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A Collection of Integer Solutions to Non-homogeneous Ternary Higher Degree Diophantine Equation $3\left(x^2+y^2\right)-5xy=\left(k^2+11\right)z^{2s+1}$

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ABSTRACT: This paper aims at determining non-zero distinct integer solutions to non-homogeneous ternary higher degree Diophantine equation $3\left(x^2+y^2\right)-5xy=\left(k^2+11\right)z^{2s+1}$. Transformation techniques and factorization methods are utilized to obtain the same.

Keywords: Ternary higher degree equation, non-homogeneous higher degree equation, integer solutions, transformation technique, factorization method.

Contents

1. Introduction

The theory of Diophantine equations is an ancient subject that typically involves solving, polynomial equation in two or more variables or a system of polynomial equations with the number of unknowns greater than the number of equations, in integers and occupies a pivotal role in the region of mathematics. The subject of Diophantine equations has fascinated and inspired both amateurs and mathematicians alike and so they merit special recognition. Solving higher degree Diophantine equations can be challenging as they involve finding integer solutions that satisfy the given polynomial equation. Learning about the various techniques to solve these higher power Diophantine equation in successfully deriving their solutions help us understand how numbers work and their significance in different areas of mathematics and science. For the
sake of clear understanding by the readers, one may refer the varieties of Diophantine equations with multi variables [1-36]. This paper aims at determining many integer solutions to nonhomogeneous polynomial higher degree equation with three unknowns given by $3\left(x^2+y^2\right)-5xy=\left(k^2+11\right)z^{2s+1}$. Transformation techniques and factorization methods are utilized to obtain the same.

2. Method of Analysis

The non-homogeneous ternary heptic Diophantine equation to be solved is

$$3\left(x^2+y^2\right)-5xy=\left(k^2+11\right)z^{2s+1} \tag{2.1}$$

The insertion of the linear transformations

$$x=u+v, y=u-v, u\neq\pm v\neq 0 \tag{2.2}$$

in (2.1) leads to the equation

$$u^2+11v^2=\left(k^2+11\right)z^{2s+1} \tag{2.3}$$

The process of obtaining many integer solutions to (2.1) is illustrated below:
Pattern 2.1
To start with, the choice

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$$u = kv, k \neq \pm 1 \tag{2.4}$$

in (2.3) gives

$$v^2 = z^{2\ s+1}$$

which is satisfied by

$$v = \beta^{(2s+1)t}, z = \beta^{2t}, \beta \neq \pm 1$$

From (2.4), we have

$$u = k\beta^{(2\ s+1)t}$$

In view of (2.2), one has

$$x = (k+1)\beta^{(2s+1)t}, y = (k-1)\beta^{(2s+1)t}$$

Thus, when $k \neq \pm 1$, the above values of x, y, z represent the non-zero distinct integer solutions to (2.1).

Remark 2.1

In (2.4), assume

$$u = \alpha v, \alpha \neq k \tag{2.5}$$

In this case, (2.3) is written as

$$\left(\alpha^2 + 11\right) v^2 = \left(k^2 + 11\right) z^{2\ s+1}$$

which is satisfied by

$$\begin{aligned} v &= \left(\alpha^2 + 11\right)^s \left(k^2 + 11\right)^{s+1} t^{2\ s+1} \\ z &= \left(\alpha^2 + 11\right) \left(k^2 + 11\right) t^2 \end{aligned} \tag{2.6}$$

From (2.5), one has

$$u = \alpha \left(\alpha^2 + 11\right)^s \left(k^2 + 11\right)^{s+1} t^{2\ s+1}$$

In view of (2.2), one has

$$\begin{aligned} x &= (\alpha + 1) \left(\alpha^2 + 11\right)^s \left(k^2 + 11\right)^{s+1} t^{2\ s+1} \\ y &= (\alpha - 1) \left(\alpha^2 + 11\right)^s \left(k^2 + 11\right)^{s+1} t^{2\ s+1} \end{aligned} \tag{2.7}$$

Thus, when $\alpha \neq \pm 1$, the above values of x, y, z represent the non-zero distinct integer solutions to (2.1).

Remark 2.2

In (2.4), assume

$$v = \alpha u, \alpha \neq k \tag{2.8}$$

In this case, (2.3) is written as

$$\left(1 + 11\alpha^2\right) u^2 = \left(k^2 + 11\right) z^{2\ s+1}$$

which is satisfied by

$$\begin{aligned} u &= \left(1+11\alpha^2\right)^s \left(k^2+11\right)^{s+1} t^{2\ s+1} \\ z &= \left(1+11\alpha^2\right)\left(k^2+11\right) t^2 \end{aligned} \tag{2.9}$$

From (2.8), one has

$$v = \alpha\left(1+11\alpha^2\right)^s \left(k^2+11\right)^{s+1} t^{2s+1}$$

In view of (2.2), one has

$$\begin{aligned} x &= (1+\alpha)\left(1+11\alpha^2\right)^s \left(k^2+11\right)^{s+1} t^{2\ s+1} \\ y &= (1-\alpha)\left(1+11\alpha^2\right)^s \left(k^2+11\right)^{s+1} t^{2\ s+1} \end{aligned} \tag{2.10}$$

Thus, when $\alpha \neq \pm 1$, the above values of x, y, z represent the non-zero distinct integer solutions to (2.1).

Pattern 2.2

After performing a few calculations, it is observed that (2.3) is satisfied by

$$\begin{aligned} u &= p\left(p^2+11q^2\right)^s \left(k^2+11\right)^{s+1} \\ v &= q\left(p^2+11q^2\right)^s \left(k^2+11\right)^{s+1} \\ z &= \left(p^2+11q^2\right)\left(k^2+11\right) \end{aligned} \tag{2.11}$$

In view of (2.2), we obtain

$$\begin{aligned} x &= (p+q)\left(p^2+11q^2\right)^s \left(k^2+11\right)^{s+1} \\ y &= (p-q)\left(p^2+11q^2\right)^s \left(k^2+11\right)^{s+1} \end{aligned} \tag{2.12}$$

Thus, when $p \neq \pm q$, the values of x, y, z given by (2.11) & (2.12) represent non-zero distinct integer solutions to (2.1).

Pattern 2.3

Let

$$z = a^2 + 11\ b^2 = (a + i\sqrt{11}\ b)(a - i\sqrt{11}\ b) \tag{2.13}$$

Write

$$\begin{aligned} k^2 + 11 &= (k + i\sqrt{11})(k - i\sqrt{11}) \\ u^2 + 11v^2 &= (u + i\sqrt{11}v)(u - i\sqrt{11}v) \end{aligned} \tag{2.14}$$

Substituting (2.13) & (2.14) in (2.3) and equating the positive factors, we have

$$\begin{aligned} (u + i\sqrt{11}v) &= (k + i\sqrt{11})(a + i\sqrt{11}\ b)^{2\ s+1} \\ &= (k + i\sqrt{11})[f(a,\ b) + i\sqrt{11}\ g(a,\ b)] \end{aligned} \tag{2.15}$$

where

$$f(a,b) = \frac{(a+i\sqrt{11}b)^{2s+1} + (a-i\sqrt{11}b)^{2s+1}}{2}$$
$$g(a,b) = \frac{(a+i\sqrt{11}b)^{2s+1} - (a-i\sqrt{11}b)^{2s+1}}{i2\sqrt{11}} \quad (2.16)$$

On equating the coefficients of corresponding terms, we get

$$\mathrm{u} = \mathrm{kf(a,\ b)} - 11\ \mathrm{g(a,\ b)}$$
$$\mathrm{v} = \mathrm{f(a,\ b)} + \mathrm{kg(a,\ b)}$$

In view of (2.2), one obtains

$$x = (k+1)f(a,b) + (k-11)g(a,b) \quad (2.17)$$
$$y = (k-1)f(a,b) - (k+11)g(a,b)$$

Thus, (2.13) & (2.17) satisfy (2.1).

Pattern 2.4

Rewrite (2.3) as

$$\mathrm{u}^2 + 11\mathrm{v}^2 = \left(\mathrm{k}^2 + 11\right)\mathrm{z}^{2\ \mathrm{s}+1} * 1 \quad (2.18)$$

Assume the integer 1 on the R.H.S. of (2.18) as

$$1 = \frac{(\mathrm{p(u)} + \mathrm{i}\sqrt{11}\mathrm{q(u)})(\mathrm{p(u)} - \mathrm{i}\sqrt{11}\mathrm{q(u)})}{(\mathrm{r(u)})^2} \quad (2.19)$$

where

$$p(u) = \left(2u^2 - 2u - 5\right), q(u) = (2u-1), r(u) = \left(2u^2 - 2u + 6\right)$$

Substituting (2.13), (2.14) & (2.19) in (2.18) and equating the positive factors, we have

$$(\mathrm{u} + \mathrm{i}\sqrt{11}\mathrm{v}) = (\mathrm{k} + \mathrm{i}\sqrt{11})(\mathrm{f(a,\ b)} + \mathrm{i}\sqrt{11}\ \mathrm{g(a,\ b)})\frac{(\mathrm{p(u)} + \mathrm{i}\sqrt{11}\mathrm{q(u)})}{\mathrm{r(\ s)}}$$
$$= [\mathrm{F(k,u)} + \mathrm{i}\sqrt{11}\mathrm{G(k,u)}]\frac{(\mathrm{f(a,\ b)} + \mathrm{i}\sqrt{11}\ \mathrm{g(a,\ b)})}{\mathrm{r(u)}} \quad (2.20)$$

where

$$\mathrm{F(k,u)} = (\mathrm{kp(u)} - 11\mathrm{q(u)}) \quad (2.21)$$
$$\mathrm{G(k,u)} = (\mathrm{p(u)} + \mathrm{kq(u)})$$

On comparing the coefficients of corresponding terms in (2.20), one has

$$\mathrm{u} = \frac{[\mathrm{F(k,u)f(a,\ b)} - 11\mathrm{G(k,u)g(a,\ b)}]}{\mathrm{r(u)}}$$
$$\mathrm{v} = \frac{[\mathrm{G(k,u)f(a,\ b)} + \mathrm{F(k,u)g(a,\ b)}]}{\mathrm{r(u)}} \quad (2.22)$$

As our aim is to obtain integer solutions, replacing a by r(u)A and b by r(u)B in (2.13) and (2.22), we have

$$z = (r(u))^2 \left(A^2 + 11\ B^2 \right) \tag{2.23}$$

and

$$u = (r(u))^{2\ s}[\ F(k,u)f(\ A,\ B) - 11G(k,u)g(\ A,\ B)]$$
$$v = (r(u))^{2\ s}[G(k,u)f(\ A,\ B) + F(k,u)g(\ A,\ B)]$$

In view of (2.2), we get

$$x = (r(u))^{2\ s}\{f(\ A,\ B)[F(k,u) + G(k,u)] + g(\ A,\ B)[F(k,u) - 11G(k,u)]\}$$
$$y = (r(u))^{2\ s}\{f(\ A,\ B)[F(k,u) - G(k,u)] - g(\ A,\ B)[F(k,u) + 11G(k,u)]\} \tag{2.24}$$

Thus, (2.23) & (2.24) satisfy (2.1).
Note 2.1
Apart from (2.19), one may have the following representations for the integer 1:
(i) $1 = \frac{\left(11r^2 - u^2 + i\sqrt{11}(2ru)\right)\left(11r^2 - u^2 - i\sqrt{11}(2ru)\right)}{(11r^2 + u^2)^2}$
(ii) $1 = \frac{\left(r^2 - 11u^2 + i\sqrt{11}(2ru)\right)\left(r^2 - 11u^2 - i\sqrt{11}(2ru)\right)}{(r^2 + 11u^2)^2}$
(iii) $1 = \frac{(p(u) + i\sqrt{11}q(u))(p(u) - i\sqrt{11}q(u))}{(r(u))^2}$
where

$$p(u) = \left(22u^2 - 22u + 5\right), q(u) = (2u - 1), r(u) = \left(22u^2 - 22u + 6\right)$$

Following the above procedure, three more sets of integer solutions to (2.1) are obtained.
Pattern 2.5
Taking

$$v = z^s \tag{2.25}$$

in (2.3), we have

$$u^2 = z^{2\ s}\left[\left(k^2 + 11\right) z - 11\right] \tag{2.26}$$

After some algebra, it is seen that (2.26) is satisfied by the values of z & u given by

$$z_n = 1 + 2nk + \left(k^2 + 11\right) n^2 \tag{2.27}$$
$$u_n = \left(1 + 2nk + \left(k^2 + 11\right) n^2\right)^s \left[k + \left(k^2 + 11\right) n\right]$$

From (2.25), one has

$$v_n = \left(1 + 2nk + \left(k^2 + 11\right) n^2\right)^s$$

In view of (2.2), we have

$$x_n = \left(1 + 2nk + \left(k^2 + 11\right) n^2\right)^s \left[k + 1 + \left(k^2 + 11\right) n\right] \tag{2.28}$$
$$y_n = \left(1 + 2nk + \left(k^2 + 11\right) n^2\right)^s \left[k - 1 + \left(k^2 + 11\right) n\right]$$

Thus, the values of x, y, z given by (2.28) & (2.27) satisfy (2.1).
Pattern 2.6
Insertion of transformations

$$\begin{aligned} \mathrm{v} &= \left(\mathrm{k}^2+11\right)^{\mathrm{s}+1} \ \mathrm{A}^{\mathrm{s}} \\ \mathrm{z} &= \left(\mathrm{k}^2+11\right) \mathrm{A} \end{aligned} \tag{2.29}$$

in (2.3) gives

$$\mathrm{u}^2 = \left(\mathrm{k}^2+11\right)^{2\ \mathrm{s}+2} \ \mathrm{A}^{2\ \mathrm{s}}(\ \mathrm{A}-11) \tag{2.30}$$

It is seen that the expression (A-11) is a perfect square when

$$\mathrm{A} = \alpha^2 + 11$$

and from (2.30), one obtains

$$\mathrm{u} = \left(\mathrm{k}^2+11\right)^{\mathrm{s}+1} \left(\alpha^2+11\right)^{\mathrm{s}} (\alpha) \tag{2.31}$$

From (2.29), we have

$$\begin{aligned} \mathrm{v} &= \left(\mathrm{k}^2+11\right)^{\mathrm{s}+1} \left(\alpha^2+11\right)^{\mathrm{s}} \\ \mathrm{z} &= \left(\mathrm{k}^2+11\right) \left(\alpha^2+11\right) \end{aligned} \tag{2.32}$$

Using (2.31) and (2.32) in (2.2), we get

$$\begin{aligned} x &= \left(k^2+11\right)^{s+1} \left(\alpha^2+11\right)^{s} (\alpha+1) \\ y &= \left(k^2+11\right)^{s+1} \left(\alpha^2+11\right)^{s} (\alpha-1), \alpha \neq \pm 1 \end{aligned} \tag{2.33}$$

Thus, the values of x, y, z given by (2.33) & (2.32) satisfy (2.1).

Pattern 2.7

Taking

$$\mathrm{u} = \mathrm{kz}^{\mathrm{s}} \tag{2.34}$$

in (2.3), we have

$$11v^2 = z^{2s}\left[\left(k^2+11\right)z - k^2\right] \tag{2.35}$$

After some algebra, it is seen that (2.35) is satisfied by the values of z&v given by

$$\begin{aligned} \mathrm{z_n} &= 1 + 22\mathrm{n} + 11\left(\mathrm{k}^2+11\right)\mathrm{n}^2 \\ \mathrm{v_n} &= \left(1 + 22\mathrm{n} + 11\left(\mathrm{k}^2+11\right)\mathrm{n}^2\right)^{\mathrm{s}} \left[1 + \left(\mathrm{k}^2+11\right)\mathrm{n}\right] \end{aligned} \tag{2.36}$$

From (2.34), one has

$$u_n = k\left(1 + 22n + 11\left(k^2+11\right)n^2\right)^s$$

In view of (2.2), we have

$$\begin{aligned} x_n &= \left(1 + 22n + 11\left(k^2+11\right)n^2\right)^s \left[k + 1 + \left(k^2+11\right)n\right] \\ y_n &= \left(1 + 22n + 11\left(k^2+11\right)n^2\right)^s \left[k - 1 - \left(k^2+11\right)n\right] \end{aligned} \tag{2.37}$$

Thus, the values of x, y, z given by (2.37) & (2.36) satisfy (2.1).

3. Conclusion

Solving the higher degree Diophantine equation and finding the non-zero distinct integer solutions are used in various fields like cryptography, Number patterns. The concepts of Diophantine equations are encouraging the young researchers to discover new ideas in various fields like some mentioned above. To conclude, one may search for different types of higher degree Diophantine equations and their solutions.

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