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ORIGINAL ARTICLE

INTRODUCTION OF THE CONCEPT OF FRACTION AND EQUIVALENT FRACTIONS IN 20TH-CENTURY BRAZILIAN MATHEMATICS TEXTBOOKS

Introdução da ideia de fração e frações equivalentes em livros didáticos brasileiros do século XX

Introducción de la idea de fracción y de las fracciones equivalentes en libros de texto brasileños de matemáticas del siglo XX

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Abstract: This article examines the treatment of the concept of fractions in Brazilian mathematics textbooks published across different periods of the 20th century. The objective is to understand how fractions are introduced and how equivalent fractions are presented in these materials. The study adopts a historical research perspective, using textbooks as primary sources. The findings indicate that textbooks changed over time, particularly in how content was made explicit, with an increasing incorporation of visual elements, color, geometric representations, and everyday contextual situations. The ways in which introductory notions of fractions and of equivalent fractions are addressed reflect the curricular transformations underway or in effect at the time of publication—for example, the emphasis on rules for calculating the least common multiple (LCM) in textbooks from the 1960s, the treatment of fractions through equivalence classes during the Modern Mathematics Movement (MMM), and a more intuitive, context-based approach in the most recent period analyzed.
Keywords: mathematics education; Modern Mathematics Movement (MMM); equivalent fractions.

Resumo: Este artigo trata da abordagem do conteúdo de frações em livros didáticos brasileiros de matemática publicados em diferentes períodos no século XX. Assim, o objetivo é compreender como as frações são introduzidas e como as frações equivalentes são apresentadas em livros didáticos brasileiros de matemática. Trata-se de uma pesquisa histórica, que teve os livros didáticos como fonte. As análises revelam que os livros sofreram mudanças ao longo do tempo, especialmente na forma de explicitação dos conteúdos, como o uso crescente de imagens, cores, representações geométricas e a inserção de situações do cotidiano. As abordagens das noções introdutórias das frações e das frações equivalentes refletem as mudanças curriculares que ocorreram ou vigoravam no período em que os livros foram publicados, como as regras para cálculo do m.m.c. nos livros da década de 1960, a abordagem das frações por classe de equivalência no período do MMM e uma abordagem mais intuitiva e contextualizada nos livros do último período analisado.
Palavras-chave: educação matemática; Movimento da Matemática Moderna (MMM); frações equivalentes.

Resumen: Este artículo analiza el tratamiento del contenido de las fracciones en libros de texto brasileños de matemáticas publicados en distintos períodos del siglo XX. El objetivo es comprender cómo se introducen las fracciones y cómo se presentan las fracciones equivalentes en estos materiales. Se trata de una investigación de carácter histórico que utiliza los libros de texto como fuente principal. Los resultados muestran que estos libros cambiaron a lo largo del tiempo, especialmente en la explicitación de los contenidos, con una incorporación creciente de elementos visuales, colores, representaciones geométricas y situaciones contextualizadas de la vida cotidiana. Las formas de abordar las nociones introductorias de las fracciones y de las fracciones equivalentes reflejan las transformaciones curriculares en curso o vigentes en el momento de su publicación; entre ellas, el énfasis en las reglas para el cálculo del mínimo común múltiplo (m.c.m.) en los libros de la década de 1960, el tratamiento de las fracciones mediante clases de equivalencia durante el Movimiento de la Matemática Moderna (MMM) y un enfoque más intuitivo y contextualizado en el período más reciente analizado.
Palabras clave: educación matemática; Movimiento de la Matemática Moderna (MMM); fracciones equivalentes.

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INTRODUCTION

This article is the result of studies developed in a course entitled "Aspects of the History and Historiography of Mathematics Education" in the *stricto sensu* graduate program, Science Education and Mathematics Education (PPGECEM), at the State University of Western Paraná – Unioeste, Cascavel campus.

The Modern Mathematics Movement (MMM) was an important event that sparked discussions about mathematics teaching and "[...] promised to overcome elitist and ineffective teaching, promoting interest, curiosity, and learning" (Búrigo, 2006, p. 42). This movement stemmed from debates about the necessary renewal of mathematics teaching in Brazil and worldwide, beginning in the late 1950s (Alves, 2020).

Aspects of the MMM that impacted mathematics teaching can be seen in textbooks. According to Valente (2008), these materials, from other eras, serve as a means for researching the history of mathematics education. Thus, this study aims to understand how fractions are introduced and how equivalent fractions are presented in Brazilian mathematics textbooks, seeking to highlight possible common characteristics and how they were addressed before, during, and after the MMM.

The books were organized and selected in chronological order, respecting parameters from *before* the MMM: *1ª Série Ginasial da Coleção Matemática Curso Ginasial* ("1st Series of the Mathematics Collection - High School Course"), from Osvaldo Sangiorgi (1955) and *Primeira Série Ginasial da Coleção Matemática* ("First Series of the High School Mathematics Collection"), from Ary Quintella (1959); *during* the MMM: *Matemática 1: Curso Moderno, volume 1 para os ginásios* ("Mathematics 1: Modern Course, volume 1 for high schools"), from Osvaldo Sangiorgi (1966) and *Matemática, Para a Primeira Série Ginasial* ("Mathematics, for the First Series of High School"), from Ary Quintella (1966); and *after* the MMM: *Matemática Atual* ("Current Mathematics"), from Antônio José Lopes Bigode (1994) and *A Conquista da Matemática* ("The Conquest of Mathematics"), by José Ruy Giovanni, Benedito Castrucci and José Ruy Giovanni Jr (1998).

Regarding the nature of the study, this is a historical investigation based on an exploratory analysis of how fractions are presented in the selected works. The study seeks to relate the historical trajectory of the MMM in Brazil to its influence on textbooks, highlighting key characteristics reflected in the organization and treatment of fraction content. According to Pinto (2009, p. 64), the MMM in Brazil significantly restructured the "syllabus of secondary schools," impacting the production of mathematics textbooks.

The choice for fractions was based on interest in the topic, since this content is covered across several grades. Regarding the choice of textbooks, the availability of access to these works was the determining factor. Furthermore, the selected books and their authors marked their respective eras. About this, Valente (2008, p. 152) highlights that both Ary Quintella and Osvaldo Sangiorgi "became bestsellers in Brazilian mathematics education."

In relation to Ary Quintella, Valente (2008, p. 154) mentions that, "in the early 1950s, his works for high school and college reached several dozen editions", in addition, "[...] he participated in the organization of mathematics programs for basic commercial and technical courses, at the invitation of the Minister of Education, in addition to serving on numerous committees and boards for mathematics teacher competitions". According to Búrigo (2008, p. 43), the records of the MMM in Brazil and "[...] the references to Osvaldo Sangiorgi are so frequent that the understanding of his protagonism and of the movement itself [...] tend to merge". Furthermore, Sangiorgi acted as an author, organizer and disseminator of proposals for the renewal of mathematics teaching.

According to Alves (2005), Antônio José Lopes, known as *Bigode*[1] ("mustache"), associates Mathematics Education with the late 1970s, considering this period as the consolidation of the movement in Brazil, rather than its beginning. Brazilian Mathematics Education then gained international recognition and acquired an identity as a field of knowledge.

About Castrucci, Duarte (2007) comments that his adherence to the MMM led to the publication of several works, such as *Geometria: curso moderno* ("Geometry: modern course"), from 1967 and 1969. And, in partnership with the Study Group on Mathematics Teaching (GEEM), he published *Introdução à lógica matemática* ("Introduction to Mathematical Logic", 1973) and *Elementos da teoria dos conjuntos* ("Elements of Set Theory", 1967). The GEEM of São Paulo was responsible for topics related to Modern Mathematics (MM), from which his work *Matemática moderna para o ensino secundário* ("Modern Mathematics for Secondary Education") sought to introduce the ideas of sets and structures, which constitute the foundation of Modern Mathematics.

José Ruy Giovanni and his son, José Ruy Giovanni Júnior, are authors of mathematics textbooks in Brazil. With extensive experience teaching the subject, their works have been widely used in elementary and secondary education institutions throughout the country. One of their most recognized collections is "A Conquista da Matemática", intended for elementary school. This series covers grades 6-9 and is known for its didactic approach that facilitates students' understanding of mathematical concepts.

For the analysis, we chose to organize the works into pairs and by periods. We named and separated the periods: before, during, and after the MMM, classifying them based on the time period in which they were produced. Table 1 details the selected works.

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[1] The name "Bigode" was adopted when he wrote his textbooks. According to the website "O Baricentro da Mente", "the name 'Bigode' was adopted when he published his first books. His editor at the time suggested that using the name Antônio José Lopes wouldn't catch on and that it would be better to simply use *Bigode*, and he has been using it in his publications ever since." (O Baricentro da Mente, 2018, online).

Chart 1 – List of works analyzed

Before MMM	During MMM	After MMM
1ª Série Ginasial da Coleção Matemática Curso Ginasial, 11th edition, published by Editora Nacional in 1955, by Osvaldo Sangiorgi.	*Matemática 1: Curso Moderno*, volume 1 for high schools, 5th edition, published by Editora Nacional in 1966, by Osvaldo Sangiorgi.	*Matemática Atual*, published by Atual in 1994, by Antônio José Lopes Bigode.
Primeira Série Ginasial da Coleção Matemática, 68th edition, published by Editora Nacional in 1959, by Ary Quintella.	*Matemática, Para a Primeira Série Ginasial,* 121st edition, published by Editora Nacional in 1966, by Ary Quintella.	*A Conquista da Matemática*, published by FTD in 1998, by authors José Ruy Giovanni, Benedito Castrucci and José Ruy Giovanni Jr.

Source: Organized by the authors (2024).

Corroborating the study in question, Valente (2008, p. 151) states that textbooks represent important sources for analyzing mathematics teaching from a historical perspective:

> It is perhaps possible to say that mathematics is the discipline whose historical trajectory is most closely tied to textbooks. From its origins as military technical knowledge, through its rise to general school culture, the historical trajectory of the constitution and development of school mathematics in Brazil can be read in textbooks.

Therefore, textbooks are one of the sources responsible for the constitution of knowledge that, historically, has been transmitted through manuals throughout the school years of children and young people.

Brief Contextualization of the Modern Mathematics Movement (MMM) in Brazil

At the end of World War II, in the United States and some European countries, committees and conferences were established to discuss reforms in secondary mathematics education[2] as it was necessary to adapt its teaching to emerging scientific, technical, and economic advances permeated by new technologies. Thus, initiatives to reform mathematics education were developed between the mid-1950s

[2] "Art. 1. Secondary education has the following purposes: 1. To develop, in continuation of the educational work of primary education, the integral personality of adolescents; 2. To emphasize and elevate, in the spiritual formation of adolescents, patriotic consciousness and humanistic consciousness; 3. To provide a general intellectual preparation that can serve as a basis for higher studies of special training for two parallel courses: the classical course and the scientific course" (Ministério da Educação, 2020, online).

and early 1970s, taking the form of an international movement known as Modern Mathematics (Matos & Valente, 2010).

In Brazil, the strand of these changes became known as the Modern Mathematics Movement (MMM). Initially, its promoters intended to reconfigure the mathematics curriculum in secondary schools[3], later expanding its scope to include primary school. The MMM sought to bring the mathematics taught in primary school closer to that developed by researchers and taught in higher education. According to Sangiorgi (1966, p. 1), "it is necessary to overcome, with honest and constructive work, the legacy of an anachronistic teaching of Mathematics".

Changes to the curriculum and teaching system were primarily based on international publications, such as books and materials produced by the School Mathematics Study Group (SMSG), translated into several languages, beginning in 1958 (D'Ambrosio, 1987).

One of the important characteristics of the early MMM period in Brazil, that is, the 1960s, was the formation of teacher groups, such as GEEM[4], NEDEM[5] and GEEMPA[6], which played a significant role by coordinating teacher training courses, creating written materials, and influencing the government, aiming for changes in mathematics teaching programs (Fischer, 2008).

Among the prominent figures in this process are Sangiorgi, Castrucci, Jacy Monteiro, Barbosa, Caroli, and D'Ambrosio, names directly associated with GEEM. This group was the main promoter of the MMM in Brazil (Wielewski, 2008).

Some emphases can be identified in different aspects of the program proposals submitted by the GEEM and approved at the IV Brazilian Congress on Mathematics Teaching, held in 1962 in Belém, Pará, such as the lack of significance in the nullification of the denominator of algebraic fractions, for example (Búrigo, 2006).

Around 1966, several projects, seminars, and conferences were organized to address issues related to teacher training, curriculum reform, and instructional materials, highlighting the role of textbooks as a central medium for disseminating the principles of the MMM (D'Ambrosio, 1987).

On the other hand, by the mid-1970s, the MMM was receiving strong criticism. According to Búrigo (1989, p. 211), "[...] the contrast between algebra and Euclidean geometry in modern mathematics programs" created comprehension problems and, consequently, prevented solutions from being made using set theory.

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[3] "Art. 3. The high school course lasted four years and was intended for adolescents, providing access to fundamental elements of secondary education" (Ministério da Educação, 2020, online).

[4] Grupo de Estudos do Ensino da Matemática ("Mathematics Teaching Study Group", GEEM), founded in October 1961 in São Paulo.

[5] Center for Studies and Dissemination of Mathematics Teaching in Paraná.

[6] Mathematics Teaching Study Group in Porto Alegre, founded in 1970.

Soares (2001) mentions that the content of set theory became so popular that, at certain times, terms such as "Modern Mathematics" and "Set Theory" merged, assuming similar meanings, making set theory the main characteristic of Modern Mathematics.

Thus, the study of didactic works provides clues for understanding MMM, understood as articulation and organized movement, especially with regard to the changes expressed in textbooks (Santos, 2015).

Fractions in Textbooks Before, During, and After the MMM

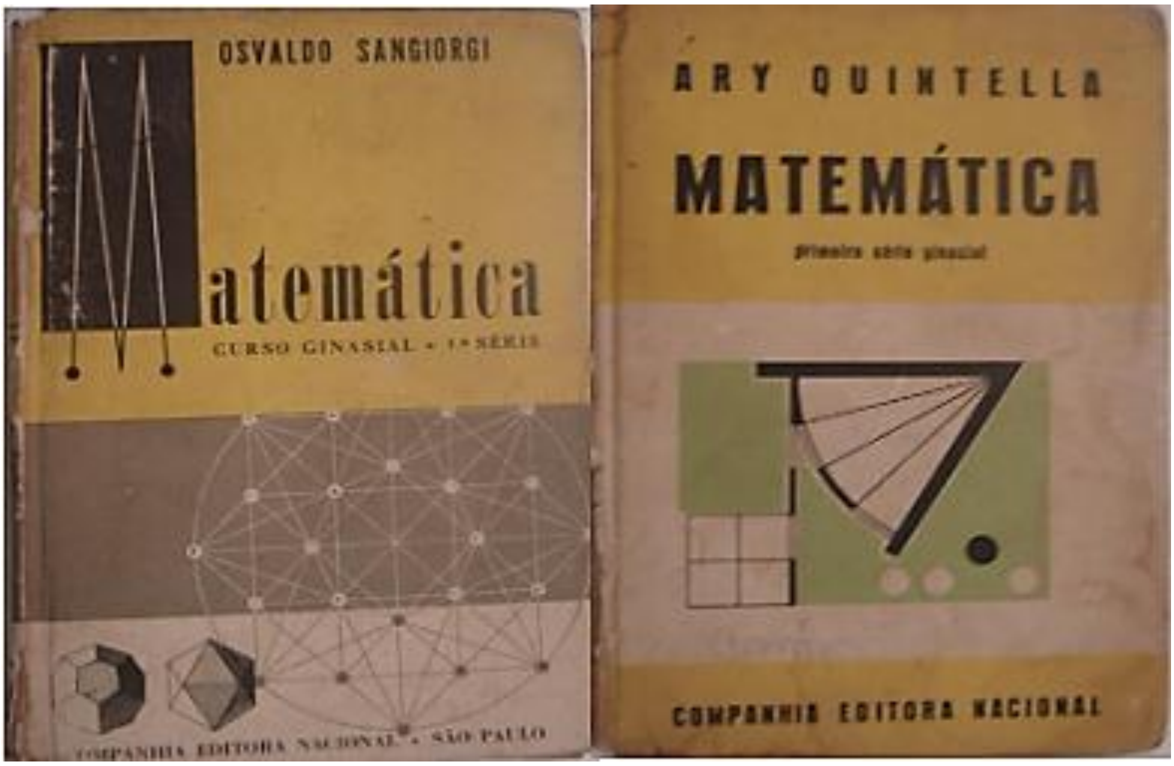
In this section, we will present a description of the selected works, listing characteristics and aspects relevant to our investigation of the content of fractions, highlighting similarities and differences. The order of the books follows the date of publication. The books were sometimes produced by a group of authors, sometimes by a single author.

Books published *before* the Modern Mathematics Movement in Brazil

For this initial period, we selected books by authors Ary Quintella and Osvaldo Sangiorgi. We will analyze the following books: *1ª Série Ginasial da Coleção Matemática Curso Ginasial*, 11th edition, published by Editora Nacional in 1955, by Osvaldo Sangiorgi, and *Primeira Série Ginasial da Coleção Matemática*, 68th edition, published by Editora Nacional in 1959, by Ary Quintella, shown in Figure 1. It was not possible to determine the exact page numbers of both works, as we consulted digitized materials and found copies with different numbers.

Regarding appearance, both feature similar color palettes, shared internal and external layouts, graphic representations on the cover, and the word "mathematics" is prominent. They also feature illustrations of geometric figures. Regarding content, both present their topics divided into four chapters: Whole Numbers, Fundamental Operations, Relative Numbers; Arithmetic Divisibility, Prime Numbers; Fractional Numbers; Legal System of Measurement Units, Usual Units and Measurements. According to Santos (2015), this organization corresponds to the topics established by the Minimum Program for the first grade of middle school.

Figure 1 – Book Cover

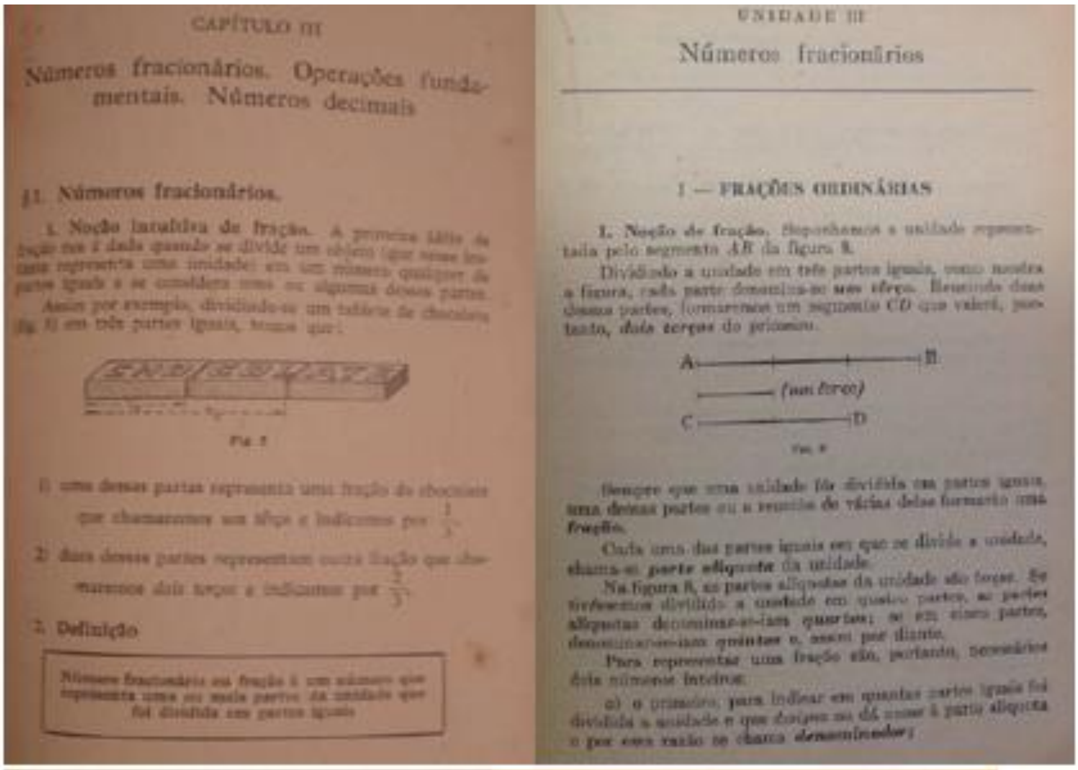


Source: Sangiorgi (1955) and Quintella (1959).

Sangiorgi (1955) presents the idea of a fraction as part of an object, as illustrated in Figure 2. Thus, the author associates a fraction with the representation of a part of a whole that can be equally divided. Based on the idea of a fraction, the author defines a fractional number or fraction as "a number that represents one or more parts of the unit that has been divided into equal parts" (Sangiorgi, 1955, p. 97).

Taking a unit and its equal division (aliquot parts), Quintella (1959, p. 117) presents each of these parts and defines the terms of the fraction, numerator and denominator. According to the author (1959, p. 118), one should write "the numerator and to its right or below, the denominator, separated by a dash", exemplifying that the representation takes one of two forms, $\frac{7}{9}$ or 7/9. Only after presenting the initial ideas and concepts of fraction, Quintella (1959, p. 120) presents its definition: "fraction is the number formed by one or more aliquot parts of the unit" and emphasizes that, for this definition, the "denominator must be at least equal to 2", since, in the case of a denominator equal to 1, its value is, "by definition, equal to the numerator".

Figure 2 – Representation of fraction - Sangiorgi (1955) and Quintella (1959)


CAPÍTULO III

Números fracionários. Operações fundamentais. Números decimais

UNIDADE III

Números fracionários

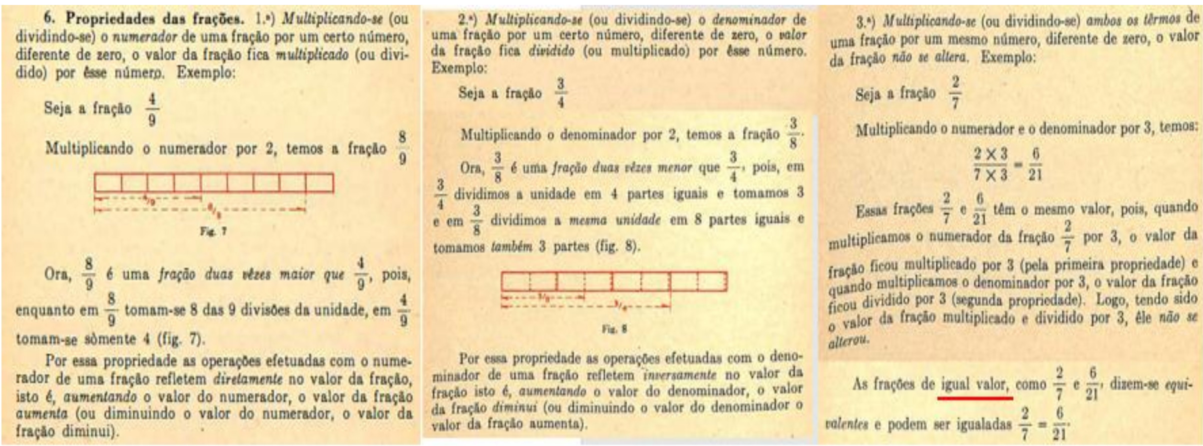
I — FRAÇÕES ORDINÁRIAS

Source: Sangiorgi (1955, p. 97) and Quintella (1959, p. 117).

Sangiorgi (1955) and Quintella (1959) define the simplification of a fraction as the process of dividing its terms by the same number to obtain an equivalent fraction.

Based on the properties illustrated in Figure 3, Sangiorgi (1955) names equivalent fractions as those with *equal value*. According to Santos (2015), in the 1950s, publications described the teaching of fractional numbers through operational techniques and some problem-solving.

Figure 3 – Illustrations of the properties of fractions - Sangiorgi (1955)


6. Propriedades das frações. 1.ª) *Multiplicando-se* (ou dividindo-se) o *numerador* de uma fração por um certo número, diferente de zero, o valor da fração fica *multiplicado* (ou dividido) por êsse número. Exemplo:

Seja a fração $\frac{4}{9}$

Multiplicando o numerador por 2, temos a fração $\frac{8}{9}$

Fig. 7

Ora, $\frac{8}{9}$ é uma *fração duas vêzes maior que* $\frac{4}{9}$, pois, enquanto em $\frac{8}{9}$ tomam-se 8 das 9 divisões da unidade, em $\frac{4}{9}$ tomam-se sòmente 4 (fig. 7).

Por essa propriedade as operações efetuadas com o numerador de uma fração refletem *diretamente* no valor da fração, isto é, *aumentando* o valor do numerador, o valor da fração *aumenta* (ou diminuindo o valor do numerador, o valor da fração diminui).

2.ª) *Multiplicando-se* (ou dividindo-se) o *denominador* de uma fração por um certo número, diferente de zero, o *valor* da fração fica *dividido* (ou multiplicado) por êsse número. Exemplo:

Seja a fração $\frac{3}{4}$

Multiplicando o denominador por 2, temos a fração $\frac{3}{8}$.

Ora, $\frac{3}{8}$ é uma *fração duas vêzes menor* que $\frac{3}{4}$, pois, em $\frac{3}{4}$ dividimos a unidade em 4 partes iguais e tomamos 3 e em $\frac{3}{8}$ dividimos a *mesma unidade* em 8 partes iguais e tomamos *também* 3 partes (fig. 8).

Fig. 8

Por essa propriedade as operações efetuadas com o denominador de uma fração refletem *inversamente* no valor da fração isto é, *aumentando* o valor do denominador, o valor da fração *diminui* (ou diminuindo o valor do denominador o valor da fração aumenta).

3.ª) *Multiplicando-se* (ou dividindo-se) *ambos os têrmos* de uma fração por um mesmo número, diferente de zero, o valor da fração *não se altera*. Exemplo:

Seja a fração $\frac{2}{7}$

Multiplicando o numerador e o denominador por 3, temos:

$$\frac{2 \times 3}{7 \times 3} = \frac{6}{21}$$

Essas frações $\frac{2}{7}$ e $\frac{6}{21}$ têm o mesmo valor, pois, quando multiplicamos o numerador da fração $\frac{2}{7}$ por 3, o valor da fração ficou multiplicado por 3 (pela primeira propriedade) e quando multiplicamos o denominador por 3, o valor da fração ficou dividido por 3 (segunda propriedade). Logo, tendo sido o valor da fração multiplicado e dividido por 3, êle *não se alterou*.

As frações de igual valor, como $\frac{2}{7}$ e $\frac{6}{21}$, dizem-se *equivalentes* e podem ser igualadas $\frac{2}{7} = \frac{6}{21}$.

Source: Sangiorgi (1955, p. 121-123).

Quintella (1959) describes the same properties as Sangiorgi (1955), using the same presentation sequence: multiplication (or division) in the numerator and denominator.

The reduction of fractions to the least common multiple, in which Sangiorgi (1955) suggests three steps and Quintella (1959) only two, is shown in Table 2 below.

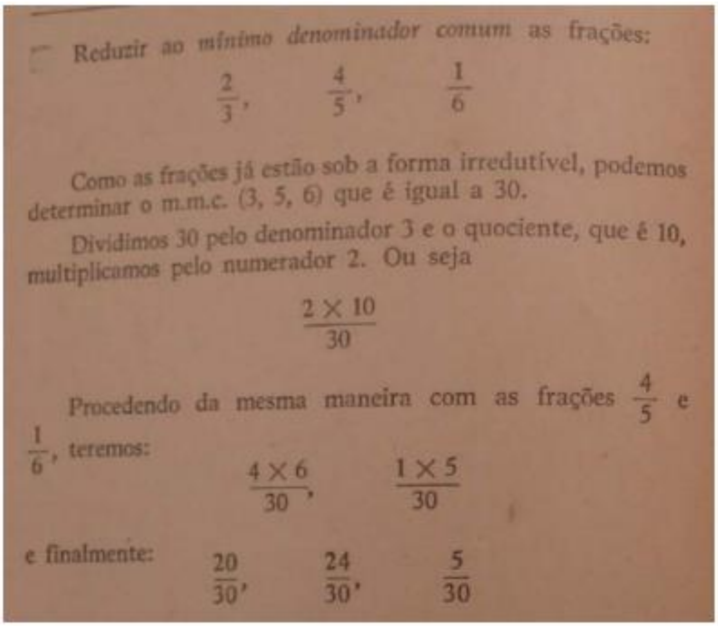
Chart 2 – Steps of reduction to the same denominator

Sangiorgi (1955)	Quintella (1959)
1. Reduce fractions to irreducible form	1. Reduce fractions to their simplest expression
2. Determine the LCM of the denominators of these fractions	2. Multiply the two terms of each by the quotient of dividing the LCM of the denominators by the corresponding denominator
3. Multiply each numerator by the quotient of dividing the LCM by the denominator and the denominator is given as LCM	

Source: Sangiorgi (1955) and Quintella (1959).

Sangiorgi (1955) presents an example, following each of these steps, from which he obtains equivalent fractions, although he does not name them that way, as seen in Figure 4.

Figure 4 – Example of using the rules to obtain equivalent fractions



Reduzir ao *mínimo denominador comum* as frações:

$$\frac{2}{3}, \quad \frac{4}{5}, \quad \frac{1}{6}$$

Como as frações já estão sob a forma irredutível, podemos determinar o m.m.c. (3, 5, 6) que é igual a 30.

Dividimos 30 pelo denominador 3 e o quociente, que é 10, multiplicamos pelo numerador 2. Ou seja

$$\frac{2 \times 10}{30}$$

Procedendo da mesma maneira com as frações $\frac{4}{5}$ e $\frac{1}{6}$, teremos:

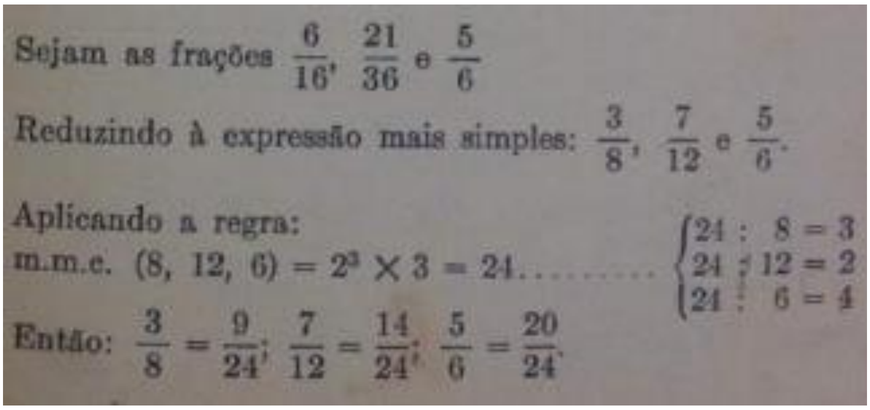
$$\frac{4 \times 6}{30}, \quad \frac{1 \times 5}{30}$$

e finalmente: $\frac{20}{30}, \quad \frac{24}{30}, \quad \frac{5}{30}$

Source: Sangiorgi (1955, p. 104).

The steps are also outlined by Quintella (1959), shown in Figure 5 below.

Figure 5 – Application of the rule to obtain equivalent fractions



Sejam as frações $\frac{6}{16}$, $\frac{21}{36}$ e $\frac{5}{6}$

Reduzindo à expressão mais simples: $\frac{3}{8}$, $\frac{7}{12}$ e $\frac{5}{6}$.

Aplicando a regra:

m.m.c. $(8,\ 12,\ 6) = 2^3 \times 3 = 24$ .... $\begin{cases} 24 : 8 = 3 \\ 24 : 12 = 2 \\ 24 : 6 = 4 \end{cases}$

Então: $\frac{3}{8} = \frac{9}{24}$; $\frac{7}{12} = \frac{14}{24}$; $\frac{5}{6} = \frac{20}{24}$.

Source: Quintella (1959, p. 129).

Therefore, in order to list the differences we highlighted, we created Table 3 below.

Chart 3 – Comparison regarding the representation of the content of fractions

Sangiorgi (1955)	Quintella (1959)
The representation of unity is illustrated by a chocolate bar	The representation of unity is illustrated by a segment
There is no correspondent in this work	Uses the concept of “aliquot part” and “fractional unit”
The book considers a fraction as a fractional number	The book considers a fraction as a fractional number and as a quotient of two integers
The book considers the simplification of a fraction as the division of its terms by the same number, obtaining a fraction equal to the given one	The book considers the simplification of a fraction as the division of its terms by the same number, obtaining a fraction equal to the given one
The book defines equivalent fractions as those of equal value and obtains them by reducing to the same denominator	To obtain equivalent fractions, reduce them to simpler expressions, and then to the same denominator

Source: Organized by the authors (2024).

Based on the summary presented in Table 3, we highlight that the approaches differ minimally. Both authors take the study of fractions as their starting point, introducing techniques and numerical examples in a preliminary manner, prior to the formal presentation of concepts.

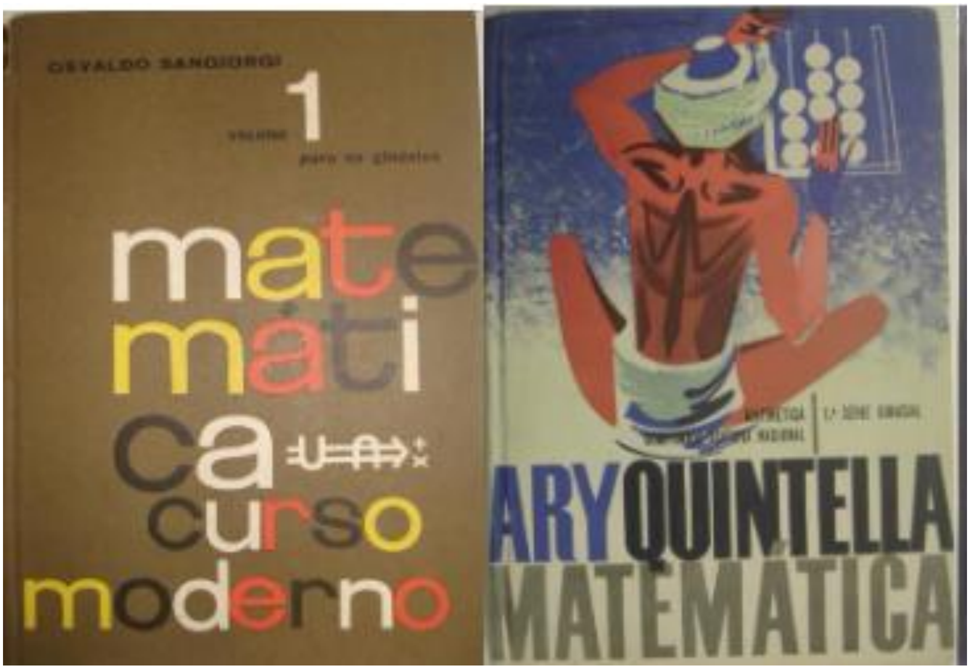
According to Valente (2008a, p. 589), during the 1950s, this more technical perspective on content gave rise to discussions about mathematics teaching, emphasizing the “pedagogical attitudes” that mathematics teachers should adopt, considering “the parallel between students’ mental structures and those of mathematical structures”.

Books Published *During* the Modern Mathematics Movement in Brazil

In this section, we present works by the same authors mentioned in the previous section, Ary Quintella and Osvaldo Sangiorgi, who published mathematics books during the Modern Mathematics Movement in Brazil in the 1960s.

We will analyze the books *Matemática 1: Curso Moderno*, volume 1, for Junior High Schools, 5th edition, published by Editora Nacional in 1966, by Osvaldo Sangiorgi (Figure 6); and *Matemática para a Primeira Série Ginasial,* 121st edition, published by the same publisher in 1966, by Ary Quintella (Figure 6). As for the previous books, we were unable to specify the exact number of pages of these two works due to inconsistencies in the digitized materials.

Figure 6 – Book Covers



Source: Sangiorgi (1966) and Quintella (1966).

Initially, the cover design already appears different from publications from the period preceding the MMM. We see a more striking color palette, with larger letters and without the geometric representations of the past. According to Santos (2015), some changes were made, such as the use of illustrations as a textual and visual resource and the variation in the internal color palette. Miorim (2005, p. 7) corroborates this, claiming that "[...] these changes concern the dimensions of the books, the characteristics of their binding, the printing quality, the gradual incorporation of color, the use of visual resources, and a better distribution of space".

Valente (2008a, p. 606) explains that there is a "new layout in the presentation of school content, the use of fonts and numbers of different sizes and shapes; the inclusion of colors on the inside pages, photographs, and drawings," and that the visual patterns of math books from the 1950s are progressively replaced by a new approach and style. In Sangiorgi's 1966 work, compared to the 1955 publication, we see changes in the "index" in the layout, selection, and presentation of contents. The index in Sangiorgi's work (1966) also differs from that of Quintella (1955 and 1966), who maintained the contents from one book to the next. Below, in Table 4, we list the contents described in the works highlighted in this subsection:

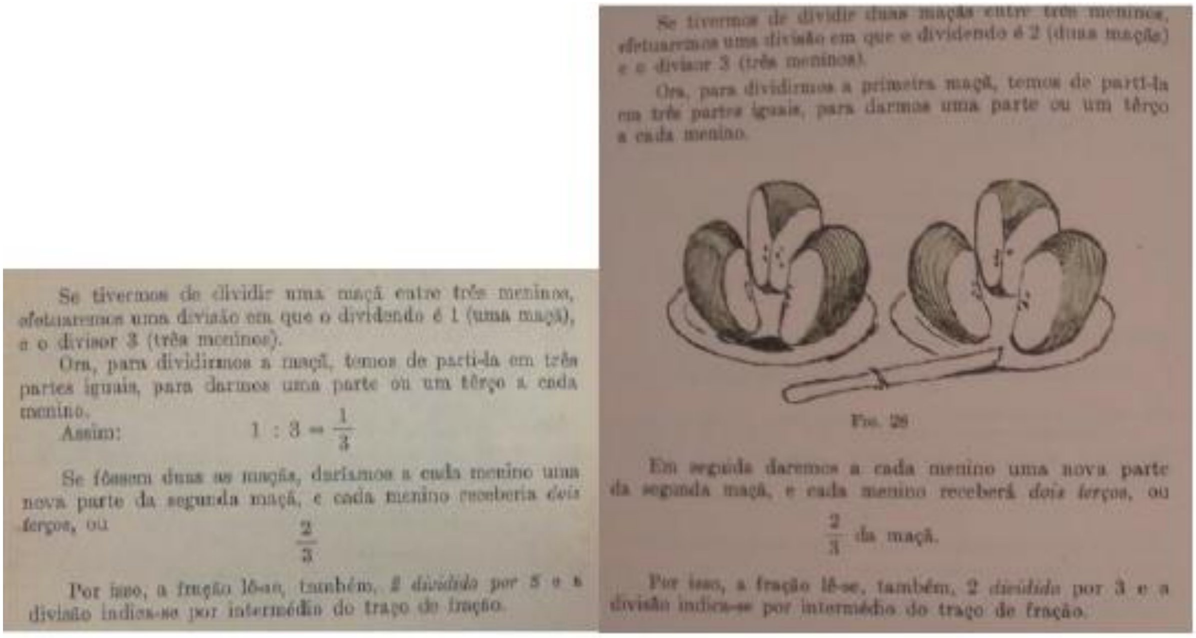
Chart 4 – Description of the indexes of the works of Sangiorgi (1966) and Quintella (1966)

Sangiorgi (1966)	Quintella (1966)
1. Number of numerals – number system – bases	1. Whole numbers
2. Operations with integers – structural properties	2. Divisibility; prime numbers
3. Divisibility – multiples and divisors; prime numbers; complete factorization	3. Fractional numbers
4. Fractional numbers – operations, structural properties	4. Metric system
5. Study of the main plane and spatial geometric figures	
6. Measurement system; decimal system; and non-decimal systems	

Source: Elaborated by the authors (2024).

Even though Quintella (1966) does not assign a unit to the study of sets, the author presents this content when explaining whole numbers, the first topic addressed in the work. As in the previous edition we analyzed, the author relates a fraction to the measurement of a quantity and to the quotient of a division; in addition, the illustrative device used by the author is highlighted (Figure 7).

Figure 7 – Comparison of Quintella's books



Se tivermos de dividir uma maçã entre três meninos, efetuaremos uma divisão em que o dividendo é 1 (uma maçã), e o divisor 3 (três meninos).

Ora, para dividirmos a maçã, temos de parti-la em três partes iguais, para darmos uma parte ou um têrço a cada menino.

Assim: $1 : 3 = \frac{1}{3}$

Se fôssem duas as maçãs, daríamos a cada menino uma nova parte da segunda maçã, e cada menino receberia *dois terços*, ou

$$\frac{2}{3}$$

Por isso, a fração lê-se, também, *2 dividido por 3* e a divisão indica-se por intermédio do traço de fração.

Se tivermos de dividir duas maçãs entre três meninos, efetuaremos uma divisão em que o dividendo é 2 (duas maçãs) e o divisor 3 (três meninos).

Ora, para dividirmos a primeira maçã, temos de parti-la em três partes iguais, para darmos uma parte ou um têrço a cada menino.

Fig. 28

Em seguida daremos a cada menino uma nova parte da segunda maçã, e cada menino receberá *dois terços*, ou

$$\frac{2}{3} \text{ da maçã.}$$

Por isso, a fração lê-se, também, 2 *dividido* por 3 e a divisão indica-se por intermédio do traço de fração.

Source: Quintella (1955, p. 120) and Quintella (1966, p. 156), respectively.

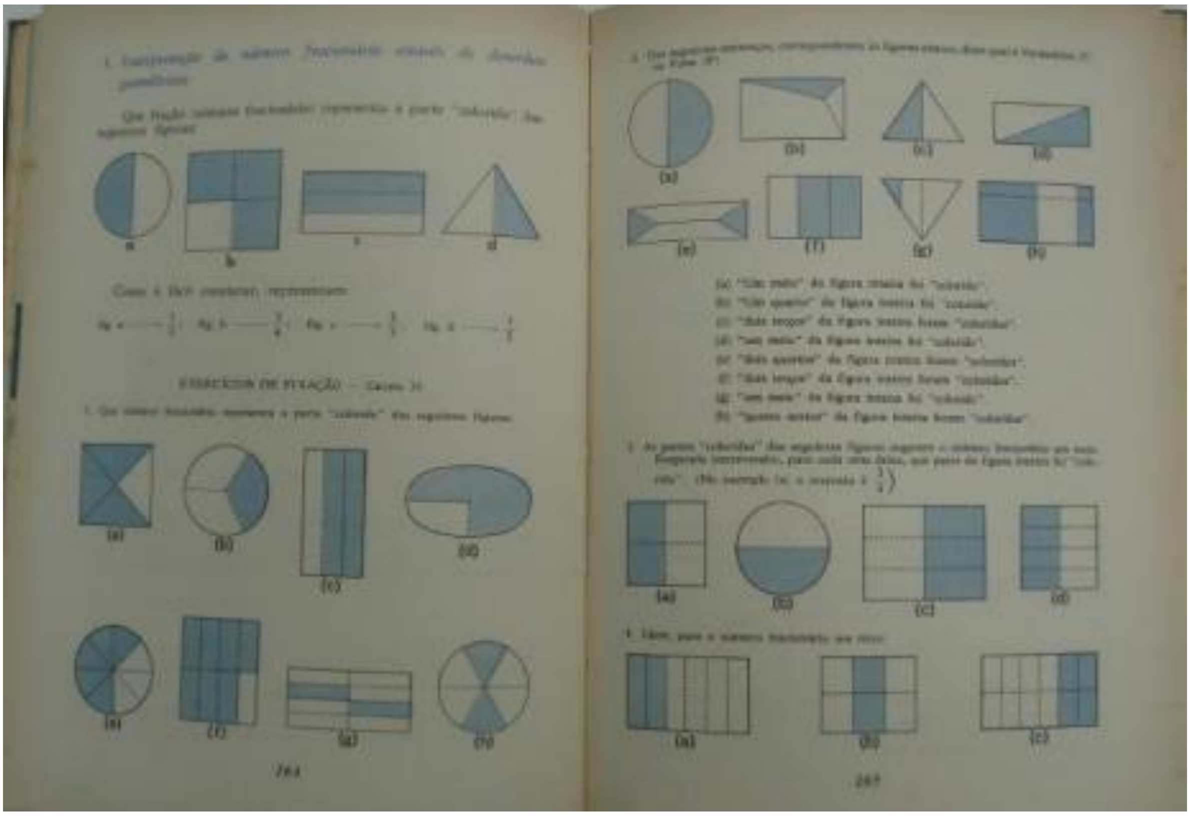
To introduce the initial ideas, as he did in the previous book (1955), Sangiorgi (1966) uses chocolate bars and Quintella (1966) uses an apple pie. Another aspect already mentioned is the visual differences in approaches, particularly the use of colorful illustrations and familiar everyday elements, as can be seen in Figure 7 (Quintella, 1955; Quintella, 1966) and Figure 8 (Sangiorgi, 1966; Quintella, 1966), which present the concept of a fractional number.

Figure 8 – Concept of a fractional number

Source: Sangiorgi (1966) and Quintella (1966), respectively.

For Sangiorgi (1966, p. 162), a fractional unit represents each of the parts obtained by dividing the unit into equal parts. Thus, $\frac{1}{4}, \frac{1}{2}, \frac{1}{5}$ are examples of fractional units, as they correspond to the unit divided into 4, 2, or 5 equal parts, respectively. Even without defining it, Quintella presents an explanation for the fractional number: "when a whole or a unit is divided into equal parts, one of these parts or the union of several forms a fraction of the whole" (Quintella, 1966, p. 153).

Figure 9 – Representation of the fractional number in Sangiorgi (1966)

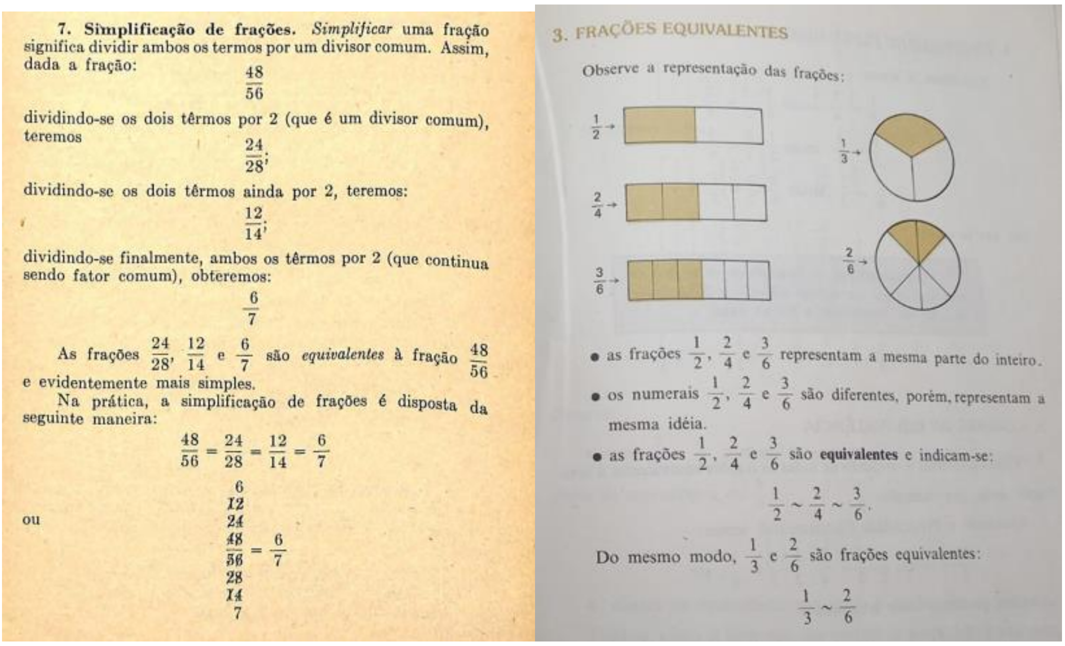


Source: Sangiorgi (1966, p. 164-165).

For the authors, the fraction can also be represented graphically, being composed of two whole numbers (numerator and denominator), called *terms of the fraction*, separated by a horizontal bar (Sangiorgi, 1966, p. 161) or a slash (Quintella, 1966, p. 154), for example: $\frac{7}{9}$ or 7/9. This "distinction" between a *fraction* and a *fractional number* is not made by Quintella; when addressing the topic as "ordinary fractions," he always refers directly to fractions, not using the term *fractional number,* as Sangiorgi does, as exemplified in Figure 9. Quintella (1966, p. 156) relates a fraction to the measurement of a quantity and to the quotient of a division, which seems to associate the fraction with the idea of number.

Regarding equivalent fractions, Quintella (1966) explains them through illustrations and presents numerical examples. Sangiorgi (1966) obtains them through explanations of simplifications, subsequently presenting a numerical example. He does not present geometric representations, as he emphasizes only the rule: "by multiplying (or dividing) both terms of a fraction by the same natural number, one obtains a fraction equivalent to the given one" (Sangiorgi, 1966, p. 176). See Figure 10 below:

Figure 10 – Equivalent Fractions



7. **Simplificação de frações.** *Simplificar* uma fração significa dividir ambos os termos por um divisor comum. Assim, dada a fração:

$$\frac{48}{56}$$

dividindo-se os dois têrmos por 2 (que é um divisor comum), teremos

$$\frac{24}{28};$$

dividindo-se os dois têrmos ainda por 2, teremos:

$$\frac{12}{14};$$

dividindo-se finalmente, ambos os têrmos por 2 (que continua sendo fator comum), obteremos:

$$\frac{6}{7}$$

As frações $\frac{24}{28}$, $\frac{12}{14}$ e $\frac{6}{7}$ são *equivalentes* à fração $\frac{48}{56}$ e evidentemente mais simples.

Na prática, a simplificação de frações é disposta da seguinte maneira:

$$\frac{48}{56} = \frac{24}{28} = \frac{12}{14} = \frac{6}{7}$$

ou

$$\begin{array}{c} 6 \\ \cancel{12} \\ \cancel{24} \\ \cancel{48} \\ \hline \cancel{56} \\ \cancel{28} \\ \cancel{14} \\ 7 \end{array} = \frac{6}{7}$$

3. FRAÇÕES EQUIVALENTES

Observe a representação das frações:

- as frações $\frac{1}{2}$, $\frac{2}{4}$ e $\frac{3}{6}$ representam a mesma parte do inteiro.
- os numerais $\frac{1}{2}$, $\frac{2}{4}$ e $\frac{3}{6}$ são diferentes, porém, representam a mesma idéia.
- as frações $\frac{1}{2}$, $\frac{2}{4}$ e $\frac{3}{6}$ são **equivalentes** e indicam-se:

$$\frac{1}{2} \sim \frac{2}{4} \sim \frac{3}{6}.$$

Do mesmo modo, $\frac{1}{3}$ e $\frac{2}{6}$ são frações equivalentes:

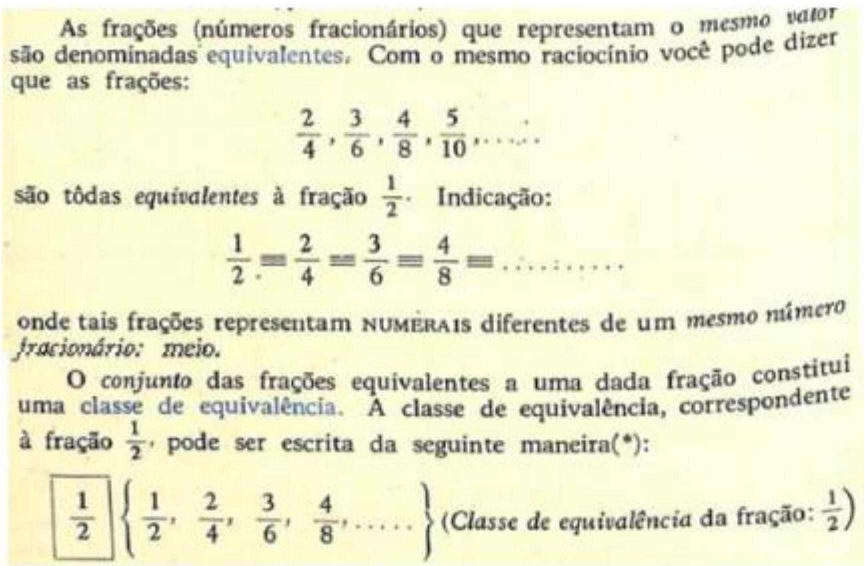
$$\frac{1}{3} \sim \frac{2}{6}$$

Source: Sangiorgi (1966, p. 123) and Quintella (1966, p. 135), respectively.

Sangiorgi (1966) defines equivalent fractions and equal fractions as distinct concepts, since the equivalence is valid when the fractions represent the same value and are only considered equal when they have the same values for numerators and denominators.

In this book, Sangiorgi (1966) includes equivalence classes between fractions. These are presented as a subset of equivalent fractions (Figure 11).

Figure 11 – Definition of equivalent fractions



As frações (números fracionários) que representam o *mesmo valor* são denominadas equivalentes. Com o mesmo raciocínio você pode dizer que as frações:

$$\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \ldots\ldots$$

são tôdas *equivalentes* à fração $\frac{1}{2}$. Indicação:

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \ldots\ldots\ldots$$

onde tais frações representam NUMERAIS diferentes de um *mesmo número fracionário: meio.*

O *conjunto* das frações equivalentes a uma dada fração constitui uma classe de equivalência. A classe de equivalência, correspondente à fração $\frac{1}{2}$, pode ser escrita da seguinte maneira(*):

$$\boxed{\frac{1}{2}} \left\{ \frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\ldots \right\} \text{(\textit{Classe de equivalência} da fração: } \tfrac{1}{2}\text{)}$$

Source: Sangiorgi (1966, p. 174).

Quintella (1966, p. 161) considers fractions of the same value to be equivalent, that is, $\frac{2}{3} = \frac{4}{6}$. He does not mention the equality of fractions and does not explain equivalence classes. To describe these differences, we constructed Table 5.

Chart 5 – Summary of differences in representations of fraction content

Sangiorgi (1966)	Quintella (1966)
The representation of a fractional unit is illustrated by a piece of a chocolate bar	The representation of a fractional unit is illustrated by a piece of apple pie.
Presents the description of fraction as a fractional number; beginning of the concept of rational numbers	Presents the description of a fraction as a measure of a quantity or quotient of a division
Presents equivalence classes as a subset of equivalent fractions	Does not address equivalence classes
Provides geometric illustrations to represent fractional numbers, but not equivalent fractions.	Presents equivalent fractions through illustrations

Source: Elaborated by the authors (2024).

Thus, we see that the content remains the same in these works, with some changes compared to the previous period, particularly in the description and approach to the content, with more diagrams and figures. Regarding the changes, the latest works analyzed represent fractions through figures, whereas the previous books did not use this means of representation. It is also worth noting that Sangiorgi develops equivalent fractions solely through simplification, abstaining from geometric representation; furthermore, he presents equivalence classes as a set of equivalent fractions.

Books *after* the Modern Mathematics Movement in Brazil

Following our historical trajectory, we will analyze the following books from the 1990s: *Matemática Atual*, by Antônio José Lopes Bigode, published by Atual in 1994, and *A Conquista da Matemática*, by José Ruy Giovanni, Benedito Castrucci, and José Ruy Giovanni Jr., published by FTD in 1998. Their covers are shown in Figure 12.

If we compare them to the previous works, we notice a visual change in the approach and color palette of both covers. Both the outside and inside are colorful, showcasing elements that are easily visible and part of everyday life.

Figure 12 – Book covers

Source: Bigode (1994) and Giovanni et al. (1998).

Bigode (1994) focuses on fractions in Unit IV, entitled *Novos Números e as Medidas* ("New Numbers and Measurements"). Within this unit, there are two chapters: *Frações* e *Operações com Calculadora* ("Fractions and Calculator Operations"), which introduces operations with fractions using a calculator. The book contains 220 pages, distributed across four units and thirteen chapters, concluding with a 16-page "teacher's manual."

The "teacher's manual" presents the collection's theoretical assumptions, defines the characteristics of the teaching project, provides information on classroom management, textbook and notebook use, and covers homework, group activities, laboratory work, projects, environmental studies, cross-cutting themes, teaching resources, calculators, and assessment.

Unit IV (Figure 13) covers fractions, decimals, percentages, measurement systems, and calculator operations, demonstrating how to use a calculator to find prime numbers, powers, and fractions. At the end of each concept, it presents some activities with consolidation/repetition exercises and contextualized exercises.

Figure 13 – Unit IV in the book's index

UNIDADE IV — NOVOS NÚMEROS E AS MEDIDAS

CAPÍTULO 9: FRAÇÕES

1. As partes do Tangram 141
2. As frações 143
3. Nomenclatura e representação de frações 144
4. Fração maior que o todo 155
5. Frações equivalentes 158
6. Comparação de frações 161
7. Simplificação de frações 163
8. Frações decimais 164

CAPÍTULO 13: OPERAÇÕES COM CALCULADORA

1. Múltiplos na máquina de calcular 214
2. Caçando números primos com a calculadora 216
3. Potências 218
4. Frações na calculadora 220

Source: Bigode (1994).

In the first pages of *A Conquista da Matemática*, the reader finds a short introductory text, the content of which seeks to emphasize that the abstract nature of the subject may cause some disappointment. Giovanni et al. (1998) concentrate the content on fractions in unit 5 (Figure 14), entitled *A forma fracionária dos números racionais* ("The Fractional Form of Rational Numbers"). The chapter begins on page 102 and ends on page 142, and it is divided in 16 subsections.

Figure 14 – Index – Unit 5

A forma fracionária dos números racionais 5

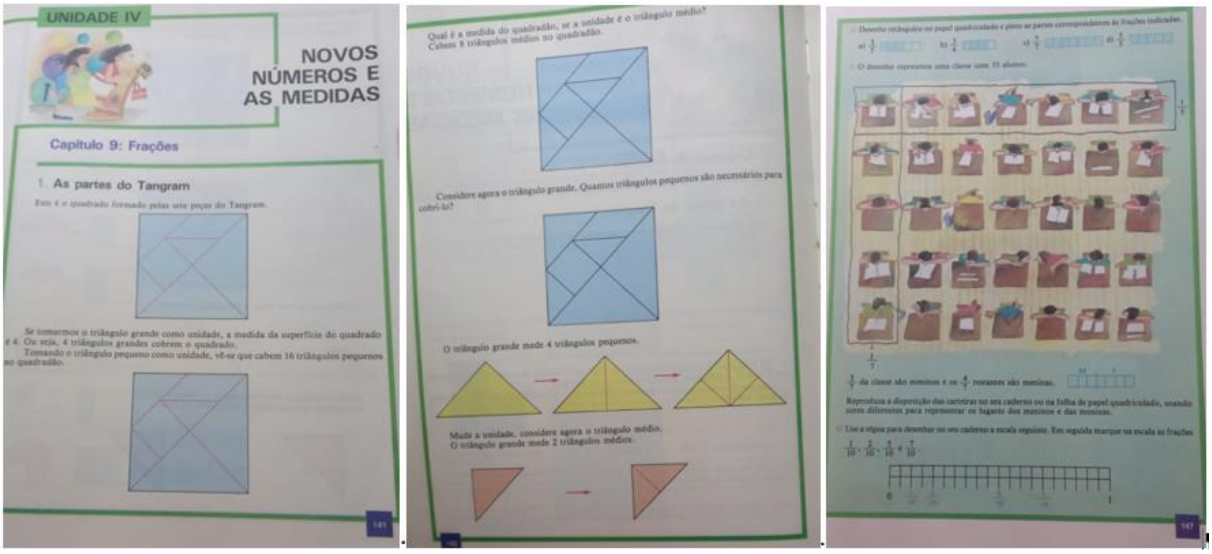
20. A idéia de fração ________ 102
21. Trabalhando com frações de numerador 1 ________ 106
22. Trabalhando com as frações ________ 108
23. Como podem ser as frações ________ 111
24. Frações equivalentes ________ 114
25. Simplificação de frações ________ 116
26. Reduzindo duas ou mais frações ao mesmo denominador ________ 118
27. As frações e a porcentagem ________ 120
28. Comparação de números racionais na forma fracionária ________ 122
29. Adição e subtração ________ 125
30. A forma mista ________ 130
31. Multiplicação ________ 131
32. Divisão ________ 135
33. Potenciação ________ 139
34. Raiz quadrada exata ________ 141
35. Resolução de problemas ________ 142

Source: Giovanni et al. (1998).

At the end of the book is the "teacher's manual," which presents the authors' concerns regarding how the work will be evaluated by teachers, as well as questions such as: "[...] are we providing our students with challenging situations? [...] Are games problem-solving situations? Do they develop reasoning and mental calculation skills?" (p. 1). The manual explores other aspects, such as specific objectives, methodological guidelines, and so-called enrichment situations, which address problems involving curiosities for each unit.

Regarding the introductory content of fractions, each work uses different approaches. Bigode introduces them through tangrams and various situations of everyday application (Figure 15), such as "The drawing represents the class with 35 students, 3/7 of the class are boys and the remaining 4/7 are girls" (Bigode, 1994, p. 147).

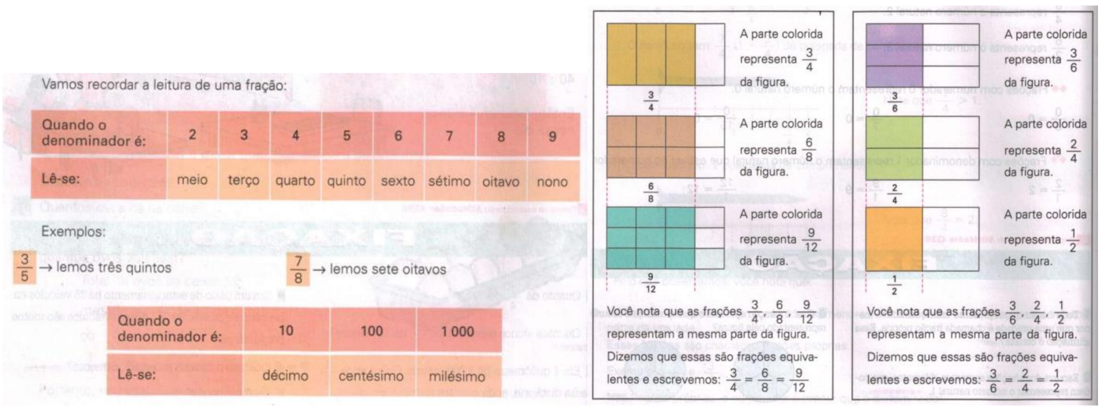
Figure 15 – Introduction to the content of fractions



Source: Bigode (1994, p. 141-142; 147).

In the work of Giovanni et al. (1998), contextualization and the use of historical information about the content are evident. Furthermore, there is a concern with rigor, writing, and the correct way to read fractions (Figure 16), whether in geometric representation or in drawings.

Figure 16 – Reading fractions and geometric representation



Source: Giovanni et al. (1998, p. 108; 114).

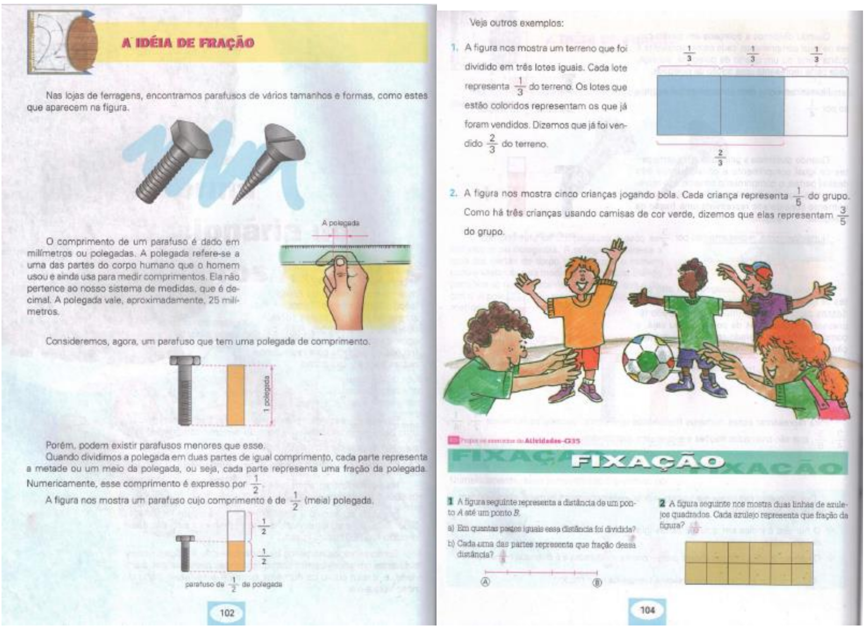
Visually, the works are colorful (Figure 16) with striking and eye-catching tones, using the history of mathematics (Figure 17) as a means of contextualizing and configuring the concept.

Figure 17 – Historical context, use of bright colors, and example exercises

Source: Giovanni et al. (1998, p. 100-101).

The introductory concept of fractions presented by Giovanni et al. (1998) uses familiar and easily accessible elements, such as screws. The content is presented in detail, drawing the student's attention to the concepts, listing possible solutions—that is, solved situations—always before the exercise lists, titled "reinforcement." The activities and examples use everyday actions, such as playing with a ball (Figure 18).

Figure 18 – Concept of fractions; Contextualized activities and reinforcement

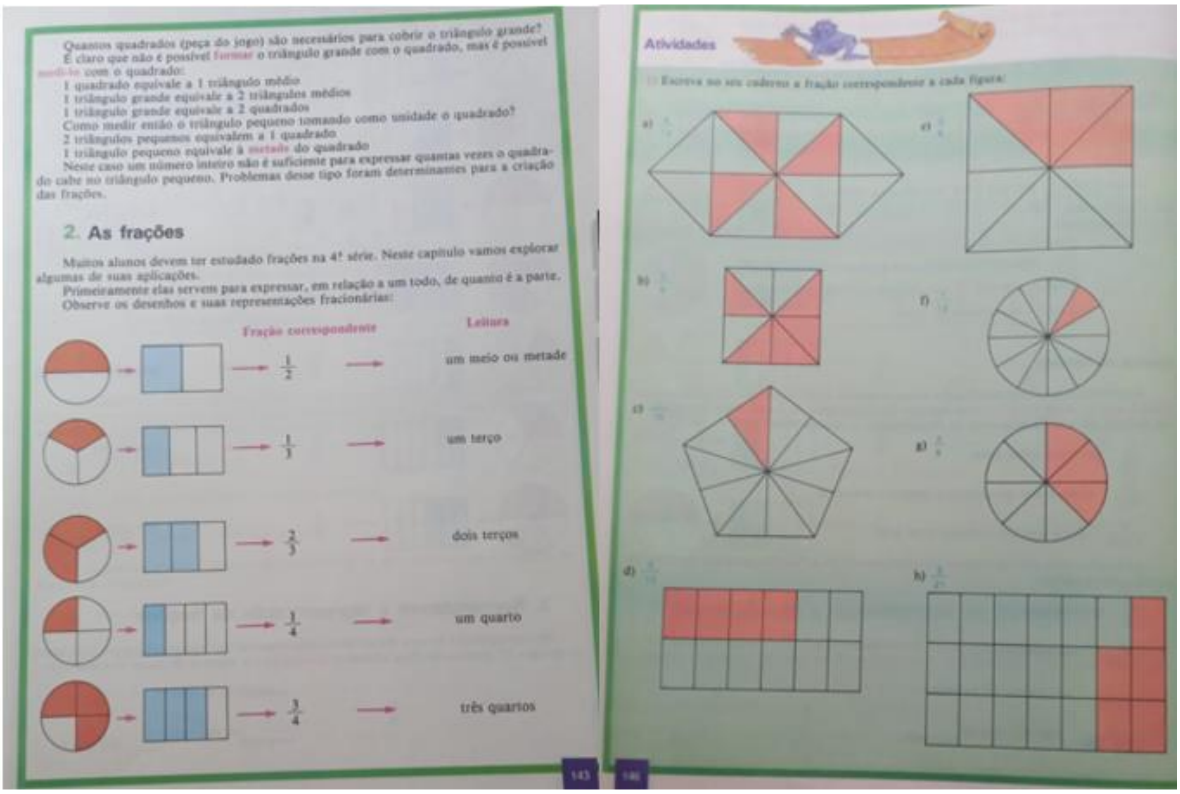


Source: Giovanni et al. (1998, p. 102; 104).

Bigode (1994) explores a more descriptive presentation in his chapters, using examples from a diverse bibliography; For example, in the representation of fractions, "according to Malba Tahan, author of the classic *The Man Who Counted*, the forms $\frac{a}{b}$ and a/b indicate fractions, or the division of a by b, and are attributed to the Arabs" (Bigode, 1994, p. 145).

Figure 19 shows images that present the introductory idea of fractions using geometric representation and proposed exercises/activities.

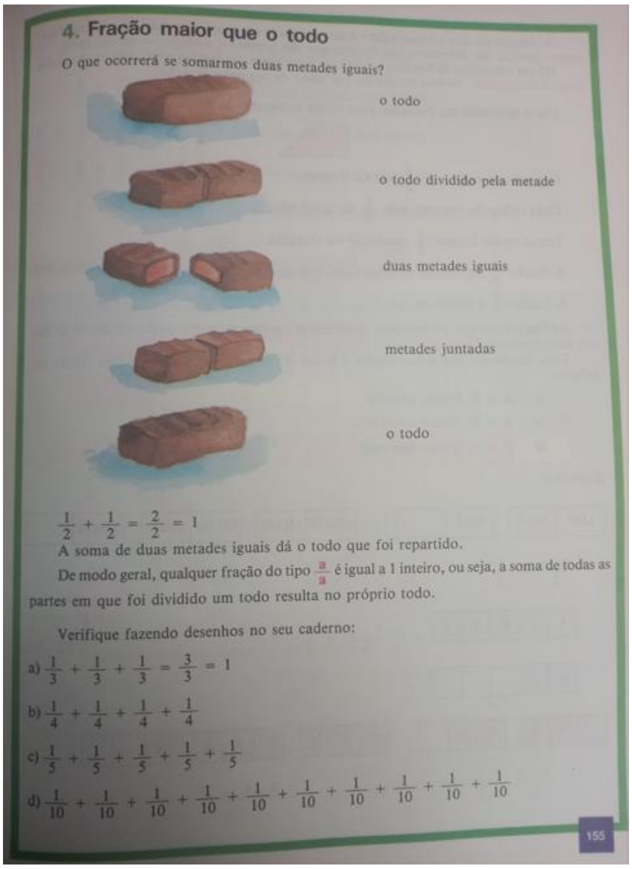
Figure 19 – Geometric representation and activities



Source: Bigode (1994, p. 143; 146).

Similar to the previously analyzed editions, common objects/foods are used, that is, the representation of the fraction as part/whole, with the example represented by a chocolate (Figure 20).

Figure 20 – Part/whole representation



4. Fração maior que o todo

O que ocorrerá se somarmos duas metades iguais?

$\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$

A soma de duas metades iguais dá o todo que foi repartido.

De modo geral, qualquer fração do tipo $\frac{a}{a}$ é igual a 1 inteiro, ou seja, a soma de todas as partes em que foi dividido um todo resulta no próprio todo.

Verifique fazendo desenhos no seu caderno:

a) $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1$

b) $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

c) $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$

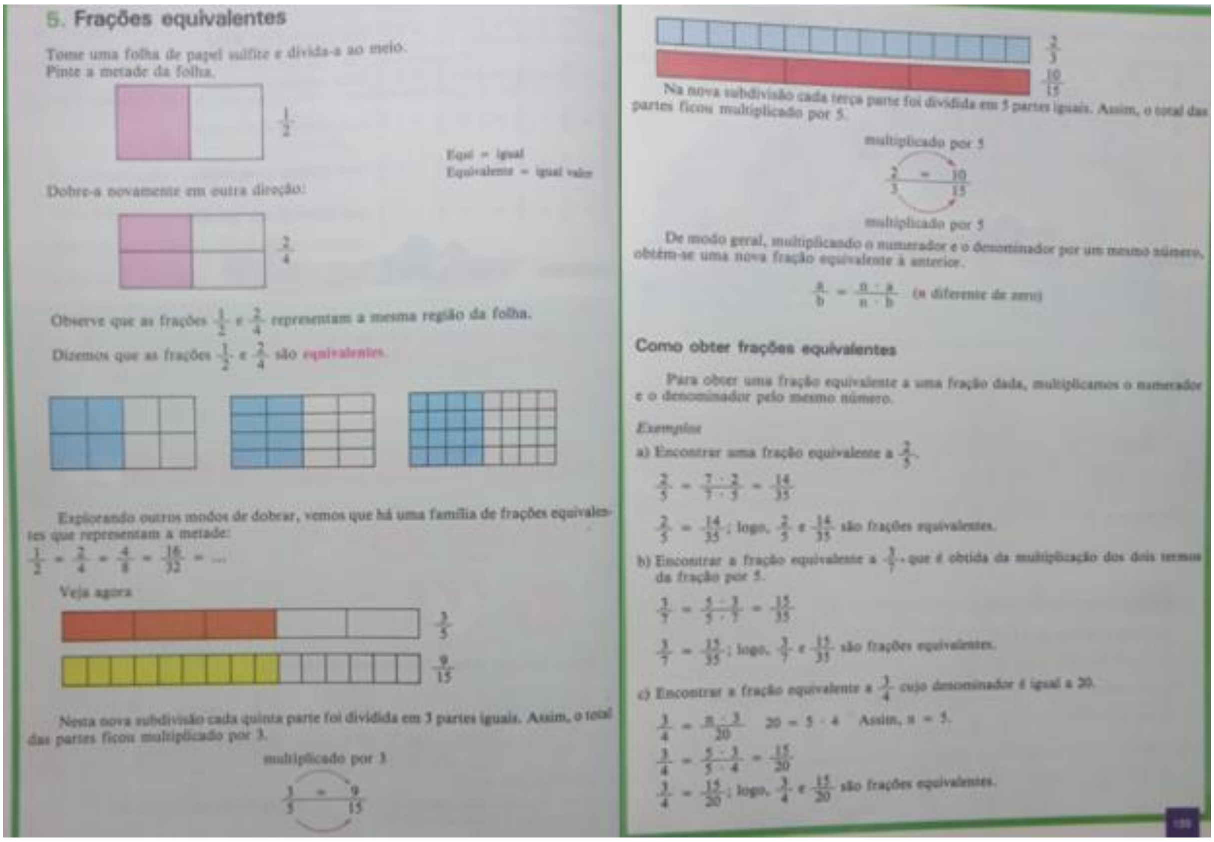
d) $\frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10}$

155

Source: Bigode (1994, p. 155).

The meaning of fractions is expressed through examples in colorful drawings. There is a representation of part/whole and the location of fractional numbers on the number line. Furthermore, at the end of the page, there is a list of activities to reproduce the introduced idea. Regarding equivalent fractions (Figure 21), although the color palette and layout are more diverse and colorful, the mathematical resources for explaining the content are similar when compared to the books by Sangiorgi (1966) and Quintella (1966).

Figure 21 – Equivalent Fractions



5. Frações equivalentes

Tome uma folha de papel sulfite e divida-a ao meio.
Pinte a metade da folha.

$\frac{1}{2}$

Equi = igual
Equivalente = igual valor

Dobre-a novamente em outra direção:

$\frac{2}{4}$

Observe que as frações $\frac{1}{2}$ e $\frac{2}{4}$ representam a mesma região da folha.

Dizemos que as frações $\frac{1}{2}$ e $\frac{2}{4}$ são equivalentes.

Explorando outros modos de dobrar, vemos que há uma família de frações equivalentes que representam a metade:

$\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{16}{32} = \ldots$

Veja agora

$\frac{3}{5}$

$\frac{9}{15}$

Nesta nova subdivisão cada quinta parte foi dividida em 3 partes iguais. Assim, o total das partes ficou multiplicado por 3.

multiplicado por 3

$\frac{3}{5} = \frac{9}{15}$

$\frac{2}{3}$

$\frac{10}{15}$

Na nova subdivisão cada terça parte foi dividida em 5 partes iguais. Assim, o total das partes ficou multiplicado por 5.

multiplicado por 5

$\frac{2}{3} = \frac{10}{15}$

multiplicado por 5

De modo geral, multiplicando o numerador e o denominador por um mesmo número, obtém-se uma nova fração equivalente à anterior.

$\frac{a}{b} = \frac{n \cdot a}{n \cdot b}$ (n diferente de zero)

Como obter frações equivalentes

Para obter uma fração equivalente a uma fração dada, multiplicamos o numerador e o denominador pelo mesmo número.

Exemplos

a) Encontrar uma fração equivalente a $\frac{2}{5}$.

$\frac{2}{5} = \frac{7 \cdot 2}{7 \cdot 5} = \frac{14}{35}$

$\frac{2}{5} = \frac{14}{35}$; logo, $\frac{2}{5}$ e $\frac{14}{35}$ são frações equivalentes.

b) Encontrar a fração equivalente a $\frac{3}{7}$, que é obtida da multiplicação dos dois termos da fração por 5.

$\frac{3}{7} = \frac{5 \cdot 3}{5 \cdot 7} = \frac{15}{35}$

$\frac{3}{7} = \frac{15}{35}$; logo, $\frac{3}{7}$ e $\frac{15}{35}$ são frações equivalentes.

c) Encontrar a fração equivalente a $\frac{3}{4}$ cujo denominador é igual a 20.

$\frac{3}{4} = \frac{n \cdot 3}{20}$ $20 = 5 \cdot 4$ Assim, $n = 5$.

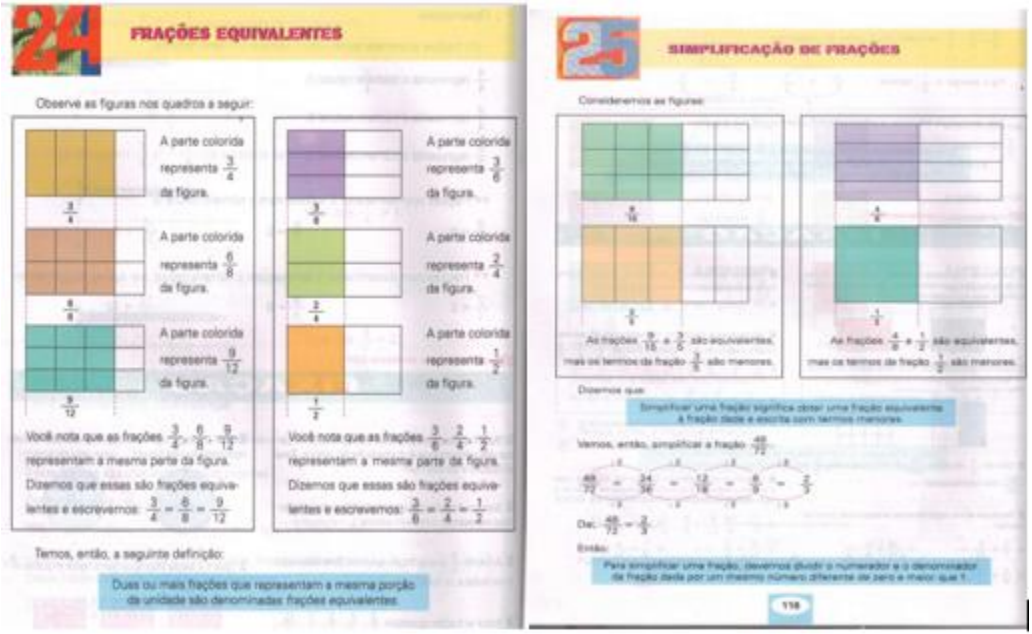
$\frac{3}{4} = \frac{5 \cdot 3}{5 \cdot 4} = \frac{15}{20}$

$\frac{3}{4} = \frac{15}{20}$; logo, $\frac{3}{4}$ e $\frac{15}{20}$ são frações equivalentes.

Source: Bigode (1994, p. 158 e 159).

To obtain these equivalent fractions, the same ideas already seen in the books from the 1960s are used, in which Sangiorgi (1966) uses simplification and Quintella (1966) also explains it with geometric representations. It can be seen that Bigode (1994) addresses equivalent fractions with geometric representation and addresses the idea of multiplication to obtain them. Authors Giovanni et al. (1998), like Bigode (1994), present equivalent fractions before introducing simplification and maintain the format of geometric representations, alluding to the meaning of fraction as part/whole (Figure 22):

Figure 22 – Equivalent fractions before fraction simplification



Source: Giovanni et al. (1998, p. 114; 116).

In order to explain the differences and similarities, we summarize them in Table 6:

Chart 6 – Representation of the content of fractions

Bigode (1994)	Giovanni et al. (1998)
Numerical and geometric representation of fractions	Accuracy with writing and geometric representation of fractions
Introduces content with tangram and everyday elements	Historical contextualization of the contents and presentation of everyday elements
Uses measuring objects	They use measuring objects
Exercises at the end of each chapter	Consolidation activities with everyday elements, arranged among the contents
Addresses equivalent fractions with geometric representation and uses the idea of multiplication to obtain them	They present equivalent fractions with geometric representations in a separate topic, before introducing simplification.

Source: Elaborated by the authors (2024).

Final remarks

According to Valente (2008), textbooks have a prominent place in the written record of the history of school mathematics in our country, reinforcing the potential of this research source. Over the decades, changes were gradually implemented in textbooks, ranging from the most visual aspects to the written representations, including changes in the approaches. According to Fiorentini (2009, p. 21), actions such as "representing by image, making comparisons between the imagined representation and the object of its real action" comprise what is known as "interactionist constructivism." Specifically, regarding fractions, the first two books analyzed still used few illustrations. The introductory strategy adopted was a sequential presentation of rules, followed by a few examples and exercises, with minor variations in this order and, in very few cases, problems being used to introduce the content to be covered, such as the apple example in Quintella's (1959) book. In the works of the 1960s, the use of illustrations as a textual and visual resource and an increase in the number of internal colors, exercises, and examples were already evident. In the works of Sangiorgi (1955; 1966), the most significant change was the inclusion of equivalence classes between fractions, as a section of the chapter devoted to fractional numbers. The changes in Quintella's (1959; 1966) works were slightly modest compared to Sangiorgi's.

In the textbooks published after the MMM, by Bigode (1994) and Giovanni et al. (1998), there is a contextualized approach, connected to everyday life and teaching linked to other areas of knowledge, for example, the use of history to explain concepts. Specifically regarding equivalent fractions, books prior to the MMM (Sangiorgi, 1955; Quintella, 1959) addressed them with a technical emphasis, presenting fixed rules for simplification and reduction to the same denominator. During the MMM, visual changes in the books were accompanied by a greater effort to make the content more understandable and relevant to everyday life. Sangiorgi's work (1966) introduced the concept of fraction equivalence classes, aligning with the MMM's proposal to incorporate more formal mathematical structures into basic education, inspired by set theory and the language of modern mathematics. On the other hand, Quintella (1966), despite presenting the content with illustrations, did not adopt this formalization. In works published after the MMM (Bigode, 1994; Giovanni et al., 1998), equivalent fractions continue to be presented based on the ideas of multiplication and simplification. However, these operations are often preceded by geometric representations, historical contextualization, and everyday situations. One example is the presentation of equivalent fractions even before the introduction of simplification, as in the work of Giovanni et al. (1998).

In summary, it is clear that the approaches to the introductory notions of fractions and equivalent fractions in the books reflect the curricular changes that occurred or were in effect during the period in which the books were published. For instance, textbooks prior to the MMM include rules for calculating the LCM; during the MMM period, fractions are approached through equivalence classes; and in later textbooks, a more intuitive and contextualized treatment of the topic is observed.

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