Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2026 (44) 12** : 1–16.
ISSN-0037-8712
doi:10.5269/bspm.84528

An Approach to Transportation Problems Using Intuitionistic Quadripartitioned Neutrosophic Sets

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ABSTRACT: In today's world, unpredictable economic and environmental factors make it difficult to estimate supply, demand, and transportation costs accurately. This study focuses on solving transportation problems where such uncertain data is expressed using intuitionistic quadripartitioned neutrosophic numbers (IQPNNs). These numbers offer a better way to handle complex uncertainty compared to traditional fuzzy numbers, which often fall short in dealing with high levels of indeterminacy and contradiction. To address this, we use intuitionistic quadripartitioned neutrosophic sets (IQPNSs), which provide a more powerful mathematical approach to manage unclear and conflicting information. A new algorithm is proposed to find the best transportation plan using operations on IQPNNs. A practical example is provided to show how the method works and to compare it with existing techniques. The results show that our IQPNS-based model improves accuracy, adaptability, and dependability in uncertain transportation situations. This work also opens up new possibilities for future research on improving transportation systems in uncertain environments.

Keywords: Fuzzy set, intuitionistic set, neutrosophic set, quadri-partitioned neutrosophic set.

Contents

2020 *Mathematics Subject Classification:* 03E72, 03F55, 03E75.
Submitted April 04, 2026. Published June 09, 2026.

1. Introduction

Fuzzy logic and its extensions have played a pivotal role in enhancing decision-making processes across various transportation modes—land, maritime, and aerial. These developments have addressed vital issues such as cost optimization, risk mitigation, environmental sustainability, and operational efficiency. Researchers have increasingly focused on integrating uncertainty into core transportation parameters like cost, supply, and demand to better reflect real-world dynamics.

Muthunandhini and Palanivel [14] combined fuzzy risk analysis with Multi-Criteria Decision Making (MCDM) techniques for optimizing multimodal transportation systems. Kokila and Deepa [12] employed triangular and interval-valued trapezoidal fuzzy numbers to handle parameter uncertainties. For fully fuzzy transportation problems, heuristic and metaheuristic techniques such as Particle Swarm Optimization (PSO) [5], Genetic Algorithms (GA), and Adaptive Neuro-Fuzzy Inference Systems (ANFIS) [2] have been proposed. Singh et al. [15] emphasized the use of defuzzification and ranking functions to transform fuzzy models into crisp equivalents suitable for optimization.

More recent developments include the use of advanced fuzzy set extensions. Gahlawat et al. [17] introduced Pythagorean fuzzy frameworks, while Zulqarnain et al. [18] applied interval-valued q-rung orthopair fuzzy soft sets (IVq-ROFSS) for transportation-related decision-making. Anjum et al. [3] incorporated T-spherical fuzzy sets for evaluating cooperative intelligent transportation systems (C-ITS). These innovations reflect a growing trend of integrating fuzzy logic with AI, soft computing, and hybrid optimization methods.

Despite these contributions, limited attention has been given to neutrosophic set theory particularly quadri-partitioned neutrosophic numbers which can model higher degrees of uncertainty and inconsistency. Traditional fuzzy and intuitionistic frameworks often fail to capture the full spectrum of vagueness and contradiction inherent in dynamic transportation systems.

This study aims to bridge that gap by introducing a novel methodology that incorporates intuitionistic quadri-partitioned neutrosophic numbers into the transportation problem framework. These numbers extend the expressive power of uncertainty modeling by decomposing each value into four components. The proposed model uses score and accuracy functions to convert the neutrosophic values into a crisp linear programming structure. This enables the application of standard optimization tools, such as Excel Solver, to real-life logistics and supply chain scenarios.

The primary objectives of this research are: 1) To enrich classical transportation models with an intuitionistic quadri-partitioned neutrosophic framework that captures deeper uncertainty in parameters. 2) To formulate and solve transportation problems-balanced and unbalanced-under this framework using crisp optimization techniques. 3) To demonstrate the applicability of the proposed method through a practical numerical illustration.

The motivation lies in addressing the inadequacies of existing models when dealing with fluctuating transport costs, unpredictable demands, and volatile supply chains—commonplace in modern transportation networks. This contribution sets the stage for future work in neutrosophic modeling and decision-making under extreme uncertainty.

2. Preliminaries

Definition 2.1 [19] Let X be a non-empty universe. A *fuzzy set* A^* on X is defined by assigning to each element $x \in X$ a membership value $\mu_{A^*}(x) \in [0,1]$. It is represented as:

$$A^* = \{(x, \mu_{A^*}(x)) \mid x \in X\},$$

where $\mu_{A^*}(x)$ indicates the degree to which x belongs to the set A^*. A value of 1 denotes full membership, 0 implies no membership, and any value between 0 and 1 reflects partial association.

Definition 2.2 [6] An *intuitionistic fuzzy set* (IFS) $A^{\#}$ over the universe X is defined as:

$$A^{\#} = \{(x, \mu_{A^{\#}}(x), \nu_{A^{\#}}(x)) \mid x \in X\},$$

where $\mu_{A^{\#}}(x) \in [0,1]$ and $\nu_{A^{\#}}(x) \in [0,1]$ are the membership and non-membership degrees, respectively, satisfying the condition:

$$0 \leq \mu_{A^{\#}}(x) + \nu_{A^{\#}}(x) \leq 1.$$

The hesitation or uncertainty degree is given by:

$$\delta_{A^\#}(x) = 1 - \mu_{A^\#}(x) - \nu_{A^\#}(x).$$

Definition 2.3 [16] A *neutrosophic set* $\mathbb{N}$ defined over a universe X is represented as:

$$\mathbb{N} = \{(x, T_{\mathbb{N}}(x), I_{\mathbb{N}}(x), F_{\mathbb{N}}(x)) \mid x \in X\},$$

where $T_{\mathbb{N}}(x)$, $I_{\mathbb{N}}(x)$, and $F_{\mathbb{N}}(x) \in [0,1]$ denote the truth-membership, indeterminacy-membership, and falsity-membership values, respectively. These components are independent, and their sum is not bounded by a fixed constant, allowing a flexible description of uncertain information.

Definition 2.4 [10] A *quadri-partitioned neutrosophic set* (QPNS) $\mathcal{S}$ is an extension of neutrosophic sets, including an additional component for contradiction. It is expressed as:

$$\mathcal{S} = \{(x, \tau_{\mathcal{S}}(x), \iota_{\mathcal{S}}(x), \chi_{\mathcal{S}}(x), \phi_{\mathcal{S}}(x)) \mid x \in X\},$$

where $\tau_{\mathcal{S}}(x)$, $\iota_{\mathcal{S}}(x)$, $\chi_{\mathcal{S}}(x)$, and $\phi_{\mathcal{S}}(x) \in [0,1]$ represent the degrees of truth, indeterminacy, contradiction, and falsity, respectively. This structure is useful in capturing situations with conflicting or ambiguous data.

Definition 2.5 [4] An *intuitionistic quadri-partitioned neutrosophic set* (IQPNS), denoted by $\mathcal{I}$, is defined over a universe X as:

$$\mathcal{I} = \{(x, T_{\mathcal{I}}(x), F_{\mathcal{I}}(x), I_{\mathcal{I}}(x), C_{\mathcal{I}}(x)) \mid x \in X\},$$

where each function maps X into the interval $[0,1]$, and: $T_{\mathcal{I}}(x)$: degree of truth, $F_{\mathcal{I}}(x)$: degree of falsity, $I_{\mathcal{I}}(x)$: degree of indeterminacy and $C_{\mathcal{I}}(x)$: degree of contradiction. These values satisfy the following conditions for all $x \in X$:

$$T_{\mathcal{I}}(x) + F_{\mathcal{I}}(x) \leq 1, \quad I_{\mathcal{I}}(x) \in [0,1], \quad C_{\mathcal{I}}(x) \in [0,1],$$

and collectively:

$$T_{\mathcal{I}}(x) + F_{\mathcal{I}}(x) + I_{\mathcal{I}}(x) + C_{\mathcal{I}}(x) \leq 3.$$

This model captures a more detailed spectrum of uncertainty, accommodating truth-falsity complementarity and independent measures of ambiguity and contradiction.

3. Mathematical Formulation of the Transportation Model

Suppose an organization operates p distribution centers, each responsible for supplying a uniform product to q retail outlets. Every warehouse has a fixed supply capacity, and each outlet requires a specified quantity of the product. The unit cost of shipping from a warehouse to a retail outlet is known and is assumed to be directly proportional to the quantity transported. The objective is to determine a transportation plan that meets all supply and demand requirements while minimizing the total cost of distribution.

Objective Function

The overall transportation cost is minimized by solving the following objective function:

$$\min \ Z = \sum_{u=1}^{p} \sum_{v=1}^{q} y_{uv} \cdot t_{uv}, \tag{3.1}$$

where:

- p: Number of supply points (warehouses),
- q: Number of demand points (retail outlets),

- u: Index for supply locations, $u = 1, 2, \ldots, p$,
- v: Index for demand locations, $v = 1, 2, \ldots, q$,
- S_u: Supply available at warehouse u,
- R_v: Demand required at retail outlet v,
- y_{uv}: Quantity shipped from warehouse u to outlet v,
- t_{uv}: Cost per unit for transportation from warehouse u to outlet v.

The aim is to identify the shipment quantities y_{uv} that result in the least possible transportation cost while fully meeting the supply and demand constraints.

Model constraints

The model must satisfy the following set of constraints:
Supply capacity constraints:
The total quantity shipped from each warehouse should not exceed its supply capacity:

$$\sum_{v=1}^{q} y_{uv} = S_u, \quad \forall u = 1, 2, \ldots, p \tag{3.2}$$

Demand satisfaction constraints:
Each retail outlet must receive an amount that meets its demand:

$$\sum_{u=1}^{p} y_{uv} = R_v, \quad \forall v = 1, 2, \ldots, q \tag{3.3}$$

Non-negativity constraints:
Negative quantities of transportation are not feasible, hence:

$$y_{uv} \geq 0, \quad \forall u \in \{1, \ldots, p\},\ v \in \{1, \ldots, q\} \tag{3.4}$$

This formulation provides a structured approach for minimizing the total cost of transporting goods while ensuring that all demands are met and supply limits are respected.

4. Mathematical Representation of the Fuzzy Transportation Problem

In real-world logistics systems, key parameters such as unit transportation costs, supply capacities, and customer demands are frequently uncertain. This uncertainty may arise from market fluctuations, data inaccuracy, or subjective expert assessments. To incorporate such imprecision, fuzzy set theory provides a suitable modeling approach by expressing these uncertain parameters as fuzzy numbers [**?**].

Fuzzy Objective Function

The goal remains to minimize the overall transportation cost, but in a fuzzy environment, the unit cost coefficients are modeled as fuzzy values. The fuzzy objective function is formulated as:

$$\min \ \tilde{Z} = \sum_{u=1}^{p} \sum_{v=1}^{q} y_{uv} \otimes \tilde{c}_{uv}, \tag{4.1}$$

where:

- $\tilde{c}_{uv}$: Fuzzy cost of transporting one unit from warehouse u to outlet v,
- y_{uv}: Quantity delivered from u to v,
- $\otimes$: Denotes fuzzy multiplication.

Fuzzy Model Constraints

To reflect the uncertain nature of supply and demand, the traditional constraints are adjusted using fuzzy quantities.

Fuzzy Supply Limits:

Each warehouse u has an approximate supply capacity represented by a fuzzy number $\tilde{S}_u$. The total goods sent from that warehouse must not exceed this fuzzy value:

$$\sum_{v=1}^{q} y_{uv} \leq \tilde{S}_u, \quad \text{for all } u = 1, 2, \ldots, p \tag{4.2}$$

Fuzzy Demand Requirements:

Each retail outlet v has a demand level represented by the fuzzy quantity $\tilde{D}_v$. The total supply received by the outlet must meet or surpass this fuzzy demand:

$$\sum_{u=1}^{p} y_{uv} \geq \tilde{D}_v, \quad \text{for all } v = 1, 2, \ldots, q \tag{4.3}$$

Non-Negativity Condition:

All transportation quantities must remain non-negative and real-valued:

$$y_{uv} \geq 0, \quad \forall u \in \{1, \ldots, p\},\ v \in \{1, \ldots, q\} \tag{4.4}$$

This fuzzy adaptation of the classical transportation model allows for a more realistic and flexible representation of uncertainty in critical parameters. As a result, it enhances the model's applicability and reliability in decision-making processes under ambiguous or fluctuating environments.

5. Proposed Mathematical Model for Intuitionistic Quadri-Partitioned Neutrosophic Transportation Problems

Conventional transportation models generally rely on the assumption that parameters such as unit cost, supply availability, and demand quantities are precisely known. However, this is rarely the case in real-world scenarios, where such data are often influenced by uncertainty arising from economic shifts, changing customer needs, and unpredictable logistics conditions. To accommodate this imprecision, fuzzy transportation models have been proposed in which these key parameters are expressed through fuzzy numbers.

Expanding on this direction, the current study incorporates a more refined uncertainty modeling framework based on the *intuitionistic quadri-partitioned neutrosophic set*. This enhanced structure captures uncertainty through four distinct membership components—truth, indeterminacy, contradiction, and falsity—allowing a more comprehensive representation of ambiguity in transportation parameters.

Objective Function

The optimization goal is to minimize the total transportation cost, represented using intuitionistic quadri-partitioned neutrosophic numbers as follows:

$$\min \{T_{\mathcal{Z}}, I_{\mathcal{Z}}, C_{\mathcal{Z}}, F_{\mathcal{Z}}\} = \sum_{u=1}^{p} \sum_{v=1}^{q} w_{uv} \cdot \{T_{\mathcal{T}_{uv}}, I_{\mathcal{T}_{uv}}, C_{\mathcal{T}_{uv}}, F_{\mathcal{T}_{uv}}\}, \tag{5.1}$$

where:

- w_{uv}: Quantity of goods transported from source u to destination v,
- $\{T_{\mathcal{T}_{uv}}, I_{\mathcal{T}_{uv}}, C_{\mathcal{T}_{uv}}, F_{\mathcal{T}_{uv}}\}$: Neutrosophic cost per unit, consisting of truth (T), indeterminacy (I), contradiction (C), and falsity (F) components.

Constraint Set

Neutrosophic supply constraints:
Each warehouse u has a supply limit represented by an intuitionistic quadri-partitioned neutrosophic value:

$$\sum_{v=1}^{q} w_{uv} = \{T_{\mathcal{S}_u}, I_{\mathcal{S}_u}, C_{\mathcal{S}_u}, F_{\mathcal{S}_u}\}, \quad \forall u = 1, 2, \ldots, p \tag{5.2}$$

Neutrosophic demand constraints:
Each retail outlet v has an associated demand modeled similarly:

$$\sum_{u=1}^{p} w_{uv} = \{T_{\mathcal{D}_v}, I_{\mathcal{D}_v}, C_{\mathcal{D}_v}, F_{\mathcal{D}_v}\}, \quad \forall v = 1, 2, \ldots, q \tag{5.3}$$

Non-negativity condition:
Shipment quantities must be non-negative:

$$w_{uv} \geq 0 \tag{5.4}$$

Validity of membership components:
All neutrosophic membership values must satisfy the following upper-bound condition to ensure consistency:

$$0 \leq T_{\mathcal{Z}} + I_{\mathcal{Z}} + C_{\mathcal{Z}} + F_{\mathcal{Z}} \leq 3, \tag{5.5}$$

$$0 \leq T_{\mathcal{S}_u} + I_{\mathcal{S}_u} + C_{\mathcal{S}_u} + F_{\mathcal{S}_u} \leq 3, \quad 0 \leq T_{\mathcal{D}_v} + I_{\mathcal{D}_v} + C_{\mathcal{D}_v} + F_{\mathcal{D}_v} \leq 3, \tag{5.6}$$

$$0 \leq T_{\mathcal{T}_{uv}} + I_{\mathcal{T}_{uv}} + C_{\mathcal{T}_{uv}} + F_{\mathcal{T}_{uv}} \leq 3, \tag{5.7}$$

for all $u = 1, 2, \ldots, p$ and $v = 1, 2, \ldots, q$.

This formulation provides a robust extension of the traditional transportation model by incorporating a richer mathematical structure to capture uncertainty. It is particularly effective in decision environments characterized by incomplete, vague, or contradictory information.

Balanced vs. Unbalanced Problem Formulation

A transportation problem is considered to be in a *balanced state* if the total available supply equals the total required demand, evaluated under the intuitionistic quadri-partitioned neutrosophic addition operator $\oplus$:

$$\sum_{u=1}^{p} \oplus \{T_{\mathcal{S}_u}, I_{\mathcal{S}_u}, C_{\mathcal{S}_u}, F_{\mathcal{S}_u}\} = \sum_{v=1}^{q} \oplus \{T_{\mathcal{D}_v}, I_{\mathcal{D}_v}, C_{\mathcal{D}_v}, F_{\mathcal{D}_v}\}. \tag{5.8}$$

In this formulation:

- p: Number of origins or supply nodes (e.g., warehouses, factories),
- q: Number of destinations or demand nodes (e.g., retail outlets, distribution centers),
- u: Index for supply points, $u = 1, 2, \ldots, p$,
- v: Index for demand points, $v = 1, 2, \ldots, q$,
- $\{T_{\mathcal{S}_u}, I_{\mathcal{S}_u}, C_{\mathcal{S}_u}, F_{\mathcal{S}_u}\}$: Neutrosophic representation of supply at node u,
- $\{T_{\mathcal{D}_v}, I_{\mathcal{D}_v}, C_{\mathcal{D}_v}, F_{\mathcal{D}_v}\}$: Neutrosophic representation of demand at node v,
- w_{uv}: Quantity transported from source u to destination v,

- $\{T_{\mathcal{T}_{uv}}, I_{\mathcal{T}_{uv}}, C_{\mathcal{T}_{uv}}, F_{\mathcal{T}_{uv}}\}$: Unit transportation cost modeled via intuitionistic quadri-partitioned neutrosophic numbers between u and v,
- $\{T_{\mathcal{Z}}, I_{\mathcal{Z}}, C_{\mathcal{Z}}, F_{\mathcal{Z}}\}$: Aggregate cost incurred for all shipments under the quadri-partitioned neutrosophic representation,
- $\oplus$: Represents the addition operator defined for IQPNS numbers.

If the above equality is not satisfied, the transportation problem is said to be *unbalanced*. In such scenarios, artificial or dummy supply/demand points are incorporated, assigned with appropriately constructed neutrosophic values, in order to equalize the total supply and demand within the problem domain.

This extended modeling technique using intuitionistic quadri-partitioned neutrosophic numbers (IQPNS) enhances the classical transportation framework by accommodating multiple layers of uncertainty, including conflicting and indeterminate information. As a result, the proposed approach provides a powerful and realistic tool for handling uncertain transportation data in dynamic supply chain and logistics environments.

6. Proposed Approach for Solving Transportation Problems within an Intuitionistic Quadri-Partitioned Neutrosophic Framework

Several methodologies have been developed to solve transportation problems under uncertain conditions. Traditional approaches, including the Northwest Corner Rule, Row and Column Minima methods, and Vogel's Approximation Method (VAM), are commonly used for generating initial feasible solutions. These are often followed by optimality tests such as the Modified Distribution (MODI) method. In recent years, heuristic and evolutionary algorithms have also been incorporated to improve computational efficiency and solution quality. Despite these advancements, the application of intuitionistic quadri-partitioned neutrosophic numbers (IQPNNs) for solving transportation problems involving ambiguous cost, supply, and demand parameters remains largely unexplored.

To address this gap, we present a novel solution framework that embeds IQPNNs into the classical transportation model. This enhanced method utilizes the expressiveness of IQPNNs to model uncertainty more comprehensively and applies the MODI technique for determining optimality. The proposed algorithm is particularly suited for decision-making in uncertain environments, such as hospital logistics, where precise information is often unavailable.

Step 1: Problem Setup

- Construct the transportation cost matrix, where each entry is expressed using an IQPNN.
- Define supply and demand values as crisp quantities.
- If the total supply and total demand are unequal, introduce a dummy destination or origin to balance the system.

Step 2: Transformation of IQPNN Costs

- Use a suitable score or accuracy function to convert each IQPNN cost into a scalar value.
- Generate a crisp equivalent cost matrix from these values to support further analysis.

Step 3: Initial Solution via Least Cost Method (LCM)

- Identify the cell with the lowest transportation cost in the crisp matrix.
- Allocate the minimum of the remaining supply or demand to that cell.
- Update the supply and demand values accordingly.
- Continue the process until all allocations are completed.

Step 4: Optimality Evaluation Using MODI

- Assign dual variables U_i to supply points and V_j to demand points.
- Compute the opportunity cost (also called reduced cost) for each unallocated cell as:

$$\Delta_{ij} = C_{ij} - (U_i + V_j),$$

  where C_{ij} is the crisp cost from Step 2.

- If all $\Delta_{ij} \geq 0$, the solution is optimal.
- If any $\Delta_{ij} < 0$, perform the following:
  - Locate the cell with the most negative Δ_{ij}.
  - Form a closed path (loop) connecting basic variables.
  - Adjust allocations within the loop to reduce the total cost.
  - Repeat this improvement process until all opportunity costs are non-negative.

Step 5: Determination of Total Neutrosophic Cost

- Multiply the final shipment quantities with their corresponding IQPNN cost values.
- Sum these neutrosophic products to obtain the final transportation cost Z^*, represented in the IQPNN format:

$$Z^* = \{T_{Z^*}, I_{Z^*}, C_{Z^*}, F_{Z^*}\}.$$

Step 6: Interpretation of Results

- Analyze the final cost in terms of its four components—truth (T), indeterminacy (I), contradiction (C), and falsity (F).
- Identify the routes that offer maximum reliability by examining the distribution of these neutrosophic components.

The proposed methodology advances classical transportation modeling by embedding a more expressive uncertainty representation. By leveraging IQPNNs, this algorithm offers a refined decision-making tool for applications in uncertain and dynamic domains such as supply chain logistics and healthcare resource allocation.

7. Application in Emergency Logistics under Uncertainty

In humanitarian logistics particularly during disaster relief operations timely and efficient distribution of critical items such as food, medical kits, and clean water is crucial. However, practical relief efforts often encounter substantial uncertainty arising from various dynamic and unpredictable conditions:

- **Unstable demand:** The number of individuals requiring aid can vary rapidly depending on the evolving severity and reach of the disaster, making accurate demand forecasting difficult.
- **Limited supply availability:** Disruptions in procurement, warehouse limitations, and delays in transport may restrict the supply of essential items.
- **Fluctuating transportation costs:** Cost variability arises due to fuel price changes, blocked or damaged routes, inclement weather, and urgency of delivery.
- **Decision-making ambiguity:** Incomplete or conflicting data, rapidly changing conditions, and human judgment introduce additional uncertainty in planning and execution.

To model such multifaceted uncertainty in transportation costs more effectively, we employ *intuitionistic quadri-partitioned neutrosophic numbers* (IQPNNs). This framework offers a richer representation compared to classical fuzzy or probabilistic approaches by capturing four distinct aspects of uncertainty for each cost entry. An IQPNN is denoted by a tuple $\langle T, I, C, F \rangle$, where:

- T: Degree of truth or confidence in the transportation cost estimate.
- I: Level of indeterminacy or ambiguity due to incomplete or unknown information.
- C: Measure of contradiction reflecting conflicting data or opinions.
- F: Degree of falsity or risk of infeasibility (e.g., blocked routes or inaccessible locations).

By adopting IQPNNs, the proposed model captures the underlying vagueness and inconsistency associated with emergency logistics, enabling more resilient and informed transportation decisions.

Study Objective

This study seeks to develop a decision-support model that enables efficient resource allocation in disaster relief scenarios while explicitly considering uncertainty. The key goals include:

1. Minimizing the overall transportation cost using an IQPNN-based cost structure.
2. Ensuring all affected zones receive required supplies, despite possible disruptions.
3. Identifying cost-effective and reliable transportation routes by analyzing the neutrosophic components associated with uncertainty.

This framework strengthens emergency planning by balancing cost-efficiency and operational robustness in uncertain environments.

Neutrosophic Transportation Cost Matrix

In emergency logistics systems, the estimation of transportation costs is inherently uncertain due to rapidly changing external conditions. To illustrate this, Table 1 presents an IQPNN-based transportation cost matrix involving three supply centers and three destination hospitals. While costs are modeled using IQPNNs, the supply and demand quantities are assumed to be crisp.

Suppliers \ Hospitals	D_1	D_2	D_3	Supply
S_1	$\langle 0.6, 0.2, 0.1, 0.3 \rangle$	$\langle 0.5, 0.3, 0.2, 0.4 \rangle$	$\langle 0.7, 0.1, 0.2, 0.3 \rangle$	30
S_2	$\langle 0.4, 0.4, 0.2, 0.5 \rangle$	$\langle 0.6, 0.2, 0.3, 0.3 \rangle$	$\langle 0.5, 0.3, 0.1, 0.4 \rangle$	25
S_3	$\langle 0.3, 0.5, 0.3, 0.4 \rangle$	$\langle 0.4, 0.3, 0.2, 0.5 \rangle$	$\langle 0.5, 0.4, 0.2, 0.4 \rangle$	20
Demand	5	25	20	50

Table 1: Transportation cost matrix represented using IQPNNs with crisp supply and demand values.

8. Problem Balancing and Score-Based Cost Evaluation

In the illustrated transportation setup, the total supply from all sources is $30 + 25 + 20 = 75$, whereas the cumulative demand across destinations is only $5 + 25 + 20 = 50$. This disparity creates an unbalanced transportation problem.

To resolve this and ensure the problem is solvable using standard allocation methods, an artificial (dummy) demand point, labeled D_4, is introduced with a demand of 25 units. This adjustment equalizes the total supply and demand at 75 units each, thereby enabling the application of classical solution techniques. The transportation costs from each supplier to this dummy destination are assigned the neutrosophic value $\langle 0, 0, 0, 0 \rangle$, which signifies that no actual cost or uncertainty is incurred when allocating resources to the dummy node. This balancing approach is essential when modeling logistics problems under uncertain conditions.

9. Solution Strategy Using Score Function

To incorporate intuitionistic quadri-partitioned neutrosophic numbers (IQPNNs) into traditional optimization frameworks, a numerical transformation of neutrosophic values is necessary. This is achieved using a specially defined score function, given by:

$$S = \frac{T - F + 1 - I - C}{3},$$

where T, F, I, and C represent the degrees of truth, falsity, indeterminacy, and contradiction, respectively.

IQPNNs provide a refined model for capturing complex uncertainty. However, their direct inclusion in optimization procedures requires a crisp equivalent. The proposed score function serves this purpose by evaluating the net reliability of each IQPNN, assigning a single numerical value that reflects the influence of all four components.

The score function penalizes falsity, contradiction, and indeterminacy while favoring truth, thus producing a scalar value that facilitates comparison among alternatives. Once the IQPNN values in the cost matrix are converted to crisp scores, standard optimization techniques can be applied. Table 2 illustrates this conversion for the transportation cost matrix.

Suppliers \ Hospitals	D_1	D_2	D_3
S_1	$\langle 0.6, 0.2, 0.1, 0.3\rangle$ ($S = 0.3333$)	$\langle 0.5, 0.3, 0.2, 0.4\rangle$ ($S = 0.2000$)	$\langle 0.7, 0.1, 0.2, 0.3\rangle$ ($S = 0.3667$)
S_2	$\langle 0.4, 0.4, 0.2, 0.5\rangle$ ($S = 0.1000$)	$\langle 0.6, 0.2, 0.3, 0.3\rangle$ ($S = 0.2667$)	$\langle 0.5, 0.3, 0.1, 0.4\rangle$ ($S = 0.2333$)
S_3	$\langle 0.3, 0.5, 0.3, 0.4\rangle$ ($S = 0.0333$)	$\langle 0.4, 0.3, 0.2, 0.5\rangle$ ($S = 0.1333$)	$\langle 0.5, 0.4, 0.2, 0.4\rangle$ ($S = 0.1667$)

Table 2: Crisp score transformation of IQPNN-based transportation cost matrix.

10. Initial Feasible Solution Using the Least Cost Method (LCM)

To establish a starting solution for the IQPNN-based transportation model, the Least Cost Method (LCM) is employed. This technique prioritizes cells with the minimum crisp cost values (derived from the IQPNN score function) and allocates supplies accordingly. The steps followed in this allocation process are outlined below:

- Locate the cell with the minimum crisp score.
- Allocate the minimum value between the available supply and corresponding demand to that cell.
- Update the supply and demand after the allocation.
- Repeat the process until all supply and demand values are exhausted.

Step-by-Step Allocation Procedure

1. The cell (S_3, D_1) has the smallest score: $\langle 0.3, 0.5, 0.3, 0.4\rangle$, with a score value of 0.0333.

$$\text{Supply at } S_3 = 20,$$
$$\text{Demand at } D_1 = 5.$$

   Allocation: 5 units to (S_3, D_1).

2. Next, the lowest available cost is in (S_1, D_2): $\langle 0.5, 0.3, 0.2, 0.4\rangle$, score $= 0.2000$.

$$\text{Supply at } S_1 = 30,$$
$$\text{Demand at } D_2 = 25.$$

   Allocation: 25 units to (S_1, D_2).

3. The next minimum score is found at (S_1, D_3): $\langle 0.7, 0.1, 0.2, 0.3\rangle$, with a score of 0.3667.

$$\text{Remaining Supply at } S_1 = 5,$$
$$\text{Demand at } D_3 = 20.$$

Allocation: 5 units to (S_1, D_3).

4. Then, allocate to (S_2, D_3), where the score is 0.2333.

$$\text{Supply at } S_2 = 25,$$
$$\text{Remaining Demand at } D_3 = 15.$$

Allocation: 15 units to (S_2, D_3).

5. After fulfilling D_3, S_2 still has 10 units of supply left. These are assigned to a dummy destination node (created earlier to balance the problem).

$$\text{Dummy Demand} = 25.$$

Allocation: 10 units from S_2 to Dummy.

6. The remaining dummy demand of 15 units is assigned to S_3, which has a matching available supply:

$$\text{Remaining Supply at } S_3 = 15.$$

Allocation: 15 units from S_3 to Dummy.

Final Allocation Table

Source	D_1	D_2	D_3	Dummy
S_1	0	25	5	0
S_2	0	0	15	10
S_3	5	0	0	15

Transportation Flow Diagram

The allocations presented above satisfy all supply and demand conditions, including dummy adjustments. This solution provides a solid starting point for the subsequent optimality evaluation using techniques such as the Modified Distribution Method (MODI) under the IQPNN framework.

11. Optimality Evaluation Using the MODI Technique

To verify whether the initial feasible solution derived via the Least Cost Method (LCM) is optimal, the Modified Distribution Method (MODI) is applied. If the current solution fails to satisfy optimality conditions, the MODI algorithm iteratively adjusts allocations to reduce overall cost under the IQPNN-based framework.

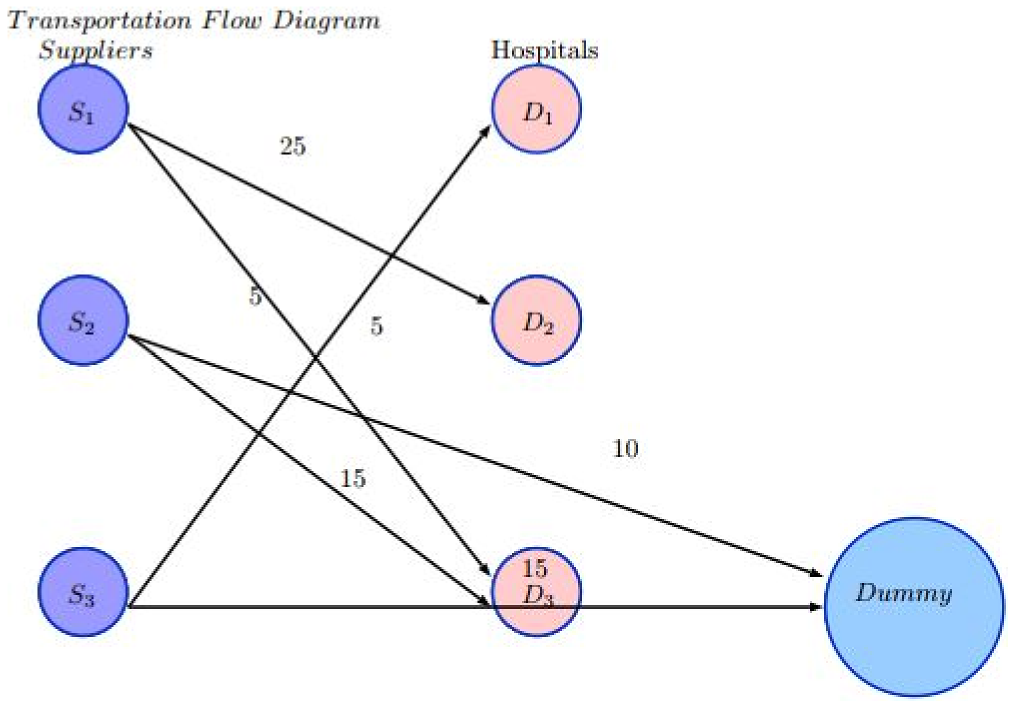
Step 1: Determination of Dual Variables

Let U_i and V_j denote the dual variables corresponding to each supply point and demand location, respectively. For every basic (allocated) cell (i, j), the relationship is given by:

$$U_i + V_j = C_{ij},$$

where C_{ij} is the crisp cost score derived from the IQPNN score function for the corresponding route.

We begin by setting a reference dual value, typically $U_1 = 0$, and proceed to determine other dual variables using the values from allocated cells. For example:



- From the allocation (S_1, D_2), we get:

$$U_1 + V_2 = 0.6 \Rightarrow V_2 = 0.6.$$

- From allocation (S_2, D_3), we derive:

$$U_2 + V_3 = 0.6 \Rightarrow U_2 = 0.6 - V_3.$$

- If we assume $V_3 = 0.8$, then:

$$U_2 = 0.6 - 0.8 = -0.2.$$

Step 2: Calculation of Reduced Costs

For each non-basic (unallocated) cell (i, j), the opportunity cost (also called the reduced cost) is computed using the formula:

$$\Delta_{ij} = C_{ij} - (U_i + V_j).$$

Illustrative calculations:

- For cell (S_1, D_1):

$$\Delta_{11} = 0.7 - (0 + 0.5) = 0.2.$$

- For cell (S_2, D_2):

$$\Delta_{22} = 0.7 - (-0.2 + 0.6) = 0.3.$$

Step 3: Checking Optimality Conditions

If all reduced costs $\Delta_{ij} \geq 0$, the current solution is considered optimal. If any $\Delta_{ij} < 0$, it indicates that further improvements are possible. For instance, if we identify a reduced cost such as $\Delta_{12} = -0.1$, the solution must be improved by reallocating shipments.

Step 4: Iterative Enhancement Process

When negative reduced costs are present:

1. Identify the cell with the most negative Δ_{ij}.
2. Construct a closed loop path involving that cell and existing allocations, alternating between + and - signs.
3. Adjust the allocations along the loop by transferring units to reduce the overall transportation cost.
4. Recalculate dual variables and reduced costs.
5. Repeat this procedure until all $\Delta_{ij} \geq 0$, indicating optimality.

12. Computation of Total IQPNN-Based Transportation Cost

Step 1: Tabulation of Allocated Routes and Corresponding IQPNN Costs

Route	**Quantity**	**IQPNN Cost** $\langle T, I, C, F\rangle$
$S_1 \to D_2$	25	$\langle 0.6, 0.3, 0.2, 0.5\rangle$
$S_1 \to D_3$	5	$\langle 0.8, 0.1, 0.2, 0.7\rangle$
$S_2 \to D_3$	15	$\langle 0.6, 0.3, 0.1, 0.5\rangle$
$S_2 \to$ Dummy	10	$\langle 0, 0, 0, 0\rangle$
$S_3 \to D_1$	5	$\langle 0.4, 0.5, 0.3, 0.2\rangle$
$S_3 \to$ Dummy	15	$\langle 0, 0, 0, 0\rangle$

Table 3: IQPNN Cost Components for Allocated Shipments

Step 2: Aggregation of Weighted Neutrosophic Components

To compute the total IQPNN transportation cost, each allocated quantity is multiplied by its respective cost components and summed:

$$Z^* = \sum w_{ij} \cdot \langle T_{ij}, I_{ij}, C_{ij}, F_{ij}\rangle = \langle 30.0, 15.0, 9.0, 24.5\rangle.$$

Step 3: Normalization and Interpretation

To better interpret the result, the total IQPNN cost vector is normalized by the grand total of transported units (75):

$$Z^*_{\text{normalized}} = \left\langle \frac{30}{75}, \frac{15}{75}, \frac{9}{75}, \frac{24.5}{75} \right\rangle = \langle 0.4, 0.2, 0.12, 0.33\rangle.$$

For clearer understanding and policy decisions, the components can be scaled to a 0–100 percentage range:

$$Z^*_{\text{scaled}} = \langle 40, 20, 12, 32.67\rangle.$$

This final result summarizes the overall transportation performance, with:

- 40% contribution from reliable (truth) information,
- 20% from indeterminate factors,
- 12% from contradictory inputs,
- 32.67% indicating potential risk or failure.

This breakdown aids decision-makers in assessing not only cost efficiency but also the uncertainty embedded within the logistics plan.

13. Interpretation of Final Cost Vector Z^*

- **Truth (T = 40)**: Represents a moderate assurance level regarding the reliability of the estimated transportation costs under stable operating conditions.
- **Indeterminacy (I = 20)**: Indicates a measurable degree of uncertainty arising from unpredictable variables or incomplete information.
- **Contradiction (C = 12)**: Reflects a relatively small extent of conflicting or contradictory data related to cost estimations.
- **Falsity (F = 32.67)**: Denotes a moderate-to-high possibility of cost deviations or unanticipated expenses in the logistics process.

14. Supplier-Based Performance Assessment

To evaluate supplier effectiveness under uncertainty, the IQPNN cost components are interpreted to assess both reliability and potential risk. The table below summarizes the relative standing of each supplier based on neutrosophic indicators: **Most suitable:** S_2 — Offers balanced performance, with

Supplier	T	I	C	F	**Reliability Level**	**Cost Risk Level**
S_1	Moderate	Low	Low	High	Moderate	High
S_2	Moderate	Moderate	Low	Moderate	Balanced	Medium
S_3	Low	High	Moderate	Low	Low	Low

Table 4: Supplier Evaluation Based on Neutrosophic Cost Attributes

moderate values across all dimensions.
Runner-up: S_3 — Lower falsity suggests reliability, though high indeterminacy signals data ambiguity.
Least recommended: S_1 — Elevated falsity indicates a higher likelihood of cost uncertainty or disruptions.

15. Comparative Overview of Set-Theoretic Approaches

The table below contrasts various uncertainty modeling frameworks—including Fuzzy Sets, Intuitionistic Fuzzy Sets (IFS), Neutrosophic Sets, Quadri-Partitioned Neutrosophic Sets (QPNS), and Interval-Valued Quadri-Partitioned Neutrosophic Sets (IQPNS)—based on their ability to model uncertainty and optimize transportation decisions. IQPNS emerges as the most versatile and nuanced model, especially in representing contradictory and vague data scenarios.

Criteria	**Fuzzy Set**	**IFS**	**Neutrosophic Set**	**QPNS**	**IQPNS**
Capability to Model Uncertainty	Moderate	High	Very High	Excellent	Outstanding
Membership Structure	Single (μ)	Pair (μ, ν)	Triple (T, I, F)	Quadruple (T, I, C, F)	Interval Quadruple $\langle T, I, C, F \rangle$
Contradiction Representation	Absent	Absent	Not Explicit	Supported	Fully Captured with Intervals
Decision Accuracy	Limited	Improved	High	Very High	Exceptional
Adaptability to Real-World Logistics	Low	Moderate	Strong	Excellent	Superior
Computational Demand	Low	Moderate	High	Elevated	Highest
Handling Uncertain Transportation Costs	Weak	Fair	Robust	Stronger	Optimal
Conflict Modeling	Not Handled	Not Handled	Partial	Comprehensive	Comprehensive with Interval Precision
Cost Modeling Flexibility	Constrained	More Flexible	Highly Adaptable	Maximum Flexibility	Interval-Enhanced Flexibility
Optimization Capability	Basic	Average	Strong	Advanced	Highly Effective and Reliable

Table 5: Comparison of Set-Theoretic Frameworks for Solving Transportation Problems under Uncertainty

Conclusions

This research introduces a novel approach for solving transportation problems where key parameters—such as transportation expenses, supply availability, and demand requirements—are subject to uncertainty. Unlike conventional crisp models that fall short in handling real-world ambiguity, the proposed framework leverages intuitionistic quadri-partitioned neutrosophic sets (IQPNS) to more accurately represent uncertain information.

By applying arithmetic operations tailored to IQPNS, the method derives optimal transportation strategies that remain stable and reliable under imprecise conditions.

A specialized score function is constructed to evaluate and rank IQPNS values, significantly improving the decision-making process by accounting for truth, indeterminacy, contradiction, and falsity. Computational comparisons highlight the superiority of this model over traditional fuzzy and intuitionistic fuzzy systems, offering a more comprehensive and flexible mechanism for dealing with uncertainty and inconsistency in logistics environments.

To validate the effectiveness of the approach, a case-based numerical illustration is presented. The results show that the proposed method is not only computationally viable and easy to apply but also highly effective for uncertain transportation scenarios. The study paves the way for future extensions into more complex contexts, such as dynamic or multi-stage logistics models, further enhancing its relevance in real-world decision support systems.

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