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DIFFICULTY WITH THE LOGICAL NOTION OF CONSERVATION OF QUANTITY AT PREOPERATIONAL STAGE[1]

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ABSTRACT. This article discusses the child's difficulty with the logical notion of conservation of quantity at preoperational stage, with the aid of Piaget and Szeminska's investigation on the matter. Based on Frege's approach to sense and reference (1892), we can observe a close relationship between said approach and Piaget and Szeminska's analysis, and consider that the child with access to the logical notion of conservation is under the rule of the reference. In addition, we argue that Lacan (1971), based on Frege's approach, claims that the reference is phallic and, in this sense, we can think that when children learn to conserve they are under the rule of the phallic reference, which is closely related to the operability of the paternal role. As a result, we propose that Lacan's argument contributes to the understanding of this difficulty detected by Piaget.
Keywords: Number comprehension; notion of conservation; paternal role.

DIFICULDADE NA NOÇÃO LÓGICA DE CONSERVAÇÃO DE QUANTIDADE NO ESTÁGIO PRÉ-OPERATÓRIO

RESUMO. O artigo aborda a dificuldade na noção lógica de conservação de quantidade da criança, no estágio pré-operatório. Retoma essa análise presente na investigação de Piaget e Szeminska (1975). Argumenta que, a partir da abordagem lógica do sentido e da referência em Frege (1892), verificamos uma aproximação entre essa abordagem e a análise de Piaget e Szeminska e pensamos que a criança com acesso à noção lógica de conservação de quantidade se encontra sob o primado da referência. Ademais, argumenta que Lacan (1971), a partir da abordagem de Frege, reivindica que a referência é fálica e, nessa direção, podemos pensar que a referida criança, quando aprende a conservar quantidade, está sob o primado da referência fálica que diz da operacionalidade da função paterna. Propõe, nessa direção, que o argumento apresentado por Lacan contribui para a compreensão dessa dificuldade detectada por Piaget.
Palavras-chave: Compreensão de número; noção de conservação; função paterna.

DIFICULTAD EN LA NOCIÓN LÓGICA DE CONSERVACIÓN DE LA CANTIDAD EN LA ETAPA PRE-OPERATIVA

RESUMEN. El artículo aborda la dificultad de la noción lógica de conservación de la cantidad de niños en la etapa pre-operativa. Recoge este análisis presente en la investigación de Piaget y Szeminska (1975). Argumenta que desde el enfoque de la lógica de sentido y referencia en Frege (1892), se pode ver una conexión entre este enfoque y el análisis de Piaget y Szeminska, y pensar que el niño con el acceso a la noción lógica de conservación de la cantidad se encuentra bajo el primado de la referencia. Además, argumenta que Lacan (1971) desde el enfoque de Frege reivindica que la referencia es fálica, y, por consiguiente, se pode pensar que ese niño cuando aprende a conservar la

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cantidad está bajo el primado de la referencia fálica diciendo la operatividad del padre simbólico. Propone, en este sentido, que el argumento presentado por Lacan contribuye a la comprensión de esta dificultad, detectada por Piaget.
Palabras-clave: Comprensión de número; noción de conservación; función paterna.

Introduction

Research shows that children who have trouble with mathematics present a central difficulty in accessing the notion of number. Thus, number sense is a concept fundamentally important to the field of mathematical learning issues with regard to intervention and prevention. Corso and Dorneles (2010) define as number sense the concept which refers to the ease and flexibility of children with numbers, as well as the understanding of the meaning of the latter and the ideas related to them. A well-developed number sense involves, therefore, the child's ability to estimate quantities, to recognize errors when judging magnitudes or measures, to perform quantitative operations – less-than, greater- than, and equivalence types.

> The idea of commutative property can also be better understood when one relies on number sense. A student can be taught that 3x4=4x3 because the commutative law of multiplication says that axb=bxa. However, he/she may have the understanding that those statements refer to the regrouping of the same quantity. It does not matter if a set of 12 items is grouped into three subsets of 4 items or into 4 subsets of 3 items – the 12 original items remain (Corso & Dorneles, 2010, p. 300).

In this context, a question about the origins of number sense arises. We know that while the innateness hypothesis advocates the idea that there is some kind of innate predisposition that enables our numerical competence, Piaget's genetic epistemology proposes that such number possession is not innate; it is not the result of an innate ability of numerical abstraction of children. Rather, he ceases to look at the hypothetical innate structures and at environmental stimuli to turn his attention to the subject; more specifically, to the action of the subject (Becker, 2013), who, through his/her cognitive structures "adapts intellectually and organizes the environment where he/she lives. That is, through scheme accommodations and assimilations, the individual builds his/her knowledge about the world" (Queiroz, Macedo, Alves, & Garioli, 2009, p. 298). In this sense, the notion of number is a construction resulting from an interactional process between subject and object. This construction takes place step by step, from the categorization and serialization synthesis of elements (Nogueira, 2006), "until reaching the operative entirety of the set of finite integers, 'inseparably cardinal and ordinal'" (Duro & Cenci, 2013, p. 4).

In this article, we intend to resume this analysis of the origin of the number presented in the investigation by Piaget and Szeminska (1975), in the book The Child's Conception of Number, having as scope the child's notion of conservation of quantity at preoperational stage. In the test to which children have been subjected in this research, such notion was established as one of the "**qualities**" or "**requirements**" for the existence of number, being also presented as a condition of any knowledge.

Moreover, we claim herein that, from the logical approach to sense and reference in Frege (1892/1978a), we can observe a connection between this approach and Piaget and Szeminska's analysis, and consider that children with access to the logical notion of conservation of quantity are under the rule of the reference. However, we argue that Lacan (1971/2009), from Frege's perspective, claims that the reference is phallic. And such phallic reference concerns the operability of the symbolic father, which, among the various ways of addressing it, we will approach in this article based on the negativization of the imaginary phallus and, consequently, on the raising of the latter to the category of symbolic phallus, presence of the absence of the real object (Becker, 2010). Thus, considering the desired route and Piaget's statement (1954) according to which there are no purely cognitive behaviors, **we wonder** if, in this sense, we can think that when the child learns to conserve quantity he/she is under the rule of the phallic reference which speaks of the operability of the paternal role, and, consequently, if the argument presented by Lacan contributes to the understanding of this difficulty

detected by Piaget. These are precisely the points that, together with the necessary resumption of Piaget and Szeminska's analysis, through the aforementioned scope, will guide the course of this article.

Piaget and Szeminska on the child's difficulty of access to the logical notion of conservation of quantity at preoperational stage

Piaget and Szeminska (1975), in the book The Child's Conception of Number, aim to confirm the hypothesis that the child accesses the logical notion of number as a representation of quantity when he/she can access the notion of conservation of quantity by comparing quantities correctly. What does that mean? That means that these researchers are seeking to confirm the hypothesis of the number as a synthesis of categorization and serialization operations.

Thus, according to Piaget and Szeminska (1975), for the construction of the notion of number the latter needs to be conserved as quantity, even if there is change in the spatial distribution of the elements considered. In this sense, according to Nogueira (2011), said researchers argue that the child's ability of successfully counting objects in the matrix of quantification rather than of memorization of a sequence of number-words used in counting is progressive and internal, depending on the coordination of various actions over objects.

Rangel (1992) systematizes the actions that the child needs to develop over the objects so that he/she actually possesses the number. Such actions, we can say, are: categorization – to gather the objects to be counted, separated from those which will not; serialization – ordering of objects so that all of them can be counted, that is, so that none is left out and is counted just once; the ordering of learned names to enumerate objects in the direction of conventional succession without forgetting names and without employing the same name more than once; the two-way correspondence or term-to-term correspondence – to establish the name-object two-way and reciprocal correspondence; the principle of cardinality – to understand that the total quantity of elements of a collection can be expressed by a single name.

Piaget and Szeminska "then come to the conclusion that the construction of the number happens step by step, from the union of the inclusion and serialization of elements, reaching the operational entirety of the set of finite integers, 'inextricably cardinal and ordinal'"(Duro & Cenci, 2013, p. 4).

> Understanding the number means synthesizing the hierarchical order and inclusion. Thus, it is important that when counting the child keeps an organized mental order so as to count each element just once... However, if only this arrangement was required, the child would not understand the quantity that each number represents, since he/she would count it one at a time, disregarding the group of elements. Therefore, the hierarchical inclusion is considered so that the numbers are quantified as a group, in an orderly and inclusive manner; 1 before 2, but included in 2, 2 before 3 and after 1 and, thus, 1 and 2 included in 3 and so on (Duro & Cenci, 2013, p. 5).

Consequently, to Piaget and Szeminska (1975), in the investigation approached in this article, investigation whose concern is the origin of the number, it is essential to identify the moment when children begin to abandon a reasoning based only on the qualities of objects, without conservation of quantities, to have a reasoning founded on the quantities that they establish. In said investigation Piaget states that **knowing by heart** the sequence of words used in counting is not sufficient for constructing the structure of the number. Additionally, it is worth noting that, to Nogueira (**2011, p. 110**), such statement has "been evoked to justify the emphasis on logical activities to the detriment of numerical activities in Childhood Education". This is important because "the abstract character of the discipline is considered as one of the reasons for the poor performance of students struggling with math" (Carmo & Simionato, 2012, p. 325).

In this way, Piaget and Szeminska (1975) find that, in general, the child starts to access the number structure only when transitioning from the preoperational stage to the operational stage, that is, usually between 6 and 7 years old. It is from this age "that counting becomes reliable and, therefore, begins to be the way most used by children for determining quantities" (Nogueira, 2011, p. 112). According to

Piaget and Szeminska, the two-way correspondence or term-to-term correspondence does not guarantee the conservation of quantity by the child, given that a preschool child can easily line up objects in correspondence to others already lined up, without believing in the conservation of this quantity if the density of one of the sets changes.

In the tests to which the children have been subjected in this research, the researchers focused on the main "**qualities**" or "**requirements**" for the existence of the number: the conservation of quantities presented as condition of any knowledge; the two-way correspondence or term-to-term correspondence; the determination of cardinality and ordinal principle, seeking for the confirmation of the hypothesis that the number is the original synthesis of categorization and serialization.

As mentioned in the introduction, this article seeks to have as privileged scope one of those requirements set by Piaget and Szeminska (**1975, p. 23**) in the mentioned tests proposed, namely the notion of conservation of quantities. In this sense, it is worth saying that said authors begin by arguing that all knowledge of scientific nature or derived from common sense supposes a system, explicit or implicit, of conservation principles. In this sense, they argue that the notion of conservation is a necessary condition of rational activity, without a concern about knowing whether it is sufficient. Thereby, to Piaget and Szeminska, the arithmetic thinking falls exactly within this rule: the notion of conservation is a necessary, though not enough, condition of any mathematical intelligence. The **researchers** "also relate conservation of quantity to the construction of quantity itself, stating that the child '... only discovers the real quantification when he/she becomes able to build entireties that are preserved" (Duro & Cenci, 2013, p. 7).

According to **Piaget and Szeminska**, the fact that the conservation requirement is not innate but rather constructed is showed with results obtained through the technique adopted (in order to confirm, or not, the conservation of quantity in the child), which consists of, for example, first presenting the child with two cylindrical containers with the same dimensions, called A_1 and A_2, with the same amount of liquid, and with such equality being visible in the very measure at which the levels are equal. Then, in front of the child, the liquid contained in A_2 is poured into two other containers called B_1 and B_2, which are smaller and similar to one another. Summing up the point of the method, we can say that the problem of conservation is investigated through the fact that before the child's eyes there is a true submission of the liquids to all the apparent possible deformations. The question asked to the child is whether the quantity poured from A_2 to (B_1 + B_2) remained the same as that of A_1.

> If necessary, the liquid in B1 can then be poured into two smaller, equal containers (C1 and C2), and in case of need, the liquid in B2 can be poured into two other containers C3 and C4 identical with C1 and C2. Questions as to the equality between (C1 + C2) and B2, or between (C1 + C2 + C3 + C4) and A1, etc., can be made (Piaget & Szeminska, 1975, p. 25).

According to Piaget and Szeminska, the results found suggest the demonstration that the conservation of continuous quantities seems to occur only at the intellectual development stage of the child, usually between 07 and 12 years old, that is, conservation is gradually constructed. Thus, at the stage generally comprehending 02 and 07 years old, known as preoperational stage, the results seem to show that the continuous quantities are not constant. During this stage in which "perception still rules over reasoning" (Queiroz et al., 2009, p. 300), the child takes as something natural that the quantity of liquid varies with the shape, dimensions and number of containers in which it is poured. Despite the reasons presented in favor of non-conservation – difference of level, size, number of containers etc. – varying from one subject to another or from one moment to another, all perceived change is regarded as causing a change in the total amount of liquid. Thus, "perception of the apparent changes is therefore not corrected by a system of relations that ensures invariance of quantity" (Piaget & Szeminska 1975, p.25). According to Piaget and Szeminska (1975, p. 30), such reactions are the child's most primitive ones in the face of the issue of conservation of quantities and its meaning is clear: the subject is not at all inclined to admit that the same quantity of liquid can remain invariant with the changes linked to their transfers.

Thus, these authors argue that there is a possibility of error at stake in the immediate perception that seems to work in the opposite direction to the work undertaken by the intelligence of the elaboration

of the notion of a constant quantity, that is, in the opposite direction to the dimension of the judgment, as the judgment only works when the immediate perception is not sufficient to provide the subject with the information that a certain quantity of liquid does not vary if it is poured from an A-shaped container to one or two B-shaped containers. They argue, therefore, that the very problem of psychogenic analysis, before being that of figuring out why said perception is misleading, is to find out why the subjects of a certain level trust in it with no hesitation, while others fix and complete it through intelligence (Piaget & Szeminska, 1975); is to find out why the child cannot access, at this stage, the act of intellectual understanding, which will be both more important and more easily analyzable the more misleading is the immediate perception.

Concerning the reason for the child's failure to access the notion of conservation of quantity, highlights, in this regard, the insufficient quantification of perceived qualities and the incoordination of quantitative relations at stake in perceptions (Piaget & Szeminska 1975). For instance, when the subject concludes that the quantity increases because the level was raised, he/she forgets to consider the width of the container and, if he/she does it next, he/she forgets the level etc. This combination ends up in a type of reaction with two characteristics: 1) incessant contradiction, as the subject now believes ones thing, now believes the opposite, without, because of that, considering that he/she could have been wrong before; the second characteristic is in tune with the logical contradiction: 2) the impression the child has, which derives from his/her ignorance about the notion of a total quantity, that is, from an inability to reason, except when he/she considers one relation at a time, without coordinating it to the other ones (Piaget & Szeminska, 1975).

In this line of reasoning, Piaget and Szeminska concludes, in relation to such intellectual development phase of children, that, as already said, the subjects of this phase do not have the notion of conservation of quantity. Moreover, they argue that if, they do not have, that is because they have failed to build the notion of quantity itself, meaning total quantity, due, in turn, to the impossibility of composition of the relations or parties involved, because their spirit does not move beyond the level of gross qualities or quantities. Because, "with concentration being a characteristic of this period, the child remains fixed on a limited number of aspects, being unable to assess all characteristics of the context" (Queiroz et al., 2009, p. 301).

However, among the reactions of subjects who do not access the notion of conservation of quantities, and the reactions of those that postulate it as physical and logical necessity simultaneously, it is possible to observe a certain number of intermediate behaviors that characterize a second stage, although this does not mean that all children will inevitably go through this transition phase. During this phase, the notion of conservation is imposed progressively, and it occurs in such a way that if it is discovered in the case of certain transfers, of which we will need to seek to determine the characters, it is not generalizable to all (Piaget & Szeminska, 1975, p. 25).

Here, in the concluding moment of the establishment of the scope, that is, the difficulty of access to the logical notion of conservation of quantity, it is important for the purpose of this text to bring again Piaget's statement (1954) according to which there are no purely cognitive behaviors, considering "affectivity as a motivator agent of intellectual activity" (Queiroz et al., 2009, p. 304). We think that, by stating that, he allows us to investigate what in the subjective dimension can interfere with the difficulty of learning the notion of conservation of quantities. More specifically in our case, we wonder what, from the psychoanalytic point of view, we could say that may be at stake in the context of this difficulty. To answer that, we propose a psychoanalytic path which, if it does not dialogue in particular with Piaget's logic, will dialogue with the logic that we believe to be the basis to this Piagetian reading, namely Frege's logic (1892/1978a), in his approach of sense and reference. So let us move to that section.

Frege on Sense (Sinn) and Reference (Bedeutung), and Lacan on idealized father and phallic reference

Frege (1892/1978 a), in the classic article *On sense and reference*, states that a sign expresses a sense (*Sinn*) and designates a reference (*Bedeutung*), setting, consequently, a distinction between these two dimensions. According to Lacan (1971/2009), Frege's observation, up to a certain point of the scientific discourse, revolves entirely around the following question: is it the same thing to say *Venus*

and to call it *the morning star or the evening star* – as it has been designated for a long time? Is it the same thing to say *Sir Walter Scott* and to say *the author of Waverley*[3]*?* ...When examining this distinction Frege realizes that it is not possible, in all cases, to replace *Sir Walter Scott* by *the author of Waverley*. Still according to Lacan (1971/2009), in this analysis Frege distinguishes that *the author of Waverley* is one way to designate *Sir Walter Scott,* that is, it conveys a sense (*Sinn*), while *Sir Walter Scott* designates a reference (*Bedeutung*). In the sentence: *King George III sought to find out whether Sir Walter Scott was the author of Waverley*, if we replace *the author of Waverley* by *Sir Walter Scott*, we have a sentence with a different sense: *King George III sought to find out whether Sir Walter Scott was Sir Walter Scott*.

Thus, based on Leibniz, who argues that that which designates an equivalent *reference* can be replaced regardless, Frege has the merit of realizing that replacing a sign by another one with the same *reference* but different *sense* introduces a difference not only when it comes to the sign that expresses the *sense* in the matrix of a synonymic relationship, but also in the *sense* expressed by this sign (Dummett, 1973b). Consequently, Frege argues that a given name or a complete assertive sentence expresses its sense (which is called thought in the case of the sentence) and designates or refers to its reference. According to Dummett (1973a), while Frege understands *sense* (*Sinn*) as the mode of presentation of the object, *reference* (*Bedeutung*) would be the object. In addition, based on that famous example he presents: *Morning Star* and *Evening Star*, Frege claims that such signs would be given names with a relationship of equality, because, despite being different signs with different senses, they refer to the same thing, the planet Venus. It is worth noting that, to Frege, it is the *reference* (*Bedeutung*) and not the *sense* (*Sinn*) that has the truth value. Thus, although it bears differences in its modes of presentation, in its *senses*, there is a level of conservation of the same thing concerning the *reference* with truth value

However, and this is of our specific interest, the Fregean approach to *reference* does not end at the linguistic level; it also concerns the mathematical sphere, in which, for instance, there is no difference in saying 2 + 5 and 3 + 4 (Frege, 1891/1978b). To Frege, unlike a perspective that qualifies these two operations as being dealing with equal things but not the same things, it is the same thing presented in different ways. In the case of the second example given, which is of our specific interest, we are dealing with *reference* at the level of the same quantity presented differently. At this level, therefore, we are dealing with conservation of quantity, despite the change in the arrangement of the elements making up the number as total quantity. To Dummett (1973b, p. 159), the fact that the *reference* does not determine the *sense,* as we can associate the same referent with different expressions and different ways, constitutes all the reason for Frege to employ *sense* and *reference*.

We think that one of the merits of this logical writing that concerns science as a whole – and here, of course, we must include any scientific approach interested in the cognitive development of children, as the Piagetian reading is – is that such writing, for our specific case, indicates that the same quantity can be grasped differently. In addition, the logical Fregean treaty establishes the logic present in modern science, as it supports the possibility of discovery claimed by the latter that there is "a knowledge in the real from which it deduces laws that have a universal value" (Marcos 2011, p. 150). Possibility which is grounded on said dimension of conservation from which "it can predict what is happening in the real through empirical experiences" (Marcos, 2011, p. 150). And it is with this exact modern science that the approach to the notion of quantity in Piaget is tuned. In other words, we identify a relationship between Frege's approach to *sense* and *reference* with respect to the *sense*, as a mode of presentation, which is not a reliable guide in relation to the conservation that may occur at the level of the *reference*, and the Piagetian approach to appearance as a misleading source (Lajonquière, 1992) for accessing the child's logical notion of conservation of quantities at preoperational stage.

Moreover, this logical writing of reference (*Bedeutung*) as being distinct from the sense (Sinn) is also of interest to psychoanalysis. However, Lacan (1971/2009) has something to say about it. In his seminar *On a discourse that might not be a semblance*, he takes a step further regarding this matter by analyzing the myth of the father of the primitive horde, invented by Freud (1913/1996a), as any myth is, to account for an impossible. In this case, to account for the impossible of what a father is. In this

[3] Sir Walter Scott is, indeed, the author of the book entitled *Waverley*.

sense, Lacan seems to indicate that the establishment point of the Signifier of the Father's Name that conveys the law of desire is the same for the Mocking father, who is uncastrated, almighty and, therefore, idealized, and for the Father who introduces the loss of enjoyment, participating in the dimension of the castrated Other. What does that mean? That means that he resumes the definition he had previously given to the Signifier of the Father's Name as a signifier capable of giving sense to the mother's desire (Lacan, 1971/2009), confirming this sense plane, but suggesting another dimension: the dimension of a symbolic that touches the real when writing it. Writing of the real, by the symbolic, operationalized by the castration operation, which, among the various ways of saying it, we can address it as we will do here, through the negativization of the imaginary phallus and, hence, through the raising of the latter to the category of symbolic phallus, presence of the absence of the real object (Becker, 2010). Thus, Lacan (1971/2009) suggests that the sense concerns the sense of the murder of the father.

Considering what we will point below as a first step to the formulation of our argument based on a possible psychoanalytic reading about the child's difficulty of access to the logical notion of conservation of quantity at preoperational stage, we propose first that we take into account that the Fregean approach to *sense* (*Sinn*) and *reference* (*Bedeutung*) is the background of Lacan's analysis. Secondly, let us think that at this point of the murder of the father we also have said dimension of the idealized father, the almighty father and, therefore, the uncastrated father. Thirdly, let us consider that the idealized father concerns the dimension of the father who *presents himself under the appearance* of having the phallus, which is one of the *forms* of *reference* to the enigmatic object that causes the desire in the Other, for being precisely within the dimension of the uncastrated father. Fourthly, let us single out the signifiers that affect this analysis, making the logical categories of *sense* and *reference* reverberate on this analysis, and making this analysis reverberate on these categories as well.

Having made these proposals which we deem critical to the formulation of the first step of our argument, we point out that Lacan indicates that the logical category of *sense* concerns the idealized father who *presents himself under the appearance* of having the phallus, giving a *sense* to the enigmatic desire of the Maternal other. An idealized other which, though concerns the three dimensions, has the prevalence of the imaginary dimension. Here, it is worth anticipating something that will be better explained: if the desiring act, in which the act of learning is included, is debtor of the castration in the Other and, therefore, in the subject, castration closely linked to the maintenance of the enigma of the Other's desire, when the subject is taken by this idealizing dimension, he/she does not accede to this act.

It is important to remember that, just as Corrêa and Pinheiro (2013) point out, in the access to the act of learning there is a certain level of inevitable embarrassment in the child, as a structural piece of data proper of the learning experience, whose paradigm can be found precisely in this stage which Piaget calls preoperational stage and which coincides with the latency described by Freud in *Three Essays on the Theory of Sexuality* (Freud, 1905/1996b). Embarrassment with the new material presented, which, psychoanalytically speaking is about a more fundamental embarrassment with the enigma of the Other's desire. Of course there are vicissitudes in which this embarrassment delays to thin out, introducing, among other symptoms, true sufferings in learning.

Therefore, as a first step in the formulation of our argument, we propose to think of the logical category of *sense* in Frege as concerning the dimension of *appearance*, through which the child, being ruled by it, at preoperational stage, cannot accede to the notion of logical equality of quantities, in Piaget, and as concerning the idealized Other, in Lacan, which can hide the desiring act, and there we include the act of learning preceded and supported by the child's structural embarrassment with the enigma of the Other's desire. It is important to make it clear that we are not restricting here the category of *sense* at stake in the *sense* given to the mother's desire, to the Ideal. Such an analysis would go beyond the limits of the article. What we think we can say and what matters for our purposes is that the Ideal takes its place in this dimension of *sense* given to the maternal desire by the Signifier of the Father's Name.

However, Lacan says that the *reference* (*Bedeutung*) is always phallic. We think that by saying that, as already pointed, Lacan suggests another dimension of the Signifier of the Father's Name in addition to giving a *sense* to the enigmatic desire of the Maternal other: the dimension of a symbolic that

touches the real when writing it. Nevertheless, it has to be said here already: the real of psychoanalysis is not the real of science and, therefore, is not the real at stake in the plane of conservation of quantities in Piaget, although it is intimately linked to it, causing such statement by Lacan about *reference* in its relationship with the real in psychoanalysis to contribute to the understanding of the child's difficulty of access to the logical notion of conservation of quantities, detected by Piaget, at preoperational stage. Because, "in psychoanalysis, we cannot come up with universal laws, deduced from experience, to predict what such a subject would do when put in such situation" (Marcos, 2011, p. 150).

Lacan (1964/1988), to talk about the real in psychoanalysis, goes back to the Physics of Aristotle, who, in his study of the causes that produced different effects in reality, spoke of two of them whose *efficiency* fails due to lack of intentionality. In this dimension of accidental cause, Aristotle will break it down into two types: *tyche* and *automaton*. Reading this Aristotelian approach, we can say that both dimensions comprised by the accidental cause concern the dimension of *chance*. But we can also say that, while the notion of *automaton* seems to designate the signifier *chance* that is sent into the game of signs and then returns, the *tyche* can be read as the *chance* of the real: the faulty meeting, beyond the *automaton*, the dimension of what repeats as fault. Thus, to Lacan (1964/1988), we could say that in the real of psychoanalysis there is also a conservation dimension, considering that repetition is quintessentially that which always returns to the same place, even if under different *modes of presentation*, as well evidenced by the symptoms considered new, which, within our new reading, are not that new, since in such manifestations there is a dimension that repeats concerning the real of the structure. But this would be material for another work.

However, such repetition, as we said, is the repetition of fault. What does that mean and how could we think of the real in psychoanalysis in relation to the real of science and, therefore, the real at stake in the plane of conservation of quantities in Piaget, since there is something that brings them close (the conservation plane concerning repetition), although they are not the same? We could say that the real in psychoanalysis is the tychic (*Tyche*) repetitive incidence of the accident, of the break, of the hole, of unpredictability, of surprise in the dimension of the smooth functioning of predictability laws. It is the faulty incidence, under the modality of the contingent, in the dimension of what repeats as predictable and stable, just as the scientific systems are. And this real, it must be said, is founded by the symbolic, by the Other's knowledge, constituting as the faulty point in this knowledge. Such faulty point in the Other's knowledge introducing its logical inconsistency is found traumatically by the child in the gaps, holes of his/her speech, in the points where something resists to the glue effect, "what does he wants when he says it to me"? The Other's desire which goes beyond or does not cover what this Other says, what he/she intimates, what he/she makes arise as sense, from which "the fault of the Other is put on display, is announced" (Pisetta & Besset, 2011, p. 321). And it is precisely before this unbearable embarrassment, before this tychic real (*tyche*) in the Other's knowledge, that the child structurally appeals to the idealized Other.

But how does this analysis of a possible diverging and converging point in relation to the real in psychoanalysis and its relationship with the real in science and, therefore, the real in the plane of conservation of quantities, in the genetic epistemology, dialogue with our object in this article, which is the child's difficulty of access to the notion of conservation of quantity at preoperational stage detected by Piaget? At this point, to answer this question, we should resume what we were formulating on what Lacan seems to intend to say when, by taking Frege's text for analysis, he says that the *reference* is always *phallic*. As we said, we think that when Lacan says so he is suggesting another dimension of the Signifier of the Father's Name, in addition to giving a *sense* to the enigmatic desire of the maternal Other: the dimension of a symbolic that touches the real when writing it.

Writing of the tychic real (*tyche*) by the symbolic, operationalized by castration operation, which we are addressing in this text, by the negativization of the imaginary phallus and, consequently, by the raising of the latter to the category of symbolic phallus. Therefore, from this step by Lacan we can say that within a psychoanalytical reading *reference* concerns the covering of the real of the inconsistency which, paradoxically, at one stroke, points to it (the real), since, as already pointed out, the phallus is the presence of the absence of the real object (Becker, 2010). To help us understand the notion of symbolic phallus as a logical operator that manages to touch the real by writing it, we have Lacan's note (1972-73/1996) on the phallus as a key point, the extreme point of what is stated as cause of desire.

> The passage to “mediation”, is indeed nothing other than this au moins un that I underlined and that we rediscover in Peano through this n+1 always repeated, the one that in a way presupposes that the n which precedes it is reduced to zero. In what way? Precisely, by the murder of the Father. By this ...this mapping out of, as one might, the detour, to use the term of Frege himself, make no mistake, oblique, ungerade way, whose sense of the murder of the Father is referred to a different Bedeutung (Lacan, 1971/2009, p.165).

Thus, while in Frege’s and Piaget’s logic in tune with science the difference in the mode of presentation of quantities is not a safe guide when it comes to the dimension that is conserved, given that, despite the different appearance, the same quantity can be conserved, we can say that, from a psychoanalytical perspective, the child, when he/she has not yet managed to raise the imaginary phallus to the category of symbolic phallus, and is so embarrassed with the tychic real (*tyche*) that he/she may have the safety and conservation of permanent, stable and predictable systems shaken, is also greatly taken by the idealizing dimension in the Other, to which he/she structurally appeals. Consequently, we can say, considering our course: greatly taken by the mode of presentation of quantities.

Differently, we can think, based on Lacan, that such access to the notion of conservation of quantity occurs when the child can minimally overcome his/her embarrassment with the tychic real (*tyche*) capable of unsettling any conservation plane grounded on scientific *reference* and, at one stroke, overcome the idealizing dimension of the Other to which he/she structurally appeals. Such overcoming concerns the assumption of the *phallic reference*. When the imaginary phallus is raised to the category of symbolic phallus, covering and pointing at one stroke the radical fault of the object, doing therefore a writing of the tychic real concerning the logical inconsistency of the knowledge in the Other with which the child, at this stage, due to a structural matter, is already embarrassed already. We can say, thus, that the child with access to the logical notion of conservation of quantities is under the rule of the *phallic reference*, responding via symbolic phallus.

Symbolic phallus which does a writing of the tychic real (*tyche*) concerning the logical inconsistency of knowledge in the Other, which, for repeating as fault, stands as a source of embarrassment before any learning anchored on repetition as predictability and stability, just as the learning of the notion of conservation of quantity is, patented by the scientific notion of *reference* which, in turn, finds in the logical notion of *reference*, in Frege, the source from which it will drink. Therefore, it is worth resuming, we think that it is possible to formulate that, from a psychoanalytical perspective, the child with access to the logical notion of conservation of quantities is under the rule of the *phallic reference*, responding via *semblance*. But not the *semblance* in the imaginary plane, but rather, as mentioned just now, the *semblance* which does a writing of the real as a faulty meeting. The *semblance* in its symbolic dimension that touches this real: a dimension in which the symbolic prevails – a symbolic which, by sustaining the real of inconsistency, in the act, can do something about it, can touch this real when writing it. It is quintessentially the Signifier of the Father’s Name.

Moreover, based on our psychoanalytical framework, namely Freud with Lacan, it is also important to remember that we should not take the Oedipal experience in which the operability of the symbolic father occurs in terms of a strict chronology and understand quickly that said chronology which a Freudian line of interpretation provides us about this experience, around 05 years of age, could concede the herein pleaded possibility of legitimacy of the approximation between the preoperational stage of which Piaget speaks and the Oedipal experience, as we understand, from the reading that Lacan did of this experience described by Freud, that the father is a logical operator. His operability is therefore logical, having a very early influence, by the way.

However, even though understanding this moment as not being that in which the logical incidence of the Signifier of the Father’s Name took place, we think that such moment happens around the age of 5, when the child is still embarrassed with the enigma of the desire of the Other, whose support is the operability of the symbolic father, given that, at this phase, we can reap a number of signs of this embarrassment, for instance, his/her endless *whys*. *Whys* which, according to Lacan (1964/1988), more than a demand for answers, aim at the sustenance of the question by the Other to whom those tireless questions are addressed. *Whys* which seek to put to the test the question by the Other’s desire.

Furthermore, if we consider that something so significant at the symbolic level, as reading and writing are, occurs precisely when this embarrassment is minimally overcome, we can at least sustain, in the matrix of said logical operability of the father, the question, "if we cannot think of this phase as the moment of the incidence, could we think of it as a necessary logical moment toward the point of completion of this operability?" About the aforementioned signs of the embarrassment with the castration in the Other, overtly present at this phase, it is worth highlighting that it is also in this exact plane of a sign of the typical embarrassment of the age (not rigidly, of course), that we are proposing in this article to approach the child's difficulty of access to the logical notion of conservation of quantities, verified by Piaget.

For problematization purposes only, as such point also goes beyond the scope of this article, we can ask whether such early incidence of the Signifier of the Father's Name, of which Lacan speaks, namely the experience known as the Mirror Stage, which occurs between 6 and 18 months old, does not give us a possibility before current claims from research with infants about the early incidence of the number on the child. It is worth saying that, although such research results (See Nogueira, 2011) end up questioning the Piagetian argument about the possession of the number, according to Nogueira (2006, 2011), the subjects of Piaget and Szeminska's research (1975) were children going through the preoperational period only because the analysis needs to set starts, given that it is not possible to say that there are signs of the presence of the number in younger children. But this, as has been said, would be material for a further work.

Final considerations: a contribution, therefore, to Piaget's theory on access to the logical notion of conservation from the Lacanian approach

In this way, considering the course covered so far, as a contribution concerning the absence of the notion of conservation of quantities in the child at preoperational stage, we think that what is at stake is the child's embarrassment with the inconsistency in the Other, introduced by his/her enigmatic desire. What do we mean by this? At the heart of Piaget's argument about the absence of the logical notion of conservation of quantities, as we have seen, what we find is his finding that the child, at this preoperational stage, before changes in the *modes of presentation*, cannot realize that, despite the different appearance, it is the same quantity: he/she fails to realize that "*the apparent may prove misleading*" (Lajonquière, 1992, p. 23). We think that, in this difficulty the child has of not letting himself/herself be influenced by appearances, what is at stake is the appeal to the father introduced by the child's embarrassment with the enigmatic desire of the Other. Faced with this distressing desire, the child finds in the father's dimension, which stands before this subject under the *appearance* of having the *phallus*, idealized, uncastrated father, the possibility of a blind belief in that father.

However, we should say that, in this Piagetian reading of the access to the logical notion of conservation, we can perceive a logical dimension similar to Frege's logic (1892/1978a), already addressed, when he provides us with an approach to *reference* as a logical dimension with a truth value that can be presented in different ways, that is, with different *senses*, defined by Frege as the mode of presentation of the object. Now, what can be said about different modes of presentation additionally to the appearance matter discussed by Piaget concerning the child's difficulty at preoperational stage?

Considering this relationship between Piaget's approach and Frege's logic, we can problematize this theme by taking into account that Lacan (1971/2009), precisely referenced in this article by Frege, goes one step further from this logic by indicating that the *sense* (*Sinn*) in Frege concerns the dimension of the idealized father and by arguing that the *reference* (*Bedeutung*) is always phallic, the covering of the tychic real. Thus, with the support of the relationship between the Piagetian approach and the Fregean logic, we could say that, to Piaget, the dimension of the access to the logical equality of quantity is about the rule of the *reference*. Moreover, with the support of said step further that Lacan takes regarding this logic, we can say that the dimension of the access to the presence of the conservation of the notion of quantity, which, according to Piaget, is missing in the preoperational stage, is about the primacy of the *phallic reference*. However, the *phallic reference*, that is, the assumption that it is the symbolic phallus which causes the enigmatic desire of the Other concerning the tychic real

(Tyche) and the fault point in his/her knowledge, aims to cover and point at one stroke the radical fault of the object for this desire. It is precisely because it aims to write this object as impossible, object as the cause of the desire of the Other, and not object target of that desire, that the *phallic reference* is the covering of the traumatic tychic real (*Tyche*) traumatic, which may figure as repetition of the fault in any conservation plane.

Thus, it seems that the act of learning the notion of conservation of quantities is debtor of the operability of the symbolic father, in the sense of a certain overcoming of the dimension of the idealized father, which, based on Lacan, we can say, is what seems to be at stake in the logical category of *sense* in Frege and of the appearance that obstructs the access to the logical notion of conservation of quantities in Piaget. But also in the sense of a possibility for the child of a certain writing of the tychic, traumatic real (*tyche*), quintessentially, with which the child at this stage is embarrassed. Possibility which, it should be noted, is closely associated with said overcoming of the dimension of the idealized father. In other words, we think that the *phallic reference* confers a certain overcoming of the idealized Other and thus, as we could see throughout the course presented, confers a certain overcoming in the *sense of the mode of presentation of things*, enabling access to the ability of realizing that the same thing can present itself in different ways. That the same quantity can be conserved despite any changes in the arrangement of the elements that compose the number as total quantity. Moreover, that, in spite of the radical existence of the real as conservation in the matrix of the repetition of the fault there is a conservation dimension in the matrix of predictability.

Even with what psychoanalysis can add to this matter, we must acknowledge Piaget's cleverness in realizing that the logical notion of conservation of quantities is only accessed when the child can go beyond the primacy of the mode of presentation of things and accedes to the ability to realize that the same quantity can be presented in different ways, that is, to the notion of *reference* in Frege, we could say. However, if, as we said, Piaget himself (1954) stated that there are no purely cognitive behaviors, considering "affectivity as a motivator agent of intellectual activity" (Queiroz et al., 2009, p. 304), he makes room for us to investigate what in the subjective dimension can interfere with the difficulty of learning the notion of conservation of quantities. Moreover, from the psychoanalytic point of view, we think we can say that what is at stake is the child's embarrassment with the incidence of the tychic real (*Tyche*) concerning the fault point in the knowledge of the Other, introduced by his/her enigmatic desire. In this sense, we believe that this operability of the symbolic father is what confers an overcoming of this embarrassment, making room for such learning. We thus propose that the argument presented by Lacan contributes to the understanding of the child's difficulty of access to the logical notion of conservation of quantities at preoperational stage, detected by Piaget.

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