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A COMPARISON BETWEEN AE AND NPV IN THE REPLACEMENT PROBLEM

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Abstract

When maintenance cost rise and market value of physical assets decreases, it is a sign that the moment to replace the equipment (physical asset) is close. In the traditional literature the main criteria in engineering economics are the net present value (NPV) and the annuity equivalent (AE). However, the differences between both criteria are not explicit. So, the goal of this paper is to compare and contrast the models of NPV and AE, as well as their advantages and disadvantages, in the replacement problem. On conclusion, for the decision-maker, the AE is the best criteria, as it can provide the asset's economic life and conduct the replacement between two assets. The other criterion, the NPV, as it doesn't provide the economic life, but only used for replacement between two assets, it is not as useful as the AE.

Keywords: *maintenance cost, replacement, assets management, annuity equivalent, net present value..*

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1. Introduction

Many authors, in classical papers and books, like Alchian (1958), Park (2002), Park and Sharp-Bette in (1990), Grant (1990) and Thuesen (1971), have used principles of engineering economic, as net present value (NPV) and annuity equivalent (AE), in the physical assets replacement problem. However, in the literature, the differences between both models are not showed explicit, including the advantages and disadvantages of each one.

In the case of replacement of equipment, both criteria are effective for making the decision, as we shall see along this paper. There are several situations in which problems of cash flow must be solved through VPL and others in which the criterion of AE is more convenient. Finally, the goal of this paper is to compare and contrast the models of NPV and AE, as well as their advantages and disadvantages, in the replacement problem. To accomplish this, it will be considered a discrete model with no technological improvement.

1.1. The Physical Assets Replacement Problem

The economic value of physical assets, such as industrial equipment, depends on physical deterioration. So, it is in function of the asset's age (Leatham, Baker, 1981), (Perry et al, 1990), (Park, 2002). Therefore, industrial equipment are vulnerable to devaluation, which can decrease the market value of the assets (Park, Sharpe-Bette, 1990).

Although age represents the effects of physical deterioration of assets, it should take into account how the asset is used. The level of use is a link between the present and the future. The industry manager, when deciding the level of production scale, chooses between using the equipment now and preserving it to use it later (Keynes, 1936). Thus, preservation – which can decrease the effects of age on physical deterioration in assets – means the use of preventive and corrective maintenance in assets. Therefore, such conservation measures can be considered as an economic decision made (Parks, 1979). So, with the proper maintenance, physical assets can be used for much more time than its physical nature allow. For example, it is possible to see vintage cars driving on the streets. However, to make it possible, the companies must be willing to pay a higher price. Finally, the manager should employ the assets in the present, as it is more profitable, so that the more it is used, the shorter the remaining life of the asset, thereby reducing their market value.

This reduction can be viewed under an economic perspective. For economists, the devaluation of assets is a problem of resource allocation, which is based on expectations of future events (Edward & Bell, 1961). In this paper, the physical assets will be studied regarding the economic point of view. Along with that comes the concept of Economic Life. The asset's Economic Life is the length of its usefulness, in a way that the expenses, i.e., annually sum of the maintenance costs and capital costs, are minimum. Therefore, the Economic Life of the asset is the optimum moment to replace the asset.

Because of that, if the asset is kept longer than its economic life, the expenses of maintenance will have increased a lot. Meanwhile, if the asset is replaced before its economic life, the capital cost will not have been fully fiscal depreciated. Therefore, part of the investment, in the acquisition cost of the asset will be lost. So, physical assets in general are always used for a limited time. The questions are: (1) when is the right moment to replace a physical asset and (2) how to determine it?

The procedure in traditional literature is to incorporate the costs in decision making, which is done in a few steps. First, you take the physical asset that you want to determine the economic life. Then, collect the maintenance cost, acquisition cost and resale value of the equipment. Thus, organize all these values in a cash flow. According to Hillier et al (2010), the cash flow helps to explain the changes in balances sheets.

Therefore, it is used to facilitate the evaluation of the impact of cash inflows and outflows in different years.

Later, it is necessary that all in and out cash flow values over the years of the life of the asset are accounted for in the value of money, that is, interest rates and inflation are taken into account. The justification to account for these fees is to show the comparison between present and future actions. Then, it is possible to conduct a replacement between two assets, a new and an old one (in operation).

2. Theoretical Framework

2.1 NPV and AE in replacement problem

The NPV criterion is a comparison between two projects – for the replacement problem – two assets. For this, given all the inflows and outflows of projects over the years, it is chosen a minimum rate of return determined by the company, which represents the performance of the best alternative investment.

To achieve the NPV of a series of inflows and outflows D_1, D_2, D_3, ..., D_N, such that N represents the year, it is calculated as follows, as described by the equation (1):

$$NPV(n) = \frac{D_0}{(i+1)^0} + \frac{D_1}{(i+1)^1} + \frac{D_2}{(i+1)^2} + \cdots + \frac{D_n}{(i+1)^n} \qquad (1)$$

To illustrate the criterion, it will be given as an example, the decision of whether to invest in a project, which will have regular monthly income of US$ 3,500.00 over the next few years, but the initial cost is US$ 30,000.00 and the company minimum rate return is 2.00% per month. Table 1 shows the cash flow of this project.

Table 1 – Illustration of the NPV of a cash flow

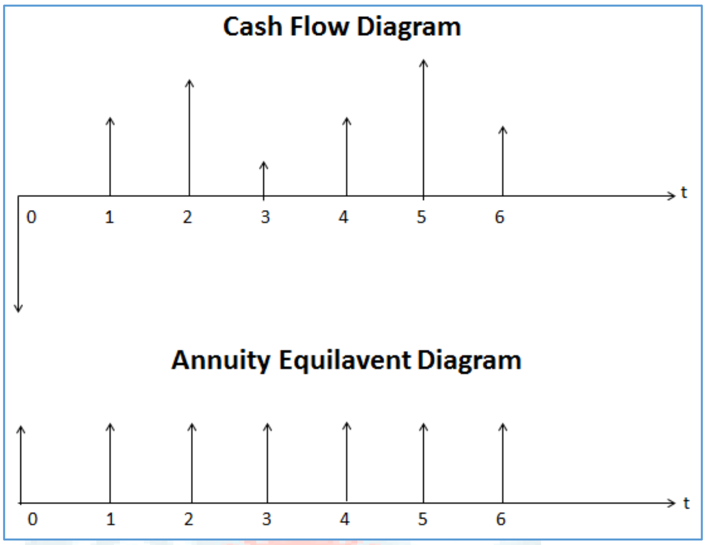
Month	Inflows	Outflows	Total	NPV
0		-US$ 30.000,00	-US$ 30.000,00	-US$ 30.000,00
1	US$ 3.500,00		US$ 3.500,00	-US$ 26.568,63
2	US$ 3.500,00		US$ 3.500,00	-US$ 23.204,54
3	US$ 3.500,00		US$ 3.500,00	-US$ 19.906,41
4	US$ 3.500,00		US$ 3.500,00	-US$ 16.672,95
5	US$ 3.500,00		US$ 3.500,00	-US$ 13.502,89
6	US$ 3.500,00		US$ 3.500,00	-US$ 10.394,99
7	US$ 3.500,00		US$ 3.500,00	-US$ 7.348,03
8	US$ 3.500,00		US$ 3.500,00	-US$ 4.360,81
9	US$ 3.500,00		US$ 3.500,00	-US$ 1.432,17
10	US$ 3.500,00		US$ 3.500,00	US$ 1.439,05
11	US$ 3.500,00		US$ 3.500,00	US$ 4.253,97
12	US$ 3.500,00		US$ 3.500,00	US$ 7.013,69

Source: Authors

In Table 1 there are all inflows (monthly income), outflows (cost of entry to the project), the full amount month-to-month and the NPV for each month, which is calculated according to equation (1). From the tenth month, the NPV becomes positive, i.e., using the company's minimum rate of return, the project is worth investing.

In AE criterion, the idea is to convert the values of a series of unequal cash flow in a series of identical periodically payments, i.e., the annuity equivalent. In Figure 1 there is an unequal series of a project as well as their equivalent constant values along the time

Figure 1 - Compariss on between diagrams



Source: Authors

In Figure 1, the Cash Flow Diagram displays the inflows (with arrows pointing upwards) and outflows (arrows pointed downward) in cash over time. The diverse size of each arrow indicates that there are different values of in and outflows. The Annuity Equivalent Diagram display all arrows in the same direction and with the same size, it means that the annuity equivalent criterion achieve a series of equal amounts along the years.

So, the AE is calculated through the equation (2).

$$AE(n) = NPV * \left[\frac{i * (1+i)^n}{(1+i)^n - 1}\right] \quad (2)$$

To illustrate, given an irregular cash flow, in a hypothetical case, as shown in Table 2, the NPV is calculated and then the AE according to the equation (2).

Table 2 – Exemplo do critério do AE

Year	Inflow	NPV	AE
0	US$ 15.000,00		
1	US$ 1.000,00		
2	US$ 5.000,00		
3	US$ 7.000,00	US$ 24.861,29	US$ 10.350,97

Source: Authors

In Table 2, it is possible to see all the irregular inflows along the four years. In the column NPV, there is the final value with a minimum rate of return of 12%, as it is calculated with equation (1) and, in the last column AE, it is calculated the value of the AE with equation (2).

3. Methodology

In this section two example are solved with methods of engineering economics, in the same way as in traditional literature. The first is a case of replacement of an asset for a new one, using both the NPV criterion and AE, but giving greater focus to the first. The second example, an analysis is made of the economic life of an asset with AE and NPV, but here giving greater focus on the AE criteria.

For the first, it is considered a gas compressor from an oil refinery that was purchased two years ago for US$35,000.00. Maintenance costs started in year 1 with a value of US$5,000.00 and increase US$1,500.00 per year. Currently, in year 2, the market value of this equipment is US$25,000.00. The expectation is that in 5 years, that amount is US$8,000.00. However, the company analyzes the alternative of selling the current equipment and buying a new one for US$30,000.00, which the value of initial maintenance is US$5,000.00 and increases by US$500.00 over the next years. For comparison, the value of this asset in year 5 is US$10,000.00. The problem is to consider whether the company should perform the replacement of equipment in order to reduce costs, given the opportunity to perform the exchange.

In Table 3 and Table 4 there are the cash flow of the old and the new equipment.

Table 3 – Cash flow of the equipment in operation

Year	Maintenance	Acquisition Cost	Market Value	Total	NPV
0		35.000,00			
1	- 5.000,00				
2	- 6.500,00		25.000,00	- 25.000,00	
3	- 8.000,00			- 8.000,00	
4	- 9.500,00			- 9.500,00	
5	- 11.000,00		8.000,00	- 3.000,00	- 41.851,54

Source: Authors

In Table 3 you can see the values for the cost of acquisition, maintenance and market value, but until Year 2 the values have already occurred and the following are forecasts. Furthermore, it is important to note on the line of Year 2 that the value indicated in the Total column (-US$25,000.00) is not the sum of maintenance costs with the market value. This value represents the opportunity cost of selling the equipment at market value at the end of Year 2. So, the problem consists in deciding whether or not to replace the equipment in Year 2.

Calculating the NPV from the column Total, then we have:

$$NPV = \frac{-25000}{(1+0{,}12)^0} + \frac{-8000}{(1+0{,}12)^1} + \frac{-9500}{(1+0{,}12)^2} + \frac{-3000}{(1+0{,}12)^3} = -41851{,}54$$

Thus, the NPV of the cost of gas compressor is -US$41,851.54. Then, let's calculate the NPV of the new asset, considered for replacement.

In Table 4 there are the values of the cash flow of the new equipment, which is the candidate for replacement.

Table 4 – Cash flow of the new equipment

Year	Maintenance	Acquisition Cost	Market Value	Total	NPV
0					
1					
2		30.000,00		- 30.000,00	
3	- 5.000,00			- 5.000,00	
4	- 5.500,00			- 5.500,00	
5	- 6.000,00		10.000,00	4.000,00	- 36.001,73

Source: Authors

Again, calculating the NPV of the column *Total*, then we have:

$$NPV = \frac{-30000}{(1+0{,}12)^0} + \frac{-5000}{(1+0{,}12)^1} + \frac{-5500}{(1+0{,}12)^2} + \frac{4000}{(1+0{,}12)^3} = -36001{,}73$$

Comparing the NPV of the two cash flows is possible to see that in year 5, the NPV of the current equipment is US$41,851.54, and the new equipment is -US$36,001.73. Thus, the new equipment's NPV is the highest value. Thus, given the opportunity to carry out the replacement of the equipment at the end of year two, the company should perform it.

However, to better understand the methodology, it is necessary to question how, in fact, is spent if the equipment is replaced or not. To better visualize this, in Table 5 all expenses are listed from the end of year two (time to make the decision to replace or not the equipment) until year 5.

Table 5 – Comparison between the expenses of both assets along the years

Year	Current Equipment		New Equipment	
	Expenditures	NPV	Expenditures	NPV
2			US$ 5.000,00	US$ 5.000,00
3	US$ 8.000,00	US$ 7.142,86	US$ 5.000,00	US$ 4.464,29
4	US$ 9.500,00	US$ 7.573,34	US$ 5.500,00	US$ 4.384,57
5	US$ 11.000,00	US$ 7.829,58	US$ 6.000,00	US$ 4.270,68
Total	US$ 28.500,00		US$ 21.500,00	

Source: Authors

Table 5 is divided into two parts, such that in the first one there are the expenditures with their NPV if the equipment is not replaced (*Current Equipment*) and in the other part, there are the expenditures and the NPV of *New Equipment*. Thus, in the *Current Equipment*, the expenditures are the maintenance costs. On the other hand, in the *New Equipment*, at the year two the expenses are regarding the replacement cost, such that this value is the difference between the current equipment sold for US$25,000.00 and the purchase of new equipment for US$30,000.00. In the following years, the expenditures are related to maintenance costs only. Finally, if the assets were not exchanged at the end of three years, the decision-maker would have to spend US$28,500.00, whereas with the exchange would spend US$21,500.00, i.e., a difference of US$7,000.00.

In this case, as the expenditures for both equipment happened in different years, the present value should be calculated to quantify the amount of initial investment if it equipment is replaced or not. Thus, at the end of year 2, as the replacement occurs, the decision-maker saves the amount of US$4,426.25 calculated for values of year 2.

So, the NPV criterion does not report that the monetary difference between both alternatives, i.e., between replace and not the asset. Therefore, this is its main drawback. To overcome this difficulty, there is the criterion of AE. The main advantage in using the AE is that with it you can see how much is spent on the asset on a periodic basis (monthly, annual kWh, tones, cycles, etc.).

Thus, in the latter example, it can be solved with AE. So, the AE calculated for the current and new equipment verifies that are respectively -US$13,778.97 and -US$11,853.01. Consequently, the equipment should be replaced. So, you can see the difference in annual expenditures with each asset.

Regarding the second example, the aim is to clarify the concept of Economic Life and the application of criteria AE. So, consider, for example, an asset that is acquired for US$10,000,000.00. The expected maintenance costs and market values over the years are in Table 6.

Table 6 – Maintenance costs and Market values for the Economic Life example

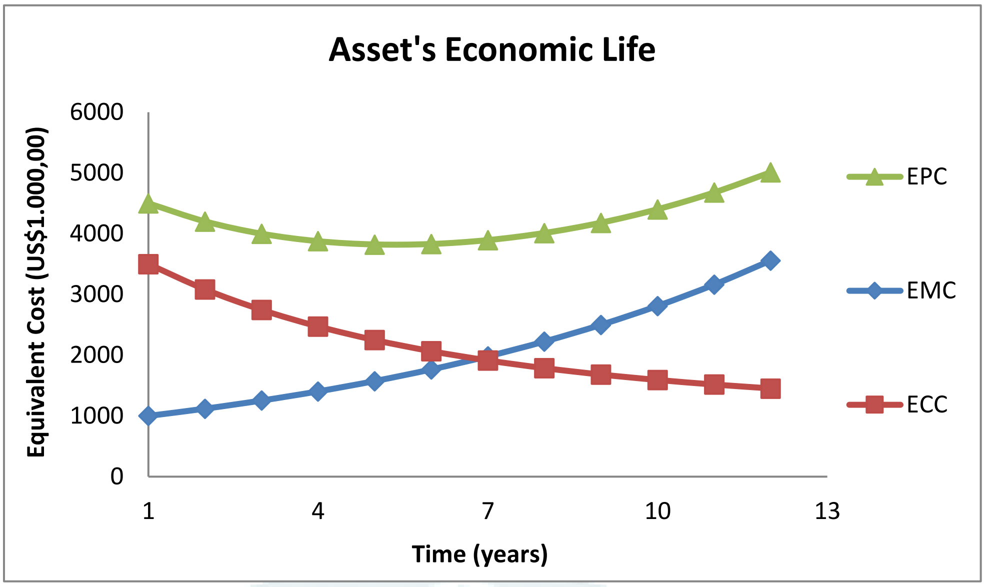
Year	Maintenance (US$1.000,00)	Market Value (US$1.000,00)
0	-	10.000,00
1	1.000,00	7.500,00
2	1.250,00	5.625,00
3	1.562,50	4.218,75
4	1.953,13	3.164,06
5	2.441,41	2.373,05
6	3.051,76	1.779,79
7	3.814,70	1.334,84
8	4.768,37	1.001,13
9	5.960,46	750,85
10	7.450,58	563,14
11	9.313,23	422,35
12	11.641,53	316,76

Source: Authors

In Table 6 we can see the evolution of the maintenance costs and the market value of the asset over the years. As maintenance costs increase and the market value decreases, on question arises: At what time the asset has the lowest life cycle cost?

For the values in Table 6, it is possible to watch in Figure the result in this example

Figure 2 – Asset's Economic Life

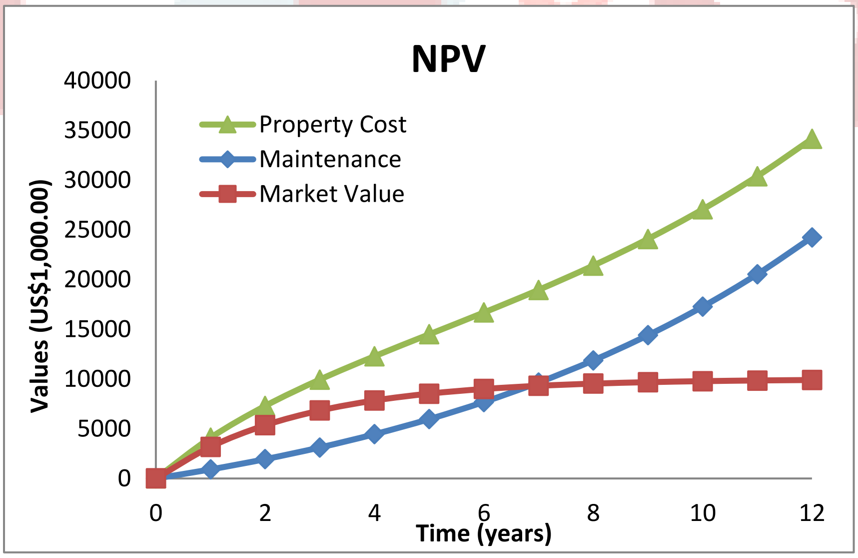


Source: Authors

On Figure there are the equivalent costs (vertical axis) of the three variables over time (horizontal axis). As you can see, we calculated the AE of the maintenance (EMC – Equivalent Maintenance Costs), the market value (ECC – Equivalent Capital Cost) and the AE of both added together, thus obtaining the equivalent property cost (EPC). In this curve, the point of minimum value represents the optimal period of use of the asset, i.e., the E*conomic Life* of the asset, which, in the example, is five years. So, this is the optimal period in which an asset should be used.

On the other hand, in case it was calculated the NPV of the values in Table 6, the result would be as it is shown in Figure 3.

Figure 3 – Attempt to model asset's economic life with NPV



Source: Authors

On Figure 3, there is an attempt to model economic life with NPV. It is possible to see that the data in Table 6, when applied to the NPV method, does not result in a minimum point in the curve of equivalent property cost. For this reason, it does not use NPV to calculate the asset's economic life

4. Results

Finally, it can summarize the main characteristics of both criterion towards physical assets replacement and economic life:

- The AE informs the year whose expenditure is minimal, i.e., the point of the economic life of the asset in operation. Not only, but also tells how much it is spent per year with each asset, in order to compare which is the best option. Besides, it can be used in replacement of two assets.
- The NPV does not inform the asset's economic life. This criterion is only used for asset replacement, i.e., given the opportunity to replace an asset in operation for a new, NPV determines the best alternative. Furthermore, it can be used to quantify the life cycle of the asset.

5. Discussions

As it was shown in the previous section, the AE was the best indicator for physical assets replacement. Nevertheless, for this paper, it was taken a few assumptions: (1) No technological innovation happened and (2) it was only taken a discrete analysis. Recently, many authors have worked with technological innovation (Yatsenko, Hritonenko, 2011), (Rogers, Hartman, 2005), (Regnier et al, 2004), (Bethuyne, 1998) as well as with a continuous analysis (Yatsenko, Hritonenko, 2010), (Cesca, Novaes, 2012). However, it was not found a discussion between AE and NPV. So, for a first analysis, it was considering a simplified model with no technology improvement, nor continuous analysis. For further researches and papers, those two assumptions will be undertaken.

6. Conclusions

Regarding the goal of this paper, as a result, for the decision-maker, in a discrete model with no technological improvement, the AE is the best criteria, as it can provide the asset's economic life and conduct the replacement between two assets. The other criterion, the NPV, as it doesn't provide the economic life, but only used for replacement between two assets, it is not as useful as the AE.

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