\mathcal{TO}$ if and only if for each subset B of X_2 and for each $\mathfrak{F}\mathcal{F}ro$ set U containing $h^{-1}_{\mathfrak{F}}(B)$, there is a $\mathfrak{F}\mathcal{F}Mclo$ set V of X_2 such that $B \subseteq V$ and $h^{-1}_{\mathfrak{F}}(V) \subseteq U$.*

Proof: Suppose $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is a surjective and $\mathcal{AFF}M\mathcal{TO}$ function and $B \subseteq V$. Let U be $\mathfrak{F}\mathcal{F}ro$ set of X_1 such that $h^{-1}_{\mathfrak{F}}(B) \subseteq U$. Since $h_{\mathfrak{F}}$ is $\mathcal{AFF}M\mathcal{TO}$ function, $h_{\mathfrak{F}}(U) = [h_{\mathfrak{F}}(U^c)]^c$ is $\mathfrak{F}\mathcal{F}Mclo$ set. Then $V = [h_{\mathfrak{F}}(U^c)]^c$ is $\mathfrak{F}\mathcal{F}Mclo$ set of X_2 containing B such that $h^{-1}_{\mathfrak{F}}(V) \subseteq U$. □

Theorem 5.4 *A map $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathcal{AFF}M\mathcal{TO}$ if and only if for each subset A of X_2 and each $\mathfrak{F}\mathcal{F}rc$ set U containing $h^{-1}_{\mathfrak{F}}(A)$ there is a $\mathfrak{F}\mathcal{F}Mclo$ set V of X_2 such that $A \subseteq V$ and $h^{-1}_{\mathfrak{F}}(V) \subseteq U$.*

Proof: Suppose $h_{\mathfrak{F}}$ is $\mathcal{AFF}M\mathcal{TO}$. Let $A \subseteq Y$ and U be a $\mathfrak{F}\mathcal{F}rc$ set of X_1 such that $h^{-1}_{\mathfrak{F}}(A) \subseteq U$. Now U^c is $\mathfrak{F}\mathcal{F}ro$ and $h_{\mathfrak{F}}$ is $\mathcal{AFF}M\mathcal{TO}$, $h_{\mathfrak{F}}(U^c)$ is $\mathfrak{F}\mathcal{F}Mclo$ set in X_2. Then $V = (h_{\mathfrak{F}}(U^c))^c$ is a $\mathfrak{F}\mathcal{F}Mclo$ set in X_2. Note that $h^{-1}_{\mathfrak{F}}(A) \subseteq U$ implies $A \subseteq V$ and $h^{-1}_{\mathfrak{F}}(V) = (h^{-1}_{\mathfrak{F}}(h_{\mathfrak{F}}(U^c)))^c \subseteq (U^c)^c = U$. That is $h^{-1}_{\mathfrak{F}}(V) \subseteq U$.

Conversely, let F be a $\mathfrak{F}\mathcal{F}ro$ set of X_1. Then $h^{-1}_{\mathfrak{F}}(h_{\mathfrak{F}}(F)^c) \subseteq F^c$ and F^c is $\mathfrak{F}\mathcal{F}rc$ set in X_1. By hypothesis, there exist a $\mathfrak{F}\mathcal{F}Mclo$ set V in X_2 such that $h_{\mathfrak{F}}(F^c) \subseteq V$ and $V^c \subseteq h_{\mathfrak{F}}(F)$ and so $F \subseteq (h^{-1}_{\mathfrak{F}}(V))^c$. Hence $h_{\mathfrak{F}}(F) \subseteq h_{\mathfrak{F}}((h^{-1}_{\mathfrak{F}}(V))^c)$ which implies $h_{\mathfrak{F}}(F) \subseteq V^c$. Since V^c is $\mathfrak{F}\mathcal{F}Mclo$, $h_{\mathfrak{F}}(F)$ is $\mathfrak{F}\mathcal{F}Mclo$. That is $h_{\mathfrak{F}}(F)$ is $\mathfrak{F}\mathcal{F}Mclo$ in X_2. Therefore $h_{\mathfrak{F}}$ is $\mathcal{AFF}M\mathcal{TO}$. □

Corollary 5.2 *A map $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathcal{AFF}M\mathcal{TC}$ if and only if for each subset A of X_2 and each $\mathfrak{F}\mathcal{F}ro$ set U containing $h^{-1}_{\mathfrak{F}}(A)$, there is a $\mathfrak{F}\mathcal{F}Mclo$ set V of X_2 such that $A \subseteq V$ and $h^{-1}_{\mathfrak{F}}(V) \subseteq U$.*

Theorem 5.5 *???? If $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathcal{AFF}M\mathcal{TC}$ and A is $\mathfrak{F}\mathcal{F}rc$ subset of X_1 then $h_{\mathfrak{F}} : (X_A, \tau_{\mathfrak{F}}(F_1)) \to (X_2, \tau_2)$ is $\mathcal{AFF}M\mathcal{TC}$.*

Proof: Consider the function $h_{\mathfrak{F}} : (X_A, \tau_{\mathfrak{F}}(F_A)) \to (X_2, \tau_2)$ and let V be any $\mathfrak{F}\mathcal{F}Mclo$ set in X_2. Since $h_{\mathfrak{F}}$ is $\mathcal{AFF}M\mathcal{TC}$, $h^{-1}_{\mathfrak{F}}(V)$ is $\mathfrak{F}\mathcal{F}rc$ subset of X_1. Since A is $\mathfrak{F}\mathcal{F}rc$ susset of X_1 and $h^{-1}_{\mathfrak{F}}(V) = A \cap h^{-1}_{\mathfrak{F}}(V)$ is $\mathfrak{F}\mathcal{F}rcs$ in A, it follows $h^{-1}_{\mathfrak{F}}(V)$ is $\mathfrak{F}\mathcal{F}rcs$ in A. Hence $h_{\mathfrak{F}}$ is $\mathcal{AFF}M\mathcal{TC}$. □

Remark 5.1 *$\mathcal{AFF}M\mathcal{T}clO$ mapping is $\mathcal{AFF}M\mathcal{TO}$ and $\mathcal{AFF}M\mathcal{TC}$ map.*

Remark 5.2 *Theorems 5.1 to 5.5, Corollaries 5.1 and 5.2 and Remark 5.1 are holds for $\mathfrak{F}\mathcal{F}o$, $\mathfrak{F}\mathcal{F}\theta o$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}o$ & $\mathfrak{F}\mathcal{F}\delta\mathcal{P}o$ sets.*

6. Almost Fermatean fuzzy M totally continuous functions

In this section, some new continuous functions are introduced and discussed their characterizations.

Definition 6.1 A map $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is said to be

(i) Fermatean fuzzy (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) totally continuous (briefly, $\mathfrak{F}\mathcal{F}\mathcal{T}Cts$ (resp. $\mathfrak{F}\mathcal{F}\theta\mathcal{T}Cts$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}\mathcal{T}Cts$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}\mathcal{T}Cts$ and $\mathfrak{F}\mathcal{F}M\mathcal{T}Cts$)) if $h_{\mathfrak{F}}^{-1}(V)$ is $\mathfrak{F}\mathcal{F}clo$ (resp. $\mathfrak{F}\mathcal{F}\theta clo$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}clo$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}clo$ and $\mathfrak{F}\mathcal{F}Mclo$) set in X_1 for each $\mathfrak{F}\mathcal{F}o$ (resp. $\mathfrak{F}\mathcal{F}\theta o$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}o$ and $\mathfrak{F}\mathcal{F}Mo$) set V in X_2.

(ii) Almost Fermatean fuzzy (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) totally continuous (briefly, $\mathcal{A}\mathfrak{F}\mathcal{F}\mathcal{T}Cts$ (resp. $\mathcal{A}\mathfrak{F}\mathcal{F}\theta\mathcal{T}Cts$, $\mathcal{A}\mathfrak{F}\mathcal{F}\theta\mathcal{S}\mathcal{T}Cts$, $\mathcal{A}\mathfrak{F}\mathcal{F}\delta\mathcal{P}\mathcal{T}Cts$ and $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}Cts$)) if $h_{\mathfrak{F}}^{-1}(V)$ is $\mathfrak{F}\mathcal{F}clo$ (resp. $\mathfrak{F}\mathcal{F}\theta clo$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}clo$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}clo$ and $\mathfrak{F}\mathcal{F}Mclo$) set in X_1 for each $\mathfrak{F}\mathcal{F}ro$ set V in X_2.

(iii) Almost Fermatean fuzzy (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) totally clopen continuous (briefly, $\mathcal{A}\mathfrak{F}\mathcal{F}\mathcal{T}cloCts$ (resp. $\mathcal{A}\mathfrak{F}\mathcal{F}\theta\mathcal{T}cloCts$, $\mathcal{A}\mathfrak{F}\mathcal{F}\theta\mathcal{S}\mathcal{T}cloCts$, $\mathcal{A}\mathfrak{F}\mathcal{F}\delta\mathcal{P}\mathcal{T}cloCts$ and $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}cloCts$)) if $h_{\mathfrak{F}}^{-1}(V)$ is $\mathfrak{F}\mathcal{F}clo$ (resp. $\mathfrak{F}\mathcal{F}\theta clo$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}clo$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}clo$ and $\mathfrak{F}\mathcal{F}Mclo$) set in X_1 for each $\mathfrak{F}\mathcal{F}rclo$ set V in X_2.

Theorem 6.1 *A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}Cts$ function if the inverse image of every $\mathfrak{F}\mathcal{F}rc$ set of X_2 is $\mathfrak{F}\mathcal{F}Mclo$ set in X_1.*

Proof: Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}Cts$ and F be any $\mathfrak{F}\mathcal{F}rc$ set in X_2. Then F^c is $\mathfrak{F}\mathcal{F}ro$ set in X_2. Since $h_{\mathfrak{F}}$ is $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}Cts$, $h_{\mathfrak{F}}^{-1}(F^c)$ is $\mathfrak{F}\mathcal{F}Mclo$ set in X_1. That is $(h_{\mathfrak{F}}^{-1}(F))^c$ is $\mathfrak{F}\mathcal{F}Mclo$ set in X_1. This implies that $h_{\mathfrak{F}}^{-1}(F)$ is $\mathfrak{F}\mathcal{F}Mclo$ set in X_1. □

Theorem 6.2 *A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}Cts$ is an $\mathcal{A}\mathfrak{F}\mathcal{F}MCts$ function.*

Proof: Suppose $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}Cts$ and U is any $\mathfrak{F}\mathcal{F}ro$ subset of X_2. Since $h_{\mathfrak{F}}$ is $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}Cts$, $h_{\mathfrak{F}}^{-1}(U)$ is $\mathfrak{F}\mathcal{F}Mclo$ in X_1. This implies that $h_{\mathfrak{F}}^{-1}(U)$ is $\mathfrak{F}\mathcal{F}Mo$ in X_1. Therefore the function $h_{\mathfrak{F}}$ is $\mathcal{A}\mathfrak{F}\mathcal{F}MCts$. □

Theorem 6.3 *For any bijective map $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ the following statements are equivalent:*

(i) $h_{\mathfrak{F}}^{-1} : (X_2, \tau_2) \to (X_1, \tau_1)$ *is* $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}Cts$.

(ii) $h_{\mathfrak{F}}$ *is* $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}O$.

(iii) $h_{\mathfrak{F}}$ *is* $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}C$.

Proof: (i) → (ii): Let U be a $\mathfrak{F}\mathcal{F}ro$ set of X_1. By assumption, $(h_{\mathfrak{F}}^{-1})^{-1}(U) = h_{\mathfrak{F}}(U)$ is $\mathfrak{F}\mathcal{F}Mclo$ in X_2 and so $h_{\mathfrak{F}}$ is $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}O$.

(ii) → (iii): Let F be a $\mathfrak{F}\mathcal{F}rc$ set of X_1. Then F^c is $\mathfrak{F}\mathcal{F}ro$ set in X_1. By assumption $h_{\mathfrak{F}}(F^c)$ is $\mathfrak{F}\mathcal{F}Mclo$ set in X_2. Hence $h_{\mathfrak{F}}$ is $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}C$.

(iii) → (i): Let F be a $\mathfrak{F}\mathcal{F}rc$ set of X_1. By assumption, $h_{\mathfrak{F}}(F)$ is $\mathfrak{F}\mathcal{F}Mclo$ set in X_2. But $h_{\mathfrak{F}}(F) = (h_{\mathfrak{F}}^{-1})^{-1}(F)$ and therefore $h_{\mathfrak{F}}^{-1}$ is $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}Cts$. □

Remark 6.1 *Theorems 6.1 to 6.3 are holds for $\mathfrak{F}\mathcal{F}o$, $\mathfrak{F}\mathcal{F}\theta o$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}o$ & $\mathfrak{F}\mathcal{F}\delta\mathcal{P}o$ sets.*

7. Super Fermatean fuzzy M clopen continuous functions

In this section, we introduce the concept of super $\mathfrak{F}\mathcal{F}Mclo$ continuous in $\mathfrak{F}\mathcal{F}ts$.

Definition 7.1 A map $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is said to be super Fermatean fuzzy (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) clopen continuous (briefly, $\mathcal{SU}\mathfrak{F}\mathcal{F}cloCts$ (resp. $\mathcal{SU}\mathfrak{F}\mathcal{F}\theta cloCts$, $\mathcal{SU}\mathfrak{F}\mathcal{F}\theta\mathcal{S}cloCts$, $\mathcal{SU}\mathfrak{F}\mathcal{F}\delta\mathcal{P}cloCts$ and $\mathcal{SU}\mathfrak{F}\mathcal{F}MCloCts$)) if for each $x_\alpha \in X_1$ and for each $\mathfrak{F}\mathcal{F}clo$ (resp. $\mathfrak{F}\mathcal{F}\theta clo$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}clo$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}clo$ and $\mathfrak{F}\mathcal{F}Mclo$) set V containing $h_{\mathfrak{F}}(x_\alpha)$ in X_2, there exist a $\mathfrak{F}\mathcal{F}ro$ set U containing x_α such that $h_{\mathfrak{F}}(U) \subseteq V$.

Theorem 7.1 *Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}\mathcal{O}$. Then $h_{\mathfrak{F}}$ is $\mathcal{SU}\mathfrak{F}\mathcal{F}MCloCts$ if $h_{\mathfrak{F}}(x_\alpha)$ is $\mathfrak{F}\mathcal{F}Mclo$ in X_2.*

Proof: Let G be $\mathfrak{F}\mathcal{F}Mclo$ set in X_2. Now $h_{\mathfrak{F}}^{-1}(G)$ is $\mathfrak{F}\mathcal{F}ros$ in X_1. Since the intersection of $\mathfrak{F}\mathcal{F}Mclo$ set is $\mathfrak{F}\mathcal{F}Mclo$ set in $X_2, h_{\mathfrak{F}}(h_{\mathfrak{F}}^{-1}(G)) = G \wedge h_{\mathfrak{F}}(x_\alpha)$ is $\mathfrak{F}\mathcal{F}Mclo$ in X_2. Therefore, $h_{\mathfrak{F}}^{-1}(G)$ is $\mathfrak{F}\mathcal{F}ro$ in X_1. Hence $h_{\mathfrak{F}}$ is $\mathcal{SU}\mathfrak{F}\mathcal{F}MCloCts$ function. □

Theorem 7.2 *If $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is surjective and $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}\mathcal{O}$, then $h_{\mathfrak{F}}$ is $\mathcal{SU}\mathfrak{F}\mathcal{F}MCloCts$.*

Proof: Let G be $\mathfrak{F}\mathcal{F}Mclo$ set in X_2. Take $A = h_{\mathfrak{F}}^{-1}(G)$. Since $h_{\mathfrak{F}}(A) = G$ is $\mathfrak{F}\mathcal{F}Mclo$ set in X_2, by the Theorem 7.1, A is $\mathfrak{F}\mathcal{F}ro$ set in X_1. Therefore $h_{\mathfrak{F}}$ is $\mathcal{SU}\mathfrak{F}\mathcal{F}MCloCts$. □

Definition 7.2 A map $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is said to be Fermatean fuzzy (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) clopen irresolute function (briefly, $\mathfrak{F}\mathcal{F}cloIrr$ (resp. $\mathfrak{F}\mathcal{F}\theta cloIrr$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}cloIrr$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}cloIrr$ and $\mathfrak{F}\mathcal{F}MCloIrr$)) if $h_{\mathfrak{F}}^{-1}(V)$ is $\mathfrak{F}\mathcal{F}clo$ (resp. $\mathfrak{F}\mathcal{F}\theta clo$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}clo$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}clo$ and $\mathfrak{F}\mathcal{F}Mclo$) set in X_1 for each $\mathfrak{F}\mathcal{F}clo$ (resp. $\mathfrak{F}\mathcal{F}\theta clo$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}clo$, $\mathfrak{F}\mathcal{F}\delta\mathcal{P}clo$ and $\mathfrak{F}\mathcal{F}Mclo$) set V in X_2.

Theorem 7.3 *Let (X_1, τ_1), (X_2, τ_2) and (X_3, τ_3) be $\mathfrak{F}\mathcal{F}ts$. Then the composition $g_{\mathfrak{F}} \circ h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_3, \tau_3)$ is $\mathcal{SU}\mathfrak{F}\mathcal{F}MCloCts$ function where $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathcal{SU}\mathfrak{F}\mathcal{F}MCloCts$ function and $g_{\mathfrak{F}} : (X_2, \tau_2) \to (X_3, \tau_3)$ is $\mathfrak{F}\mathcal{F}MCloIrr$ function.*

Proof: Let A be a $\mathfrak{F}\mathcal{F}rc$ set of X_1. Since $h_{\mathfrak{F}}$ is $\mathcal{SU}\mathfrak{F}\mathcal{F}MCloCts$, $h_{\mathfrak{F}}(A)$ is $\mathfrak{F}\mathcal{F}Mclo$ set in X_2. Then by hypothesis, $h_{\mathfrak{F}}(A)$ is $\mathfrak{F}\mathcal{F}Mclo$ set. Since $g_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}MCloIrr$, $g_{\mathfrak{F}}(h_{\mathfrak{F}}(A)) = (g_{\mathfrak{F}} \circ h_{\mathfrak{F}})(A)$. Therefore $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}$ is $\mathcal{SU}\mathfrak{F}\mathcal{F}MCloCts$. □

Theorem 7.4 *If $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ and $g_{\mathfrak{F}} : (X_2, \tau_2) \to (X_3, \tau_3)$ are two mappings such that their composition $g_{\mathfrak{F}} \circ h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_3, \tau_3)$ is $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}C$ mapping then the following statements are true.*

(i) If $h_{\mathfrak{F}}$ is $\mathcal{SU}\mathfrak{F}\mathcal{F}MCloCts$ and surjective, then $g_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}MCloIrr$ function.

(ii) If $g_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}MCloIrr$ function and injective, then $h_{\mathfrak{F}}$ is an $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}C$ function.

Proof: (i) Let A be a $\mathfrak{F}\mathcal{F}Mclo$ set of X_2. Since $h_{\mathfrak{F}}$ is $\mathcal{SU}\mathfrak{F}\mathcal{F}MCloCts$, $h_{\mathfrak{F}}^{-1}(A)$ is $\mathfrak{F}\mathcal{F}rcs$ in X_1. Since $(g_{\mathfrak{F}} \circ h_{\mathfrak{F}})(h_{\mathfrak{F}}^{-1}(A))$ is $\mathfrak{F}\mathcal{F}Mclo$ set in M. Since $h_{\mathfrak{F}}$ is surjective, $g(A)$ is $\mathfrak{F}\mathcal{F}Mclo$ set in X_3. Therefore $g_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}MCloIrr$ function.

(ii) Let B be $\mathfrak{F}\mathcal{F}rc$ set of X_1. Since $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}$ is $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}C$, $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}(B)$ is $\mathfrak{F}\mathcal{F}Mclo$ set in X_3. Since $g_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}MCloIrr$ function, $g_{\mathfrak{F}}^{-1}((g_{\mathfrak{F}} \circ h_{\mathfrak{F}})(B))$ is $\mathfrak{F}\mathcal{F}Mclo$ set in X_2. That is $h_{\mathfrak{F}}(B)$ is $\mathfrak{F}\mathcal{F}Mclo$ set in X_2. Since $h_{\mathfrak{F}}$ is injective, $h_{\mathfrak{F}}$ is an $\mathcal{A}\mathfrak{F}\mathcal{F}M\mathcal{T}C$ function. □

Remark 7.1 *Theorems 7.1 to 7.4 are holds for $\mathfrak{F}\mathcal{F}o$, $\mathfrak{F}\mathcal{F}\theta o$, $\mathfrak{F}\mathcal{F}\theta\mathcal{S}o$ & $\mathfrak{F}\mathcal{F}\delta\mathcal{P}o$ sets.*

8. Application

Entropy as a measure of fuzziness was first proposed by Zadeh [30]. Later many mathematicians defined several entropy measures. In this section, we focus on defining an entropy measure for $\mathfrak{F}\mathcal{F}s$ that connects the degree of membership and non-membership. As an example, we have applied the proposed entropy measure in the field of seasons.

Definition 8.1 Let $A = \{< x, \mu_A(x), \lambda_A(x) | x \in X\}$ be a $\mathfrak{F}\mathcal{F}s$ in X. The new entropy measure for A denoted by $\varepsilon_{\mathfrak{F}\mathcal{F}s}(A)$, is a function, $\varepsilon_{\mathfrak{F}\mathcal{F}s} : \tau_{\mathfrak{F}\mathcal{F}s}(X) \to [0,1]$ and is defined as $\varepsilon_{\mathfrak{F}\mathcal{F}s}(A) = 1 - \frac{1}{n}\sum_{i=1}^{n}(\alpha_A - \gamma_A)^2$; for every $x_i \in A$, where $\tau_{\mathfrak{F}\mathcal{F}s}(X)$ denote the family of all $\mathfrak{F}\mathcal{F}s$'s on X.

Example 8.1 *This example illustrate the entropy measure by considering a real-world issue involving the selection of a reputable national auditing agency. The ministry of finance of a developing nation provides quotes to choose a reputable auditing agency in order to obtain an objective and equitable audit report that will keep the state's economic development on track and guarantee the accountability and transparency of state-run institutions. Experts make up the committee that reviews the quotations and determines which of them are successful. Eligible quotations are those that the committee finds successfully; the others are rejected. An expert panel is asked to rank the auditing companies* $\{A1, A2, A3\}$ *and choose the top one based on established criteria* $\{C1, C2, C3\}$. *Table 1 provides criteria descriptions, while Table 2 displays the weights assigned to the criteria. Table 2 shows that, even when the alternatives are different, the weights of the criteria determined by the entropy are consistently the same. Table 3 displays the outcomes. Furthermore, the relative closeness degrees of each alternative to ideal solution are shown in Table 4.*

Criteria	**Description of Criteria**
Required experience and capability to make independent decisions (C1)	Certification and required knowledge of accounting, business, and taxation law; understanding of management systems; auditor should not be influenced by anyone; actions, decisions, and reports based on careful analysis.
Effective communication skills (C2)	Excellent mastery of communication skills; well-versed in compelling report writing and convincing presentation skills; patient enough to elaborate points to auditee satisfaction.
Capability to comprehend different business needs (C3)	Ability to work with diverse company setups; strong analytical ability; effective planning and strategy formulation.

Table 1: Key Criteria for Auditor Selection

Table 2. Entropy measure of each patient through their symptoms.

	$C1$	$C2$	$C3$
$A1$	$< A1, C1; 0.1, 0.8 >$	$< A1, C2; 0.1, 0.4 >$	$< A1, C3; 0.5, 0.2 >$
$A2$	$< A2, C1; 0.7, 0.9 >$	$< A2, C2; 0.3, 0.2 >$	$< A2, C3; 0.3, 0.5 >$
$A3$	$< A3, C1; 0.7, 0.2 >$	$< A3, C2; 0.1, 0.5 >$	$< A3, C3; 0.8, 0.2 >$

9. Conclusion

In this paper, we have continued to study the properties of Fermatean fuzzy M open and Fermatean fuzzy M closed mappings in Fermatean fuzzy topological spaces. Also, we study about Fermatean fuzzy M Homeomorphism, almost Fermatean fuzzy M totally mappings, almost Fermatean fuzzy M totally continuous mappings and super Fermatean fuzzy M clopen continuous functions and established the relations between them we obtain some new characterizations of these mappings in Fermatean fuzzy topological spaces.

References

1. K. T. Atanassov (1983), *Intuitionistic fuzzy sets*, VII ITKR's Session, Sofia.
2. K. T. Atanassov (1986), *Intuitionistic fuzzy sets*, Fuzzy Sets Syst. **20**, 87-96.
3. K. T. Atanassov (1989), *Geometrical interpretation of the elements of the intuitionistic fuzzy objects*, Preprint IM-MFAIS-1-89, Sofia.
4. K. T. Atanassov (1999), *Intuitionistic fuzzy sets: theory and applications*, Physica, Heidelberg.
5. K. T. Atanassov (2012), *On intuitionistic fuzzy sets theory*, Springer, Berlin.
6. K. Atanassov (2016), *Review and new results on intuitionistic fuzzy sets*, International Journal Bioautomation. **20**, S17-S26.
7. C. L Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl., **24** (1968), 182-190.
8. D. Coker, *An introduction to intuitionistic fuzzy topological spaces*, Fuzzy sets and systems, **88** (1997), 81-89.
9. A. I. El-Maghrabi and M. A. Al-Juhani, *M-open sets in topological spaces,* Pioneer J. Math. Sci., **4** (2) (2011), 213–230.
10. Hariwan Z. Ibrahim(2022), *Fermatean fuzzy Topological Spaces*, J. Appl. Math. and Informatics. **40**, 85-98.
11. X. He, Y. Du and W. Liu (2016), *Pythagorean fuzzy power average operators*, Fuzzy Syst. Math. **30** (6), 116-124.
12. M. Lellis Thivagar, S. Jafari, V. Sutha Devi and V. Antonysamy, *A novel approach to nano topology via neutrosophic sets*, Neutrosophic Sets and Systems, **20** (2018), 86-94.
13. Murat Olgun, Mehmet Unver and Seyhmus Yardimci (2019), *Pythagorean fuzzy topological spaces*, Complex & Intelligent Systems. https://doi.org/10.1007/s40747-019-0095-2.
14. A. Padma, M. Saraswathi, A. Vadivel and G. Saravanakumar, *New Notions of Nano M-open Sets*, Malaya Journal of Matematik, **S** (1) (2019), 656-660.
15. V. Pankajam and K. Kavitha, *δ-open sets and δ-nano continuity in δ-nano topological spaces*, International Journal of Innovative Science and Research Technology, **2** (12) (2017), 110-118.
16. S. Saha, *Fuzzy δ-continuous mappings*, Journal of Mathematical Analysis and Applications, **126** (1987), 130-142.
17. T.Senapati and R.R.Yager(2020), *Fermatean Fuzzy Sets*, Journal of Ambient Intelligence and Humanized Computing **11**, 663-674.
18. R. Thangammal, M. Saraswathi, A. Vadivel and C. John Sundar, *Fuzzy nano Z-open sets in fuzzy nano topological spaces*, Journal of Linear and Topological Algebra, **11** (01) (2022), 27-38.
19. R. Thangammal, M. Saraswathi, A. Vadivel, Samad Noeiaghdam, C. John Sundar, V. Govindan, Aiyared Iampan, *Fuzzy nano Z-locally closed sets, extremally disconnected spaces, normal spaces, and their application*, Advances in Fuzzy Systems, (2022), 3364170.
20. A. Vadivel, M. Seenivasan and C. John Sundar, *An Introduction to δ-open sets in a Neutrosophic Topological Spaces*, Journal of Physics: Conference Series, 1724 (2021), 012011.
21. A. Vadivel, C. John Sundar, K. Kirubadevi and S. Tamilselvan, *More on Neutrosophic Nano Open Sets*, International Journal of Neutrosophic Science (IJNS), **18** (4) (2022), 204-222.
22. A. Vadivel, C. John Sundar, K. Saraswathi and S. Tamilselvan, *Neutrosophic Nano M Open Sets*, International Journal of Neutrosophic Science, **19** (1) (2022), 132-147.
23. A. Vadivel, V. Sagunthaladevi and S. Priya (2025), *More on Open Sets in Fermatean Fuzzy Topological Spaces and its Application*, accepted in J. Appl. Math. & Informatics.
24. A. Vadivel, V. Sagunthaladevi and S. Priya (2025), *Continuous and Irresolute Maps Via δ-open Sets in Fermatean Fuzzy Topological Spaces And Application of MCDM Techniques*, accepted in Communications on Applied Nonlinear Analysis.
25. A. Vadivel, V. Sagunthaladevi and S. Priya (2025), *Homeomorphism via $\delta\beta$-open Sets in Fermatean Fuzzy Topological Spaces and Application in Entropy Measure*, accepted in Communications on Applied Nonlinear Analysis.
26. R. R. Yager (2013), *Pythagorean membership grades in multicriteria decision making*, In: Technical report MII-3301. Machine Intelligence Institute, Iona College, New Rochelle.
27. R. R. Yager (2013), *Pythagorean fuzzy subsets*, In: Proceedings of the joint $IFSA$ world congress $NAFIPS$ annual meeting, 57-61.
28. R. R. Yager (2014), *Pythagorean membership grades in multicriteria decision making*, $IEEE$ Trans Fuzzy Syst. **22** (4), 958-965.
29. L. A. Zadeh (1965), *Fuzzy sets*, Inf. Control, **8**, 338-353.
30. L. Zadeh (1965), *Fuzzy Sets and Systems*, in Proc. Symp. on Systems Theory, Polytechnic Institute of Brooklyn, New York.

G.Bhuvaneswari,
Department of Mathematics,
Srivijay Vidyalaya College of Arts and Science,
Bargur (T.K.), Krishnagiri- 635 104, India.
Department of Mathematics,
Government Arts College (A), Salem- 636 007, India.
E-mail address: buviphd@gmail.com, buvielango09@gmail.com

and

S.Mehar Banu,Department of Mathematics,
Government Arts College (A), Salem- 636 007, India.
E-mail address: meharizh@gmail.com