**Some properties of Euler type integral involving extended Mittag-leffler function**

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**Abstract:** The Object of the present paper is to establish some interested theorems on Euler type integral involving extended Mittag-Leffler function. Further, we reduce some special cases involving various known functions like Wiman function, Prabhakar function, exponential and Binomial functions.

**Key Word:** Mittag-Leffler function, Hyper geometric function, exponential function etc

**1. Introduction**

Swedish mathematician Gosta Mittag-Leffler (1903) introduced the function

 (1.1)

Where *z* is a complex variable and is a Gamma function *α* ≥ 0. The Mittag-Leffler function is a direct generalization of the exponential function to which it reduces for *α* = 1. For 0 *< α <*1, it interpolates between the pure exponential and hyper geometric function. Its importance realized during the last two decades due to its involvement in the problems of physics, chemistry, biology, engineering and applied sciences. Mittag-Leffler function naturally occurs as the solution of fractional order differential or fractional order integral equation.

The generalization ofwas studied by Wiman in 1905 and he defined the function as  (1.2)

Which is known as Wiman function.

Later In 1971, Prabhakar introduced the function in the form of

  (1.3)

whereis the Pochhammer symbol (Rainsville (1960)).



In the year 2007, Shukla and Prajapati introduced the function and in the year 2009 Tariq O. Salim introduced the functionwhich are defined for

 and for α, β, γ , δ ∈ C; Re (α) > 0, Re (β) > 0, Re (γ) > 0, Re (δ) > 0 are as follows

  (1.4)

  (1.5)

Finally in 2012, a new generalization of Mittag-Leffler function was defined by Salim as

 , (1.6)

Where and 

  (1.7)

Where



**2. Main result**

**Theorem2.1:**Ifthen



**Proof:** we consider L.H.S. of Theorem 1 by 



using the change of order of integration, we get



Which is required result.

**Corollary:** for A = 0, then we have



**Theorem2.2:**If:and ,then



Proof:   (2.1)

using the integral [4, p. 263], we have

,

Again, using above result in (2.1), we have

**Corollary:** put A= 0 then Theorem 2 reduces to the following interesting results





**Corollary:** for a = 0, b = 1 then Theorem 2 reduces to

 

**Theorem3:** If



 then

**Proof:** We Starts from Left Hand Sides of theorem 3,



and using the definition of the generalized Mittage-Leffler formula, we have

  (2.3)

By using the integral [2, Eq.(3.5)]



Using above result in (2.3), we get



**Corollary:** for a = 0, we have



**Corollary:** for then theorem 3 reduces to



**3. Special cases**

**Case-1**

(i) Put and  in theorem 1,



(ii) Puttingand then by theorem 1,



(iii) Puttingand, in theorem 1.We get



Whereis prabhakar function.

(iv) Puttingand  in theorem 1, we get



Where is Wiman function.

**Case-2**

(i)When and in the theorem 2, we have



(ii) When and in theorem (2), we have





(iii) When  and  and  in theorem 2, then



Where is Prabhakar function.

(iv) Puttingandand in theorem 2, we have





Where is binomial function.

**Case-3**

(i) When and  in theorem 3, then

 

(ii)  When and  in theorem 3, we have



(iii) Whenand in theorem 3,

, where  is Prabhaker function.

(iv) Whenand  in theorem 3, we get

, where exp (zu) is an exponential function.

(v) when 

, where  is a binomial function.

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