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Note on the Nonlinear Stability of Inviscid Swirling Flows

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ABSTRACT: In this work, we investigate the nonlinear stability of an inviscid, incompressible, homogeneous fluid in an annular region bounded by two coaxial cylinders. Using Arnold's method, we derive two general stability theorems for steady basic flows subjected to two-dimensional normal-mode disturbances. The theoretical findings are illustrated with representative examples of swirling flows.

Keywords: Hydrodynamic stability, nonlinear stability, two-dimensional disturbances, swirling flows.

Contents

1. Introduction

Hydrodynamic stability theory examines the conditions under which laminar flows become unstable, how this instability develops, and how the flow ultimately transitions to turbulence. Analyzing a stability problem typically requires investigating the behavior of solutions to nonlinear partial differential equations. The linear stability of inviscid homogeneous swirling flows with respect to two-dimensional disturbances is governed by

$$(\Omega - c)\left(D_*D - \frac{m^2}{r^2}\right)\phi \;-\; \frac{(DZ)}{r}\,\phi = 0, \tag{1.1}$$

where Ω is the angular velocity, $\phi(r)e^{im(\theta-ct)}$ is the perturbed variable, $D = \dfrac{d}{dr}$ is the derivative with respect to the radial variable r, and $D_* = D + \frac{1}{r}$. This equation is similar to that of the Rayleigh equation for parallel flows. When the flow is in the annular region satisfying the condition at the boundary walls as $\phi = 0$ at $r = R_1$, and $r = R_2$, Rayleigh [1] derived the necessary condition for instability with respect to two-dimensional disturbances as gradient of the vorticity $DZ = rD^2\Omega + 3D\Omega$, must change sign at least once in the flow domain. This result is similar to that of the Rayleigh inflexion point theorem in parallel flow theory. The normal modes are unstable when $mc_i > 0$, where m is the azimuthal wave number and c_i the imaginary part of the complex phase speed. Lalas [2] showed that complex wave velocity of unstable modes lie on the semicircular region with center $(\frac{\Omega_{min}+\Omega_{max}}{2}, 0)$, and radius $(\frac{\Omega_{max}-\Omega_{min}}{2})$. This problem was further taken into consideration by Subbiah [3]. Subbiah [3] proved the Howards conjecture on this problem showing that $mc_i \to 0$ as azimuthal wave number $m \to \infty$. In the case of Couette flow, where $DZ \equiv 0$, it is well established that the spectrum contains no discrete modes. Instead, the system exhibits a continuous spectrum consisting of stable singular modes for which $\Omega - c$ becomes zero at some point within the interval $R_1 < r < R_2$. In the case of a particular swirling flow, the Rankine vortex, these continuous spectrum solutions have been examined extensively in the work of Roy and Subramanian [4]. Furthermore, the stability characteristics of inviscid, incompressible, homogeneous swirling flows with velocity field $(0, V(r), 0)$, under two-dimensional disturbances that extend beyond classical normal-mode perturbations, have been addressed in the studies of Padmini and Subbiah [5] by following the work of

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Barston [6]. Their results show that the basic flow remains stable as long as (DZ) does not change sign in this general case as well. For a neutral mode, the phase speed c is real, and the layer $r = r_c$ satisfying $\Omega(r_c) = c$ is known as the critical layer; it represents a singular point of equation (1). This singularity can be removed either by introducing viscosity or by considering nonlinear effects.

The inclusion of viscosity into the analysis changes the equation as

$$\frac{\nu}{im}\left(D_*D - \frac{m^2}{r^2}\right)^2 \phi = (\Omega - c)\left(D_*D - \frac{m^2}{r^2}\right)\phi - \frac{(DZ)}{r}\phi, \tag{1.2}$$

(cf. Drazin and Reid [7], page (103)), with boundary conditions,

$$\phi = 0 \quad \text{and} \quad D\phi = 0 \quad \text{at} \quad r = R_1, R_2. \tag{1.3}$$

In the case of Couette flow, the relation $DZ = rD^2\Omega + 3D\Omega = 0$ holds, and the structure of equation (1.2) becomes very similar to the equation that governs the stability of plane Couette flow. Since plane Couette flow is known to be stable, Drazin and Reid [7] suggested that circular Couette flow should also be stable with respect to two-dimensional disturbances. This conjecture was later verified by Ali and Herron [8], who demonstrated that viscous circular Couette flow is indeed stable under such disturbances.

All the previously discussed results on the stability of swirling flows in an annular region apply only to infinitesimal normal-mode disturbances and are based on linear analysis. However, to properly examine the stability of swirling motion in an annulus when the disturbances have finite amplitude, it is necessary to develop a nonlinear stability framework. The non-linear stability of inviscid incompressible parallel flows was examined by Arnold [9]. The development of methods for nonlinear stability analysis of inviscid incompressible flows progressed through the work of Arnold. An initial variational technique was later superseded by a more robust method introduced in Arnold [10], which successfully provided rigorous stability theorems. Known as Arnold's second method, this framework has become a fundamental tool, extensively employed in studying the stability of geophysical flows, as seen in the works of Arnold and Khesin [11], Marchioro and Pulvirenti [12], and Holm et al. [13].

Here, we consider the non-linear stability of inviscid incompressible swirling flows and obtain some general analytical results on the stability of basic flows. In Section 2, we formulate the problem by deriving the nonlinear partial differential equation that governs the motion of an inviscid, incompressible, homogeneous fluid within the annular region between two coaxial cylinders, along with the associated boundary conditions. We also derive a conserved quantity for this system. This invariant is subsequently employed to establish the nonlinear stability results. The nonlinear stability conditions for steady flows depend on the relationship between the stream function $\Psi(r, \theta)$ and the vorticity measure $Q(r, \theta)$. Stability against zero-circulation perturbations is assured if $\Psi'(Q) > 0$, or if $\Psi'(Q) < 0$ within a confined radial domain. This yields a useful corollary for axisymmetric, azimuthal flows: a monotonic radial vorticity profile, signified by $Q'(r)$ being either strictly positive or negative, is a sufficient condition for stability. The obtained results are illustrated with examples.

2. Formulation of the Problem

Let (r, θ, z) be the cylindrical polar coordinate system. Consider the inviscid fluid between the annular region $0 < R_1 < R_2 < \infty$ with velocity (u_r, u_θ, u_z), pressure p. We consider the two-dimensional flow, that is flow with velocity $\vec{u} = (u_r, u_\theta, 0)$, $\frac{\partial \vec{u}}{\partial z} = 0$.
The governing equations are

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} = -\frac{\partial p}{\partial r}, \tag{2.1}$$

$$\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} = -\frac{1}{r}\frac{\partial p}{\partial \theta}, \tag{2.2}$$

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} = 0. \tag{2.3}$$

Now, we introduce a stream function $\psi(r, \theta, t)$

$$r u_r = -\frac{\partial \psi}{\partial \theta}, \tag{2.4}$$

$$u_\theta = \frac{\partial \psi}{\partial r}. \tag{2.5}$$

This automatically satisfies the continuity equation (2.3).

$$\frac{D}{Dt}(\zeta) = 0, \tag{2.6}$$

where $\zeta = \frac{1}{r}\left[\frac{\partial}{\partial r}(r u_\theta) - \frac{\partial}{\partial r}(u_\theta)\right] = \Delta\psi$ is the scalar vorticity, and $\frac{D}{Dt} = \frac{\partial}{\partial t} + u_r \frac{\partial}{\partial r} + \frac{u_\theta}{r}\frac{\partial}{\partial \theta}$ is the Eulerian derivative of the fluid flow.
In terms of the stream function ψ the equation (2.6) becomes

$$\frac{D}{Dt}\left(\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{\partial \psi}{\partial \theta}\right)\right]\right) = 0,$$

$$\frac{D}{Dt}\left[\frac{1}{r}\left(\frac{\partial \psi}{\partial r} + r\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial^2 \psi}{\partial \theta^2}\right)\right] = 0,$$

i.e.,

$$\frac{D}{Dt}\Delta\psi = 0, \tag{2.7}$$

where $\Delta\psi = \frac{1}{r}\left[\frac{\partial \psi}{\partial r} + r\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial^2 \psi}{\partial \theta^2}\right]$.
From (2.1), (2.2), and (2.3), one obtains the conservation of kinetic energy,

$$\frac{d}{dt}\iint \left[\frac{u_r^2 + u_\theta^2}{2}\right] r\, dr\, d\theta = 0.$$

In terms of the stream function, this can be stated as

$$\frac{d}{dt}\iint \left[\frac{\frac{1}{r^2}\left(\frac{\partial \psi}{\partial \theta}\right)^2 + \left(\frac{\partial \psi}{\partial r}\right)^2}{2}\right] r\, dr\, d\theta = 0. \tag{2.8}$$

The statement of the conservation of kinetic energy given in equation (2.8) is given by

$$\frac{d}{dt}\iint_D \frac{\|\nabla \psi\|^2}{2}\, r\, dr\, d\theta = 0.$$

Consider a function $\Phi(\Delta\psi)$

$$\begin{aligned}
\frac{d}{dt}\iint_D \Phi(\Delta\psi)\, r\, dr\, d\theta &= \iint_D \left[\frac{D}{Dt}\Phi(\Delta\psi) + \Phi(\Delta\psi)\,(\nabla . u)\right] r\, dr\, d\theta \\
&= \iint_D \left[\Phi'(\Delta\psi)\frac{D}{Dt}(\Delta\psi)\right] r\, dr\, d\theta \\
&= 0.
\end{aligned}$$

Equation (2.6) can be written in the form

$$\frac{\partial \Delta\psi}{\partial t} + J(\Delta\psi, \psi) = 0, \tag{2.9}$$

where $J(\Delta\psi, \psi)$ is the Jacobian.

We denote by $Q(r, \theta)$ the steady basic-state vorticity and by $q(r, \theta, t)$ the total (time-dependent) vorticity field. Thus, capital letters refer to steady basic-state quantities, whereas lowercase letters denote perturbation or time-dependent quantities.

We describe the stability problem. Consider a steady, incompressible flow in a circular domain, represented by the stream function $\Psi = \Psi(r, \theta)$. The corresponding vorticity field is $Q(r, \theta) = \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\,\frac{\partial \Psi}{\partial r}\right) + \frac{1}{r}\frac{\partial^2 \Psi}{\partial \theta^2}\right]$. For a steady Euler flow, Ψ and Q satisfy the relation $J(Q, \Psi) = 0$, Whenever $\nabla Q \neq 0$, this implies that Ψ can be regarded as a function of Q: $\Psi = \Psi(Q)$. Suppose that a disturbance is introduced so that the perturbed fields become $\Psi \to \Psi + \phi(r, \theta, t),\ Q \to Q + q(r, \theta, t)$. We aim to show that the basic flow (Ψ, Q) is nonlinearly stable to all disturbances that satisfy the zero-circulation condition on the boundary.

3. Stability Results

Lemma 3.1 Consider the steady basic flow with stream function. For any flow, the quantity

$$H(\psi(r, \theta, t)) = \iint_D \left[\frac{\|\nabla\psi\|^2}{2} + \Phi(\Delta\psi)\right] r\,dr\,d\theta.$$

is conserved for perturbations satisfying the zero-circulation condition on ∂D,

Suppose the disturbed flow is given by $\Psi \to \Psi + \phi$, $Q \to Q + q$. Then

$$H(\Psi + \phi) = \iint_D \left[\frac{\|\nabla(\Psi + \phi)\|^2}{2} + \Phi(Q + q)\right] r\,dr\,d\theta.$$

Hence, the perturbation energy becomes

$$\begin{aligned} H(\phi) &= H(\psi + \phi) - H(\psi) \\ &= \iint_D \left[\frac{(\|\nabla\phi\|)^2}{2} + \Phi(Q + q) - \Phi(Q) - \Phi'(Q)\,q\right] r\,dr\,d\theta \end{aligned}$$

Thus,

$$H(\phi(r, \theta, t) = H(\phi(r, \theta, 0)).$$

for perturbations satisfying the zero-circulation boundary condition.
Now we shall present the first nonlinear stability theorem.

Theorem 3.1 (First Stability Theorem) If the basic flow with stream function Ψ and vorticity-per-width function Q satisfies

$$0 < c \leq \Psi'(Q) \leq C < \infty,$$

for real constants c and C, then the flow is stable in the norm $\|\ \|_+$ given by

$$\|\phi\|_+^2 = \iint_D \left[\frac{(\|\nabla\phi\|)^2}{2} + c\,q^2\right] r\,dr\,d\theta.$$

Proof: Define

$$\Phi(\eta) = \int_0^\eta \Psi(s)\,ds,\ \textit{where}\ Q_{min} \leq \eta \leq Q_{max}.$$

By hypothesis of the theorem we have

$$0 < c \leq \Phi''(\eta) \leq C < \infty, \tag{3.1}$$

in the range of $Q_{min} \leq \eta \leq Q_{max}$.
integrating (3.1) over $(\xi, \xi + h_1)$

$$ch_1 \leq [\Phi'(\xi + h_1) - \Phi'(\xi) \leq Ch_1]. \tag{3.2}$$

Integrating (3.2) w.r.to h_1 over $(0, h)$

$$\frac{ch^2}{2} \leq [\Phi(\xi + h) - \Phi(\xi) - \Phi'(\xi)h] \leq \frac{Ch^2}{2}.$$

So we have

$$\frac{cq^2}{2} \leq [\Phi(Q + q) - \Phi(Q) - \Phi'(Q)q] \leq \frac{Cq^2}{2}. \tag{3.3}$$

Consequently, adding $\frac{\|\nabla\phi\|^2}{2}$ and integrating over the flow domain D, we obtain

$$\begin{aligned}
\iint_D \left[\frac{\|\nabla\phi\|^2}{2} + \frac{cq^2}{2}\right] r\,dr\,d\theta &\leq \iint_D \left[\frac{\|\nabla\phi\|^2}{2} + \big(\Phi(Q+q) - \Phi(Q) - \Phi'(Q)q\big)\right] r\,dr\,d\theta \qquad \text{(by (3.3))} \\
&= H(\phi) = H(\phi_0) \qquad \text{(by Lemma 3.1)} \\
&\leq \iint_D \left[\frac{\|\nabla\phi_0\|^2}{2} + \big(\Phi(Q+q_0) - \Phi(Q) - \Phi'(Q)q_0\big)\right] r\,dr\,d\theta \\
&\leq \iint_D \left[\frac{\|\nabla\phi_0\|^2}{2} + \frac{Cq_0^2}{2}\right] r\,dr\,d\theta \qquad \text{(by (3.3))} \\
&\leq \frac{C}{c} \times \frac{c}{C} \iint_D \left[\frac{\|\nabla\phi_0\|^2}{2} + \frac{Cq_0^2}{2}\right] r\,dr\,d\theta \\
&\leq \frac{C}{c} \iint_D \left[\|\nabla\phi_0\|^2 + c\,q_0^2\right] r\,dr\,d\theta \\
&\leq \frac{C}{c}\,\|\phi_0\|_+^2.
\end{aligned}$$

Given $\varepsilon > 0$, choose $\delta(\varepsilon)$ defined by $\delta^2 = \frac{c}{C}\,\varepsilon^2$. So that $\|\phi_0\|_+^2 < \delta^2$ implies $\|\phi\|_+^2 \leq \varepsilon^2$.
The proof of the theorem is completed. □

Example 3.1 Consider the basic flow with velocity $U_0(r) = \frac{1}{r} + 0.2,\ 1 \leq r \leq 2$, and $\Psi'(Q) = \frac{d\psi}{dQ} > 0$. It follows that this basic flow is stable by the first stability theorem for any flow domain.

Now, we shall derive two corollaries of this theorem for swirling flows.

Corollary 3.1 If the basic flow is such that $\Psi = \Psi(r)$ and $Q = Q(r)$ with $Q'(r) \neq 0$, then under the conditions $0 < c \leq \Psi'(Q) \leq C < \infty$, we have the following bounds.

$$\iint_D \left[(\nabla\phi)^2 + \mu q^2\right] r\,dr\,d\theta \ \leq\ \frac{\left(\frac{\Psi'(r)}{Q'(r)}\right)_{\max}}{\left(\frac{\Psi'(r)}{Q'(r)}\right)_{\min}} \iint_D \left[(\nabla\phi_0)^2 + \mu q_0^2\right] r\,dr\,d\theta,$$

For any μ in the range $\left(\frac{\Psi'(r)}{Q'(r)}\right)_{\min} \leq \mu \leq \left(\frac{\Psi'(r)}{Q'(r)}\right)_{\max}$

Proof: Let $c = \left(\frac{\Psi'(r)}{Q'(r)}\right)_{\min}$ and $C = \left(\frac{\Psi'(r)}{Q'(r)}\right)_{\max}$ then under the condition $0 < c \leq \Psi'(Q) \leq C < \infty$, we have inequalities

$$\iint_D \left[(\nabla\phi)^2 + \mu q^2\right] r\,dr\,d\theta \ \leq\ \iint_D \left[(\nabla\phi_0)^2 + \mu q_0^2\right] r\,dr\,d\theta,$$

is true. Therefore, for any μ in the range $c \le \mu \le C$, we have

$$\begin{aligned}\iint_D \left[(\nabla\phi)^2 + \mu q^2\right] \, r\,dr\,d\theta &\le \frac{\mu}{c}\iint_D \left[(\nabla\phi)^2 + cq^2\right] \, r\,dr\,d\theta \\ &\le \frac{\mu}{c}\iint_D \left[(\nabla\phi_0)^2 + Cq_0^2\right] \, r\,dr\,d\theta \\ &\le \frac{C}{c}\iint_D \left[(\nabla\phi)^2 + \mu q_0^2\right] \, r\,dr\,d\theta.\end{aligned}$$

Hence, the result proved. □

Corollary 3.2 If the basic flow is swirling flow with $\Psi = \Psi(r)$ and $Q = Q(r)$ with monotonic, Q then under the conditions $0 < c \le \Psi'(Q) \le C < \infty$, we have the following bounds:

$$\iint_D q^2\, r\,dr\,d\theta \le \frac{|Q'(r)|_{\max}}{|Q'(r)|_{\min}}\iint_D q_0^2\, r\,dr\,d\theta.$$

Example 3.2 Consider the basic flow with velocity $U_0 = 3 - r,\ 1 \le r \le 2$, and $U_0(1) = 2,\ U_0(2) = 1$ and $Q'(r) = \frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(rU_0)\right] = -\frac{3}{r^2} < 0\ \forall\ r \in [1,2]$ and $\Psi'(r) = \frac{\frac{d\psi}{dr}}{\frac{dQ}{dr}} = \frac{r^2}{3}(3-r) > 0$ in $1 \le r \le 2$, this flow is stable by corollary (3.2).
For the flows we have $Q = \frac{1}{r}\left[\frac{d}{dr}(rU_0)\right]$. For monotonic Q we have $Q'(r) > 0$ or $Q'(r) < 0$ throughout the domain. This means that $Q'(r) = \frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(rU_0)\right]$ is positive throughout or negative throughout. Hence, by corollary 3.2, it follows that the flow is nonlinearly stable $\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(rU_0)\right] < 0$ or > 0, throughout the flow domain.

Lemma 3.2 If $\psi(r,\theta,t)$ is a function vanishing on the boundary of the domain $D : 0 < R_1 < r < R_2 < \infty$, and $D > 0$ are constants, then

$$\iint_D (\Delta\psi)^2\, r\,dx\,d\theta \ \ge\ \lambda_1\iint_D (\nabla\psi)^2\, r\,dr\,d\theta,$$

where λ_1 is the smallest Eigen value of $-\Delta\phi = \lambda_1\Phi$.

Lemma 3.3

$$\iint_D q^2\, r\,dr\,d\theta \ \ge\ \lambda_1\iint_D (\nabla\phi)^2\, r\,dr\,d\theta,$$

where ϕ vanishes in the flow domain.

Theorem 3.2 (Second Stability Theorem) If the basic flow with stream function Ψ and vorticity divided by width function Q satisfies

$$0 < c \le -\Psi'(Q) \le C < \infty,$$

then the basic flow is nonlinearly stable in the norm given by

$$\|\phi\|_-^2 = \iint_D q^2\, r\,dr\,d\theta$$

whenever the domain is sufficiently small.

Proof: Instead of (3.1) we have $0 < c \le -\Phi''(Q) \le C < \infty$.
Proceeding as in the proof of Theorem 3.1, we have

$$\frac{ch^2}{2} \le -\left[\Phi(\xi + h) - \Phi(\xi) - \Phi'(\xi)h\right] \le \frac{Ch^2}{2}.$$

Hence we have

$$\begin{aligned}
\iint_D \left[\frac{-\|\nabla\phi\|^2}{2} + \frac{cq^2}{2}\right] r\,dr\,d\theta &\le \iint_D \left[\frac{-\|\nabla\phi\|^2}{2} - \big(\Phi(Q+q) - \Phi(Q) - \Phi'(Q)q\big)\right] r\,dr\,d\theta \\
&= -H(\phi) = -H(\phi_0) \\
&\le \iint_D \left[\frac{-\|\nabla\phi_0\|^2}{2} - \big(\Phi(Q+q_0) - \Phi(Q) - \Phi'(Q)q_0\big)\right] r\,dr\,d\theta \\
&\le \iint_D \left[\frac{-\|\nabla\phi_0\|^2}{2} + \frac{Cq_0^2}{2}\right] r\,dr\,d\theta.
\end{aligned}$$

Consequently, we have

$$\iint_D \left[\frac{-\|\nabla\phi\|^2}{2} + \frac{cq^2}{2}\right] r\,dr\,d\theta \le C\iint_D \left[\frac{q_0^2}{2}\right] r\,dr\,d\theta. \tag{3.4}$$

By Poincaré's inequality, we obtain

$$\iint_D q^2\, r\,dr\,d\theta \;\ge\; \lambda_1 \iint_D (\nabla\phi)^2\, r\,dr\,d\theta.$$

Using this in (3.4), gives

$$\left(c - \frac{1}{\lambda_1}\right)\iint_D q^2\, r\,dr\,d\theta \le C\iint_D q_0^2\, r\,dr\,d\theta.$$

Now, if $c - \frac{1}{\lambda_1} > 0$, then

$$\iint_D q^2\, r\,dr\,d\theta \le \frac{C}{c - \dfrac{1}{\lambda_1}}\iint_D q_0^2\, r\,dr\,d\theta,$$

$$\text{i.e.,}\qquad \|\phi\|_-^2 \le \frac{C}{c - \dfrac{1}{\lambda_1}}\|\phi_0\|_-^2.$$

Given $\varepsilon > 0$, $\exists\;\; \delta(\varepsilon) > 0$ such that $\delta^2 = \frac{c - \frac{1}{\lambda_1}}{C}\,\varepsilon^2$, so that the basic flow is stable in the norm $\|\;\|_-$. □

Example 3.3 Consider a basic flow with velocity $U_0 = r^2$ and $\Psi'(r) = -\frac{r^2}{2} < 0$ where, $c = (-\Psi'(Q))_{\min} = \frac{1}{3}$, $C = (-\Psi'(Q))_{\max} = \frac{4}{3}$. So, the flow domain lies in the range, that is, $0.333 \le -\Psi'(Q) \le 1.333$. By the second stability theorem 3.2 the flow is stable.

4. Concluding Remarks

The nonlinear stability of inviscid swirling flows is examined in the present work. Using the first stability theorem, it is shown that if $\Psi(r,\theta)$denotes the stream function and $Q(r,\theta)$ the vorticity, then flows satisfying the condition $\Psi'(Q) > 0$ are stable. The second stability theorem establishes the stability of basic flows satisfying the condition $\Psi'(Q) < 0$. For the special case in which $\Psi = \Psi(r)$ and $Q = Q(r)$, the nonlinear stability of the flow is ensured when $Q(r)$ is a monotonic function of r. The obtained results are illustrated through suitable examples.

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