**ON NEW APPLICATIONS OF FRACTIONAL CALCULUS**

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**Abstract**

In the present paper author derive a number of integrals concerning various special functions which are applications of the one of Osler result. Osler provided extensions to the familiar Leibniz rule for the nth derivative of product of two functions.

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1. **Introduction**

One of the most frequently encountered tools in the theory of fractional calculus (that is, differentiation and integration of an arbitrary real or complex order) is furnished by the familiar differintegral operator  defined and represented by Oldham and Spanier [7]:

 … (1)

and

 … (2)

where n is the least positive integer such that n>q.

provides a generalization of the familiar differential and integral operator, viz.,  and  }.For a=0 the operator is given by

 … (3)

corresponding essentially to the classical Riemann-Liouville fractional derivative (or integral) of order (or –). Moreover, when, Equation (1) may be identified with the definition of the familiar Weyl fractional derivative (or integral) of order (or –).

In recent years there has appeared a great deal of literature discussing the application of the aforementioned fractional calculus operators in a number of areas of mathematical analysis (such as ordinary and partial differential equations, integral equations, summation of series, et cetera) and now stands on fairly firm footing through the research contribution of various authors (cf., e.g., [3], [4], [5], [6], [9] and [10]). In the present paper the aim of the author to derive a number of summations series concerning generalized hypergeometric function which are applications of the one of Samko result.

The integral analog of the generalized Leibnitz rule in the theory of fractional calculus was given by Osler [8] in the following form :

 … (4)

The generalized hypergeometric function of one variable viz., defined and represented as follows (see e.g. [11, p.19]) is also required here:

 … (5)

1. **The Main results**

The following summations of series are established here:

(i) For 

 … (6)

(ii) 

 … (7)

(iii) 

 … (8)

(iv) 

 … (9)

(v)For 

  … (10)

**Method of derivation**

If we take



in (3),then

L.H.S. = 



and using known result [2, p.189, eqn. (32)], we get

 For R.H.S., we similarly have



putting these on (3), (5) is established.

Summations of series from (6) to (10) are similarly established on choosing:



in (3) and using known results [2, p.189, Eqs. (26) and (32); p.193, Eq. (51), p.192, Eq. (50) and p.187, Eq. (14); p.190, Eq. (35); p.188, Eq. (21)] respectively.

1. **Special Cases**

In view of the large number of parameters involved in the integrals established above, these integrals are capable of yielding a number of known results and new integrals including integrals due to Arora and Koul [1, p.932, Eqs. (2.1) and (2.2)]. These are not record here for

lack of space.

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