



Solution of Linear Diophantine Fuzzy Linear Programming Problems by Separation and Addition Method

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ABSTRACT: There are many real-world applications for the widely used concepts of intuitionistic fuzzy sets (IFSs), Pythagorean fuzzy sets (PFSs), and q-rung orthopair fuzzy sets (q-ROFSs). Unfortunately, these theories have limitations of their own regarding membership and non-membership grades. We present the novel idea of a linear Diophantine fuzzy set (LDFS) with the addition of reference parameters to get rid of these limitations. The decision maker (DM) can freely select the grades without any restrictions due to this concept, which eliminates the limitations of existing methodologies. By modifying the physical meaning of reference parameters, this structure also categorises the problem. To address triangular fully linear Diophantine fuzzy linear programming (TFLDFLP) problem, we developed a technique separation and addition approach. The main advantage of ranking functions is that the linear Diophantine variables nature was not altered to become crisp. It is divided into linear programming (LP) strategies by using arithmetic operations and ordering relations. Numerical examples are presented in detail to illustrate the suggested methodology.

Keywords: Intuitionistic fuzzy numbers, linear programming problems, fully fuzzy linear programming problem, linear diophantine fuzzy sets, linear diophantine fuzzy numbers.

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1. Introduction

In operations research, the linear programming (LP) strategy is the most significant and successful. In everyday life, we frequently deal with hazy or unclear information. Accessible information can occasionally be ambiguous or lacking. Fuzziness is typically described linguistically by terms like intervals, fuzzy numbers, or intuitionistic fuzzy set (IFS) numbers. In fuzzy set theory, Zadeh [23] developed the idea of ambiguous data in 1965. According to Zadeh, a linguistic variable is a variable whose values are phrases or words in either a synthetic or natural language. The variable is known as a fuzzy linguistic variable if these words are expressed by fuzzy sets defined over a universal set [23]. The fuzzy linguistic technique offers positive results in a number of domains, whose descriptions are generally of high quality. The use of sentences or words rather than numbers is encouraged since linguistic classifications or categorizations are typically less absolute than algebraic or arithmetic ones. Decision-making frequently involves issues with unknown conditions, yet these issues are difficult to solve because of how difficult it is to predict and manage these uncertainties. Atanassov [2] presented the idea of intuitionistic fuzzy set (IFS). The components of degree of membership and non-membership are divided in a particular set. The geometrical explanation of the components of intuitionistic fuzzy objects was proposed by Atanassov [3]. The total of membership and non-membership degrees to which an alternative satisfying an attribute specified by the decision maker (DM) may be greater than 1, but their sum of squares is less than or equal to 1, applies to several practical difficulties. Yager [20,21] created a Pythagorean fuzzy set (PFS) as a result,

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which satisfies the requirement that the sum of squares of its membership degree and non-membership degree is less than or equal to 1. Many scholars Ali [1], Liu [10] and Yager [22] have looked at the q-rung orthopair fuzzy set (q-ROFS) model to expand the scope of IFS and PFS.

Riaz and Hashmi [17] rigorously analysed the limits on membership and non-membership functions in the FS, IFS, PFS, and q-ROFS structures in 2019 and identified these restrictions statistically. To get rid of these limitations, they added reference parameters to the IFS structure and created the linear Diophantine fuzzy set (LDFS). They claimed that the LDFS concept would eliminate limitations in the current techniques of other sets and permit the practise of freely selecting data. Additionally, they demonstrated using the arbitrary property of the reference parameters that the space of this set is greater than those of the FS, IFS, PFS, and q-ROFS.

Dubey and Mehra [4] describes LP with triangular IF numbers by varying it into crisp set and solving it afterward. The nonlinear programming method was devised by Parvathi and Malathi [15]. A novel ranking function was proposed by Nachammai and Thangaraj [12], and it was solved by Nagoorgani and Ponnalagu [13,14] without being converted into a crisp LP problem. By using LP, Jayalakshmi and Pandian [6] established the separation and bound approach.

Dubey and Mehra [4] described intuitionistic fuzzy linear programming (IFLP) using the principles of value and ambiguity. To create a more realistic alternative, Parvathi and Malathi [15,16] combined the bisection approach and the two-phase phase method. A common sorting index method was developed by Seikh et al. [18] to solve IFLP with interval numbers. In IFLP, Mitchell [11] created fuzzy ranking. In order to address the IF transportation problem, Gani and Abbas [5] presented a novel strategy. For the representative interval parameters of the IFLP, Wan and Dong [19] provided a probability degree technique. Later, Kabiraj et al. [8] developed the Zimmermann technique, which transformed IFLP into crisp LP. Moreover, to address IFLP, Jayalakshmi et al. [7] introduced another technique called the separation and addition approach.

In the present attempt, we suggest a method for solving TFLDIFLP using separation and addition. Here, we first build an LP and then use the simplex process to solve it. The key advantage is that it deals with nonnegative linear Diophantine fuzzy (LDF) solutions rather than the current approaches, ranking functions are not employed, and is particularly suitable for LP solver because it solely depends on LP methodology.

2. Preliminaries and Basic Definitions

In this section we defined some basic definitions of linear Diophantine fuzzy numbers.

Definition 2.1 [17] *Let X be the universe. A linear Diophantine fuzzy set (LDFS) R on X is defined as follows:*

$$R = \{(\theta, \langle \varkappa_R^T(\theta), \chi_R^V(\theta) \rangle, \langle \alpha(\theta), \beta(\theta) \rangle) : \theta \in X\},$$

where $\varkappa_R^T(\theta), \chi_R^V(\theta), \alpha(\theta), \beta(\theta) \in [0, 1]$ such that

$$\begin{aligned} 0 &\leq \alpha(\theta)\varkappa_R^T(\theta) + \beta(\theta)\chi_R^V(\theta) \leq 1, \forall \theta \in X, \\ 0 &\leq \alpha(\theta) + \beta(\theta) \leq 1. \end{aligned}$$

The hesitation part can be written as

$$\varrho_{\pi_R} = 1 - (\alpha(\theta)\varkappa_R^T(\theta) + \beta(\theta)\chi_R^V(\theta)),$$

where ϱ is the reference parameter.

Definition 2.2 [9] *Let $\mathcal{L}_{\mathfrak{R}}$ be a LDFS on \mathbb{R} with the following membership functions ($\varkappa_{\mathfrak{R}}^T$ and α) and non-membership functions ($\chi_{\mathfrak{R}}^V$ and β)*

$$\varkappa_{\mathfrak{R}}^T(x) = \begin{cases} \frac{x-\vartheta_1}{\vartheta_3-\vartheta_1} & \vartheta_1 \leq x \leq \vartheta_3 \\ \frac{\vartheta_5-x}{\vartheta_5-\vartheta_3} & \vartheta_3 \leq x \leq \vartheta_5 \\ 0 & \text{otherwise} \end{cases}, \quad \chi_{\mathfrak{R}}^V(x) = \begin{cases} \frac{\vartheta_3-x}{\vartheta_3-\vartheta_2} & \vartheta_2 \leq x \leq \vartheta_3 \\ \frac{x-\vartheta_3}{\vartheta_4-\vartheta_3} & \vartheta_3 \leq x \leq \vartheta_4 \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\alpha(x) = \begin{cases} \frac{x-\vartheta'_2}{\vartheta'_3-\vartheta'_2} & \vartheta'_2 \leq x \leq \vartheta'_3 \\ \frac{\vartheta'_4-x}{\vartheta'_4-\vartheta'_3} & \vartheta'_3 \leq x \leq \vartheta'_4 \\ 0 & \text{otherwise} \end{cases}, \quad \beta(x) = \begin{cases} \frac{\vartheta'_3-x}{\vartheta'_3-\vartheta'_1} & \vartheta'_1 \leq x \leq \vartheta'_3 \\ \frac{x-\vartheta'_3}{\vartheta'_5-\vartheta'_3} & \vartheta'_3 \leq x \leq \vartheta'_5 \\ 0 & \text{otherwise,} \end{cases}$$

where $\vartheta'_1 \leq \vartheta'_2 \leq \vartheta'_3 \leq \vartheta'_4 \leq \vartheta'_5$ for all $x \in \mathbb{R}$. Then $\mathcal{L}_{\mathfrak{R}}$ is called a triangular LDFN if $\vartheta_3 = \vartheta'_3$ and $\vartheta_1 \leq \vartheta_2 \leq \vartheta_3 \leq \vartheta_4 \leq \vartheta_5$.

Definition 2.3 [9] A TLDFN $\mathcal{L}_{\mathfrak{R}TLDFN} = \left\{ (\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \vartheta_5) \right\}_{(\vartheta'_1, \vartheta'_2, \vartheta_3, \vartheta'_4, \vartheta'_5)}$ is said to be positive if and only if $\vartheta_1 \geq 0$ and $\vartheta'_1 \geq 0$.

Definition 2.4 [9] Two TLDFNs $\mathcal{L}_{\mathfrak{R}TLDFN} = \left\{ (\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \vartheta_5) \right\}_{(\vartheta'_1, \vartheta'_2, \vartheta_3, \vartheta'_4, \vartheta'_5)}$ and $\varrho_{\mathfrak{R}TLDFN} = \left\{ (\delta_1, \delta_2, \delta_3, \delta_4, \delta_5) \right\}_{(\delta'_1, \delta'_2, \delta_3, \delta'_4, \delta'_5)}$ are said to be equal if and only if $\vartheta_1 = \delta_1, \vartheta_2 = \delta_2, \vartheta_3 = \delta_3, \vartheta_4 = \delta_4, \vartheta_5 = \delta_5, \vartheta'_1 = \delta'_1, \vartheta'_2 = \delta'_2, \vartheta'_4 = \delta'_4$ and $\vartheta'_5 = \delta'_5$.

We now define the arithmetic operations on TLDFNs using the concept of interval arithmetic.

Definition 2.5 [9] Consider two positive TLDFNs

$$\mathcal{L}_{\mathfrak{R}TLDFN} = \left\{ (\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \vartheta_5) \right\}_{(\vartheta'_1, \vartheta'_2, \vartheta_3, \vartheta'_4, \vartheta'_5)} \quad \text{and} \quad \varrho_{\mathfrak{R}TLDFN} = \left\{ (\delta_1, \delta_2, \delta_3, \delta_4, \delta_5) \right\}_{(\delta'_1, \delta'_2, \delta_3, \delta'_4, \delta'_5)},$$

then

$$\begin{aligned} (i) \quad \mathcal{L}_{\mathfrak{R}TLDFN} + \varrho_{\mathfrak{R}TLDFN} &= \left\{ (\vartheta_1 + \delta_1, \vartheta_2 + \delta_2, \vartheta_3 + \delta_3, \vartheta_4 + \delta_4, \vartheta_5 + \delta_5) \right\}_{(\vartheta'_1 + \delta'_1, \vartheta'_2 + \delta'_2, \vartheta_3 + \delta_3, \vartheta'_4 + \delta'_4, \vartheta'_5 + \delta'_5)}; \\ (ii) \quad \mathcal{L}_{\mathfrak{R}TLDFN} - \varrho_{\mathfrak{R}TLDFN} &= \left\{ (\vartheta_1 - \delta_5, \vartheta_2 - \delta_4, \vartheta_3 - \delta_3, \vartheta_4 - \delta_2, \vartheta_5 - \delta_1) \right\}_{(\vartheta'_1 - \delta'_5, \vartheta'_2 - \delta'_4, \vartheta_3 - \delta_3, \vartheta'_4 - \delta'_2, \vartheta'_5 - \delta'_1)}; \\ (iii) \quad \mathcal{L}_{\mathfrak{R}TLDFN} \times \varrho_{\mathfrak{R}TLDFN} &= \left\{ (\vartheta_1 \delta_1, \vartheta_2 \delta_2, \vartheta_3 \delta_3, \vartheta_4 \delta_4, \vartheta_5 \delta_5) \right\}_{(\vartheta'_1 \delta'_1, \vartheta'_2 \delta'_2, \vartheta_3 \delta_3, \vartheta'_4 \delta'_4, \vartheta'_5 \delta'_5)}; \\ (iv) \quad \mathcal{L}_{\mathfrak{R}TLDFN} \div \varrho_{\mathfrak{R}TLDFN} &= \left\{ \left(\frac{\vartheta_1}{\delta_5}, \frac{\vartheta_2}{\delta_4}, \frac{\vartheta_3}{\delta_3}, \frac{\vartheta_4}{\delta_2}, \frac{\vartheta_5}{\delta_1} \right) \right\}_{\left(\frac{\vartheta'_1}{\delta'_5}, \frac{\vartheta'_2}{\delta'_4}, \frac{\vartheta_3}{\delta_3}, \frac{\vartheta'_4}{\delta'_2}, \frac{\vartheta'_5}{\delta'_1} \right)}; \\ (v) \quad k \times \mathcal{L}_{\mathfrak{R}TLDFN} &= \begin{cases} \left\{ (k\vartheta_1, k\vartheta_2, k\vartheta_3, k\vartheta_4, k\vartheta_5) \right\}_{(k\vartheta'_1, k\vartheta'_2, k\vartheta_3, k\vartheta'_4, k\vartheta'_5)} & \text{if } k > 0 \\ \left\{ (k\vartheta_5, k\vartheta_4, k\vartheta_3, k\vartheta_2, k\vartheta_1) \right\}_{(k\vartheta'_5, k\vartheta'_4, k\vartheta_3, k\vartheta'_2, k\vartheta'_1)} & \text{if } k < 0. \end{cases} \end{aligned}$$

3. Separation and Addition Method

The mathematical procedure of separation and addition method taking in the following steps. Consider the fully linear Diophantine fuzzy linear programming problem (FLDFLPP) in general form as given below:

$$\begin{aligned} \text{Max (or Min)} \quad & \left\{ \begin{array}{l} (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) \\ (\theta'_1, \theta'_2, \theta_3, \theta'_4, \theta'_5) \end{array} \right\} = \sum_{j=1}^n \left(\left\{ \begin{array}{l} (c_j^1, c_j^2, c_j^3, c_j^4, c_j^5) \\ (c_j^{1'}, c_j^{2'}, c_j^3, c_j^{4'}, c_j^{5'}) \end{array} \right\} \otimes \left\{ \begin{array}{l} (x_j^1, x_j^2, x_j^3, x_j^4, x_j^5) \\ (x_j^{1'}, x_j^{2'}, x_j^3, x_j^{4'}, x_j^{5'}) \end{array} \right\} \right) \\ \text{subject to} \quad & \sum_{j=1}^n \left(\left\{ \begin{array}{l} (a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4, a_{ij}^5) \\ (a_{ij}^{1'}, a_{ij}^{2'}, a_{ij}^3, a_{ij}^{4'}, a_{ij}^{5'}) \end{array} \right\} \otimes \left\{ \begin{array}{l} (x_j^1, x_j^2, x_j^3, x_j^4, x_j^5) \\ (x_j^{1'}, x_j^{2'}, x_j^3, x_j^{4'}, x_j^{5'}) \end{array} \right\} \right) \{ \leq, \approx, \geq \} \\ & \left\{ \begin{array}{l} (b_i^1, b_i^2, b_i^3, b_i^4, b_i^5) \\ (b_i^{1'}, b_i^{2'}, b_i^{3'}, b_i^{4'}, b_i^{5'}) \end{array} \right\} \\ & \left\{ \begin{array}{l} (x_j^1, x_j^2, x_j^3, x_j^4, x_j^5) \\ (x_j^{1'}, x_j^{2'}, x_j^3, x_j^{4'}, x_j^{5'}) \end{array} \right\} \geq 0, \quad \forall i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n. \end{aligned}$$

Step 1: From the FLDFLP problem, first take two separate subproblems for $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)$ and $(\theta'_1, \theta'_2, \theta_3, \theta'_4, \theta'_5)$. The subproblem for $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)$ is

$$\text{Max (or Min)} \quad (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) = \sum_{j=1}^n ((c_j^1, c_j^2, c_j^3, c_j^4, c_j^5) \otimes (x_j^1, x_j^2, x_j^3, x_j^4, x_j^5))$$

$$\text{subject to} \quad \sum_{j=1}^n ((a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4, a_{ij}^5) \otimes (x_j^1, x_j^2, x_j^3, x_j^4, x_j^5)) \{ \leq, \approx, \geq \}$$

$$(b_i^1, b_i^2, b_i^3, b_i^4, b_i^5)$$

$$(x_j^1, x_j^2, x_j^3, x_j^4, x_j^5) \geq 0, \quad \forall i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n,$$

and the subproblem for $(\theta'_1, \theta'_2, \theta_3, \theta'_4, \theta'_5)$ is

$$\text{Max (or Min)} \quad (\theta'_1, \theta'_2, \theta_3, \theta'_4, \theta'_5) = \sum_{j=1}^n ((c_j^{1'}, c_j^{2'}, c_j^3, c_j^{4'}, c_j^{5'}) \otimes (x_j^{1'}, x_j^{2'}, x_j^3, x_j^{4'}, x_j^{5'}))$$

$$\text{subject to} \quad \sum_{j=1}^n ((a_{ij}^{1'}, a_{ij}^{2'}, a_{ij}^3, a_{ij}^{4'}, a_{ij}^{5'}) \otimes (x_j^{1'}, x_j^{2'}, x_j^3, x_j^{4'}, x_j^{5'})) \{ \leq, \approx, \geq \}$$

$$(b_i^{1'}, b_i^{2'}, b_i^{3'}, b_i^{4'}, b_i^{5'})$$

$$(x_j^{1'}, x_j^{2'}, x_j^3, x_j^{4'}, x_j^{5'}) \geq 0, \quad \forall i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n.$$

Step 2: First we solve the subproblem $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)$. For this, using the arithmetic operations and partial ordering relations, we write the above subproblem $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)$ as LP problem. We construct the objective function as follows:

$$\text{Max (or Min)} \quad \theta = \sum_{j=1}^n c_j^1 x_j^1 + \sum_{j=1}^n c_j^2 x_j^2 + \sum_{j=1}^n c_j^3 x_j^3 + \sum_{j=1}^n c_j^4 x_j^4 + \sum_{j=1}^n c_j^5 x_j^5.$$

and the constraints as follows:

$$\sum_{j=1}^n a_{ij}^1 x_j^1 \{ \leq, \approx, \geq \} b_i^1, \quad i = 1, 2, \dots, m;$$

$$\sum_{j=1}^n a_{ij}^2 x_j^2 \{ \leq, \approx, \geq \} b_i^2, \quad i = 1, 2, \dots, m;$$

$$\sum_{j=1}^n a_{ij}^3 x_j^3 \{ \leq, \approx, \geq \} b_i^3, \quad i = 1, 2, \dots, m;$$

$$\sum_{j=1}^n a_{ij}^4 x_j^4 \{ \leq, \approx, \geq \} b_i^4, \quad i = 1, 2, \dots, m;$$

$$\sum_{j=1}^n a_{ij}^5 x_j^5 \{ \leq, \approx, \geq \} b_i^5, \quad i = 1, 2, \dots, m.$$

$$\theta_r \leq \theta_{r+1}, \quad r = 1, 2, 3, 4.$$

$$x_j^r \leq x_j^{r+1}, \quad r = 1, 2, 3, 4 \text{ and } j = 1, 2, \dots, n.$$

Now we get this subproblem in classical linear programming form. Then we solve the problem by simplex method and obtain the values of $(x_j^1, x_j^2, x_j^3, x_j^4, x_j^5)$ for $j = 1, 2, \dots, n$.

Step 3: Solve the subproblem for $(\theta'_1, \theta'_2, \theta_3, \theta'_4, \theta'_5)$ by using the method used in step 2.

Step 4: Combine the solutions obtained in step 2 and step 3. By simple substitutions we obtain the solution of

$$\left\{ \begin{array}{l} (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) \\ (\theta'_1, \theta'_2, \theta_3, \theta'_4, \theta'_5) \end{array} \right\} \text{ and } \left\{ \begin{array}{l} (x_j^1, x_j^2, x_j^3, x_j^4, x_j^5) \\ (x_j^{1'}, x_j^{2'}, x_j^3, x_j^{4'}, x_j^{5'}) \end{array} \right\} \text{ for } j = 1, 2, \dots, n.$$

4. Numerical Examples

In this section the above proposed method is explained with the following numerical examples.

Example 4.1 *Let us consider TriFLDFLP problem:*

$$\begin{aligned} \text{Maximize } \tilde{\theta} &= \tilde{A}\tilde{X} + \tilde{B}\tilde{Y} + \tilde{C}\tilde{Z} \\ \text{Subject to } &\tilde{A}_1\tilde{X} + \tilde{B}_1\tilde{Y} + \tilde{C}_1\tilde{Z} \leq \tilde{D}_1 \\ &\tilde{A}_2\tilde{X} + \tilde{B}_2\tilde{Y} + \tilde{C}_2\tilde{Z} \leq \tilde{D}_2 \\ &\tilde{A}_3\tilde{X} + \tilde{B}_3\tilde{Y} + \tilde{C}_3\tilde{Z} \leq \tilde{D}_3 \\ &\tilde{X}, \tilde{Y}, \tilde{Z} \geq 0. \end{aligned}$$

Where,

$$\begin{aligned} \tilde{\theta} &= \begin{cases} (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) \\ (\theta'_1, \theta'_2, \theta_3, \theta'_4, \theta'_5) \end{cases} & \tilde{X} &= \begin{cases} (x_1, x_2, x_3, x_4, x_5) \\ (x'_1, x'_2, x_3, x'_4, x'_5) \end{cases} \\ \tilde{Y} &= \begin{cases} (y_1, y_2, y_3, y_4, y_5) \\ (y'_1, y'_2, y_3, y'_4, y'_5) \end{cases} & \tilde{Z} &= \begin{cases} (z_1, z_2, z_3, z_4, z_5) \\ (z'_1, z'_2, z_3, z'_4, z'_5) \end{cases} \\ \tilde{A} &= \begin{cases} (2, 4, 5, 6, 8) \\ (1, 3, 5, 7, 9) \end{cases}, & \tilde{B} &= \begin{cases} (3, 5, 6, 7, 9) \\ (2, 4, 6, 8, 10) \end{cases}, & \tilde{C} &= \begin{cases} (4, 6, 7, 8, 10) \\ (3, 5, 7, 9, 11) \end{cases}, \\ \tilde{A}_1 &= \begin{cases} (2, 4, 5, 6, 8) \\ (1, 3, 5, 7, 9) \end{cases}, & \tilde{B}_1 &= \begin{cases} (5, 7, 8, 9, 11) \\ (4, 6, 8, 10, 12) \end{cases}, & \tilde{C}_1 &= \begin{cases} (8, 11, 12, 13, 15) \\ (8, 9, 12, 14, 16) \end{cases}, \\ \tilde{A}_2 &= \begin{cases} (8, 10, 11, 12, 14) \\ (5, 9, 11, 13, 15) \end{cases}, & \tilde{B}_2 &= \begin{cases} (6, 8, 9, 10, 12) \\ (6, 7, 9, 11, 13) \end{cases}, & \tilde{C}_2 &= \begin{cases} (11, 13, 14, 15, 17) \\ (10, 12, 14, 16, 18) \end{cases}, \\ \tilde{A}_3 &= \begin{cases} (10, 12, 13, 14, 16) \\ (9, 11, 13, 15, 17) \end{cases}, & \tilde{B}_3 &= \begin{cases} (13, 15, 16, 17, 19) \\ (12, 14, 16, 18, 20) \end{cases}, & \tilde{C}_3 &= \begin{cases} (3, 5, 6, 7, 9) \\ (2, 4, 6, 8, 10) \end{cases}, \\ \tilde{D}_1 &= \begin{cases} (19, 42, 54, 71, 95) \\ (7, 31, 54, 83, 111) \end{cases}, & \tilde{D}_2 &= \begin{cases} (32, 60, 73, 92, 120) \\ (15, 49, 73, 106, 138) \end{cases}, & \tilde{D}_3 &= \begin{cases} (49, 76, 89, 103, 131) \\ (21, 62, 89, 117, 151) \end{cases}. \end{aligned}$$

Using the arithmetic operations and partial ordering relations we defined TFLDFLP problem as:

$$\begin{aligned} \max \theta_1 &= 2x_1 + 3y_1 + 4z_1, \\ \max \theta_2 &= 4x_2 + 5y_2 + 6z_2, \\ \max \theta_3 &= 5x_3 + 6y_3 + 7z_3, \\ \max \theta_4 &= 6x_4 + 7y_4 + 8z_4, \\ \max \theta_5 &= 8x_5 + 9y_5 + 10z_5, \end{aligned}$$

subject to

$$\begin{aligned} 2x_1 + 5y_1 + 8z_1 &\leq 19, & 8x_1 + 6y_1 + 11z_1 &\leq 32, & 10x_1 + 13y_1 + 3z_1 &\leq 49, \\ 4x_2 + 7y_2 + 11z_2 &\leq 42, & 10x_2 + 8y_2 + 13z_2 &\leq 60, & 12x_2 + 15y_2 + 5z_2 &\leq 76, \\ 5x_3 + 8y_3 + 12z_3 &\leq 54, & 11x_3 + 9y_3 + 14z_3 &\leq 73, & 13x_3 + 16y_3 + 6z_3 &\leq 89, \\ 6x_4 + 9y_4 + 13z_4 &\leq 71, & 12x_4 + 10y_4 + 15z_4 &\leq 92, & 14x_4 + 17y_4 + 7z_4 &\leq 103, \\ 8x_5 + 11y_5 + 15z_5 &\leq 95, & 14x_5 + 12y_5 + 17z_5 &\leq 120, & 16x_5 + 19y_5 + 9z_5 &\leq 131, \end{aligned}$$

$$\begin{aligned} \theta_1 &\leq \theta_2, \theta_2 \leq \theta_3, \theta_3 \leq \theta_4, \theta_4 \leq \theta_5, \\ x_1 &\leq x_2, x_2 \leq x_3, x_3 \leq x_4, x_4 \leq x_5, \\ y_1 &\leq y_2, y_2 \leq y_3, y_3 \leq y_4, y_4 \leq y_5, \\ z_1 &\leq z_2, z_2 \leq z_3, z_3 \leq z_4, z_4 \leq z_5, \\ x_1, \dots, x_5; y_1, \dots, y_5, z_1, \dots, z_5 &\geq 0. \end{aligned}$$

Now consider the following LP problem

$$\max \theta = \begin{cases} 2x_1 + 3y_1 + 4z_1 \\ +4x_2 + 5y_2 + 6z_2 \\ +5x_3 + 6y_3 + 7z_3 \\ +6x_4 + 7y_4 + 8z_4 \\ +8x_5 + 9y_5 + 10z_5. \end{cases}$$

subject to

$$\begin{aligned} 2x_1 + 5y_1 + 8z_1 &\leq 19, & 8x_1 + 6y_1 + 11z_1 &\leq 32, & 10x_1 + 13y_1 + 3z_1 &\leq 49, \\ 4x_2 + 7y_2 + 11z_2 &\leq 42, & 10x_2 + 8y_2 + 13z_2 &\leq 60, & 12x_2 + 15y_2 + 5z_2 &\leq 76, \\ 5x_3 + 8y_3 + 12z_3 &\leq 54, & 11x_3 + 9y_3 + 14z_3 &\leq 73, & 13x_3 + 16y_3 + 6z_3 &\leq 89, \\ 6x_4 + 9y_4 + 13z_4 &\leq 71, & 12x_4 + 10y_4 + 15z_4 &\leq 92, & 14x_4 + 17y_4 + 7z_4 &\leq 103, \\ 8x_5 + 11y_5 + 15z_5 &\leq 95, & 14x_5 + 12y_5 + 17z_5 &\leq 120, & 16x_5 + 19y_5 + 9z_5 &\leq 131, \end{aligned}$$

$$\begin{aligned} 2x_1 + 3y_1 + 4z_1 - 4x_2 - 5y_2 - 6z_2 &\leq 0, \\ 4x_2 + 5y_2 + 6z_2 - 5x_3 - 6y_3 - 7z_3 &\leq 0, \\ 5x_3 + 6y_3 + 7z_3 - 6x_4 - 7y_4 - 8z_4 &\leq 0, \\ 6x_4 + 7y_4 + 8z_4 - 8x_5 - 9y_5 - 10z_5 &\leq 0, \end{aligned}$$

$$\begin{aligned} x_1 - x_2 &\leq 0, & x_2 - x_3 &\leq 0, \\ x_3 - x_4 &\leq 0, & x_4 - x_5 &\leq 0, \\ y_1 - y_2 &\leq 0, & y_2 - y_3 &\leq 0, \\ y_3 - y_4 &\leq 0, & y_4 - y_5 &\leq 0, \\ z_1 - z_2 &\leq 0, & z_2 - z_3 &\leq 0, \\ z_3 - z_4 &\leq 0, & z_4 - z_5 &\leq 0, \\ x_1, \dots, x_5, y_1, \dots, y_5, z_1, \dots, z_5 &\geq 0. \end{aligned}$$

Solving the above problem by simplex method, the optimal solution is:

$$\begin{aligned} (x_1, x_2, x_3, x_4, x_5) &= \left(\frac{204}{133}, \frac{759}{358}, \frac{844}{393}, \frac{479}{214}, \frac{703}{249} \right) \\ (y_1, y_2, y_3, y_4, y_5) &= \left(\frac{331}{133}, \frac{535}{179}, \frac{1292}{393}, \frac{358}{107}, \frac{851}{249} \right) \\ (z_1, z_2, z_3, z_4, z_5) &= \left(\frac{58}{133}, \frac{205}{179}, \frac{1111}{786}, \frac{226}{107}, \frac{578}{249} \right) \\ (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) &= \left(\frac{1633}{133}, \frac{5423}{179}, \frac{31721}{786}, \frac{5751}{107}, \frac{19063}{249} \right). \end{aligned}$$

Similarly,

$$\begin{aligned} \max \theta'_1 &= 1x'_1 + 2y'_1 + 3z'_1, \\ \max \theta'_2 &= 3x'_2 + 4y'_2 + 5z'_2, \\ \max \theta_3 &= 5x_3 + 6y_3 + 7z_3, \\ \max \theta'_4 &= 7x'_4 + 8y'_4 + 9z'_4, \\ \max \theta'_5 &= 9x'_5 + 10y'_5 + 11z'_5, \end{aligned}$$

subject to

$$\begin{aligned} 1x'_1 + 4y'_1 + 8z'_1 &\leq 7, & 5x'_1 + 6y'_1 + 10z'_1 &\leq 15, & 9x'_1 + 12y'_1 + 2z'_1 &\leq 21, \\ 3x'_2 + 6y'_2 + 9z'_2 &\leq 31, & 9x'_2 + 7y'_2 + 12z'_2 &\leq 49, & 11x'_2 + 14y'_2 + 4z'_2 &\leq 62, \\ 5x_3 + 8y_3 + 12z_3 &\leq 54, & 11x_3 + 9y_3 + 14z_3 &\leq 73, & 13x_3 + 16y_3 + 6z_3 &\leq 89, \\ 7x'_4 + 10y'_4 + 14z'_4 &\leq 83, & 13x'_4 + 11y'_4 + 16z'_4 &\leq 106, & 15x'_4 + 18y'_4 + 8z'_4 &\leq 117, \\ 9x'_5 + 12y'_5 + 16z'_5 &\leq 111, & 15x'_5 + 13y'_5 + 18z'_5 &\leq 138, & 17x'_5 + 20y'_5 + 10z'_5 &\leq 151, \end{aligned}$$

$$\begin{aligned}
&\theta'_1 \leq \theta'_2, \theta'_2 \leq \theta'_3, \theta'_3 \leq \theta'_4, \theta'_4 \leq \theta'_5, \\
&x'_1 \leq x'_2, x'_2 \leq x_3, x_3 \leq x'_4, x'_4 \leq x'_5, \\
&y'_1 \leq y'_2, y'_2 \leq y_3, y_3 \leq y'_4, y'_4 \leq y'_5, \\
&z'_1 \leq z'_2, z'_2 \leq z_3, z_3 \leq z'_4, z'_4 \leq z'_5, \\
&x'_1, \dots, x'_5, y'_1, \dots, y'_5, z'_1, \dots, z'_5 \geq 0.
\end{aligned}$$

Now consider the following LP problem may be written as:

$$\max \theta' = \begin{cases} 1x'_1 + 2y'_1 + 3z'_1 \\ +3x'_2 + 4y'_2 + 5z'_2 \\ +5x_3 + 6y_3 + 7z_3 \\ +7x'_4 + 8y'_4 + 9z'_4 \\ +9x'_5 + 10y'_5 + 11z'_5 \end{cases}$$

subject to

$$\begin{aligned}
1x'_1 + 4y'_1 + 8z'_1 &\leq 7, & 5x'_1 + 6y'_1 + 10z'_1 &\leq 15, & 9x'_1 + 12y'_1 + 2z'_1 &\leq 21, \\
3x'_2 + 6y'_2 + 9z'_2 &\leq 31, & 9x'_2 + 7y'_2 + 12z'_2 &\leq 49, & 11x'_2 + 14y'_2 + 4z'_2 &\leq 62, \\
5x_3 + 8y_3 + 12z_3 &\leq 54, & 11x_3 + 9y_3 + 14z_3 &\leq 73, & 13x_3 + 16y_3 + 6z_3 &\leq 89, \\
7x'_4 + 10y'_4 + 14z'_4 &\leq 83, & 13x'_4 + 11y'_4 + 16z'_4 &\leq 106, & 15x'_4 + 18y'_4 + 8z'_4 &\leq 117, \\
9x'_5 + 12y'_5 + 16z'_5 &\leq 111, & 15x'_5 + 13y'_5 + 18z'_5 &\leq 138, & 17x'_5 + 20y'_5 + 10z'_5 &\leq 151,
\end{aligned}$$

$$\begin{aligned}
1x'_1 + 2y'_1 + 3z'_1 - 3x'_2 - 4y'_2 - 5z'_2 &\leq 0, \\
3x'_2 + 4y'_2 + 5z'_2 - 5x_3 - 6y_3 - 7z_3 &\leq 0, \\
5x_3 + 6y_3 + 7z_3 - 7x'_4 - 8y'_4 - 9z'_4 &\leq 0, \\
7x'_4 + 8y'_4 + 9z'_4 - 9x'_5 - 10y'_5 - 11z'_5 &\leq 0,
\end{aligned}$$

$$\begin{aligned}
x'_1 - x'_2 &\leq 0, & x'_2 - x_3 &\leq 0, \\
x_3 - x'_4 &\leq 0, & x'_4 - x'_5 &\leq 0, \\
y'_1 - y'_2 &\leq 0, & y'_2 - y_3 &\leq 0, \\
y_3 - y'_4 &\leq 0, & y'_4 - y'_5 &\leq 0, \\
z'_1 - z'_2 &\leq 0, & z'_2 - z_3 &\leq 0, \\
z_3 - z'_4 &\leq 0, & z'_4 - z'_5 &\leq 0,
\end{aligned}$$

$$x'_1, \dots, x'_5, y'_1, \dots, y'_5, z'_1, \dots, z'_5 \geq 0.$$

Solving the above problem by simplex method, the optimal solution is:

$$\begin{aligned}
(x'_1, x'_2, x_3, x'_4, x'_5) &= \left(\frac{99}{65}, \frac{1216}{597}, \frac{844}{393}, \frac{1167}{463}, \frac{1639}{533} \right) \\
(y'_1, y'_2, y_3, y'_4, y'_5) &= \left(\frac{7}{13}, \frac{501}{199}, \frac{1292}{393}, \frac{1577}{463}, \frac{153}{41} \right) \\
(z'_1, z'_2, z_3, z'_4, z'_5) &= \left(\frac{27}{65}, \frac{649}{597}, \frac{1111}{786}, \frac{1035}{463}, \frac{1284}{533} \right) \\
(\theta'_1, \theta'_2, \theta_3, \theta'_4, \theta'_5) &= \left(\frac{50}{13}, \frac{12905}{597}, \frac{31721}{786}, \frac{30100}{463}, \frac{48765}{533} \right).
\end{aligned}$$

Hence, the optimal solution of above TriFLDFLP problem is:

$$\begin{aligned}
\begin{cases} (x_1, x_2, x_3, x_4, x_5) \\ (x'_1, x'_2, x_3, x'_4, x'_5) \end{cases} &= \begin{cases} \left(\frac{204}{133}, \frac{759}{358}, \frac{844}{393}, \frac{479}{214}, \frac{703}{249} \right) \\ \left(\frac{99}{65}, \frac{1216}{597}, \frac{844}{393}, \frac{1167}{463}, \frac{1639}{533} \right) \end{cases} \\
\begin{cases} (y_1, y_2, y_3, y_4, y_5) \\ (y'_1, y'_2, y_3, y'_4, y'_5) \end{cases} &= \begin{cases} \left(\frac{331}{133}, \frac{535}{179}, \frac{1292}{393}, \frac{358}{107}, \frac{851}{249} \right) \\ \left(\frac{7}{13}, \frac{501}{199}, \frac{1292}{393}, \frac{1577}{463}, \frac{153}{41} \right) \end{cases} \\
\begin{cases} (z_1, z_2, z_3, z_4, z_5) \\ (z'_1, z'_2, z_3, z'_4, z'_5) \end{cases} &= \begin{cases} \left(\frac{58}{133}, \frac{205}{179}, \frac{1111}{786}, \frac{226}{107}, \frac{578}{249} \right) \\ \left(\frac{27}{65}, \frac{649}{597}, \frac{1111}{786}, \frac{1035}{463}, \frac{1284}{533} \right) \end{cases} \\
\begin{cases} (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) \\ (\theta'_1, \theta'_2, \theta_3, \theta'_4, \theta'_5) \end{cases} &= \begin{cases} \left(\frac{1633}{133}, \frac{5423}{179}, \frac{31721}{786}, \frac{5751}{107}, \frac{19063}{249} \right) \\ \left(\frac{50}{13}, \frac{12905}{597}, \frac{31721}{786}, \frac{30100}{463}, \frac{48765}{533} \right) \end{cases}.
\end{aligned}$$

Example 4.2 Let us consider TFLDFLP problem:

$$\begin{aligned} \text{Minimize } \tilde{\theta} &= \tilde{A}\tilde{X} + \tilde{B}\tilde{Y} + \tilde{C}\tilde{Z} \\ \text{Subject to } &\tilde{A}_1\tilde{X} - \tilde{B}_1\tilde{Y} + \tilde{C}_1\tilde{Z} \geq \tilde{D}_1 \\ &\tilde{A}_2\tilde{X} - \tilde{B}_2\tilde{Y} + \tilde{C}_2\tilde{Z} \geq \tilde{D}_2 \\ &-\tilde{A}_3\tilde{X} - \tilde{B}_3\tilde{Y} + \tilde{C}_3\tilde{Z} \leq \tilde{D}_3 \\ &\tilde{X}, \tilde{Y}, \tilde{Z} \geq 0. \end{aligned}$$

Where,

$$\begin{aligned} \tilde{\theta} &= \begin{cases} (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) \\ (\theta'_1, \theta'_2, \theta_3, \theta'_4, \theta'_5) \end{cases} & \tilde{X} &= \begin{cases} (x_1, x_2, x_3, x_4, x_5) \\ (x'_1, x'_2, x_3, x'_4, x'_5) \end{cases} \\ \tilde{Y} &= \begin{cases} (y_1, y_2, y_3, y_4, y_5) \\ (y'_1, y'_2, y_3, y'_4, y'_5) \end{cases} & \tilde{Z} &= \begin{cases} (z_1, z_2, z_3, z_4, z_5) \\ (z'_1, z'_2, z_3, z'_4, z'_5) \end{cases} \\ \tilde{A} &= \begin{cases} (2, 3, 4, 5, 6) \\ (1, 3, 4, 4, 5) \end{cases}, & \tilde{B} &= \begin{cases} (3, 4, 5, 6, 7) \\ (2, 4, 5, 5, 6) \end{cases}, & \tilde{C} &= \begin{cases} (4, 5, 6, 7, 8) \\ (3, 5, 6, 6, 7) \end{cases}, \\ \tilde{A}_1 &= \begin{cases} (5, 6, 7, 8, 9) \\ (4, 6, 7, 7, 9) \end{cases}, & \tilde{B}_1 &= \begin{cases} (2, 3, 4, 5, 6) \\ (2, 3, 4, 4, 5) \end{cases}, & \tilde{C}_1 &= \begin{cases} (9, 10, 11, 12, 14) \\ (6, 7, 11, 13, 15) \end{cases}, \\ \tilde{A}_2 &= \begin{cases} (3, 4, 5, 6, 7) \\ (4, 5, 5, 6, 7) \end{cases}, & \tilde{B}_2 &= \begin{cases} (1, 2, 3, 4, 5) \\ (1, 2, 3, 7, 8) \end{cases}, & \tilde{C}_2 &= \begin{cases} (3, 4, 5, 6, 7) \\ (4, 5, 5, 6, 7) \end{cases}, \\ \tilde{A}_3 &= \begin{cases} (5, 6, 7, 8, 9) \\ (3, 4, 7, 8, 9) \end{cases}, & \tilde{B}_3 &= \begin{cases} (4, 5, 6, 7, 8) \\ (3, 4, 6, 6, 6) \end{cases}, & \tilde{C}_3 &= \begin{cases} (4, 5, 6, 7, 8) \\ (3, 4, 6, 7, 9) \end{cases}, \\ \tilde{D}_1 &= \begin{cases} (9, 19, 21, 33, 45) \\ (5, 15, 21, 35, 55) \end{cases}, & \tilde{D}_2 &= \begin{cases} (36, 54, 74, 90, 137) \\ (28, 62, 74, 106, 165) \end{cases}, & \tilde{D}_3 &= \begin{cases} (26, 34, 44, 58, 81) \\ (16, 29, 44, 80, 96) \end{cases}. \end{aligned}$$

Using the arithmetic operations and partial ordering relations we defined TFLDFLP problem as:

$$\begin{aligned} \min \theta_1 &= 2x_1 + 3y_1 + 4z_1, \\ \min \theta_2 &= 3x_2 + 4y_2 + 5z_2, \\ \min \theta_3 &= 4x_3 + 5y_3 + 6z_3, \\ \min \theta_4 &= 5x_4 + 6y_4 + 7z_4, \\ \min \theta_5 &= 6x_5 + 7y_5 + 8z_5, \end{aligned}$$

subject to

$$\begin{aligned} 5x_1 - 6y_1 + 9z_1 &\geq 9, & 3x_1 - 5y_1 + 3z_1 &\geq 36, & -9x_1 - 8y_1 + 4z_1 &\geq 26, \\ 6x_2 - 5y_2 + 10z_2 &\geq 19, & 4x_2 - 4y_2 + 4z_2 &\geq 54, & -8x_2 - 7y_2 + 5z_2 &\geq 34, \\ 7x_3 - 4y_3 + 11z_3 &\geq 21, & 5x_3 - 3y_3 + 5z_3 &\geq 74, & -7x_3 - 6y_3 + 6z_3 &\geq 44, \\ 8x_4 - 3y_4 + 12z_4 &\geq 33, & 6x_4 - 2y_4 + 6z_4 &\geq 90, & -6x_4 - 5y_4 + 7z_4 &\geq 58, \\ 9x_5 - 2y_5 + 14z_5 &\geq 45, & 7x_5 - y_5 + 7z_5 &\geq 137, & -5x_5 - 4y_5 + 8z_5 &\geq 81, \\ \theta_1 &\leq \theta_2, & \theta_2 &\leq \theta_3, & \theta_3 &\leq \theta_4, & \theta_4 &\leq \theta_5, \\ x_1 &\leq x_2, & x_2 &\leq x_3, & x_3 &\leq x_4, & x_4 &\leq x_5, \\ y_1 &\leq y_2, & y_2 &\leq y_3, & y_3 &\leq y_4, & y_4 &\leq y_5, \\ z_1 &\leq z_2, & z_2 &\leq z_3, & z_3 &\leq z_4, & z_4 &\leq z_5, \\ x_1, \dots, x_5, & y_1, \dots, y_5, & z_1, \dots, z_5 &\geq 0. \end{aligned}$$

Now consider the following LP problem can be written as:

$$\min \theta = \begin{cases} 2x_1 + 3y_1 + 4z_1 \\ +3x_2 + 4y_2 + 5z_2 \\ +4x_3 + 5y_3 + 6z_3 \\ +5x_4 + 6y_4 + 7z_4 \\ +6x_5 + 7y_5 + 8z_5. \end{cases}$$

subject to

$$\begin{aligned}
5x_1 - 6y_1 + 9z_1 &\geq 9, & 3x_1 - 5y_1 + 3z_1 &\geq 36, & -9x_1 - 8y_1 + 4z_1 &\geq 26, \\
6x_2 - 5y_2 + 10z_2 &\geq 19, & 4x_2 - 4y_2 + 4z_2 &\geq 54, & -8x_2 - 7y_2 + 5z_2 &\geq 34, \\
7x_3 - 4y_3 + 11z_3 &\geq 21, & 5x_3 - 3y_3 + 5z_3 &\geq 74, & -7x_3 - 6y_3 + 6z_3 &\geq 44, \\
8x_4 - 3y_4 + 12z_4 &\geq 33, & 6x_4 - 2y_4 + 6z_4 &\geq 90, & -6x_4 - 5y_4 + 7z_4 &\geq 58, \\
9x_5 - 2y_5 + 14z_5 &\geq 45, & 7x_5 - y_5 + 7z_5 &\geq 137, & -5x_5 - 4y_5 + 8z_5 &\geq 81, \\
2x_1 + 3y_1 + 4z_1 - 3x_2 - 4y_2 - 5z_2 &\leq 0, \\
3x_2 + 4y_2 + 5z_2 - 4x_3 - 5y_3 - 6z_3 &\leq 0, \\
4x_3 + 5y_3 + 6z_3 - 5x_4 - 6y_4 - 7z_4 &\leq 0, \\
5x_4 + 6y_4 + 7z_4 - 6x_5 - 7y_5 - 8z_5 &\leq 0, \\
x_1 - x_2 &\leq 0, & x_2 - x_3 &\leq 0, \\
x_3 - x_4 &\leq 0, & x_4 - x_5 &\leq 0, \\
y_1 - y_2 &\leq 0, & y_2 - y_3 &\leq 0, \\
y_3 - y_4 &\leq 0, & y_4 - y_5 &\leq 0, \\
z_1 - z_2 &\leq 0, & z_2 - z_3 &\leq 0, \\
z_3 - z_4 &\leq 0, & z_4 - z_5 &\leq 0, \\
x_1, \dots, x_5, y_1, \dots, y_5, z_1, \dots, z_5 &\geq 0.
\end{aligned}$$

Solving the above problem by simplex method, the optimal solution is:

$$\begin{aligned}
(x_1, x_2, x_3, x_4, x_5) &= \left(\frac{22}{13}, \frac{67}{26}, \frac{224}{65}, \frac{47}{13}, \frac{529}{91} \right) \\
(y_1, y_2, y_3, y_4, y_5) &= (0, 0, 0, 0, 0) \\
(z_1, z_2, z_3, z_4, z_5) &= \left(\frac{134}{13}, \frac{142}{13}, \frac{738}{65}, \frac{148}{13}, \frac{1252}{91} \right) \\
(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) &= \left(\frac{580}{13}, \frac{1621}{26}, \frac{5324}{65}, \frac{1271}{13}, \frac{13190}{91} \right).
\end{aligned}$$

Similarly,

$$\begin{aligned}
\min \theta'_1 &= 1x'_1 + 2y'_1 + 3z'_1, \\
\min \theta'_2 &= 3x'_2 + 4y'_2 + 5z'_2, \\
\min \theta'_3 &= 4x'_3 + 5y'_3 + 6z'_3, \\
\min \theta'_4 &= 4x'_4 + 5y'_4 + 6z'_4, \\
\min \theta'_5 &= 5x'_5 + 6y'_5 + 7z'_5,
\end{aligned}$$

subject to

$$\begin{aligned}
4x'_1 - 5y'_1 + 6z'_1 &\geq 5, & 4x'_1 - 8y'_1 + 4z'_1 &\geq 28, & -9x'_1 - 6y'_1 + 3z'_1 &\geq 16, \\
6x'_2 - 4y'_2 + 7z'_2 &\geq 15, & 5x'_2 - 7y'_2 + 5z'_2 &\geq 62, & -8x'_2 - 6y'_2 + 4z'_2 &\geq 29, \\
7x'_3 - 4y'_3 + 11z'_3 &\geq 21, & 5x'_3 - 3y'_3 + 5z'_3 &\geq 74, & -7x'_3 - 6y'_3 + 6z'_3 &\geq 44, \\
7x'_4 - 3y'_4 + 13z'_4 &\geq 35, & 6x'_4 - 2y'_4 + 6z'_4 &\geq 106, & -4x'_4 - 4y'_4 + 7z'_4 &\geq 80, \\
9x'_5 - 2y'_5 + 15z'_5 &\geq 55, & 7x'_5 - 1y'_5 + 7z'_5 &\geq 165, & -3x'_5 - 3y'_5 + 9z'_5 &\geq 96, \\
\theta'_1 &\leq \theta'_2, & \theta'_2 &\leq \theta'_3, & \theta'_3 &\leq \theta'_4, & \theta'_4 &\leq \theta'_5, \\
x'_1 &\leq x'_2, & x'_2 &\leq x'_3, & x'_3 &\leq x'_4, & x'_4 &\leq x'_5, \\
y'_1 &\leq y'_2, & y'_2 &\leq y'_3, & y'_3 &\leq y'_4, & y'_4 &\leq y'_5, \\
z'_1 &\leq z'_2, & z'_2 &\leq z'_3, & z'_3 &\leq z'_4, & z'_4 &\leq z'_5, \\
x'_1, \dots, x'_5, y'_1, \dots, y'_5, z'_1, \dots, z'_5 &\geq 0.
\end{aligned}$$

Now consider the following LP problem may be written as:

$$\min \theta' = \begin{cases} 1x'_1 + 2y'_1 + 3z'_1 \\ +3x'_2 + 4y'_2 + 5z'_2 \\ +4x'_3 + 5y'_3 + 6z'_3 \\ 4x'_4 + 5y'_4 + 6z'_4 \\ +5x'_5 + 6y'_5 + 7z'_5. \end{cases}$$

subject to

$$\begin{aligned}
4x'_1 - 5y'_1 + 6z'_1 &\geq 5, & 4x'_1 - 8y'_1 + 4z'_1 &\geq 28, & -9x'_1 - 6y'_1 + 3z'_1 &\geq 16, \\
6x'_2 - 4y'_2 + 7z'_2 &\geq 15, & 5x'_2 - 7y'_2 + 5z'_2 &\geq 62, & -8x'_2 - 6y'_2 + 4z'_2 &\geq 29, \\
7x_3 - 4y_3 + 11z_3 &\geq 21, & 5x_3 - 3y_3 + 5z_3 &\geq 74, & -7x_3 - 6y_3 + 6z_3 &\geq 44, \\
7x'_4 - 3y'_4 + 13z'_4 &\geq 35, & 6x'_4 - 2y'_4 + 6z'_4 &\geq 106, & -4x'_4 - 4y'_4 + 7z'_4 &\geq 80, \\
9x'_5 - 2y'_5 + 15z'_5 &\geq 55, & 7x'_5 - 1y'_5 + 7z'_5 &\geq 165, & -3x'_5 - 3y'_5 + 9z'_5 &\geq 96,
\end{aligned}$$

$$\begin{aligned}
1x'_1 + 2y'_1 + 3z'_1 - 3x'_2 - 4y'_2 - 5z'_2 &\leq 0, \\
3x'_2 + 4y'_2 + 5z'_2 - 4x_3 - 5y_3 - 6z_3 &\leq 0, \\
4x_3 + 5y_3 + 6z_3 - 4x'_4 - 5y'_4 - 6z'_4 &\leq 0, \\
4x'_4 + 5y'_4 + 6z'_4 - 5x'_5 - 6y'_5 - 7z'_5 &\leq 0,
\end{aligned}$$

$$\begin{aligned}
x'_1 - x'_2 &\leq 0, & x'_2 - x_3 &\leq 0, \\
x_3 - x'_4 &\leq 0, & x'_4 - x'_5 &\leq 0, \\
y'_1 - y'_2 &\leq 0, & y'_2 - y_3 &\leq 0, \\
y_3 - y'_4 &\leq 0, & y'_4 - y'_5 &\leq 0, \\
z'_1 - z'_2 &\leq 0, & z'_2 - z_3 &\leq 0, \\
z_3 - z'_4 &\leq 0, & z'_4 - z'_5 &\leq 0,
\end{aligned}$$

$$x'_1, \dots, x'_5, y'_1, \dots, y'_5, z'_1, \dots, z'_5 \geq 0.$$

Solving the above problem by simplex method, the optimal solution is:

$$\begin{aligned}
(x'_1, x'_2, x_3, x'_4, x'_5) &= \left(\frac{5}{12}, \frac{103}{60}, \frac{224}{65}, \frac{131}{33}, \frac{271}{28} \right) \\
(y'_1, y'_2, y_3, y'_4, y'_5) &= (0, 0, 0, 0, 0) \\
(z'_1, z'_2, z_3, z'_4, z'_5) &= \left(\frac{79}{12}, \frac{641}{60}, \frac{738}{65}, \frac{452}{33}, \frac{389}{28} \right) \\
(\theta'_1, \theta'_2, \theta_3, \theta'_4, \theta'_5) &= \left(\frac{121}{6}, \frac{1757}{30}, \frac{5324}{65}, \frac{3236}{33}, \frac{2039}{14} \right).
\end{aligned}$$

Hence, the optimal solution of above TFLDFLP problem is:

$$\begin{aligned}
\begin{cases} (x_1, x_2, x_3, x_4, x_5) \\ (x'_1, x'_2, x_3, x'_4, x'_5) \end{cases} &= \begin{cases} \left(\frac{22}{13}, \frac{67}{26}, \frac{224}{65}, \frac{47}{13}, \frac{529}{91} \right) \\ \left(\frac{5}{12}, \frac{103}{60}, \frac{224}{65}, \frac{131}{33}, \frac{271}{28} \right) \end{cases} \\
\begin{cases} (y_1, y_2, y_3, y_4, y_5) \\ (y'_1, y'_2, y_3, y'_4, y'_5) \end{cases} &= \begin{cases} (0, 0, 0, 0, 0) \\ (0, 0, 0, 0, 0) \end{cases} \\
\begin{cases} (z_1, z_2, z_3, z_4, z_5) \\ (z'_1, z'_2, z_3, z'_4, z'_5) \end{cases} &= \begin{cases} \left(\frac{134}{13}, \frac{142}{13}, \frac{738}{65}, \frac{148}{13}, \frac{1252}{91} \right) \\ \left(\frac{79}{12}, \frac{641}{60}, \frac{738}{65}, \frac{452}{33}, \frac{389}{28} \right) \end{cases} \\
\begin{cases} (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) \\ (\theta'_1, \theta'_2, \theta_3, \theta'_4, \theta'_5) \end{cases} &= \begin{cases} \left(\frac{580}{13}, \frac{5423}{179}, \frac{1621}{26}, \frac{1271}{13}, \frac{13190}{91} \right) \\ \left(\frac{121}{6}, \frac{1757}{30}, \frac{5324}{65}, \frac{3236}{33}, \frac{2039}{14} \right) \end{cases}.
\end{aligned}$$

Now consider the following fully linear Diophantine fuzzy linear programming (FLDFLP) with m constraints and n variables:

$$\text{Max (or Min)} \quad \left\{ \begin{array}{l} (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) \\ (\theta'_1, \theta'_2, \theta_3, \theta'_4, \theta'_5) \end{array} \right. = \sum_{j=1}^n \left(\left\{ \begin{array}{l} (c_j^1, c_j^2, c_j^3, c_j^4, c_j^5) \\ (c_j^{1'}, c_j^{2'}, c_j^{3'}, c_j^{4'}, c_j^{5'}) \end{array} \right\} \otimes \left\{ \begin{array}{l} (x_j^1, x_j^2, x_j^3, x_j^4, x_j^5) \\ (x_j^{1'}, x_j^{2'}, x_j^{3'}, x_j^{4'}, x_j^{5'}) \end{array} \right\} \right)$$

$$\begin{aligned}
\text{subject to} \quad & \sum_{j=1}^n \left(\left\{ \begin{array}{l} (a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4, a_{ij}^5) \\ (a_{ij}^{1'}, a_{ij}^{2'}, a_{ij}^{3'}, a_{ij}^{4'}, a_{ij}^{5'}) \end{array} \right\} \otimes \left\{ \begin{array}{l} (x_j^1, x_j^2, x_j^3, x_j^4, x_j^5) \\ (x_j^{1'}, x_j^{2'}, x_j^{3'}, x_j^{4'}, x_j^{5'}) \end{array} \right\} \right) \{ \leq, \approx, \geq \} \\
& \left\{ \begin{array}{l} (b_i^1, b_i^2, b_i^3, b_i^4, b_i^5) \\ (b_i^{1'}, b_i^{2'}, b_i^{3'}, b_i^{4'}, b_i^{5'}) \end{array} \right.
\end{aligned}$$

$$\left\{ \begin{array}{l} (x_j^1, x_j^2, x_j^3, x_j^4, x_j^5) \\ (x_j^{1'}, x_j^{2'}, x_j^{3'}, x_j^{4'}, x_j^{5'}) \end{array} \right. \geq 0, \quad \forall i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n.$$

$$\text{Maximize (or Minimize) } \tilde{Z} = \sum_{k=1}^n \tilde{c}_k \tilde{x}_k$$

subject to

$$\sum_{k=1}^n \tilde{a}_{jk} \tilde{x}_k \leq \tilde{b}_j, \quad 1 \leq j \leq m;$$

where \tilde{x}_k is unrestricted for some k .

Example 4.3 Let us consider TFLDFLP problem:

$$\begin{aligned} \text{Maximize } \tilde{\theta} &= \tilde{A}\tilde{X} + \tilde{B}\tilde{Y} + \tilde{C}\tilde{Z} \\ \text{Subject to } &\tilde{A}_1\tilde{X} + \tilde{B}_1\tilde{Y} + \tilde{C}_1\tilde{Z} \leq \tilde{D}_1 \\ &\tilde{A}_2\tilde{X} + \tilde{B}_2\tilde{Y} + \tilde{C}_2\tilde{Z} \leq \tilde{D}_2 \\ &\tilde{A}_3\tilde{X} + \tilde{B}_3\tilde{Y} + \tilde{C}_3\tilde{Z} \leq \tilde{D}_3 \\ &\tilde{X} \text{ is unrestricted, } \tilde{Y}, \tilde{Z} \geq 0. \end{aligned}$$

$$\begin{aligned} \tilde{\theta} &= \begin{cases} (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) \\ (\theta'_1, \theta'_2, \theta_3, \theta'_4, \theta'_5) \end{cases} & \tilde{X} &= \begin{cases} (x_1, x_2, x_3, x_4, x_5) \\ (x'_1, x'_2, x_3, x'_4, x'_5) \end{cases} \\ \tilde{Y} &= \begin{cases} (y_1, y_2, y_3, y_4, y_5) \\ (y'_1, y'_2, y_3, y'_4, y'_5) \end{cases} & \tilde{Z} &= \begin{cases} (z_1, z_2, z_3, z_4, z_5) \\ (z'_1, z'_2, z_3, z'_4, z'_5) \end{cases} \end{aligned}$$

$$\tilde{A} = \begin{Bmatrix} (2, 4, 5, 6, 8) \\ (1, 3, 5, 7, 9) \end{Bmatrix}, \quad \tilde{B} = \begin{Bmatrix} (3, 5, 6, 7, 9) \\ (2, 4, 6, 8, 10) \end{Bmatrix}, \quad \tilde{C} = \begin{Bmatrix} (4, 6, 7, 8, 10) \\ (3, 5, 7, 9, 11) \end{Bmatrix},$$

$$\tilde{A}_1 = \begin{Bmatrix} (1, 3, 4, 5, 7) \\ (0, 2, 4, 6, 8) \end{Bmatrix}, \quad \tilde{B}_1 = \begin{Bmatrix} (3, 5, 6, 7, 9) \\ (2, 4, 6, 8, 10) \end{Bmatrix}, \quad \tilde{C}_1 = \begin{Bmatrix} (4, 6, 7, 8, 10) \\ (3, 5, 7, 9, 11) \end{Bmatrix},$$

$$\tilde{A}_2 = \begin{Bmatrix} (3, 5, 6, 7, 9) \\ (2, 4, 6, 8, 10) \end{Bmatrix}, \quad \tilde{B}_2 = \begin{Bmatrix} (2, 4, 5, 6, 8) \\ (1, 3, 5, 7, 9) \end{Bmatrix}, \quad \tilde{C}_2 = \begin{Bmatrix} (3, 5, 6, 7, 9) \\ (2, 4, 6, 8, 10) \end{Bmatrix},$$

$$\tilde{A}_3 = \begin{Bmatrix} (4, 6, 7, 8, 10) \\ (3, 5, 7, 9, 11) \end{Bmatrix}, \quad \tilde{B}_3 = \begin{Bmatrix} (5, 7, 8, 9, 11) \\ (2, 6, 8, 10, 12) \end{Bmatrix}, \quad \tilde{C}_3 = \begin{Bmatrix} (2, 4, 5, 6, 8) \\ (1, 3, 5, 7, 9) \end{Bmatrix},$$

$$\tilde{D}_1 = \begin{Bmatrix} (21, 47, 61, 73, 101) \\ (9, 35, 61, 87, 121) \end{Bmatrix}, \quad \tilde{D}_2 = \begin{Bmatrix} (28, 54, 68, 80, 108) \\ (14, 42, 68, 94, 128) \end{Bmatrix}, \quad \tilde{D}_3 = \begin{Bmatrix} (39, 65, 79, 91, 119) \\ (19, 53, 79, 105, 139) \end{Bmatrix}.$$

Using the arithmetic operations and partial ordering relations we defined TFLDFLP problem as:

$$\begin{aligned} \max \theta_1 &= 2x_1 + 3y_1 + 4z_1, \\ \max \theta_2 &= 4x_2 + 5y_2 + 6z_2, \\ \max \theta_3 &= 5x_3 + 6y_3 + 7z_3, \\ \max \theta_4 &= 6x_4 + 7y_4 + 8z_4, \\ \max \theta_5 &= 8x_5 + 9y_5 + 10z_5, \end{aligned}$$

subject to

$$\begin{aligned} 1x_1 + 3y_1 + 4z_1 &\leq 21, & 3x_1 + 2y_1 + 3z_1 &\leq 28, & 4x_1 + 5y_1 + 2z_1 &\leq 39, \\ 3x_2 + 5y_2 + 6z_2 &\leq 47, & 5x_2 + 4y_2 + 5z_2 &\leq 54, & 6x_2 + 7y_2 + 4z_2 &\leq 65, \\ 4x_3 + 6y_3 + 7z_3 &\leq 61, & 6x_3 + 5y_3 + 6z_3 &\leq 68, & 7x_3 + 8y_3 + 5z_3 &\leq 79, \\ 5x_4 + 7y_4 + 8z_4 &\leq 73, & 7x_4 + 6y_4 + 7z_4 &\leq 80, & 8x_4 + 9y_4 + 6z_4 &\leq 91, \\ 7x_5 + 9y_5 + 10z_5 &\leq 101, & 9x_5 + 8y_5 + 9z_5 &\leq 108, & 10x_5 + 11y_5 + 8z_5 &\leq 119, \end{aligned}$$

$$\begin{aligned}
\theta_1 &\leq \theta_2, & \theta_2 &\leq \theta_3, & \theta_3 &\leq \theta_4, & \theta_4 &\leq \theta_5, \\
x_1 &\leq x_2, & x_2 &\leq x_3, & x_3 &\leq x_4, & x_4 &\leq x_5, \\
y_1 &\leq y_2, & y_2 &\leq y_3, & y_3 &\leq y_4, & y_4 &\leq y_5, \\
z_1 &\leq z_2, & z_2 &\leq z_3, & z_3 &\leq z_4, & z_4 &\leq z_5, \\
x_1, \dots, x_5 & \text{ is unrestricted, } & y_1, \dots, y_5, & z_1, \dots, z_5 & \geq 0.
\end{aligned}$$

Now consider the following LP problem can be written as:

$$\max \theta = \begin{cases} 2x_1 + 3y_1 + 4z_1 \\ +4x_2 + 5y_2 + 6z_2 \\ +5x_3 + 6y_3 + 7z_3 \\ +6x_4 + 7y_4 + 8z_4 \\ +8x_5 + 9y_5 + 10z_5. \end{cases}$$

subject to

$$\begin{aligned}
1x_1 + 3y_1 + 4z_1 &\leq 21, & 3x_1 + 2y_1 + 3z_1 &\leq 28, & 4x_1 + 5y_1 + 2z_1 &\leq 39, \\
3x_2 + 5y_2 + 6z_2 &\leq 47, & 5x_2 + 4y_2 + 5z_2 &\leq 54, & 6x_2 + 7y_2 + 4z_2 &\leq 65, \\
4x_3 + 6y_3 + 7z_3 &\leq 61, & 6x_3 + 5y_3 + 6z_3 &\leq 68, & 7x_3 + 8y_3 + 5z_3 &\leq 79, \\
5x_4 + 7y_4 + 8z_4 &\leq 73, & 7x_4 + 6y_4 + 7z_4 &\leq 80, & 8x_4 + 9y_4 + 6z_4 &\leq 91, \\
7x_5 + 9y_5 + 10z_5 &\leq 101, & 9x_5 + 8y_5 + 9z_5 &\leq 108, & 10x_5 + 11y_5 + 8z_5 &\leq 119,
\end{aligned}$$

$$\begin{aligned}
2x_1 + 3y_1 + 4z_1 - 4x_2 - 5y_2 - 6z_2 &\leq 0, \\
4x_2 + 5y_2 + 6z_2 - 5x_3 - 6y_3 - 7z_3 &\leq 0, \\
5x_3 + 6y_3 + 7z_3 - 6x_4 - 7y_4 - 8z_4 &\leq 0, \\
6x_4 + 7y_4 + 8z_4 - 8x_5 - 9y_5 - 10z_5 &\leq 0,
\end{aligned}$$

$$\begin{aligned}
x_1 - x_2 &\leq 0, & x_2 - x_3 &\leq 0, \\
x_3 - x_4 &\leq 0, & x_4 - x_5 &\leq 0, \\
y_1 - y_2 &\leq 0, & y_2 - y_3 &\leq 0, \\
y_3 - y_4 &\leq 0, & y_4 - y_5 &\leq 0, \\
z_1 - z_2 &\leq 0, & z_2 - z_3 &\leq 0, \\
z_3 - z_4 &\leq 0, & z_4 - z_5 &\leq 0,
\end{aligned}$$

$$x_1, \dots, x_5 \text{ is unrestricted, } y_1, \dots, y_5, z_1, \dots, z_5 \geq 0.$$

Solving the above problem by simplex method, the optimal solution is:

$$\begin{aligned}
(x_1, x_2, x_3, x_4, x_5) &= \left(\frac{40}{7}, \frac{360}{59}, \frac{444}{71}, \frac{520}{83}, \frac{688}{107} \right) \\
(y_1, y_2, y_3, y_4, y_5) &= \left(\frac{17}{7}, \frac{149}{59}, \frac{182}{71}, \frac{213}{83}, \frac{279}{107} \right) \\
(z_1, z_2, z_3, z_4, z_5) &= \left(2, \frac{158}{59}, \frac{209}{71}, \frac{246}{83}, \frac{348}{107} \right) \\
(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) &= \left(\frac{187}{7}, \frac{3133}{59}, \frac{4775}{71}, \frac{6579}{83}, \frac{11495}{107} \right).
\end{aligned}$$

Similarly,

$$\begin{aligned}
\max \theta'_1 &= 1x'_1 + 2y'_1 + 3z'_1, \\
\max \theta'_2 &= 3x'_2 + 4y'_2 + 5z'_2, \\
\max \theta'_3 &= 5x'_3 + 6y'_3 + 7z'_3, \\
\max \theta'_4 &= 7x'_4 + 8y'_4 + 9z'_4, \\
\max \theta'_5 &= 9x'_5 + 10y'_5 + 11z'_5,
\end{aligned}$$

subject to

$$\begin{aligned}
0x'_1 + 2y'_1 + 3z'_1 &\leq 9, & 2x'_1 + 1y'_1 + 2z'_1 &\leq 14, & 3x'_1 + 2y'_1 + 1z'_1 &\leq 19, \\
2x'_2 + 4y'_2 + 5z'_2 &\leq 35, & 4x'_2 + 3y'_2 + 4z'_2 &\leq 42, & 5x'_2 + 6y'_2 + 3z'_2 &\leq 53, \\
4x_3 + 6y_3 + 7z_3 &\leq 61, & 6x_3 + 5y_3 + 6z_3 &\leq 68, & 7x_3 + 8y_3 + 5z_3 &\leq 79, \\
6x'_4 + 8y'_4 + 9z'_4 &\leq 87, & 8x'_4 + 7y'_4 + 8z'_4 &\leq 94, & 9x'_4 + 10y'_4 + 7z'_4 &\leq 105,
\end{aligned}$$

$$8x'_5 + 10y'_5 + 11z'_5 \leq 121, \quad 10x'_5 + 9y'_5 + 10z'_5 \leq 128, \quad 11x'_5 + 12y'_5 + 9z'_5 \leq 139,$$

$$\begin{aligned} \theta'_1 &\leq \theta'_2, \quad \theta'_2 \leq \theta_3, \quad \theta_3 \leq \theta'_4, \quad \theta'_4 \leq \theta'_5, \\ x'_1 &\leq x'_2, \quad x'_2 \leq x_3, \quad x_3 \leq x'_4, \quad x'_4 \leq x'_5, \\ y'_1 &\leq y'_2, \quad y'_2 \leq y_3, \quad y_3 \leq y'_4, \quad y'_4 \leq y'_5, \\ z'_1 &\leq z'_2, \quad z'_2 \leq z_3, \quad z_3 \leq z'_4, \quad z'_4 \leq z'_5, \\ x'_1, \dots, x'_5 &\text{ is unrestricted, } y'_1, \dots, y'_5, z'_1, \dots, z'_5 \geq 0. \end{aligned}$$

Now consider the following LP problem can be written as:

$$\max \theta' = \begin{cases} 1x'_1 + 2y'_1 + 3z'_1 \\ +3x'_2 + 4y'_2 + 5z'_2 \\ +5x_3 + 6y_3 + 7z_3 \\ +7x'_4 + 8y'_4 + 9z'_4 \\ +9x'_5 + 10y'_5 + 11z'_5. \end{cases}$$

subject to

$$\begin{aligned} 0x'_1 + 2y'_1 + 3z'_1 &\leq 9, \quad 2x'_1 + 1y'_1 + 2z'_1 \leq 14, \quad 3x'_1 + 2y'_1 + 1z'_1 \leq 19, \\ 2x'_2 + 4y'_2 + 5z'_2 &\leq 35, \quad 4x'_2 + 3y'_2 + 4z'_2 \leq 42, \quad 5x'_2 + 6y'_2 + 3z'_2 \leq 53, \\ 4x_3 + 6y_3 + 7z_3 &\leq 61, \quad 6x_3 + 5y_3 + 6z_3 \leq 68, \quad 7x_3 + 8y_3 + 5z_3 \leq 79, \\ 6x'_4 + 8y'_4 + 9z'_4 &\leq 87, \quad 8x'_4 + 7y'_4 + 8z'_4 \leq 94, \quad 9x'_4 + 10y'_4 + 7z'_4 \leq 105, \\ 8x'_5 + 10y'_5 + 11z'_5 &\leq 121, \quad 10x'_5 + 9y'_5 + 10z'_5 \leq 128, \quad 11x'_5 + 12y'_5 + 9z'_5 \leq 139, \end{aligned}$$

$$\begin{aligned} 1x'_1 + 2y'_1 + 3z'_1 - 3x'_2 - 4y'_2 - 5z'_2, \\ 3x'_2 + 4y'_2 + 5z'_2 - 5x_3 - 6y_3 - 7z_3, \\ 5x_3 + 6y_3 + 7z_3 - 7x'_4 - 8y'_4 - 9z'_4, \\ 7x'_4 + 8y'_4 + 9z'_4 - 9x'_5 - 10y'_5 - 11z'_5, \end{aligned}$$

$$\begin{aligned} x'_1 - x'_2 &\leq 0, \quad x'_2 - x_3 \leq 0, \\ x_3 - x'_4 &\leq 0, \quad x'_4 - x'_5 \leq 0, \\ y'_1 - y'_2 &\leq 0, \quad y'_2 - y_3 \leq 0, \\ y_3 - y'_4 &\leq 0, \quad y'_4 - y'_5 \leq 0, \\ z'_1 - z'_2 &\leq 0, \quad z'_2 - z_3 \leq 0, \\ z_3 - z'_4 &\leq 0, \quad z'_4 - z'_5 \leq 0, \end{aligned}$$

$$x'_1, \dots, x'_5 \text{ is unrestricted, } y'_1, \dots, y'_5, z'_1, \dots, z'_5 \geq 0.$$

Solving the above problem by simplex method, the optimal solution is:

$$\begin{aligned} (x'_1, x'_2, x_3, x'_4, x'_5) &= \left(\frac{48}{11}, \frac{284}{47}, \frac{444}{71}, \frac{604}{95}, \frac{796}{119} \right) \\ (y'_1, y'_2, y_3, y'_4, y'_5) &= \left(\frac{24}{11}, \frac{118}{47}, \frac{182}{71}, \frac{246}{95}, \frac{318}{119} \right) \\ (z'_1, z'_2, z_3, z'_4, z'_5) &= \left(\frac{17}{11}, \frac{121}{47}, \frac{209}{71}, \frac{297}{95}, \frac{63}{17} \right) \\ (\theta'_1, \theta'_2, \theta_3, \theta'_4, \theta'_5) &= \left(\frac{147}{11}, \frac{1929}{47}, \frac{4775}{71}, \frac{8869}{95}, \frac{15195}{119} \right). \end{aligned}$$

Hence the optimal solution of above TFLDFLP problem is:

$$\begin{aligned} \begin{cases} (x_1, x_2, x_3, x_4, x_5) \\ (x'_1, x'_2, x_3, x'_4, x'_5) \end{cases} &= \begin{cases} \left(\frac{40}{7}, \frac{360}{59}, \frac{444}{71}, \frac{520}{83}, \frac{688}{107} \right) \\ \left(\frac{48}{11}, \frac{284}{47}, \frac{444}{71}, \frac{604}{95}, \frac{796}{119} \right) \end{cases} \\ \begin{cases} (y_1, y_2, y_3, y_4, y_5) \\ (y'_1, y'_2, y_3, y'_4, y'_5) \end{cases} &= \begin{cases} \left(\frac{17}{7}, \frac{149}{59}, \frac{182}{71}, \frac{213}{83}, \frac{279}{107} \right) \\ \left(\frac{24}{11}, \frac{118}{47}, \frac{182}{71}, \frac{246}{95}, \frac{318}{119} \right) \end{cases} \\ \begin{cases} (z_1, z_2, z_3, z_4, z_5) \\ (z'_1, z'_2, z_3, z'_4, z'_5) \end{cases} &= \begin{cases} \left(2, \frac{158}{59}, \frac{209}{71}, \frac{246}{83}, \frac{348}{107} \right) \\ \left(\frac{17}{11}, \frac{121}{47}, \frac{209}{71}, \frac{297}{95}, \frac{63}{17} \right) \end{cases} \\ \begin{cases} (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) \\ (\theta'_1, \theta'_2, \theta_3, \theta'_4, \theta'_5) \end{cases} &= \begin{cases} \left(\frac{187}{7}, \frac{3133}{59}, \frac{4775}{71}, \frac{6579}{83}, \frac{11495}{107} \right) \\ \left(\frac{147}{11}, \frac{1929}{47}, \frac{4775}{71}, \frac{8869}{95}, \frac{15195}{119} \right) \end{cases}. \end{aligned}$$

5. Conclusion

This study proposes a simple, straightforward, and easy-to-apply method for determining the value of the decision variables in LDFLP issues that uses non-negative LDF numbers. The linear programming problems with LDF parameters that occur in real-world scenarios can be simply determined by the use of a separable and bound approach.

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